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REVIEW ARTICLE

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Operational Research: methods and applications

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ABSTRACT

Throughout its history, Operational Research has evolved to include methods, models and algorithms that have been applied to a wide range of contexts. This encyclopedic article consists of two main sections: methods and applications. The first summarises the up-to-date knowledge and provides an overview of the state-of-the-art methods and key developments in the various subdomains of the field. The second offers a wide-ranging list of areas where Operational Research has been applied. The article is meant to be read in a nonlinear fashion and used as a point of reference by a diverse pool of readers: academics, researchers, students, and practitioners. The entries within the methods and applications sections are presented in alphabetical order.

The authors dedicate this paper to the 2023 Turkey/Syria earthquake victims. We sincerely hope that advances in OR will play a role towards minimising the pain and suffering caused by this and future catastrophes.

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63. This subsection was written by Kai Virtanen and Raimo P. Hämäläinen.

3.16. Military and homeland security[63]

The birth of OR is related to the use of optimisation modelling for military operations and resource planning during the Second World War. The early linear programming (§2.14) problems ranged from the efficient use of weapon systems to logistics and strategy planning. Today, the arena of defence has expanded extensively with new areas including information and cyber warfare. The need to counter terrorism has created the field of homeland security. OR has a role in all these emerging topics. One can say that all OR methods are applied in military and homeland security problems.

Optimisation methods are used in a wide range of defence and security applications. For instance, assigning weapons to targets (Kline et al., 2019) using integer programming (§2.15; §2.4) has been addressed with a variety of optimisation algorithms. Other integer programming studies include, for example scheduling of training for military personnel (Fauske & Hoff, 2016) as well as military workforce and capital planning (Brown et al., 2004). Mixed integer linear programming is utilised in diverse applications such as path planning of unmanned ground and areal vehicles including drones, mission planning, acquisition decisions of military systems as well as load planning in transportation. Optimisation of vehicles' routes is also carried out by solving network optimisation problems (§2.12) with shortest path algorithms (Royset et al., 2009). Network optimisation is also used, e.g., in developing military countermeasures. Examples of bilevel and robust optimisation (§2.21) formulations cover positioning of defensive missile interceptors (Brown et al., 2005) and design of a supply chain for medical countermeasures against bioattacks (Simchi-Levi et al., 2019). Multiobjective optimisation has been applied, for example in optimising boat resources of coast guard (Wagner & Radovilsky, 2012) and planning of airstrikes against terrorist organisations (Dillenburger et al., 2019). Inherent structures of specific military optimisation problems have motivated the development of new solution techniques (Boginski et al., 2015) including, for example, metaheuristics (§2.13). Such techniques are used, e.g., for solving nonlinear military optimisation tasks (§2.16) such as design of projectiles.

Game theoretic modelling (§2.11) is used in many defence studies. Information related topics include misinformation in warfare (Chang et al., 2022) and public warnings against terrorist attacks (Bakshi & Pinker, 2018). Examples of game theoretic strategy design problems cover the optimal use of missiles and the validation of combat simulations (Poropudas & Virtanen, 2010). Designing security and counter strategies against enemies, terrorists and adversarial countries naturally lead to the use of game models. Interdiction network game models arise in security applications (Holzmann & Smith, 2021), and they are used, e.g., in route planning through a minefield.

Military simulation models (§2.19) are classified into constructive, virtual and live simulations (Tolk, 2012). Constructive simulations do not involve real-time human participation. They are based on well-known modelling methodologies such as Monte Carlo, discrete event and agent-based simulations. Applications of constructive models cover, e.g., the development and use of weapons, sensor and communications systems, planning of operations and campaigns, improving maintenance processes of military systems, and evaluating effects of fire. In addition, cyber-defence analyses have been conducted (Damodaran & Wagner, 2020). Constructive simulations have also been used in simulation-optimisation studies such as scheduling maintenance activities of aircraft, military workforce planning, and aircraft fleet management (Mattila & Virtanen, 2014; Jnitova et al., 2017).

The complexity of modelling human behaviour generates a major challenge for constructive simulation. This issue is avoided in virtual simulations, i.e., simulators in which real people operate simulated systems and in live simulations where real people operate real systems with simulated weapon effects. These practices are typically used, e.g., in military exercises and training of personnel. An emerging trend is to combine live, virtual and constructive simulations into a single simulation activity (Mansikka et al., 2021b). Applications of this simulation type vary from training to testing large-scale systems and mission rehearsals (Hodson & Hill, 2014). In a combined simulation, new ways to measure performance are introduced (Mansikka et al., 2021a) by complementing traditional measures such as loss exchange or kill ratio by human measures such as participants' situation awareness, mental workload and normative performance (Mansikka et al., 2019).

Features of virtual simulation can be recognised in wargaming (Turnitsa et al., 2021) that has been used for military training and educating since the early 19th century. Other wargaming areas are, for example, examination of warfighting tactics as well as evaluation of military operations and scenarios. Nowadays, wargames are also applied in studies of international relations and security as well as in analyses of government policy, international trade, and supply-chain mechanics (Reddie et al., 2018). The implementations of wargames range from manual tabletop map exercises to computer-supported setups in which different OR and artificial intelligence techniques are utilised (Davis & Bracken, 2022).

