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Strategic sector coupling? Market power in heat and power markets[☆]

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ABSTRACT

Power-sector decarbonisation envisages extensive uptake of variable renewable energy (VRE) technologies. Although VRE output is intermittent, coupling between heat and power sectors via combined heat and power (CHP) plants could provide the requisite flexibility. However, strategic CHP plants could use the link between the two energy sectors to manipulate electricity prices. We use a bi-level model to investigate the incentives for the exercise of such market power. At the upper level, a firm with both heat-only and CHP plants is a Stackelberg leader when determining its heat output and is constrained by power-market operations at the lower level. Such a strategic firm produces more (less) heat from its CHP (heat-only) plant vis-à-vis the social optimum to constrain its power output, thereby boosting the electricity price. Additional market power at the lower level from power-only generation induces the strategic heat producer to reduce distortions to its operations as long as the electricity market is relatively large. In order to attenuate welfare losses from such strategic behaviour, we devise an incentive-based regulatory mechanism consisting of a subsidy to or a tax on CHP heat output. Numerical examples illustrate the properties of our analytical results, which can inform future negotiations over CHP cost allocations between regulators and producers.

1. Introduction

A central plank in the decarbonisation pathways of most OECD countries is increased output from variable renewable energy (VRE) such as solar and wind power.[1] This vision has been supported by policies that have led to substantial decreases in the capital costs of VRE. Indeed, VRE's levelised costs are now below those of established technologies such as gas-fired power plants.[2] Yet, intermittent output and the lack of suitable sites in proximity to load centres are obstacles to VRE's more widespread adoption. Thus, diverse strategies for mitigating the effects of VRE's spatio-temporal intermittency have been posited, e.g., demand response, energy storage, flexible (or, controllable) generation, and transmission expansion.

Going beyond these piecemeal measures, the ambition in the recent European Green Deal[3] and the U.S. Inflation Reduction Act of 2022[4] is to foster the electrification of the broader economy with greater coupling between energy sectors, viz., heat, industrial processes, natural gas, power, and transport.[5] Indeed, better coordination between the heat and power sectors, for example, could leverage the flexibility of heat supply and storage to accommodate VRE's intermittent output. Full integration between such sectors would be akin to a socially optimal first-best outcome from a welfare perspective (Xi et al., 2020) but would be in conflict with the decentralised manner in which distinct energy sectors are operated. Consequently, implicit coordination based on either a price or a quantity signal has been proposed to enable

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1 https://www.naturvardsverket.se/en/topics/climate-transition/sveriges-klimatarbete/swedens-climate-act-and-climate-policy-framework/.
2 https://www.eia.gov/outlooks/aeo/electricity_generation/pdf/AEO2023_LCOE_report.pdf.
3 https://ec.europa.eu/info/strategy/priorities-2019-2024/european-green-deal_en.
4 https://www.congress.gov/bill/117th-congress/house-bill/5376.
5 https://www.regeringen.se/informationsmaterial/2022/09/national-electrification-strategy—a-summary/.

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so-called market-aware dispatch of heating plants (Ordoudis et al., 2020).

Such a market-based mechanism could be especially beneficial in the context of combined heat and power (CHP) production. By co-generating heat and power, a CHP plant attains higher overall energy efficiency than separate production of the two commodities. Since a CHP plant provides a link between the two energy sectors, it can adapt its output based on the price signal from the electricity market to maintain energy balance in the power sector. In northern Europe, e.g., Denmark and Finland, CHP plays a prominent role in both the heat and power sectors. In 2019, 36% and 33% of Danish and Finnish, respectively, gross electricity generation was from CHP, cf. an EU average of 12%.[6] CHP's share of heat output in the two countries in the same year was 68% and 73%, respectively.[7,8] Targets for improving energy efficiency and integrating VRE have also prompted the EU as a whole as well as other regions to emphasise cogeneration.[9,10] Indeed, the fact that heating accounts for up to 50% of global energy consumption[11] presents enormous opportunities for CHP expansion.

While better coordination between heat and power production could offset VRE's intermittency, tighter sector coupling could also be exploited by strategic agents. In fact, the deregulated nature of the power sector (Wilson, 2002) has led to empirical evidence of market power even in well-functioning markets such as Nord Pool (Fogelberg and Lazarczyk, 2019; Lundin, 2021; Lundin and Tangerås, 2020; Tangerås and Mauritzen, 2018). As a result, there is a rich literature on strategic behaviour by flexible producers (Crampes and Moreaux, 2001), prosumers (Ramyar et al., 2020), and storage operators (Sioshansi, 2014). For example, hydro producers who are prohibited from outright "spilling" could still move water from peak to off-peak periods to manipulate prices to their advantage (Bushnell, 2003; Debia et al., 2021; Genc et al., 2020). In a similar vein, the district-heating sector could be enticed by the possibility to boost its revenues from participating in the power sector by using the flexibility of its CHP and heat-only portfolio to affect wholesale electricity prices. Although the heat sector is effectively regulated subject to long-term bilateral contracts, heat producers with sufficient leverage would still have the incentive to operate their fleet of CHP and heat-only plants to modify their power output and, thus, electricity prices.

The extant literature on the economics of CHP operations has focused on either optimal dispatch under exogenous prices or second-best coordination mechanisms in an equilibrium setting without strategic behaviour. Examples of the former strand typically take the perspective of a microgrid in devising operational policies for small-scale on-site generation with CHP applications (Marnay et al., 2008). Uncertainty in prices is tackled via either real options (Wickart and Madlener, 2007) or stochastic programming (Maurovich-Horvat et al., 2016) to determine optimal fuel switching or to manage risk, respectively. Meanwhile, equilibrium analyses use game-theoretic models to uncover efficient coordination between interdependent energy sectors, e.g., either electricity and natural gas (Ordoudis et al., 2020; Xi et al., 2020) or heat and power (Mitridati and Pinson, 2016; Mitridati et al., 2020; Wu and Rosen, 1999). With rare exceptions, this strand of the literature assumes perfect competition by market participants even if prices are determined endogenously. Furthermore, game-theoretic models that permit departures from perfect competition only identify circumstances in which market power may be exerted (Mousavian et al., 2021; Virasjoki et al., 2018) but do not propose countervailing regulatory mechanisms to alleviate its impact. Virasjoki et al. (2018), for example, conduct a Nash–Cournot analysis of the Nordic heat and power sectors to illustrate how strategic power companies would shift their heat production towards CHP plants and away from heat-only plants in order to create the possibility to withhold power output. This effect is especially pertinent for extraction plants, which comprise 83% of the installed CHP capacity in the Nordic region (Virasjoki et al., 2018) and exhibit a tradeoff between heat and power output at full load.

In effect, there is a gap in the literature on CHP operations and sector coupling because the existing works overlook the potential for strategic behaviour by a heat producer and how it may be regulated. Indeed, although such an entity's primary product (heat) is typically sold at administratively determined prices that are more amenable to regulatory scrutiny, its power output is often treated as a byproduct (Overland and Sandoff, 2014) that serves merely to compensate for (intermittent) power-only generation. For example, in Denmark, smaller CHP plants operate on a not-for-profit principle in both heat and power sales, but larger CHP plants may profit from the electricity market. Therefore, the allocation of costs between the two outputs for a large CHP plant is not clearcut and depends on negotiations.[12] In this context, sector coupling presents heat producers with the opportunity to exploit their CHP operations to manipulate the electricity price while not altering their overall heat output, which is similar to the perspective of Bushnell (2003), Debia et al. (2021), and Genc et al. (2020) in investigating incentives in a restructured electricity industry. Hence, identifying departures from the socially optimal mix of heat production due to sector coupling would be relevant to energy regulators, e.g., in informing how to allocate costs of CHP plants to power and heat outputs, as part of the transition towards a VRE-based power system.

Given this background and research gap, we develop a game-theoretic model of coupled heat and power sectors in order to address the following three research questions (RQs):

RQ 1. *How are operations and welfare outcomes affected by a profit-maximising strategic heat producer vis-à-vis a welfare maximiser?*

RQ 2. *What are the additional impacts on operations and welfare from the exertion of market power by a power-only producer in the electricity sector?*

RQ 3. *Which incentive-based regulatory mechanism(s) should be implemented as part of CHP cost-allocation negotiations to mitigate the impact of strategic behaviour in coupled heat and power sectors?*

In addressing RQ 1, we find that a strategic heat producer that acts as a Stackelberg leader relative to the electricity market would increase its CHP heat output vis-à-vis that of a benchmark welfare maximiser. This departure from the socially optimal outcome would constrain CHP power output from an extraction plant and increase the electricity price. Allowing for further market power by the power-only producer's behaving à la Cournot could actually lead to more CHP power output (and less CHP heat output) than in a perfectly competitive electricity market by a strategic heat producer as long as the electricity market were relatively large compared to the heat market (RQ 2). Intuitively, this result holds because a strategic heat producer benefits from both higher electricity prices (due to withholding by a dominant power-only producer) and greater electricity sales (due to its increased power output from CHP). Finally, we prove how regulatory intervention in the form of either a subsidy to or a tax on CHP heat output in line with existing protocols for cost allocation leads to the first-best outcome of central planning when the electricity market is perfectly competitive but merely a second-best result when the power-only sector behaves à la Cournot, thereby addressing RQ 3.

