Effects of green investing on optimal stock portfolios

Erik Backman

School of Science

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Supervisor

Prof. Ahti Salo

Advisor

Prof. Ahti Salo

Aalto University
School of Science

Aalto University
School of Science

Aalto University, P.O. BOX 11000, 00076 AALTO
www.aalto.fi
Abstract of the bachelor's thesis

Author Erik Backman	
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Abstract

As concern over global issues has grown, the ESG (Environmental, Social, Governance) framework has gained popularity among investors. Of the three factors, environment is typically ranked as the most prominent. Climate change, its regulatory consequences, and the rise of environmentally friendly, "green" investing have made incorporating sustainability into investment decisions more common.

This thesis develops a multi-objective mixed-integer quadratic programming (MIQP) model to analyze the trade-offs between risk, return and environmental friendliness of an optimal stock portfolio. The returns and risk are calculated from historical data, while the environmental objective is formulated from environmental scores provided by LSEG. We analyze both the historical efficient frontiers and the realized performance of the portfolio during the holding period.

The results of this thesis suggest that opting for a greener portfolio does not necessarily undermine returns, and may in fact reduce risks, indicating that it is possible to achieve both financial performance and a positive environmental impact.

Keywords Portfolio optimization, green investing, multi-objective optimization, Environmental Score, mixed-integer optimization,

Aalto University
School of Science

Aalto-yliopisto, PL 11000, 00076 AALTO
www.aalto.fi
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Tiivistelmä

Globaalien huolenaiheiden kasvaessa ESG-kehys (ympäristö, yhteiskuntavastuu, hyvä hallintotapa) on saavuttanut suosiota sijoittajien keskuudessa. Kolmesta tekijästä ympäristöä pidetään tyypillisesti tärkeimpänä. Ilmastonmuutos, sitä seurannut regulaatio sekä ympäristöystävällisen, vihreän sijoittamisen nousu ovat tehneet kestävän kehityksen sisällyttämisestä sijoituspäätöksiin yhä tavallisempaa.

Tämä kandidaatintyö hyödyntää monitavoitteista sekalukuoptimointimallia analysoimaan tuoton, riskin ja ympäristöystävällisyyden välisiä riippuvuuksia optimaalisissa osakeportfolioissa. Tuotto ja riski lasketaan historiallisesta datasta, kun taas ympäristötavoite muotoillaan LSEG:n tarjoamista ympäristöpisteistä. Työssä analysoidaan sekä historiallisia tehokkaita rintamia että portfolion suoriutumista tarkkailujakson aikana.

Tutkimuksen tulokset viittaavat siihen, että vihreämmän portfolion valitseminen ei välttämättä heikennä tuottoja ja voi itse asiassa vähentää riskejä. Tämä osoittaa, että on mahdollista saavuttaa sekä taloudellinen suorituskyky että positiivinen ympäristövaikutus.

Avainsanat portfolio-optimointi, vihreä sijoittaminen, monitavoiteoptimointi, ympäristöpisteet, sekalukuoptimointi

Contents

1 Introduction

As capital markets play a pivotal role in shaping the world through funding innovation, research, infrastructure, the allocation of resources has an impact on societal development. Traditionally, the primary incentive for investing has been to maximize wealth. However, due to increasing concern over global issues, such as climate change and social inequality, capital markets are evolving into platforms where the investor can impact global issues by allocating wealth responsibly.

Furthermore, the landmark report *Who Cares Wins* (UN, 2004), first coined the term ESG (Environmental, Social and Governance). The report highlights the importance of these three factors in investment decision-making. At a growing pace, investors, whether institutional or private investors, are incorporating ESG factors into their investment strategies. At the beginning of 2018, sustainable investing assets in Australia, New Zealand, Canada, Europe, Japan and U.S. combined amounted to \$30.7 trillion, reflecting a 34% growth over two years (GSIA, 2018).

Along with the rising interest of investors, governments and other institutions play a critical role in promoting the ESG framework through regulation and by introducing policies that promote transparency and standardize ESG reporting. As companies increasingly disclose more information related to ESG factors, data vendors have started to provide ESG scores to meet the growing demand from investors who wish to prioritize responsibility in decision-making.

Climate change is one of the most urgent challenges to humanity, and therefore financial markets have a great impact on the future of the planet. Global agreements, such as the *Paris Agreement* (UNFCCC, 2015), have set ambitious goals to limit the temperature increase to 1.5° above the pre-industrial levels. Of the three factors of ESG, environment is typically ranked as the most prominent. According to BlackRock, 88% of clients regard climate-related risks as the top sustainability concern regarding investments (BlackRock, 2020).

While there is ongoing debate about how much environmentally friendly, "green" investing can impact the environment, another important question from the standpoint of an investor is whether green investing affects risks and returns, and if so, in what ways.

This thesis aims to understand the effects of environmentally friendly investing on stock portfolios by constructing a multi-objective portfolio optimization model. The model incorporates environmental scores provided by LSEG alongside traditional measures of risk and return. In addition, the model is designed with practical constraints, such as limiting the number of different assets, to mimic realistic scenarios that are applicable for both private investors and fund managers. To eliminate the effects of key differences in regulation between countries, the thesis is restricted to U.S. stocks only.

The rest of the thesis is structured as follows. Section 2 provides an overview of the literature in portfolio optimization as well as responsible and green investing. Section 3 defines the asset universe for the study, constructs the optimization model, and models the trade-offs between risk, return and Environmental Score. In Section 4, the realized performances of the constructed optimal portfolios are evaluated and compared to estimates. Finally, a summary of the study, along with key remarks and a discussion of future directions is provided in Section 5.