Dynamic phenomena regarding military and defence are often represented with differential or difference equations. Examples of these models are Lanchester attrition equations that describe the evolution of strengths of opposing forces in gunfire combat (e.g., Jaiswal, 2012). There are also several modifications of these equations aiming to model, e.g., asymmetrical combat, tactical restrictions and even morale issues. Another example of simple combat models is the salvo model that represents naval combat of warships involving missiles

(Hughes, 1995). Optimal control (see also §2.6) has been utilised, for example in planning optimal paths of military vehicles as well as in guidance systems of unmanned aerial vehicles, drones and missiles (Karelahti et al., 2007). For a recent overview, see for example Israr et al. (2022). Another type of optimal control application is the assignment of resources to counter-terror policies and measures (Seidl et al., 2016). Markov decision processes and approximate dynamic programming (§2.9) have recently emerged as important techniques for analysing dynamic military decision-making problems related to, e.g., missile defence interceptors and military medical evacuation (Jenkins et al., 2021).

The need for multicriteria evaluation is common in military decision-making. Example applications of multi-criteria decision analysis (MCDA; see also §2.8) are acquisition of military systems and equipment procurement, military unit realignments and base closures, locating military bases, and assessment of future military concepts and technologies (Ewing et al., 2006; Geis et al., 2011; Harju et al., 2019). Public procurement even for the military is regulated in many countries, and directives require it to consider multiple criteria (Lehtonen & Virtanen, 2022). It is interesting to notice that the recent acquisition decision of a 5th generation multi-role fighter aircraft in Finland was, indeed, supported by MCDA (Keränen, 2018). MCDA weighting methods have also been used to create measures of mental workload in military tasks (Virtanen et al., 2022). In portfolio decision analysis problems, the goal is to find the best set of components, e.g., in weapons systems or in force mix for reconnaissance, with respect to multiple criteria (Burk & Parnell, 2011). The evaluation of the effectiveness of military systems calls also for the use of cost-benefit analysis (§2.18; Melese et al., 2015). Data envelopment analysis (§2.7) is a multicriteria approach helping to seek efficiency also in military problems such as personnel planning.

MCDA studies in homeland security is a broad area ranging from the design of countermeasure portfolios to threat analysis and cyber-security (Wright et al., 2006). The questions of interest include, e.g., identification of terrorists' goals and preferences, estimation of attacks' consequences, and comparison of countermeasure actions (Abbas et al., 2017). Cost-benefit models are also relevant in terrorism research (Hausken, 2018).

Today, we are witnessing the vast growth of the use of machine learning and artificial intelligence (§2.1) in military and security problems (Dasgupta et al., 2022; Galán et al., 2022). Such problem areas are, e.g., wargaming and simulation, command and control of autonomous unmanned vehicles including drones, air surveillance, and cyber-security only to mention a few. Data analytics (see also §2.3) is naturally also used in military OR (Hill, 2020), e.g., for supporting logistics planning. Considering uncertainty is essential, e.g., in intelligence analysis and risk analysis related to terrorism (see also §2.18) . Adversarial risk analysis (Rios Insua et al., 2021) uses Bayesian approaches (see also §2.18) for taking into account information, beliefs and goals of adversaries. A similar approach is also applied in the modelling of pilot decision-making in air combat with influence diagrams (Virtanen et al., 2004). Markov models and Bayesian networks are used to evaluate risks and conduct time dependent probabilistic reasoning related to military missions (Poropudas & Virtanen, 2011). Kaplan (2010) studies the infiltration and interdiction of terror plots using queueing theory (§2.17).

In the future, combat models need to include socio-cultural and behavioural factors (Numrich & Picucci, 2012). We are also likely to see an increase in the modelling of individual and group behaviour as well as the consideration of behavioural issues in military and homeland security contexts. Behavioural game theory can give insights into military strategy and conflict situations. Behavioural OR (§2.2), which studies the impacts of the human modeller and model users including cognitive biases in decision support, is likely to receive increasing attention in military applications as well.

For further readings, we refer to the military OR textbooks by Fox and Burks (2019) and Jaiswal (2012). The recently edited volume by Scala and Howard II (2020) describes various OR methods and how to apply them in military problems. Abbas et al. (2017) and Herrmann (2012) focus on homeland security modelling.

3.17. Natural resources[64]

Climate change and natural resource management require different quantitative and qualitative models that support public policy (Ackermann & Howick, 2022). One of the early papers on the use of modelling for natural resource utilisation describes a resource analysis simulation procedure to assess the environmental impact of human activities (Bryant, 1978). The procedure comprises a structural model to express the complex network of interacting human activity systems and a parametric model to determine the scale of the activity being modelled.

An integrated decision support system for water distribution and management was built to generate alternative water allocation and agricultural production scenarios for a semi-arid region (Datta, 1995). The model considers ground and surface water