[6] https://ec.europa.eu/eurostat/web/energy/database/additional-data.

[7] https://ens.dk/sites/ens.dk/files/Statistik/energy_statistics_2021.pdf.

[8] https://energia.fi/files/5402/District_Heating_in_Finland_2019.pdf.

[9] https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32004L0008&from=DE.

[10] https://www.buzer.de/s1.htm?g=Kraft-W%C3%A4rme-Kopplungsgesetz+-+KWKG&f=1.

[11] https://www.iea.org/reports/renewables-2019.

[12] https://ens.dk/sites/ens.dk/files/contents/material/file/regulation_and_planning_of_district_heating_in_denmark.pdf.

In terms of novelty, we address the themes of sector coupling (Fernqvist et al., 2023; Calvo García et al., 2025; Dai et al., 2025) and market dispatch (Cui et al., 2021; Huang et al., 2025; Seeger et al., 2025) explored in the recent literature. While such work tackles CHP's operational constraints in more detail, only Seeger et al. (2025) from this list allow for market power. Even so, their Nash–Cournot model does not reflect the sequential interactions between heat and power markets as our Stackelberg framework captures. Moreover, besides simply quantifying welfare losses from the exercise of market power, we explicitly propose countervailing regulation to mitigate its impact. In a similar vein, we articulate what makes our work distinct within the broader literature on equilibrium models. First, relative to Virasjoki et al. (2018) and Debia et al. (2021), we take an analytical approach as opposed to their numerical case studies. The advantage of our framework is that we can mathematically prove assertions about market power and technology utilisation. By contrast, numerical problem instances can provide only plausible explanations for observed outcomes. The tradeoff is that an analytical model necessitates abstracting away some real-world details, e.g., spatio-temporal and technological attributes, in order to obtain closed-form results. Nevertheless, the two types of models (analytical and numerical) should be viewed as complementary in informing regulation to guide the energy transition. Since CHP and sector coupling have largely received attention from the numerical perspective, we feel that our work redresses the balance. Second, Virasjoki et al. (2018) use a Nash–Cournot model in which heat and power markets are cleared simultaneously, whereas we have a Stackelberg (leader–follower) setup that more accurately reflects the sequential nature of heat and power sectors' market clearing in the Nordic region. While Mitridati and Pinson (2016) and Mitridati et al. (2020) also use the Stackelberg framework, they assume perfect competition and do not address the strategic behaviour in coupled heat and power sectors. Third, Genc et al. (2020) take imperfect competition into consideration but do not propose mitigation measures to correct for strategic behaviour as we do. Hence, the orientation of our work is orthogonal to that of the recent literature and can, therefore, complement its perspective.

The rest of this paper is structured as follows. Section 2 lays out the framework for analysis and our modelling assumptions. The main analytical results along with the welfare-enhancing regulatory mechanisms are presented in Section 3. A numerical example that illustrates the key findings is given in Section 4, while Section 5 summarises the work and charts directions for future research. Appendix A, B, and C contain the nomenclature, proofs of propositions, and a supplementary numerical example, respectively.

2. Research methodology

Our stylised setting assumes a single representative time period without uncertainty in either demand or generation. There are two types of markets, heat and power, which clear in sequence. This structure reflects the market design in the Nordic region, whereby heat dispatch occurs in advance of electricity market clearing (Mitridati and Pinson, 2016; Mitridati et al., 2020). The sequential dispatch is necessitated primarily because of electricity's treatment as a byproduct of CHP plants and the greater complexity induced by CHP plants' feasible operating regions. For example, Figure 4 in Pinson et al. (2017) illustrates the sequential decision-making timeline for the Copenhagen region, which requires cooperation among three entities via Varmelast.[13] In a similar vein, Finnish Energy reports that CHP plants in Finland are dispatched "...according to the need for district heat".[14] Electricity consumers are passively represented by a linear inverse-demand function, $P(q) = A - q$ [in \$/MW], where $A > 0$ [in \$/MW] is the consumers' maximum willingness to pay for electricity (which can also be thought of as the maximum demand for electricity) and $q \geq 0$ [in MW] is quantity demanded. It is assumed that the slope of the inverse-demand function [in \$/MW^2] is equal to -1. Market clearing is implicit, i.e., q equals total electricity generation, which yields the endogenous market-clearing price for electricity, p [in \$/MW]. All relevant notation is summarised in Appendix A.

Electricity supply consists of (i) profit-maximising power-only generation and (ii) profit-maximising power output from the CHP plant. Power-only generation is represented by a single entity that can behave as either a price taker or a Cournot player. The cost of power-only generation is $\frac{1}{2}Cx^2$ [in \$], where $C > 0$ [in \$/MW^2] is the power-only quadratic cost coefficient and $x \geq 0$ [in MW] is generation output. Power output from the CHP plant, $w \geq 0$ [in MW], has no explicit cost because its operational cost is allocated to heat output. However, CHP power output is restricted by CHP heat output, $y \geq 0$ [in MW$_{\text{th}}$], and incurs an implicit cost triggered by a shift away from the cost-minimising portfolio of heat production. The opportunity cost of marginal power generation from CHP is reflected by the shadow price, λ [in \$/MW], on the CHP plant's capacity.

Heat supply consists of (i) CHP heat output and (ii) heat-only output. Since the heat market functions on long-term contracts (Virasjoki et al., 2018), we assume that the total heat supply is $H > 0$ [in MW$_{\text{th}}$] with a fixed per-unit price of $R > 0$ [in \$/MW$_{\text{th}}$]. Heat-only output is $H - y$ and costs $\frac{1}{2}E\left(H - y - BH^{\text{CM}}\right)^2$, whereas CHP heat output costs $\frac{1}{2}BE\left(y - H^{\text{CM}}\right)^2$, where E [in \$/MW$_{\text{th}}{}^2$] is the heat-only quadratic cost coefficient and B [unitless] is a constant scaling factor. We assume $E > C$ and $B > 1$ to reflect the properties that heat-only output is more costly than power-only output in a VRE-dominated power system and CHP output is more costly than heat-only output, respectively. Here, the costs of heat production are expressed in terms of deviations from the cost-minimising heat production by heat-only and CHP units, $BH^{\text{CM}} \equiv \frac{BH}{B+1}$ and $H^{\text{CM}} \equiv \frac{H}{B+1}$, respectively. This result follows from the fact that a cost-minimising heat producer that has to meet a total heat output of H solves $\min_{y \geq 0} \frac{1}{2}BEy^2 + \frac{1}{2}E(H-y)^2$, which yields the first-order necessary condition (FONC) for an interior solution such that $BEy = E(H-y)$. Intuitively, the marginal costs of CHP and heat-only output are equated, which leads to the aforementioned solution for H^{CM}. All heat production is operated by a single profit-maximising entity that is the provider of district-heating services with sales into the electricity market.

In a centrally planned, i.e., integrated, setting, all heat and power decisions, q, x, y, and w, are treated as if they were the responsibility of a single welfare-maximising entity. This so-called central-planning (CP) paradigm serves as the benchmark because it yields the first-best outcome from a societal perspective. In a deregulated industry, however, both heat and power producers make decentralised decisions in order to maximise their profits. Consequently, the heat producer determines its CHP heat output, $0 \leq y \leq H$, at the upper level in anticipation of electricity market clearing at the lower level (through consumption, q), see Fig. 1. Note that the upper-level decision also implicitly includes heat-only dispatch, $H - y$. At the lower level, power-only generation, x, and CHP power output, w, are determined while taking y as given. We take a bi-level approach to model the problem of a strategic CHP producer, which is equivalent to a Stackelberg leader–follower game (Gabriel et al., 2013). Furthermore, the electricity market itself may be either perfectly competitive (PC) or subject to market power by the power-only producer (MP). In the former setting, both CHP power output and power-only output are price taking, whereas the power-only producer behaves à la Cournot under MP (von der Fehr, 2010). In either case, the heat producer sets y at the upper level while behaving perfectly competitively in the electricity market (via w at the lower level). Hence, the CHP producer's upper-level decision implicitly affects the lower-level market-clearing price for electricity, p, which is the result of a Nash equilibrium in the electricity market given the CHP heat output, y.

In order to obtain comparable analytical results across the three settings, we impose the following mild assumptions:

[13] https://www.varmelast.dk/lastfordeling/varmeplaner.

[14] https://energia.fi/en/energy-sector-in-finland/energy-production/combined-heat-and-power-generation/.

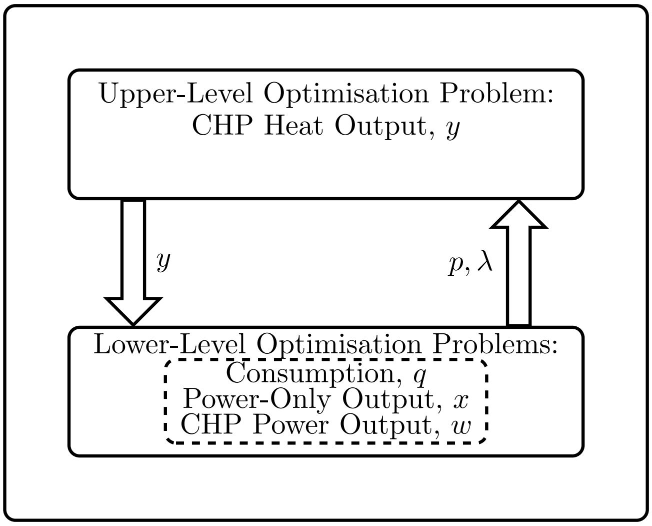


Fig. 1. Bi-level framework.