2 Background

2.1 Portfolio Optimization

Modern Portfolio Theory (MPT), introduced by Markowitz (1952), made it possible to optimize portfolios as a trade-off between expected return and risk. A cornerstone of MPT is the efficient frontier, which is a representation of the set of portfolios that provide the maximum return for a given level of risk, or minimize risk for a given level of return. These portfolios are called non-dominated portfolios, as no other portfolio can simultaneously outperform them in both return and risk.

Despite its groundbreaking impact, the mean-variance model has limitations. One key assumption of MPT is that asset returns follow a normal distribution, but empirical evidence does not fully support this assumption (Mandelbrot, 1963; Fama, 1965). However, Oldfield and Rogalski (1980) suggest that increasing the return interval from daily to weekly or monthly decreases kurtosis and skewness.

The mean-variance model traditionally uses variance or standard deviation of returns as measures of risk as they are simple to calculate, and in the case of standard deviation, easy to comprehend. However, these measures of risk also assume normality of returns. Another weakness of variance as a risk measure is that it treats deviations symmetrically.

As investors are typically more concerned about downside risk, specific measures were established. Maximum and Average Drawdown, measure either the largest single drawdown during a period, or the mean of all drawdowns (Chekhlov et al., 2005). Value-at-Risk (VaR) (Linsmeier and Pearson, 2000) and Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2002) quantify potential losses, with VaR indicating maximum expected loss for a given confidence level, while CVaR provides the average loss beyond VaR threshold.

However, these measures are not without flaws. For example, Maximum Drawdown and VaR lack subadditivity (Artzner et al., 1999), meaning that they fail to fully capture the benefits of diversification. Additionally, while subadditive, CVaR is computationally intensive and relies on distributional assumptions about losses, further complicating the model.

Even though the classical mean-variance model does not account for all factors, it is still usable as an approximation. As Markowitz (2014) states, the assumption of normality is not a necessary condition for the applications of MPT.

Although the mean-variance model reduces risk by minimizing portfolio variance for a given level of return, there are additional ways to affect portfolio composition. One such way is to impose constraints on the number of different assets in a portfolio, or restrict the minimum and maximum weights of each asset in the portfolio.

An article by Zaimovic et al. (2021) provides a review of the literature on the optimal amount of stocks in an equity portfolio. For a long period, 8-10 stocks was deemed sufficient for portfolio diversification (Evans and Archer, 1968). However, a study by Statman (1987) suggests that at least 30 stocks are required to achieve a well-diversified portfolio, while a more recent study Kryzanowski and Singh (2010) argues that on average a portfolio of 20-25 stocks is sufficient to achieve 90 % of diversification benefits. Nevertheless, in the case of smaller portfolios, fixed transaction costs incur. As a consequence, limiting portfolio size is necessary for optimal portfolio performance (Brennan, 1975).

An additional way to diversify risk is to limit the allocation of assets that belong to a specific industry or a country in the portfolio. This ensures that the portfolio is not exposed to industry-specific or country-related risks, such as economic downturns, regulatory changes and geographical risks. Although Lessard (1974) finds that country diversification is more effective than industry diversification, the latter remains relevant since our focus is on a single country.

When additional objectives are considered in forming of a portfolio, the model becomes a multi-objective problem. The ϵ-constraint method, introduced by Haimes (1971), transforms a bi-objective optimization problem into a single-objective problem, where the other objective is modeled as a constraint with a specified target level. The method can be extended for multiple objectives as shown by Ehrgott (2005). Other methods for solving multi-objective problems include evolutionary algorithms like NSGA-II (Deb et al., 2002) and weighted sum approaches.

When limiting the quantity of different assets in the portfolio, the optimization problem becomes a mixed integer quadratic programming problem. These problems are known to be NP-hard, and therefore computational effort to solve the problem increases drastically as problem size grows. Methods such as branch-and-bound (Land and Doig, 1960) or the Cutting-Plane method (Kelley, 1960) are often used to reduce computational effort.

2.2 Sustainable Investing

The recent rise in the popularity of ESG investing has motivated plenty of research. The research in this field can be divided into two segments, portfolio and non-portfolio studies (Friede et al., 2015). Portfolio studies are usually conducted on an asset-class basis. For example, Pástor et al. (2022) find that German environmentally friendly bonds outperform their non-green counterparts, while La Torre et al. (2020) identify a relationship between the ESG index of a stock and its returns.

In the context of portfolio optimization, responsibility metrics are often considered as additional objectives. These metrics can be derived from data disclosed by firms or provided by financial data services. The measures can take the form of accounting metrics, such as water consumption, or value-weighted scores defined by data vendors.

However, agency problems arise within these metrics, even when dealing with seemingly objective data. Aswani et al. (2023) find that only the emissions estimated by data vendors are correlated with stock returns, whereas the information disclosed by the firms is not. Furthermore, the robustness of the value-weighted scores has been questioned (Berg et al., 2022; Billio et al., 2021).

Examples of portfolio optimization incorporating these scores are provided by Cesarone et al. (2022); Oliver-Muncharaz (2021). These studies suggest that taking

ESG measures into account can enhance portfolio performance from the traditional perspective of an investor.

Much of the literature dissects the effects of the three ESG factors differently. When focusing only on the environmental factor, the typical measures used focus solely on carbon emissions, and do not account for resource usage and innovation. For example, Anquetin et al. (2022) introduce a carbon emission penalty in the objective to perform portfolio optimization, and show that it is possible to cut emissions by half without affecting risk-adjusted returns.

3 Data and Methodologies

3.1 Asset Universe

In this section, details about the asset selection are shared. For this thesis, we consider assets from the S&P 500. Weekly stock return data from January 2012 to December 2022 is used for the in-sample or the training period, and from 2023 until 11th of October 2024 for the out-of-sample or the holding period. Additionally, Environmental Pillar Scores are obtained from LSEG at the end of 2022 for each asset. The methodology used to calculate these scores is provided in LSEG (2023).