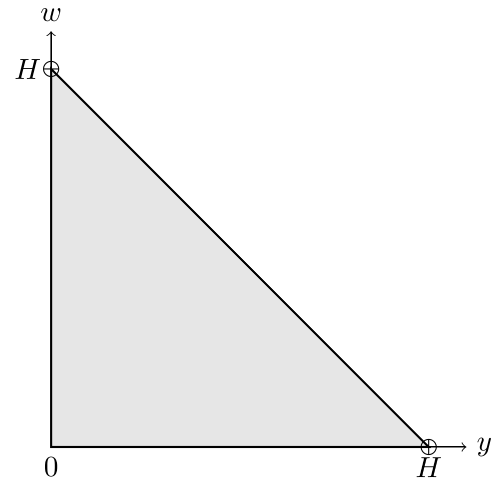


Fig. 2. Stylised feasible operating region of an extraction CHP technology.

Assumption A1. $B > 1$

Assumption A2. $E > C > 0$

Assumption A3. $A > H > 0$

Assumption A4. $A(C+1) < (C+1)H + (C+2)EH$

Assumption A5. $w + y - H \leq 0$

The aforementioned assumptions' intuition is as follows. Assumption A1 formalises that CHP heat output is more costly than heat-only output. Assumption A2 reflects the fact that heat-only output is more costly than power-only output, which is reasonable given the dominance of fossil-free electricity generation, viz., reservoir-based hydro, nuclear, and VRE, envisaged in future power systems. Assumption A3 captures the notion that the electricity market is not "too small" relative to the heat market. This is necessary to ensure that there is strictly positive power-only output and is reasonable because the power market tends to be larger than the one for district heating. For example, the Swedish net electricity supply in 2023 was approximately 170 TWh as opposed to a net supply of about 55 TWh for district heating.[15] Assumption A4 stipulates that the electricity market is not "too large" relative to the heat market. This is necessary to ensure that heat output from CHP is strictly positive. It also implies that $AC < CH + (C+1)EH$ because $A(C+1) < (C+1)H + (C+2)EH \Rightarrow AC < CH + \frac{C(C+2)EH}{C+1}$ and $C + 1 > \frac{C(C+2)}{C+1}$. This assumption is more challenging to anchor in real-world data because it relies upon parameters C and E that are less straightforward to estimate from power-only and heat-only supply functions, which tend to be piecewise-linear functions as opposed to linear functions as we assume. Nevertheless, the purpose of Assumption A4 is to rule out economically uninteresting solutions in which CHP plants do not produce any heat because the power market is "too large". From the aforementioned data, this outcome may be ruled out as the Swedish power market is about three times the district-heating sector, i.e., it is not several orders of magnitude larger. Assumption A5 is a representation of a feasible operating region for the extraction CHP technology, which comprises the majority of the installed CHP capacity in the Nordic region (Virasjoki et al., 2018). Unlike the less flexible backpressure technology, the maximum CHP power output of an extraction unit is a decreasing function of the CHP heat output, y, up to its capacity limit, which is rated to coincide with the size of the contract for heating, H (Fig. 2). Finally, note that Assumptions A3 and A4 are consistent in that $A > H \Leftrightarrow A(C+1) > (C+1)H$ does not conflict with the restriction $A(C+1) < (C+1)H + (C+2)EH$.

In reality, allocating the cost of CHP output to power and heat production is not straightforward due to the complexity of cogeneration operations. Virasjoki et al. (2018) rely on the so-called energy method, whereas Mitridati et al. (2020) refer to forecasted electricity prices to estimate the opportunity cost of heat production using the feasible operating region. Meanwhile, a survey of 33 Swedish district-heating producers reveals that most of them treat electricity as a byproduct (Overland and Sandoff, 2014). Since we aim to capture the strategic incentive to deviate from a first-best, i.e., socially optimal, production of heat under CP, we allocate all costs associated with CHP operations to heat production. Our subsequent regulatory analysis that distils either a subsidy to or a tax on CHP heat output could establish a benchmark for informing negotiations over such cost allocation. Likewise, our stylised feasible operating region abstracts away from the nonconvexities of CHP units, e.g., unit commitment and minimum-production levels, to capture the production possibility between heat and power output at the plant's rated capacity. Moreover, the tradeoff rate between heat and power production depicted in Fig. 2 will typically not be unitary. Again, since we seek to capture in a stylised manner the compromise between the two types of outputs, we normalise the gradient of the feasible operating region to -1 for analytical expediency.

3. Problem formulation and analytical results

Here, we derive the main analytical results under three market settings, CP, PC, and MP, in Sections 3.1–3.3, respectively. Appendix B contains all proofs of propositions.

3.1. Benchmark: Central planning

Under CP, a single welfare-maximising entity makes all decisions by solving the following quadratic programming (QP) problem:

$$\max_{w\geq 0, x\geq 0, y\geq 0} \; A(x+w) - \frac{1}{2}(x+w)^2 - \frac{1}{2}Cx^2 - \frac{1}{2}BE\left(y - H^{\mathrm{CM}}\right)^2 - \frac{1}{2}E\left(H - y - BH^{\mathrm{CM}}\right)^2 + RH \tag{1}$$

$$\text{s.t.} \quad w + y - H \leq 0 : \; \lambda \tag{2}$$

[15] https://www.scb.se/en/finding-statistics/statistics-by-subject-area/energy/energy-supply-and-use/annual-energy-statistics-electricity-gas-and-district-heating/.

Eq. (2) states that CHP power output is limited by a decreasing function of its heat output (y), and λ is the corresponding shadow price. Note that (1) comprises consumer surplus (CS), generator surplus (GS), and heat surplus (HS). [16]

Since (1) s.t. (2) is a QP problem with a convex objective function and a convex feasible region, it may be replaced by its Karush–Kuhn–Tucker (KKT) conditions, which are both necessary and sufficient for global optimality:

$$0 \leq w \perp -A + (x + w) + \lambda \geq 0 \tag{3}$$
$$0 \leq x \perp -A + (x + w) + Cx \geq 0 \tag{4}$$
$$0 \leq y \perp BEy - E(H - y) + \lambda \geq 0 \tag{5}$$
$$0 \leq \lambda \perp H - y - w \geq 0 \tag{6}$$

Intuitively, the KKT conditions (3)–(5) state that the marginal benefit equals the marginal cost of each primal variable, w, x, and y, in case of interior solutions. For example, if $y > 0$, then (5) indicates that the marginal benefit of infinitesimally more CHP heat output, $E(H - y)$, from reduced heat-only output equals its marginal cost of production, BEy, plus the opportunity cost of not using CHP capacity for power output, λ. Using the facts that $H^{\text{CM}} = \frac{H}{B+1}$ and $BH^{\text{CM}} = \frac{BH}{B+1}$ from Section 2, we have $H - BH^{\text{CM}} = H^{\text{CM}}$ and $(B+1)H^{\text{CM}} = H$, which is used to obtain the result in (5).

Assuming interior solutions, we solve (3)–(6) analytically to yield:

$$w^{\text{CP}} = \frac{AC + B(C+1)EH}{C + (B+1)(C+1)E} \tag{7}$$
$$x^{\text{CP}} = \frac{[A(B+1) - BH]E}{C + (B+1)(C+1)E} \tag{8}$$
$$y^{\text{CP}} = \frac{(C+1)(E+1)H - AC - H}{C + (B+1)(C+1)E} \tag{9}$$
$$\lambda^{\text{CP}} = \frac{[A(B+1) - BH]CE}{C + (B+1)(C+1)E} \tag{10}$$

Furthermore, from (3) and the definition of the inverse-demand function for electricity, $P(w + x)$, it follows that λ^{CP} is the market-clearing price of electricity, p^{CP}. It is trivial to show that (7)–(10) are, indeed, interior, i.e., $w^{\text{CP}} > 0$, $x^{\text{CP}} > 0$, $0 < y^{\text{CP}} < H$, and $\lambda^{\text{CP}} > 0$, due to Assumptions A1–A5.

In terms of comparative statics, we conduct them with respect to C in order to focus on how fluctuations in the marginal cost of power-only generation, e.g., related to water values, nuclear maintenance, and VRE intermittency, affect optimal CHP heat output. The findings are in line with intuition as formalised by Proposition P1.

Proposition P1. $\frac{\partial y^{CP}}{\partial C} < 0$.

In effect, more costly power-only generation monotonically decreases CHP heat output to produce more CHP power output as partial compensation for reduced conventional electricity generation.

3.2. Deregulated industry: Perfectly competitive generation

Under PC, both the power-only generator and the heat producer (with CHP power output) are price takers at the lower level. At the upper level, the heat producer is a Stackelberg leader when determining its CHP heat output in anticipation of electricity market clearing. Our solution approach is to proceed via backward induction by first obtaining the lower-level subgame-perfect Nash equilibrium before determining the optimal upper-level CHP output of the heat producer, while taking account of its impact on electricity-market operations.

3.2.1. Perfectly competitive generation: Lower level

The heat producer takes the electricity price, power-only generation, heat-only output, and CHP heat output as given when selecting CHP power output, w, to maximise its profit in the electricity market:

$$\max_{w \geq 0} \; pw \quad \text{s.t. (2)} \tag{11}$$

Meanwhile, the power-only producer is also a price taker when selecting the profit-maximising level of power-only output, x, given the CHP heat output, CHP power output, and heat-only output:

$$\max_{x \geq 0} px - \frac{1}{2}Cx^2 \tag{12}$$

Since each lower-level problem, i.e., (11) s.t. (2) as well as (12), is convex, it may be replaced by its KKT conditions[17]:

$$0 \leq w \perp -A + (x + w) + \lambda \geq 0 \tag{13}$$
$$0 \leq x \perp -A + (x + w) + Cx \geq 0 \tag{14}$$
$$0 \leq \lambda \perp H - y - w \geq 0 \tag{15}$$

Assuming interior solutions, we solve (13)–(15) analytically to obtain:

$$w^{\text{PC}}(y) = H - y \tag{16}$$
$$x^{\text{PC}}(y) = \frac{A + y - H}{C + 1} \tag{17}$$
$$\lambda^{\text{PC}}(y) = \frac{(A + y - H)C}{C + 1} \tag{18}$$

Furthermore, the equilibrium electricity price, $p^{\text{PC}}(y)$, is simply equal to the opportunity cost of marginal power generation from CHP, $\lambda^{\text{PC}}(y)$.

3.2.2. Perfectly competitive generation: Upper level

The heat producer's profit-maximisation problem is constrained by the lower-level problems[18]:

$$\max_{y \geq 0 \cup \{w \geq 0, x \geq 0\} \cup p} \; pw - \frac{1}{2}BE\left(y - H^{\text{CM}}\right)^2 - \frac{1}{2}E\left(H - y - BH^{\text{CM}}\right)^2 + RH \quad \text{s.t. (11) s.t. (2), (12)} \tag{19}$$

[16] Social welfare is decomposed as follows:

• CS is the gross benefit to consumers from electricity consumption, $A(x+w) - \frac{1}{2}(x+w)^2$, minus the cost of electricity purchased, $p(x+w)$.

• GS is the revenue from power-only electricity sales, px, less the cost of power-only generation, $\frac{1}{2}Cx^2$.

• HS is the revenue from CHP electricity sales, pw, plus the revenue from heat sales, RH, minus the cost of CHP heat production, $\frac{1}{2}BE\left(y - H^{\text{CM}}\right)^2$, minus the cost of heat-only production, $\frac{1}{2}E\left(H - y - BH^{\text{CM}}\right)^2$.

Since the cost of electricity purchased by the consumer, $p(x+w)$, cancels with the revenue terms accruing to the generator, px, and to the heat producer, pw, social welfare may be expressed as in (1). Here, p is the endogenous market-clearing price of electricity.

[17] Explicit representation of the consumer at the lower level entails adding the consumer-surplus-maximisation problem $\max_{q \geq 0} Aq - \frac{1}{2}q^2 - pq$ and a market-clearing condition $x + w = q$ with corresponding dual variable p. The KKT condition for the consumer-surplus-maximisation problem yields $0 \leq q \perp -A + q + p \geq 0$, which leads to $p = A - (x + w)$ from the market-clearing condition if $q > 0$. This is what enters into (13)–(14) to render $p = \lambda$ and $p = Cx$ if $w > 0$ and $x > 0$, respectively.

[18] It should be noted that p is a dual variable for the (implicit) lower-level market-clearing constraint. In spite of being a price taker at the lower level, the heat producer indirectly influences the market-clearing electricity price through its upper-level decision. As a result, the notation $y \geq 0 \cup \{w \geq 0, x \geq 0\} \cup p$ in (19) indicates that y is the heat producer's upper-level decision variable, while those influencing the upper-level problem through the constraining problems are w, x, and p.

Replacing the lower-level problems by their KKT conditions (13)–(15), we have a mathematical program with equilibrium constraints (MPEC):

(19)

s.t. (13)–(15)

By further inserting the interior solutions from the lower level (16)–(18), we obtain the following single-level unconstrained QP:

$$\max_{y\geq 0} p^{\mathrm{PC}}(y)\, w^{\mathrm{PC}}(y) - \frac{1}{2}BE\left(y - H^{\mathrm{CM}}\right)^2 - \frac{1}{2}E\left(H - y - BH^{\mathrm{CM}}\right)^2 + RH \tag{20}$$

Since the electricity price and the power output from CHP are functions of y, they cannot be ignored when taking the FONC:

$$\frac{C(2H - A - 2y)}{C+1} - BEy + E(H - y) = 0 \tag{21}$$

Note that the FONC (21) equates the marginal revenue from CHP heat production with its marginal cost and may also be expressed as $\mathcal{F}^{\mathrm{PC}}(y) = 0$. The second-order sufficient condition (SOSC) may be verified because the partial derivative of (21) with respect to y is less than zero. We can readily solve (21) to derive the optimal CHP heat output by a profit-maximising heat producer under PC:

$$y^{\mathrm{PC}} = \frac{C(2H - A) + (C+1)EH}{2C + (B+1)(C+1)E} \tag{22}$$

Again, the fact that the solutions (16)–(18) and (22) are interior, i.e., $w^{\mathrm{PC}} \equiv w^{\mathrm{PC}}\left(y^{\mathrm{PC}}\right) > 0$, $x^{\mathrm{PC}} \equiv x^{\mathrm{PC}}\left(y^{\mathrm{PC}}\right) > 0$, $\lambda^{\mathrm{PC}} \equiv \lambda^{\mathrm{PC}}\left(y^{\mathrm{PC}}\right) > 0$, and $0 < y^{\mathrm{PC}} < H$, follows from Assumptions A1–A5.

As shown by Proposition P2, a strategic heat producer uses more CHP heat output and less heat-only output in order to constrain its CHP power output while still fulfilling the obligations of its heat contract, thereby creating artificial scarcity in the electricity market.

Proposition P2. $y^{PC} > y^{CP}$.

Thus, in contrast to a welfare maximiser, a profit-maximising heat producer manipulates the electricity price by distorting the composition of its heat output, which partially addresses RQ 1.

While the socially optimal CHP heat output decreases monotonically with C in order to increase the quantity of electricity supplied, cf. Proposition P1, a profit-maximising heat producer's incentives lead to an ambiguity as summed up in Proposition P3.

Proposition P3. *$\frac{\partial y^{PC}}{\partial C} < 0$ only if $A(B+1) > 2BH$, i.e., the electricity market is "large" relative to the heat market. Otherwise, if $A(B+1) \leq 2BH$, i.e., the electricity market is relatively "small", then $\frac{\partial y^{PC}}{\partial C} \geq 0$.*

Intuitively, more costly power-only generation should monotonically decrease CHP heat output in order to enable more CHP power output. Yet, if the electricity market is relatively "small", then decreasing CHP heat output for more CHP power output will only cause the electricity price to drop precipitously without a sufficiently offsetting increase in electricity sales.[19] Hence, a strategic heat producer that participates in a perfectly competitive electricity market may actually increase its CHP heat output as power-only generation becomes more expensive if the electricity market is "small".

3.2.3. Perfectly competitive generation: Incentive-based regulatory mechanism

In contrast to a strategic (profit-maximising) heat producer, for a notional welfare-maximising heat producer, the upper-level objective function is simply social welfare as in (1). Thus, its optimal solution, $\hat{y}^{\mathrm{PC}}$, results from solving the following bi-level problem:

$$\max_{y\geq 0 \cup \{w\geq 0, x\geq 0\}\cup p} A(x+w) - \frac{1}{2}(x+w)^2 - \frac{1}{2}Cx^2 - \frac{1}{2}BE\left(y - H^{\mathrm{CM}}\right)^2 - \frac{1}{2}E\left(H - y - BH^{\mathrm{CM}}\right)^2 + RH \tag{23}$$

s.t. (11) s.t. (2)

(12)

Using the lower-level PC KKT conditions (13)–(15), the bi-level problem may again be transformed into an MPEC before being recast using the lower-level PC solutions (16)–(18) as a single-level unconstrained QP:

$$\max_{y\geq 0} A\left(x^{\mathrm{PC}}(y) + w^{\mathrm{PC}}(y)\right) - \frac{1}{2}\left(x^{\mathrm{PC}}(y) + w^{\mathrm{PC}}(y)\right)^2 - \frac{1}{2}Cx^{\mathrm{PC}}(y)^2 - \frac{1}{2}BE\left(y - H^{\mathrm{CM}}\right)^2 - \frac{1}{2}E\left(H - y - BH^{\mathrm{CM}}\right)^2 + RH \tag{24}$$

Consequently, $\hat{y}^{\mathrm{PC}}$ from solving (24) is just equal to y^{CP} from (9). It, therefore, follows that $\hat{y}^{\mathrm{PC}}$ meets the properties obtained in Section 3.1 and Proposition P1, viz., that $\hat{y}^{\mathrm{PC}}$ is an interior solution and $\frac{\partial \hat{y}^{\mathrm{PC}}}{\partial C} < 0$. Analogous to (21), we can write the corresponding FONC for (24) as $\hat{\mathcal{F}}^{\mathrm{PC}}(y) = 0$:

$$\frac{C(H - A - y)}{C+1} - BEy + E(H - y) = 0 \tag{25}$$

In contrast to (21), welfare-maximising CHP heat output is such that the marginal utility to society is equated with the marginal cost of production. From (25), it is evident that $\hat{\mathcal{F}}^{\mathrm{PC}}(y)$ is a strictly decreasing linear function of y but with a smaller vertical intercept than that of $\mathcal{F}^{\mathrm{PC}}(y)$, i.e., $\mathcal{F}^{\mathrm{PC}}(0) > \hat{\mathcal{F}}^{\mathrm{PC}}(0)$. Moreover, $\hat{\mathcal{F}}^{\mathrm{PC}}(y)$ has a smaller gradient (in absolute terms) than $\mathcal{F}^{\mathrm{PC}}(y)$ does, i.e., $\left|\frac{\partial \hat{\mathcal{F}}^{\mathrm{PC}}}{\partial y}\right| < \left|\frac{\partial \mathcal{F}^{\mathrm{PC}}}{\partial y}\right|$. Fig. 3 illustrates the difference between the two FONCs. The SOSC may also be verified because the partial derivative of (25) with respect to y is less than zero.