For an asset to be included, it must have at least one full year of return data prior to 2023 and an Environmental Pillar Score from 2022. Three companies were excluded due to lack of price data, and 11 due to a lack of Environmental Scores. As a result, the final asset universe for the optimization consists of 488 assets. As the asset universe consists of assets in the same regulatory area, comparison between different assets is more meaningful. Additionally, the assets that comprise the S&P 500 are traded actively in large volumes, and are therefore liquid.

This thesis uses weekly return data as opposed to daily or monthly returns to calculate mean returns, variances and covariances. Weekly and monthly returns are less affected by short-term price fluctuations and market noise than daily returns. Weekly returns are chosen over monthly returns to provide a larger dataset for analysis.

The assets are categorized into 11 different industry sectors according to the Global Industry Classification Standard (GICS) framework. Table 1 outlines the distribution of assets across industry sectors within our asset universe.

3.2 Mean-Variance-Environmental Model

This section describes the Mean-Variance-Environmental model used in the optimization. First, we define the variables. Next, the model is constructed by integrating additional constraints to the classical Mean-Variance model (Markowitz, 1952). Finally, we use the model to find the resulting efficient frontiers and suitable portfolios for further analysis.

For an asset universe of n assets, we define the expected return, covariance matrix and the Environmental Pillar Scores. Let $r_{i,t}$ denote the weekly return of asset i at

Table 1: Asset Counts by Sector

Sector Name	**Count**
Health Care	58
Information Technology	67
Consumer Discretionary	49
Financials	71
Consumer Staples	37
Industrials	74
Utilities	30
Materials	27
Real Estate	31
Energy	22
Communication Services	22
Total	**488**

time t. Then, the estimated weekly expected or mean return of asset i is

$$\bar{r}_i^\star = \frac{1}{T - t_i + 1} \sum_{t=t_i}^{T} r_{i,t}$$

where T denotes the length of the entire time period, and t_i the first available return for asset i. To be able to calculate covariances, we consider only the time period during which both assets have available return data. For assets i, j, when $t_i > t_j$, the estimated weekly covariance is defined as

$$\hat{\sigma}_{ij}^\star = \frac{1}{T - t_i} \sum_{t=t_i}^{T} (r_{i,t} - \bar{r}_{i,t_i}^\star)(r_{j,t} - \bar{r}_{j,t_i}^\star)$$

where $\bar{r}_{j,t_i}^\star$ is the estimated weekly mean return of asset j in the time period $t \in \{t_i, t_{i+1}, ..., T\}$. The estimated weekly variance of asset i is simply defined as

$$(\hat{\sigma}_i^\star)^2 = \frac{1}{T - t_i} \sum_{t=t_i}^{T} (r_{i,t} - \bar{r}_i^\star)^2.$$

Since at least one full year of data is required, $T > t_i + 1$, which ensures division by a positive number in all estimation formulas. For yearly comparison, we define the estimates for yearly mean return of asset i as $\bar{r}_i = 52\bar{r}_i^\star$, the yearly variance $\hat{\sigma}_i^2 = 52(\hat{\sigma}_i^\star)^2$ and the yearly covariance as $\hat{\sigma}_{ij} = 52\hat{\sigma}_{ij}^\star$. Finally, the Environmental Pillar Score, or Environmental Score for short, of asset i is simply E_i.

Let $x = (x_1, ..., x_n)$ be a vector of asset weights, where x_i is the fraction of capital invested in asset i. Then, the classical Mean-Variance portfolio optimization problem that minimizes portfolio variance for a given level of return μ reads as

$$\min_x \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{n} x_i \bar{r}_i \geq \mu \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

$$x_i \geq 0,\ \ i = 1, ..., n \tag{4}$$

where (3) denotes the budget constraint and (4) the no short-selling constraint.

To this model, we introduce a new objective function and additional constraints. Namely, we define the function of portfolio average Environmental Score, $E_P = \sum_{i=1}^{n} x_i E_i$. We also define the function for the expected or mean return of the portfolio as $r_P = \sum_{i=1}^{n} x_i \bar{r}_i$, while the variance is defined as $\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}$.

Additionally, we define additional constraints on minimum and maximum fractional weights, an industry-constraint and a constraint on the amount of non-zero asset weights for diversification purposes. To enforce bounds on the different number of stocks in a portfolio, we introduce the binary decision variables

$$y_i = \begin{cases} 0, & \text{if asset } i \text{ is not included in the portfolio} \\ 1, & \text{if asset } i \text{ is included in the portfolio.} \end{cases} \tag{5}$$

For restrictions on an asset's minimum and maximum fractional weights, we define $f_{\min}$ as the minimum fraction of capital allocated to a chosen asset, and $f_{\max}$, as the maximum fraction allocated. Additionally, for the count of different assets in the portfolio, we define m as the lower bound and M as the upper bound for the number assets. In keeping with earlier research (Kryzanowski and Singh, 2010), we set $f_{\min} = 0.005$, $f_{\max} = 0.050$, $m = 20$, $M = 30$ for the bounds of diversification. The combined weight and quantity restrictions are as follows

$$m \leq \sum_{i=1}^{n} y_i \leq M \tag{6}$$

$$y_i f_{\min} \leq x_i \leq y_i f_{\max},\ \ i = 1, ..., n. \tag{7}$$

To introduce constraints based on industry sector weights to our model, let $S \in \mathbb{R}^{11\times 488}$ represent the industry sector matrix, where

$$S_{ij} = \begin{cases} 1, & \text{if asset } j \text{ belongs to sector } i \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Then, the industry sector diversification constraint is expressed as

$$\sum_{j=1}^{n} S_{kj} x_j \leq S_{\max},\ \ k = 1, ..., s \tag{9}$$

where $S_{\max} = \frac{1}{3}$ is the maximum weight of any industry sector in the portfolio and $s = 11$ the number of different industries.