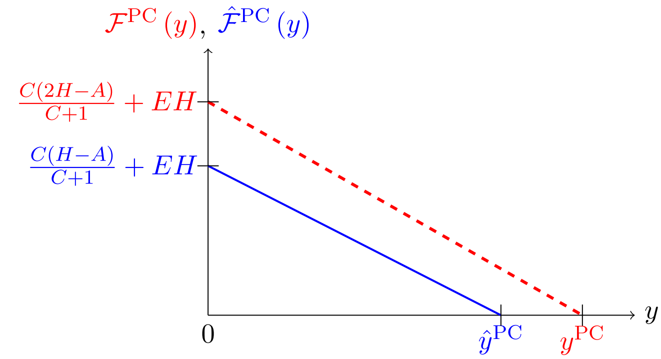


Fig. 3. PC FONCs.

In reality, a welfare-maximising entity may not directly assume operations of CHP and heat-only plants. Nevertheless, a profit-maximising heat producer's incentives may still be aligned with those of a welfare maximiser through the use of a regulatory mechanism to render a first-best outcome. In this context, the incentive-based regulatory mechanism, $\hat{S}^{\mathrm{PC}}$ [in \$/MW$_{\mathrm{th}}$], involves reducing the vertical intercept of $\mathcal{F}^{\mathrm{PC}}(y)$ as formalised in Proposition P4 and partially addressing RQ 3.

Proposition P4. *The regulatory mechanism $\hat{S}^{PC} = -\left(\frac{BCEH + \frac{AC^2}{C+1}}{C+(B+1)(C+1)E}\right) < 0$ aligns public and private incentives to yield the socially optimal CHP heat output under PC.*

In Proposition P4, the regulatory mechanism is effectively a tax that penalises CHP heat output. From a policy perspective, it may be used

[19] It is theoretically possible to have $2BH - A(B+1) \geq 0$, i.e., the electricity market is relatively "small", even though $A > H$ because $2B > B+1$ for $B > 1$. Moreover, the more expensive CHP heat output is with respect to heat-only production, the more scope there is to increase the CHP heat output as its cost-minimising operations, H^{CM}, are low.

as a benchmark in negotiations between the regulator and profit-maximising heat producers for allocating costs of CHP output. It should also be noted that the introduction of the regulatory mechanism affects the calculation of social welfare in two ways. First, heat surplus becomes $\hat{HS}^{\rm PC} = p^{\rm PC}\left(\hat{y}^{\rm PC}\right) w^{\rm PC}\left(\hat{y}^{\rm PC}\right) + \hat{S}^{\rm PC}\hat{y}^{\rm PC} - \frac{1}{2}BE\left(\hat{y}^{\rm PC} - H^{\rm CM}\right)^2 - \frac{1}{2}E\left(H - \hat{y}^{\rm PC} - BH^{\rm CM}\right)^2 + RH$. Second, regulator surplus (RS) is introduced as $\hat{RS}^{\rm PC} = -\hat{S}^{\rm PC}\hat{y}^{\rm PC}$, which is positive in case of a tax, i.e., if $\hat{S}^{\rm PC} < 0$.

3.3. Deregulated industry: Market power by power-only output

Under MP, the heat producer acts as a price taker at the lower level when selling electricity from its CHP plant, but the power-only producer exerts market power in electricity output (von der Fehr, 2010). At the upper level, the heat producer still acts as a Stackelberg leader when deciding upon its CHP heat output along with its heat-only operations. As in Section 3.2, backward induction is used to tackle the bi-level problem by first obtaining the lower-level Nash–Cournot equilibrium before solving for the upper-level CHP heat output.

3.3.1. Market power by power-only output: Lower level

Both power-only generation and CHP power production take the CHP heat output as given. CHP power production also takes the electricity price as given when maximising its operating profit, i.e., (11) s.t. (2). By contrast, while the power-only producer takes CHP power output as given, it can manipulate the electricity price via its output (26):

$$\max_{x\geq 0}\,(A-(x+w))\,x-\frac{1}{2}Cx^2 \tag{26}$$

Since each lower-level problem, i.e., (11) s.t. (2) as well as (26), is convex, it may be replaced by its KKT conditions, (13), (15), and (27), which is given as follows:

$$0 \leq x \perp -A+2x+w+Cx \geq 0 \tag{27}$$

In contrast to PC, the MP lower-level equilibrium reflected by (13), (15), and (27) leads to withholding of power-only generation for a given y because power-only output in case of an interior solution equates the marginal revenue, $A-(2x+w)$, to marginal cost, Cx.

Assuming interior solutions, we solve (13), (15), and (27) to yield the following expressions:

$$w^{\rm MP}(y) = H-y \tag{28}$$

$$x^{\rm MP}(y) = \frac{A+y-H}{C+2} \tag{29}$$

$$\lambda^{\rm MP}(y) = \frac{(A+y-H)(C+1)}{C+2} \tag{30}$$

As under PC in Section 3.2.1, the equilibrium electricity price, $p^{\rm MP}(y)$, equals the opportunity cost of marginal power generation from CHP, $\lambda^{\rm MP}(y)$. Hence, it is evident that for a given y, we have $x^{\rm MP}(y) < x^{\rm PC}(y)$ and $\lambda^{\rm MP}(y) > \lambda^{\rm PC}(y)$.

3.3.2. Market power by power-only output: Upper level

Inserting the lower-level solutions, (28)–(30), directly into the upper-level objective function, (19), we obtain a single-level QP problem similar to that in (20):

$$\max_{y\geq 0}\, p^{\rm MP}(y)\, w^{\rm MP}(y) - \frac{1}{2}BE\left(y-H^{\rm CM}\right)^2 - \frac{1}{2}E\left(H-y-BH^{\rm CM}\right)^2 + RH \tag{31}$$

Assuming interior solutions, the FONC is:

$$\frac{(C+1)(2H-A-2y)}{C+2} - BEy + E(H-y) = 0 \tag{32}$$

The first term of (32) now encapsulates the impact of power-only generation's market power at the lower level, cf. (21). From (32), the FONC may also be expressed as $\mathcal{F}^{\rm MP}(y) = 0$. The SOSC is verified because the partial derivative of (32) with respect to y is less than zero. We analytically solve (32) to determine the optimal CHP heat output by a profit-maximising heat producer under MP:

$$y^{\rm MP} = \frac{(C+1)(2H-A)+(C+2)EH}{2(C+1)+(B+1)(C+2)E} \tag{33}$$

Assumptions A1–A5 trivially ensure that the solutions (28)–(30) and (33) are interior, i.e., $w^{\rm MP} \equiv w^{\rm MP}\left(y^{\rm MP}\right) > 0$, $x^{\rm MP} \equiv x^{\rm MP}\left(y^{\rm MP}\right) > 0$, $\lambda^{\rm MP} \equiv \lambda^{\rm MP}\left(y^{\rm MP}\right) > 0$, and $0 < y^{\rm MP} < H$.

Recall that a strategic heat producer under PC optimally increases its CHP heat output vis-à-vis CP in order to restrict its CHP electricity sales, cf. Proposition P2. With additional withholding of electricity generation by the power-only producer under MP, it is not immediately apparent whether a strategic heat producer would still engage in such distortionary behaviour. Indeed, it may have the incentive to tilt its heat production away from CHP because the power-only producer is already boosting the electricity price. Thus, a strategic heat producer under MP may even be able to enjoy both higher electricity prices and sales relative to CP. As shown in Proposition P5, it reduces CHP heat output vis-à-vis CP as long as both the electricity market is "large" relative to the heat market and power-only generation is relatively inexpensive. In such a situation, the heat producer under MP has less leverage in the electricity market and is actually better off increasing the volume of its electricity sales, i.e., by restricting CHP heat output.

Proposition P5. *$y^{MP} < y^{CP}$ only if both (i) $A(B+1) > 2BH$, i.e., the electricity market is "large" relative to the heat market, and (ii) $C < C^{MP}$, i.e., power-only generation is "cheap", where C^{MP} is the positive root of the upward-facing parabola, $Q^{MP}(C)$. Otherwise, if either (i) $A(B+1) \leq 2BH$ or (ii) both $A(B+1) > 2BH$ and $C \geq C^{MP}$, then $y^{MP} \geq y^{CP}$.*

However, if either (i) the electricity market is relatively "small" or (ii) the electricity market is relatively "large" but power-only generation is relatively expensive, then the strategic heat producer under MP is incentivised to increase the electricity price by further restricting power output from CHP to prevent a collapse in the electricity price, which partially addresses RQ 2.