Now, the set of feasible portfolios is given by

$$(x, y) \in \mathcal{P} = \left\{ \begin{array}{ll} x = (x_1, ..., x_n) \in \mathbb{R}^n, & y \in \{0, 1\}^n, \\ \sum_{i=1}^{n} x_i = 1, & \\ y_i f_{\min} \leq x_i \leq y_i f_{\max}, & i = 1, \ldots, n, \\ m \leq \sum_{i=1}^{n} y_i \leq M, & \\ \sum_{j=1}^{n} S_{kj} x_j \leq S_{\max}, & k = 1, \ldots, s, \\ x_i \geq 0, & i = 1, \ldots, n. \end{array} \right\}$$

where we have combined the constraints and the definitions (3)-(9).

Now, we can model our problem as a tri-objective Mixed Integer Quadratic Programming (MIQP) problem

$$\begin{aligned} &\min_{x,y} \ \{\sigma_P^2, -r_P, -E_P\} \\ &\text{s.t. } (x, y) \in \mathcal{P}. \end{aligned} \tag{10}$$

To solve this tri-objective problem, we reformulate the problem with the standard ϵ-constraint method (Ehrgott, 2005). The reformulation leads to the following single-objective optimization problem

$$\begin{aligned} \max_{x,y} \ & \sum_{i=1}^{n} x_i \bar{r}_i \\ \text{s.t. } & \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq \omega \\ & \sum_{i=1}^{n} x_i E_i \geq \lambda \\ & (x, y) \in \mathcal{P} \end{aligned} \tag{11}$$

where ω is the maximum allowed portfolio variance (risk) and λ the minimum acceptable portfolio Environmental Score.

3.3 Mean-Variance-Environmental Frontiers

In this section, the efficient surface of the Mean-Variance-Environmental model is found from the Pareto-optimal solutions of the tri-objective MIQP problem (10). The approach follows similar methods as those presented by Cesarone et al. (2022). To solve the computationally demanding MIQP optimization, we implement the Cutting-Plane method (Kelley, 1960) in MATLAB using the solver `INTLINPROG`.

First, we begin by finding the Global Minimum Variance (GMV), Maximum Return (MR) and Maximum Environmental portfolios (ME). These portfolios are obtained by optimizing with the constraints of problem (10), but using only the corresponding objective function. For instance, the GMV portfolio is determined by the following problem

$$\begin{aligned} &\min_{x,y}\ \sigma_P^2 \\ &\text{s.t. } (x,y) \in \mathcal{P}. \end{aligned} \tag{12}$$

Table 2: Variance, return and Environmental Score of the Global Minimum Variance, Maximum Return and Maximum Environmental portfolios.

	GMV	MR	ME
σ_P^2	0.012	0.085	0.040
r_P	0.127	0.447	0.176
E_P	55.8	61.0	93.9

Table 2 lists the properties of these portfolios, and they are used to define reasonable parametric bounds for each of the frontiers.

We plot the efficient Mean-Variance frontiers for varying levels of the required Environmental Score, denoted as λ. We define the vector $\lambda = [0,\ 75,\ 82.5,\ 90]$. Furthermore, we determine a suitable interval for the portfolio variance by defining $[\omega_{min}(\lambda),\ \omega_{max}(\lambda)]$, where $\omega_{min}(\lambda) = \sigma_P^2(x_{minV}(\lambda))$ and $\omega_{max}(\lambda) = \sigma_P^2(x_{maxR}(\lambda))$, with $x_{minV}(\lambda)$ representing the portfolio that minimizes variance and $x_{maxR}(\lambda)$ the portfolio that maximizes mean returns for a given level of Environmental Score λ.

To plot the efficient Mean-Variance frontiers, we solve the optimization problem (11) for sufficiently many points within the variance interval $\omega \in [\omega_{min}(\lambda),\ \omega_{max}(\lambda)]$ and the different levels of λ as defined above.

The impact of λ on the efficient frontiers is illustrated in Figure 1. Requiring higher levels of Environmental Score affects the forming of efficient frontiers in two ways. First, with higher levels of λ, the mean returns decrease for each level of variance. Second, by increasing λ, the range of feasible portfolios with respect to variance is smaller.

To plot the efficient Environmental-Variance frontiers, we define the vector $\alpha_0 = [0,\ \frac{1}{4},\ \frac{1}{2},\ 1]$, and the required level of expected return $\mu_{\alpha_0} = \mu_{GMV} + (\mu_{MR} - \mu_{GMV})\alpha_0$, where μ_{GMV} and μ_{MR} are the expected returns of the GMV and MR portfolios. Next, an interval for portfolio variances is constructed as in the case of Mean-Variance frontiers. Then, we reformulate the MIQP problem (11) as follows

$$\begin{aligned} &\max_{x,y}\ \sum_{i=1}^{n} x_i E_i \\ &\text{s.t. } \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq \omega \\ &\qquad \sum_{i=1}^{n} x_i \bar{r}_i \geq \mu \\ &\qquad (x,y) \in \mathcal{P}, \end{aligned} \tag{13}$$

and form the efficient frontiers with varying levels of expected return μ and sufficiently many points in the suitable interval for variance ω. Figure 2 shows the effects of