In a similar vein, we can compare a strategic heat producer's incentives under PC and MP. A "large" electricity market will induce underproduction of heat from CHP vis-à-vis PC in order to exploit the possibility of both a higher electricity price and expanded electricity sales. However, a "small" electricity market will entice the strategic heat producer to create additional scarcity under MP on top of power-only generation's withholding. Proposition P6 formalises this result and also partially addresses RQ 2. Fig. 4 summarises the ordering of CHP heat output under the various market settings.

Proposition P6. *$y^{MP} < y^{PC}$ only if $A(B+1) > 2BH$, which holds for a relatively "large" electricity market.*

Given the preceding discussion and the ambiguous comparative statics under PC, cf. Proposition P3, it is not surprising that the optimal CHP heat output under MP may also either increase or decrease with C. This finding is summarised in Proposition P7.

Proposition P7. *$\frac{\partial y^{MP}}{\partial C} < 0$ only if $A(B+1) > 2BH$, i.e., the electricity market is "large" relative to the heat market. Otherwise, if $A(B+1) \leq 2BH$, i.e., the electricity market is relatively "small", then $\frac{\partial y^{MP}}{\partial C} \geq 0$.*

3.3.3. Market power by power-only output: Incentive-based regulatory mechanism

As in Section 3.2.3, it is also possible to have a welfare-maximising entity at the upper level, i.e., one that maximises social welfare (23) by selecting y subject to the lower-level problems, (11) s.t. (2) as well as (26). Proceeding directly to the QP formulation by inserting in the

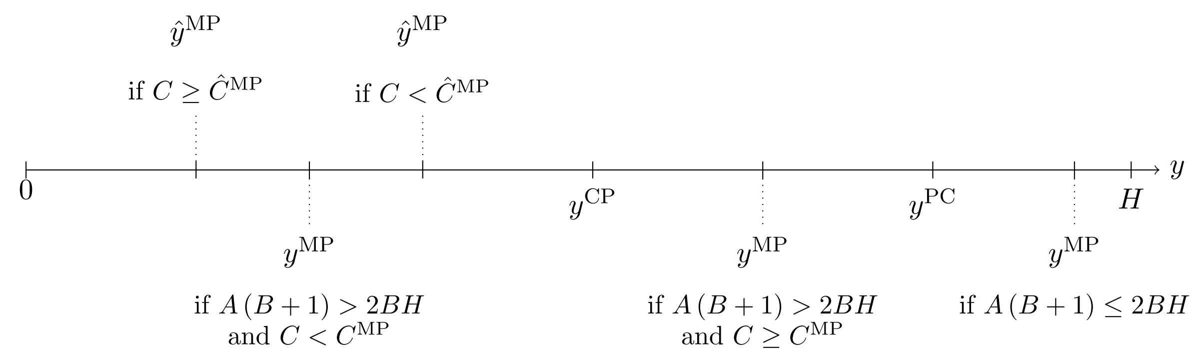


Fig. 4. Ordering of CHP heat output under various market settings.

lower-level solutions, (28)–(30), we have an unconstrained QP problem as follows:

$$\max_{y\geq 0}\; A\left(x^{\mathrm{MP}}(y)+w^{\mathrm{MP}}(y)\right)-\frac{1}{2}\left(x^{\mathrm{MP}}(y)+w^{\mathrm{MP}}(y)\right)^2-\frac{1}{2}Cx^{\mathrm{MP}}(y)^2$$
$$-\frac{1}{2}BE\left(y-H^{\mathrm{CM}}\right)^2-\frac{1}{2}E\left(H-y-BH^{\mathrm{CM}}\right)^2+RH \qquad (34)$$

Assuming an interior solution, the FONC for (34) may be expressed as $\hat{\mathcal{F}}^{\mathrm{MP}}(y)=0$, which equates the marginal utility to society from CHP heat output with its marginal cost of production, cf. (32):

$$\frac{\left(C^2+3C+1\right)(H-A-y)}{(C+2)^2}-BEy+E(H-y)=0 \qquad (35)$$

The SOSC is verified because the partial derivative of (35) with respect to y is less than zero. We can analytically solve (35) to find the optimal CHP heat output by a welfare-maximising heat producer under MP:

$$\hat{y}^{\mathrm{MP}}=\frac{(C+2)^2EH-(A-H)\left[C+(C+1)^2\right]}{C+(C+1)^2+(B+1)(C+2)^2E} \qquad (36)$$

That $\hat{y}^{\mathrm{MP}}$ is interior is verified by Proposition P8.

Proposition P8. *Under MP with a welfare-maximising heat producer at the upper level, the solution in (36) is interior, i.e.,* $0<\hat{y}^{MP}<H$.

In contrast to the ambiguous impact of C on y^{MP}, cf. Proposition P7, $\hat{y}^{\mathrm{MP}}$ monotonically decreases with C as formalised by Proposition P9.

Proposition P9. $\frac{\partial \hat{y}^{MP}}{\partial C}<0$.

Intuitively, more costly power-only generation monotonically decreases CHP heat output, thereby leading to more CHP power output. Indeed, a welfare maximiser straightforwardly attempts to compensate for lost power-only sales without narrow consideration for its own profit.

It is also possible to compare $\hat{y}^{\mathrm{MP}}$ with CHP heat output under CP, y^{CP}, PC, y^{PC}, and MP, y^{MP}. First, Proposition P10 reveals that it always the case that a welfare maximiser under MP produces less CHP heat output than under CP.

Proposition P10. $\hat{y}^{MP}<y^{CP}$.

Here, the withholding of power-only generation under MP vis-à-vis CP necessitates increased CHP power output, which a welfare-maximising heat producer is able to provide. Second, in a similar vein, Proposition P11 demonstrates that a welfare maximiser under MP always produces less CHP heat than under PC in order to mitigate the impact of power-only generation curbing.

Proposition P11. $\hat{y}^{MP}<y^{PC}$.

Third, the ordering between $\hat{y}^{\mathrm{MP}}$ and y^{MP} is ambiguous as distilled by Proposition P12 and depends on the relative cost of power-only output, cf. Proposition P5.

Proposition P12. *$\hat{y}^{MP}>y^{MP}$ as long as $C<\hat{C}^{MP}$, i.e., power-only generation is not "too expensive", where $\hat{C}^{MP}$ is the positive real root of the cubic function, $\hat{Q}^{MP}(C)$.*

The ordering of $\hat{y}^{\mathrm{MP}}$ vis-à-vis y^{MP} is indicated in Fig. 4. In effect, when power-only generation is relatively costly, the exertion of market power by the power-only producer at the lower level heavily curbs consumption. An ensuing increase in CHP power output by a strategic heat producer under MP could cause the electricity price to drop sharply without a compensating increase in sales. Thus, a strategic heat producer under MP would optimally reduce its CHP power output (or, increase its CHP heat output), which would make welfare-maximising CHP heat output under MP relatively low for $C\geq\hat{C}^{\mathrm{MP}}$. By contrast, for $C<\hat{C}^{\mathrm{MP}}$, power-only generation is cheap enough to prevent a substantial choke on consumption even with withholding by the power-only producer. Hence, a strategic heat producer under MP would actually have relatively high CHP power output (or, low CHP heat output) compared to that of a welfare-maximising heat producer under MP.[20]

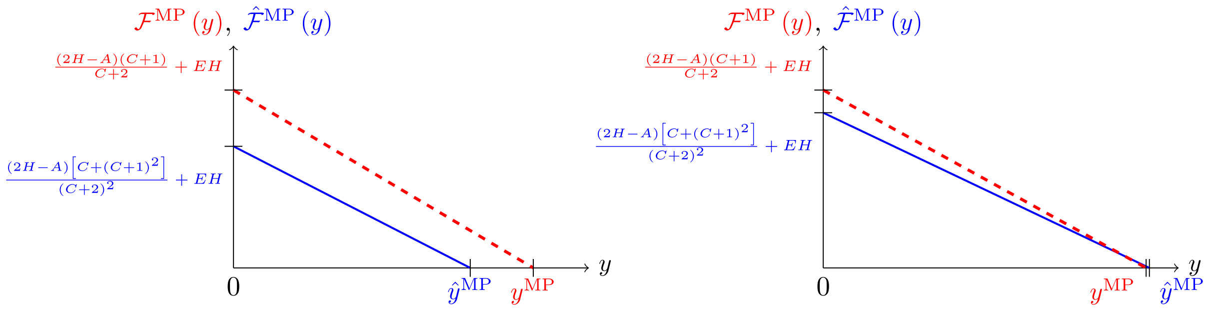
As in Section 3.2.3, a welfare-maximising heat producer may not be directly able to control CHP and heat-only plants. Instead, a regulatory mechanism, $\hat{S}^{\mathrm{MP}}$, to align public and private incentives to catalyse the socially optimal CHP heat output under MP may be devised. This is implemented in Proposition P13 and partially addresses RQ 3.

Proposition P13. *The regulatory mechanism* $\hat{S}^{MP}=-\left(\frac{B\left(C^2+3C+3\right)EH+A\left(\frac{C+1}{C+2}\left(C^2+3C+1\right)-(B+1)E\right)}{C^2+3C+1+(B+1)(C+2)^2E}\right)$ *aligns public and private incentives to yield the socially optimal CHP heat output under MP. Furthermore,* $\hat{S}^{MP}>0$ *if* $\hat{y}^{MP}>y^{MP}$.

Hence, unlike PC, the incentive-based regulatory mechanism under MP may actually be a subsidy for CHP heat output in the event that $\hat{y}^{\mathrm{MP}}>y^{\mathrm{MP}}$, i.e., for $C<\hat{C}^{\mathrm{MP}}$ as defined in Proposition P12.

Geometrically, the regulatory mechanism needed to align the FONCs (32) and (35) as given in Proposition P13 involves modifying the vertical intercept of $\mathcal{F}^{\mathrm{MP}}(y)$. Figs. 5(a) and 5(b) reveal the difference between the two FONCs depending on whether $C<\hat{C}^{\mathrm{MP}}$ or not. As under PC, the regulatory mechanism $\hat{S}^{\mathrm{MP}}$ affects heat surplus and regulator surplus in social welfare. First, $\hat{HS}^{\mathrm{MP}}=p^{\mathrm{MP}}\left(\hat{y}^{\mathrm{MP}}\right)w^{\mathrm{MP}}\left(\hat{y}^{\mathrm{MP}}\right)+\hat{S}^{\mathrm{MP}}\hat{y}^{\mathrm{MP}}-\frac{1}{2}BE\left(\hat{y}^{\mathrm{MP}}-H^{\mathrm{CM}}\right)^2-\frac{1}{2}E\left(H-\hat{y}^{\mathrm{MP}}-BH^{\mathrm{CM}}\right)^2+RH$. Second, $\hat{RS}^{\mathrm{MP}}=-\hat{S}^{\mathrm{MP}}\hat{y}^{\mathrm{MP}}$, which is negative in case of a subsidy, i.e., $\hat{S}^{\mathrm{MP}}>0$.

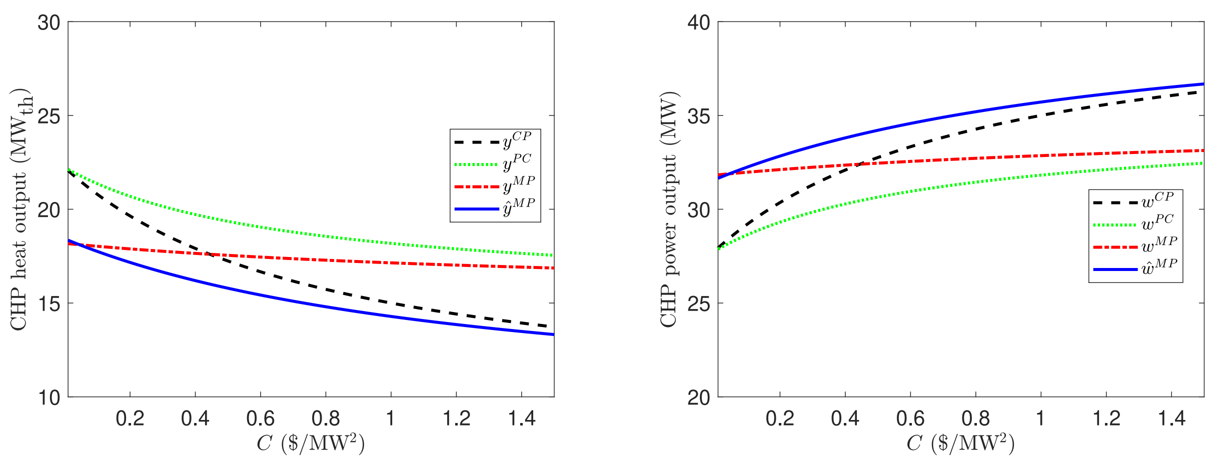
[20] It is worth noting that $\hat{y}^{\mathrm{MP}}>y^{\mathrm{MP}}$ cannot occur if $A(B+1)\leq 2BH$, e.g., in a "small" electricity market. To see this, note that $\hat{y}^{\mathrm{MP}}<y^{\mathrm{CP}}<y^{\mathrm{PC}}$ from Propositions P2, P10, and P11. Moreover, $y^{\mathrm{MP}}\geq y^{\mathrm{PC}}$ for $A(B+1)\leq 2BH$ from Proposition P6. Thus, it is impossible to have $\hat{y}^{\mathrm{MP}}>y^{\mathrm{MP}}$ in a "small" electricity market.



(a) MP FONCs for $C \geq \hat{C}^{\mathrm{MP}}$ (tax regime)

(b) MP FONCs for $C < \hat{C}^{\mathrm{MP}}$ (subsidy regime)

Fig. 5. MP FONCs.



(a) Optimal CHP heat output with respect to C

(b) Optimal CHP power output with respect to C

Fig. 6. CHP operations.

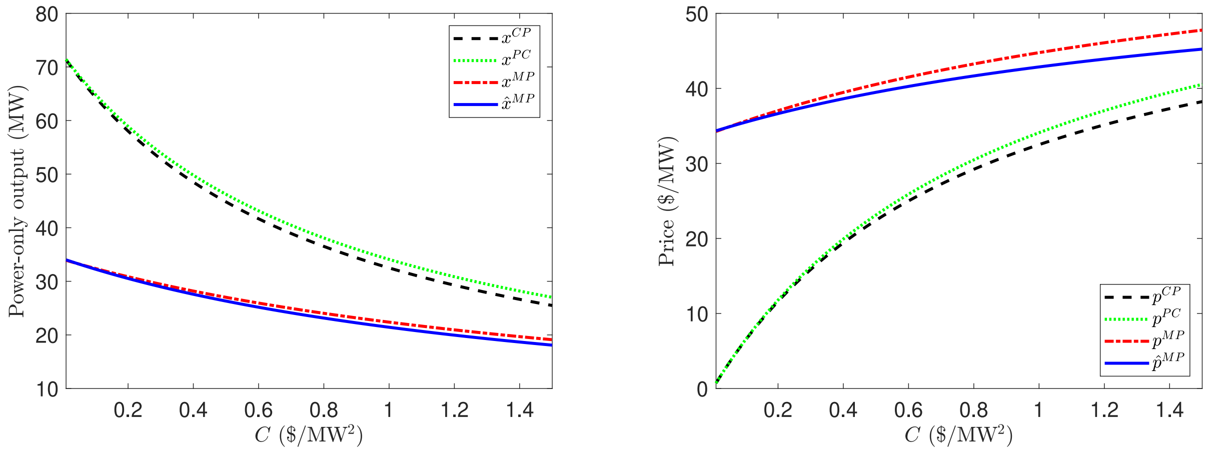
4. Numerical example

In this section, we illustrate the properties of the main analytical results from Section 3 with a numerical example based on the following parameters: $A = 100$, $B = 1.25$, $E = 2$, $H = 50$, $R = 50$, and $C \in [0.01, 1.5]$. These are in accordance with Assumptions A1–A4 and yield $H^{\mathrm{CM}} = 22.22$ as the cost-minimising CHP heat output. Note that Assumptions A3–A4 are satisfied by having A that is twice as large as H, cf. Swedish data. As we mentioned in Section 2, estimating C and E, cf. Assumption A2, is difficult because the real power-only and district-heating supply curves do not adhere to our simplifying assumptions of linearity. This is unfortunately a tradeoff of using analytical models. Meanwhile, from Table II of Virasjoki et al. (2018), it is evident that the total CHP operating cost is higher than that for heat-only plants to justify our having $B > 1$, cf. Assumption A1. Given the heterogeneity among plant types, a choice of $B = 1.25$ seems warranted for illustrative purposes. As for the heat price, R, it enters all objective functions as a constant and is, therefore, not relevant to any optimal decisions. Here, we focus on a relatively "large", i.e., $A(B+1) = 225 > 125 = 2BH$, electricity market and relegate the numerical example for a relatively "small" electricity market to Appendix C.

Fig. 6 plots CHP operations as power-only generation becomes more expensive. Under CP, the trends are straightforward: a *ceteris paribus* increase in C monotonically reduces CHP heat output, y^{CP}, cf. Proposition P1, which leads to more CHP power output, w^{CP}. At the same time, power-only operations decrease and the electricity price increases (Fig. 7). By contrast, the strategic use of CHP by the heat producer in the PC setting prompts it to increase CHP heat output vis-à-vis the socially optimal level under CP (Fig. 6(a)), cf. Proposition P2, which results in lower CHP power output (Fig. 6(b)). It should also be noted that while strategic heat production induces increased CHP heat output, it still decreases monotonically with C, i.e., $\frac{\partial y^{\mathrm{PC}}}{\partial C} < 0$, since the electricity market is "large" enough to accommodate more CHP power output without collapsing the electricity price, cf. Proposition P3. In spite of increased power-only output (Fig. 7(a)), the electricity price is still high relative to its CP level (Fig. 7(b)). As expected, this manipulation of the electricity price is to the benefit of the heat producer (Fig. 8(c)) with a concomitant loss in consumer surplus (Fig. 8(a)). Although the higher power-only output and electricity price combine to bolster generator surplus (Fig. 8(b)), the overall impact on social welfare due to strategic CHP operations in a perfectly competitive electricity industry is still negative (Fig. 8(d)).