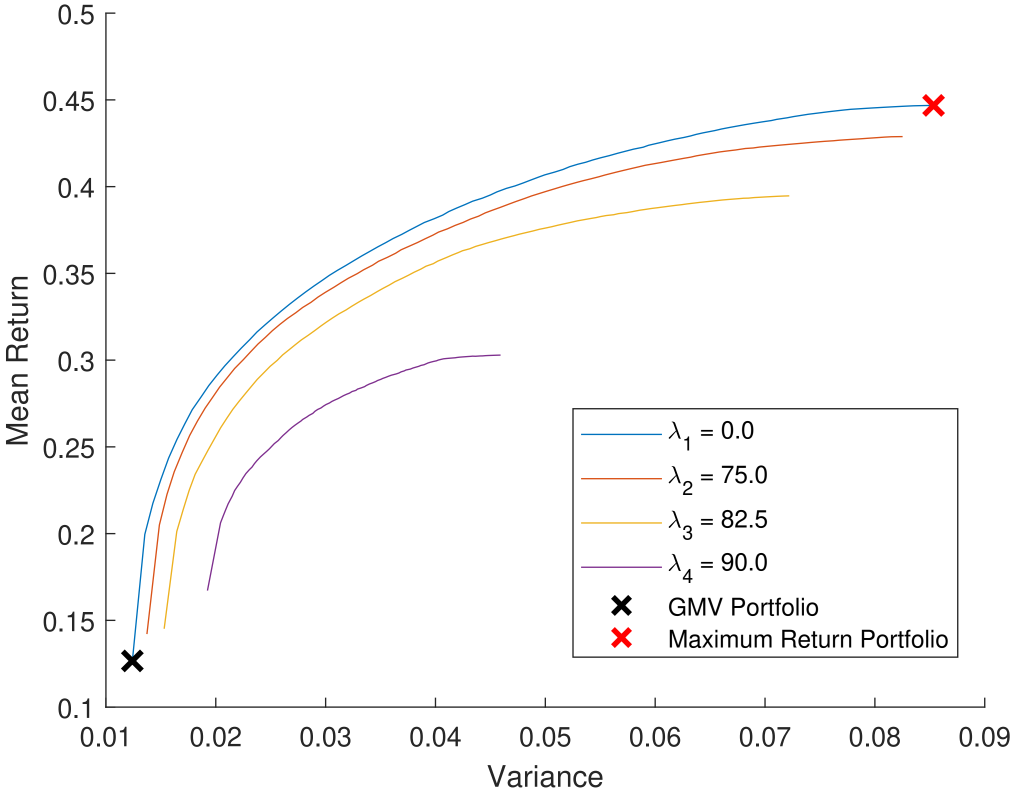

Figure 1: Efficient Mean-Variance frontiers for varying levels of Environmental Score λ.

increasing expected returns μ on the efficient Environmental-Variance frontiers. Requiring higher levels of expected return μ increases variance, while also lowering the bound of maximum Environmental Score.

Lastly, we form the efficient Environmental-Mean frontiers for varying levels of risk ω. To achieve this, let $\beta = [\frac{1}{5},\ \frac{2}{5},\ \frac{3}{5},\ \frac{4}{5}]$. The vector for different levels of risk is then defined as $\omega_\beta = \sigma^2_{GMV} + (\sigma^2_{MR} - \sigma^2_{GMV})\beta$. Then, we form the interval for the expected returns for each risk level, as in the case of the first two frontiers. We form the frontier with the optimization problem (13), but contrary to the Environmental-Variance frontier, we form the Environmental-Mean frontier with four levels of risk ω and a suitable interval for expected returns μ for each risk level.

The efficient Environmental-Mean frontiers are illustrated in Figure 3. For a given level of variance ω, increasing the mean of a portfolio return will result in lowering portfolio Environmental Score. Increasing the variance ω narrows the range of mean returns and shifts it to the right. This is in accordance with the results of the Mean-Variance frontiers.

Finally, for out-of-sample analysis, from the findings of this section, we define $\mu_\alpha(\lambda) = \mu_{minV}(\lambda) + \alpha(\mu_{maxR}(\lambda) - \mu_{minV}(\lambda))$, where $\alpha = [\frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ \frac{4}{5}]$, $\lambda = [0,\ 75,\ 82.5,\ 90]$ and $\mu_{minV}(\lambda)$ and $\mu_{maxR}(\lambda)$ represent the expected mean returns of the minimum variance and maximum return portfolios for each value of λ. Now

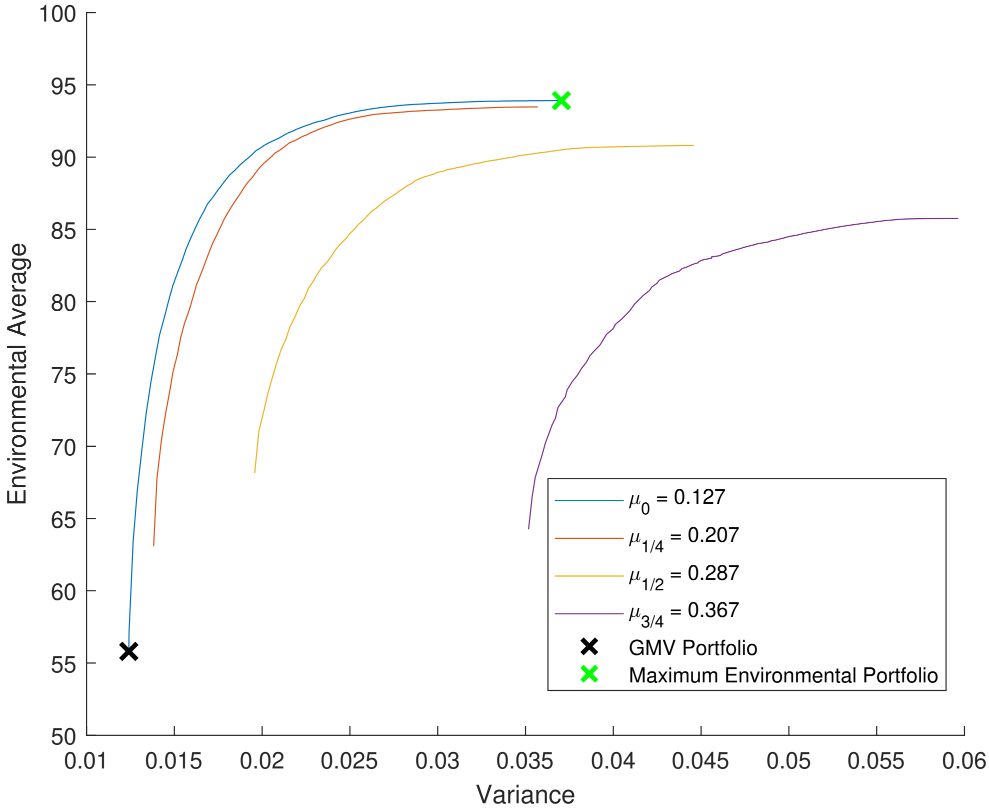

Figure 2: Efficient Environmental-Variance frontiers for varying levels of expected return μ.

we reformulate the optimization problem (13) to minimize portfolio variance for four different levels of expected return for each of the levels of required Environmental Score λ. The resulting problem is

$$\begin{aligned} \max_{x,y} \quad & \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i E_i \geq \lambda \\ & \sum_{i=1}^{n} x_i \bar{r}_i \geq \mu \\ & (x, y) \in \mathcal{P}, \end{aligned} \tag{14}$$

and we form 16 portfolios by solving for each λ and $\mu_\alpha(\lambda)$.

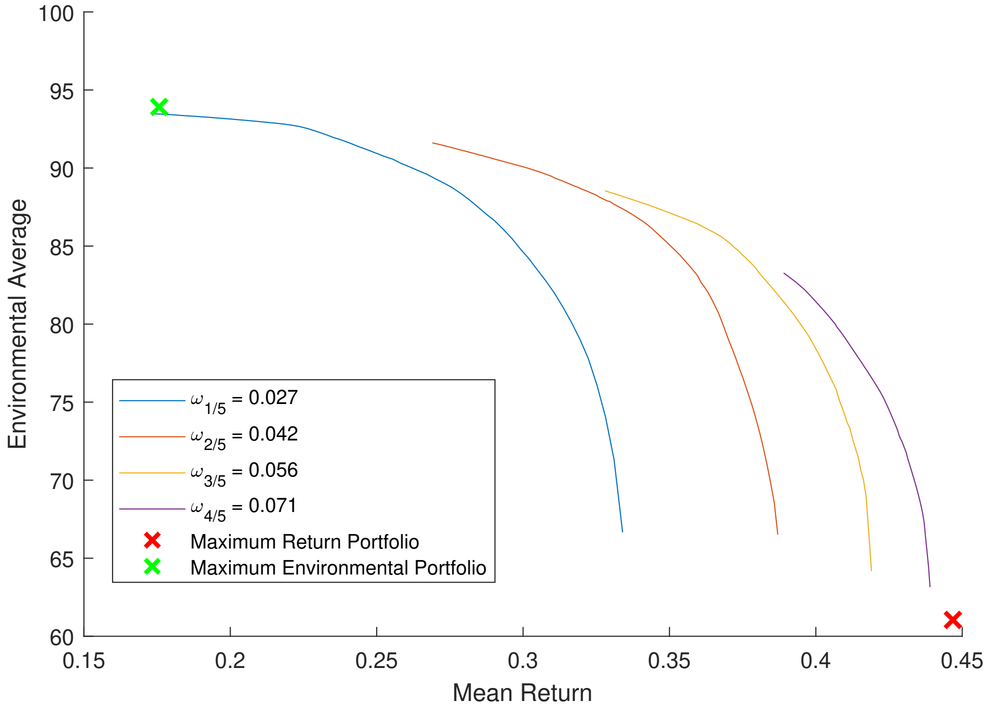

Figure 3: Efficient Environmental-Mean frontiers for varying levels of variance ω.

4 Results

This section describes the out-of-sample performance of the 16 Pareto-optimal portfolios that are optimized based on historical data. The aim is to compare the realized values to the expected values as well as the compounded returns to the S&P 500 index.

Based on the definitions of λ and $\mu_\alpha(\lambda)$, the portfolios that solve the problem (14) lie approximately on the Mean-Variance frontiers as illustrated in Figure 4. For a given Environmental Score level λ, increasing the target level of return μ, moves the portfolio up and right on the frontier.

To assess the performance of the portfolios during the out-of-sample period, in addition to the traditional measures of risk and returns, we use the Sharpe ratio (Sharpe, 1966) to measure risk-adjusted returns. The Sharpe ratio is

$$SR = \frac{r_P - r_f}{\sigma_P},$$

where $r_f = 0.03751$ is the annual yield of a US 10-year bond, and r_P, σ_P are the annualized returns and volatility (standard deviation) of portfolio P.

Table 3 reports the out-of-sample annualized mean returns, volatility and Sharpe ratios for each portfolio. Overall, increasing the target portfolio return has little effect on out-of-sample mean and volatility beyond the level $\mu_{3/4}$, while the realized return volatility remains high. Additionally, as a general trend, for each μ, raising

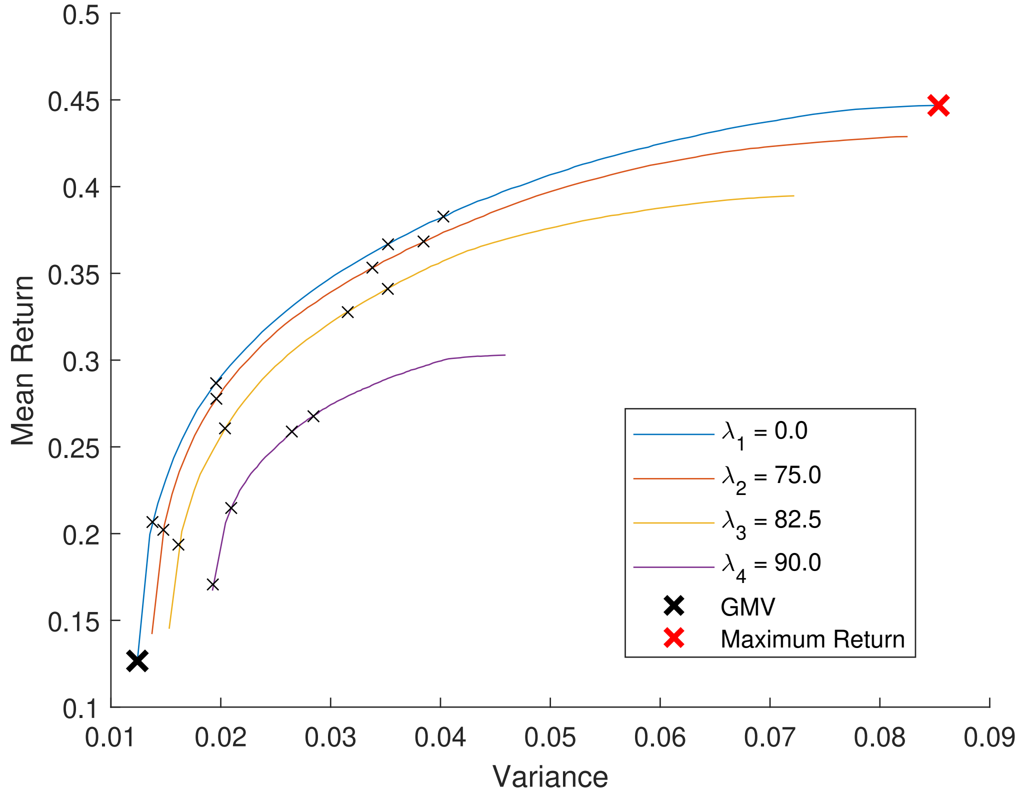

Figure 4: The location of the 16 Pareto-optimal portfolios on the Mean-Variance frontiers for different values of λ and μ

the Environmental Score requirement tends to result in lower mean returns and volatility, which is consistent with the historical efficient frontiers observed previously. However, for expected return levels $\mu_{3/4}$ and $\mu_{4/5}$, while the volatility decreases as the Environmental Score requirement increases, the drop in mean returns is smaller. Also, for each target portfolio return, the portfolios with the highest Environmental Score have the highest Sharpe ratios. This suggests that requiring higher Environmental Score from a portfolio leads to better risk-adjusted returns.

To assess the reliability of the estimations of the expected returns, the relative difference between the out-of-sample performance and expected performance is summarized in Table 4. Generally, estimations for mean returns are near the realized values, but the volatility of the realized returns is mostly higher than the estimations.

For all target return levels except $\mu_{1/4}$, the highest Environmental Score portfolios have the smallest relative difference of volatility between out-of-sample and expected returns. This suggests that estimations for volatility based on historical returns for high levels of Environmental Score are more stable. As for most portfolios the realized volatility of returns is higher than the volatility of the expected returns, but the relative differences of the Sharpe ratios are negative.

For both return levels $\mu_{3/4}$ and $\mu_{4/5}$, portfolios with the highest required Envi-

Table 3: Summary of the out-of-sample performances of the 16 Optimal Portfolios

	$\mu_{1/4}$				$\mu_{1/2}$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
Mean	0.280	0.258	0.145	0.164	0.282	0.321	0.274	0.210
Volatility	0.295	0.249	0.103	0.107	0.303	0.309	0.316	0.127
Sharpe	0.824	0.885	1.048	1.182	0.806	0.918	0.750	1.357
	$\mu_{3/4}$				$\mu_{4/5}$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
Mean	0.371	0.330	0.336	0.332	0.389	0.338	0.342	0.329
Volatility	0.363	0.336	0.337	0.287	0.368	0.344	0.338	0.257
Sharpe	0.921	0.871	0.887	1.027	0.954	0.873	0.900	1.134

Table 4: Summary of the relative difference (in %) between realized and and expected performance of the 16 Optimal Portfolios

	$\mu_{1/4}$				$\mu_{1/2}$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
Mean	35.6	27.7	-24.9	-4.1	-1.6	15.5	5.2	-2.3
Volatility	151.0	105.2	-19.0	-23.1	116.8	120.5	121.2	-12.2
Sharpe	-42.8	-34.7	-14.7	23.2	-54.7	-46.5	-52.0	10.8
	$\mu_{3/4}$				$\mu_{4/5}$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
Mean	1.3	-6.5	2.6	28.3	1.6	-8.2	0.1	23.1
Volatility	93.1	82.8	89.7	76.2	83.5	75.6	80.1	52.6
Sharpe	-47.5	-49.3	-45.7	-24.5	-44.6	-48.3	-44.4	-16.96

ronmental Score level λ_4, the realized mean return are higher than the expected values. Combined with the fact that with a choice of these parameters the Sharpe ratios are highest for all return levels, this suggests that this model can produce environmentally friendly portfolios that provide good risk-adjusted returns.

To analyze the effectiveness of the 16 different investment strategies, we calculate the compounded returns. Figure 5 shows the cumulative value of the portfolios based on the out-of-sample period compounded returns, where the S&P 500 index is used as a benchmark for comparison. When requiring at least target level $\mu_{3/4}$ for returns, all portfolios outperform the index during the out-of-sample period. For lower target levels of return, most portfolios have nearly identical value to the respective index, except for portfolios $(\lambda_3, \mu_{1/4})$, $(\lambda_4, \mu_{1/4})$ and $(\lambda_4, \mu_{1/2})$. These portfolios do not exhibit high volatility clustering, and have similar upward trend as the index. On the contrary, the rest of the portfolios drastically gain value compared to the S&P 500 starting in January 2024, but the gap narrows as time progresses.

Based on visual inspection, these portfolios gain and lose value in a similar pattern, suggesting that the optimization produces relatively similar portfolios. Even though the returns are similar, for levels $\mu_{3/4}$ and $\mu_{4/5}$, portfolios with the highest Environmental Score requirement λ_4 have lower volatility. For both $\mu_{3/4}$ and $\mu_{4/5}$,

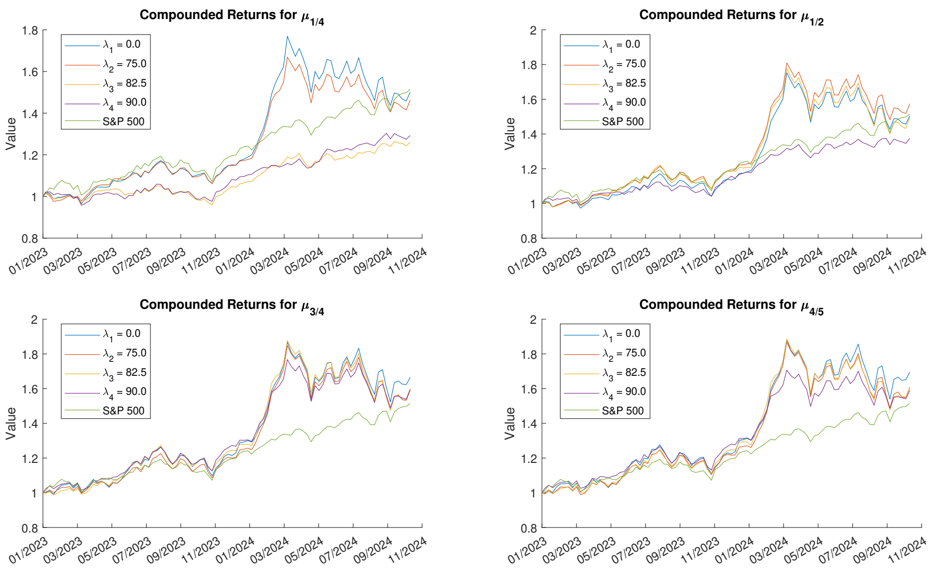

Figure 5: Cumulative returns of the S&P 500 index and the Pareto-optimal portfolios for different values of λ and μ

the best performing portfolio has the lowest environmental requirement λ_0. The other environmental level portfolios reach near identical value when compared to each other, but slightly lower than λ_0. This suggests that opting for greener portfolios does not necessarily penalize returns.

5 Summary

In this thesis, we examined the trade-offs between environmental friendliness, risk and return in optimal stock portfolios by using a portfolio optimization model that incorporates an environmental objective function. The model was applied to U.S. stocks in the S&P 500, and the environmental objective was derived from the environmental scores provided by LSEG. The thesis addressed gaps in portfolio optimization literature on environmental analysis by considering not only carbon emissions but also resource usage and innovation.

The results illustrate that, historically, there are clear relationships between the variables in the efficient frontiers. Increasing the required portfolio environmental level will decrease the maximum achievable return and increase variance. Targeting higher returns across all levels of risk tends to lower the maximum portfolio environmental level. Although the study conducted by Cesarone et al. (2022) uses ESG scores rather than environmental scores, the shapes of the efficient frontiers are similar. The similarity is reasonable as the Environmental Pillar Score constitutes 44 % of the ESG score.

The results from the out-of-sample period demonstrate that the model produced portfolios that generally outperformed their comparable index. Additionally, portfolios with higher environmental requirements typically exhibit lower volatility and higher risk-adjusted returns. This suggests that increasing the environmental sustainability of a portfolio can reduce risk while not compromising returns. This is consistent with the study conducted by Anquetin et al. (2022), but extended to a framework that includes resource usage and innovation factors.

During the out-of-sample period, the S&P 500 index appreciated significantly in value. Specifically, six companies were the cause for the majority of the upside over the past two years. This may partly explain the abnormal returns of the portfolios with higher target return levels compared to the index. It is possible that the model inadvertently selected some of these so-called "winner stocks". It may also be that these stocks performed well in terms of environmental metrics, hence returns were not drastically affected by targeting higher portfolio environmental levels.

As an average of the scores, the environmental objective of the model does not entirely exclude the possibility of including assets that perform poorly environmentally. While this approach increases diversification opportunities by increasing the number of feasible portfolios, it may reduce the green impact that the investor may wish to achieve.

The short holding period of the portfolios, especially in comparison to the training period, may explain their general tendency towards high volatility. Extending the holding period could smoothen out these extremities. In general, the short out-of-sample period is not a viable indicator for the long-term behavior of the portfolio.

To better evaluate the robustness of the results, a typical choice is to perform the optimization for multiple periods. While the findings from the historical frontiers appear robust, past performance is no guarantee for future performance. Another possibility is to replicate the study in a period of economic downturn. Such replication would help assess whether environmentally friendly portfolios are more resilient in conditions of economic turbulence. However, a problem in selecting different or multiple periods is the limited availability of historical environmental data for firms.

Additionally, by selecting a different asset universe, we could determine whether the results are applicable more generally. The results could also be affected by regulation, as performing the optimization in a region with stricter regulations, such as the EU, may provide different results than those in this study.

The model is unable to detect whether the strong performance of the green portfolios during the out-of-sample period is due to overpricing of green assets or superior financial performance in the traditional sense.

In conclusion, the thesis demonstrated that it is possible to integrate environmental objectives into portfolio optimization while maintaining competitive financial returns. Looking forward, as global attention on environmental sustainability increases, it is likely that financial markets will face more legislation and a stronger emphasis on environmental factors. As the trend of sustainable investing is still relatively young, data and literature on the subject are limited. Therefore, future research will be crucial in assessing the reliability of the findings of this thesis.

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