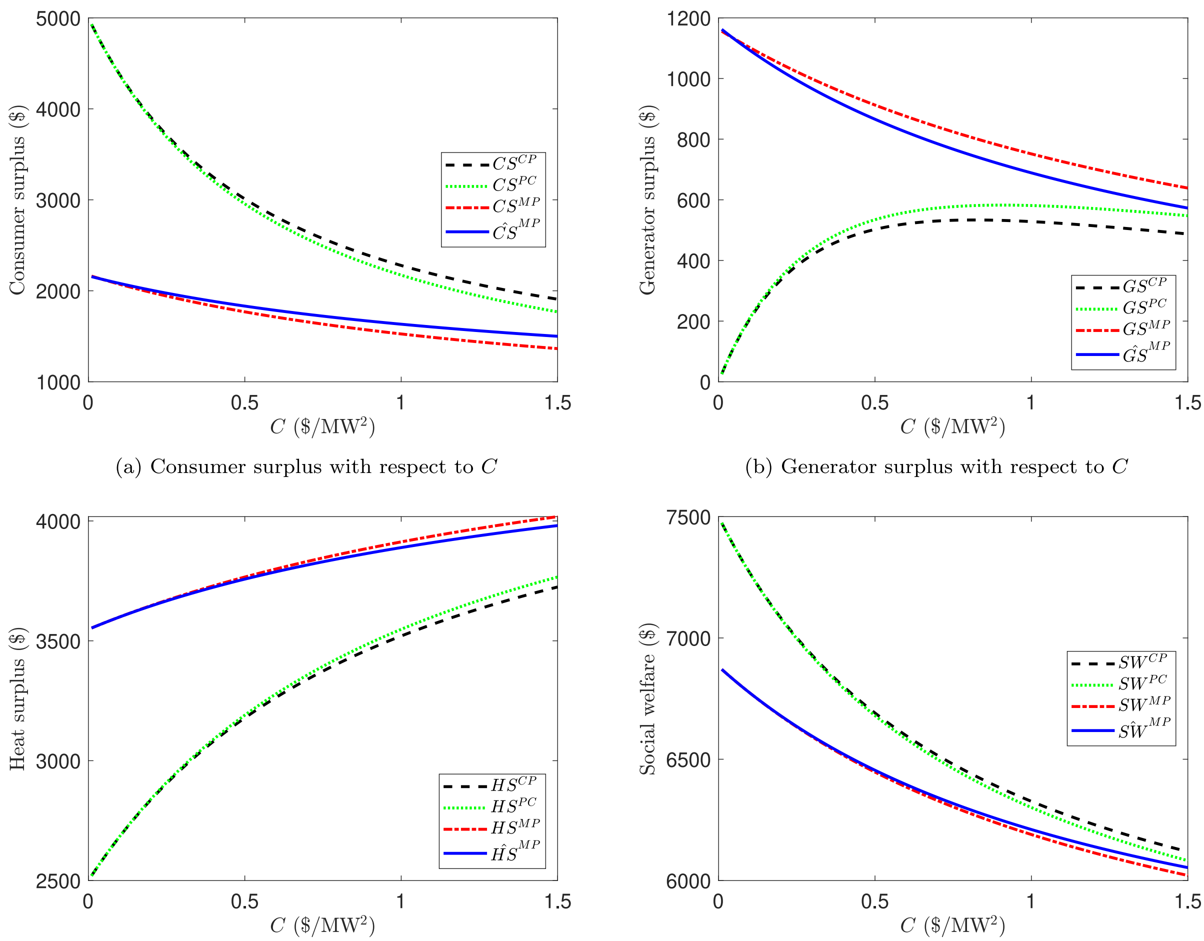
Given the economic distortion from strategic CHP operations in the PC setting, a regulatory mechanism, $\hat{S}^{\mathrm{PC}}$, may be offered in order to align private and public incentives, cf. Proposition P4. As indicated in Fig. 9, the intervention involves taxing CHP heat output. Consequently, operations under PC with a regulatory mechanism fully align with those under CP, e.g., $\hat{y}^{\mathrm{PC}} = y^{\mathrm{CP}}$, and social welfare is restored to the level attained under CP. Moreover, all welfare components under PC with an incentive-based regulatory mechanism are identical to those under CP in Fig. 8 except for heat surplus, which is reduced due to the tax, and the introduction of the regulator surplus, which is positive to reflect the revenue from the tax (Fig. 10).

Under MP, there is the additional exercise of market power by the power-only producer at the lower level. Its impact is to reduce production relative to its CP and PC levels (Fig. 7(a)) in order to increase the electricity price (Fig. 7(b)). As formalised by Proposition P6 and illustrated in Fig. 6(a), a strategic heat producer exploits this



(a) Optimal power-only output with respect to C (b) Equilibrium electricity price with respect to C

Fig. 7. Power-only operations and equilibrium electricity price.



(a) Consumer surplus with respect to C (b) Generator surplus with respect to C

(c) Heat surplus with respect to C (d) Social welfare with respect to C

Fig. 8. Social welfare and its components.

opportunity by reducing its CHP heat output vis-à-vis PC, i.e., $y^{\mathrm{MP}} < y^{\mathrm{PC}}$, given that the electricity market is assumed to be relatively "large". The intuition for such a manoeuvre is that the heat producer can simultaneously increase its CHP power output, w^{MP}, under MP relative to PC (Fig. 6(b)) and enjoy the benefits of a higher electricity price, p^{MP}, under MP relative to PC (Fig. 7(b)) without being concerned that its increased CHP power output will depress the electricity price. This is because the electricity market is large enough to absorb higher CHP power output instigated by reduced power-only output. Therefore, as indicated in Fig. 8, the heat and generator surpluses can be considerably higher than those under PC, and the corresponding depletion to consumer surplus causes overall social welfare to be lower under MP.

A similar relationship holds between MP and CP except that even in a "large" electricity market, an additional condition is required to

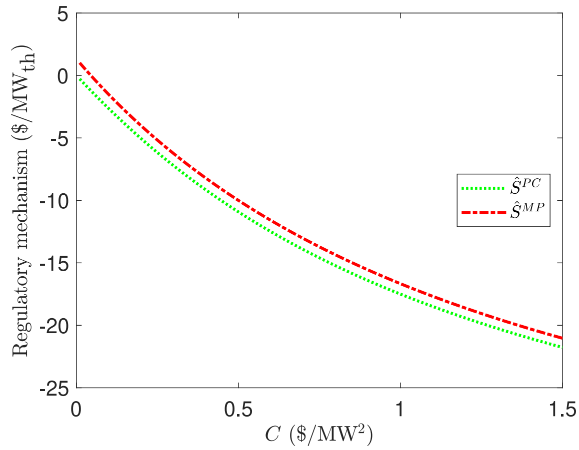


Fig. 9. Regulatory mechanism with respect to C.

ensure that a strategic heat producer reduces its CHP heat output vis-à-vis CP, viz., $C < C^{\mathrm{MP}}$. It is worthwhile for the strategic heat producer under MP to withhold CHP heat output and to increase electricity sales from CHP relative to CP only if the electricity market can absorb the additional output and power-only generation is inexpensive, cf. Proposition P5. Indeed, if power-only generation is relatively "cheap", then the strategic heat producer can take advantage of the power-only producer's withholding by increasing its CHP power output, thereby benefiting from both a higher electricity price and sales. Otherwise, if $C \geq C^{\mathrm{MP}}$, then the curb in power-only generation may be severe enough to make the electricity price highly sensitive to perturbations in CHP power output. As a result, the heat producer under MP is better off by actually boosting its CHP heat output, i.e., cutting back its CHP power output, vis-à-vis CP. The precise value of the threshold C^{MP} is 0.4444 as denoted by the positive root of the characteristic quadratic, $Q^{\mathrm{MP}}(C)$, in Fig. 11(a). Finally, as per Proposition P7, in a "large" electricity market, optimal CHP heat output under MP is monotonically decreasing with C, i.e., $\frac{\partial y^{\mathrm{MP}}}{\partial C} < 0$, as more CHP power output can be absorbed without depressing the electricity price.

While the strategic heat producer's incentives to manipulate the electricity price under PC could be fully alleviated via a regulatory mechanism, such an intervention is only partially effective under MP. As Proposition P13 proves and Fig. 9 illustrates, the incentive-based regulatory mechanism under MP, $\hat{S}^{\mathrm{MP}}$, actually subsidises CHP heat output for $C < \hat{C}^{\mathrm{MP}} = 0.0446$, cf. Proposition P12 and Fig. 11(b). Unlike the result with a regulatory mechanism under PC, here, it is not generally the case that the socially optimal level of welfare is obtained under MP with either a subsidy or a tax. This is because there are distortions at both the upper and lower levels to mitigate, and the best that the regulator can do is to use the mechanism to replicate the output that results from a welfare maximiser's operations subject to market power by the power-only producer. As formalised by Propositions P10 and P11, the welfare-maximising CHP heat output under MP, $\hat{y}^{\mathrm{MP}}$, is lower than either the socially optimal CHP heat output, y^{CP}, or that by a strategic heat producer under PC, y^{PC}.

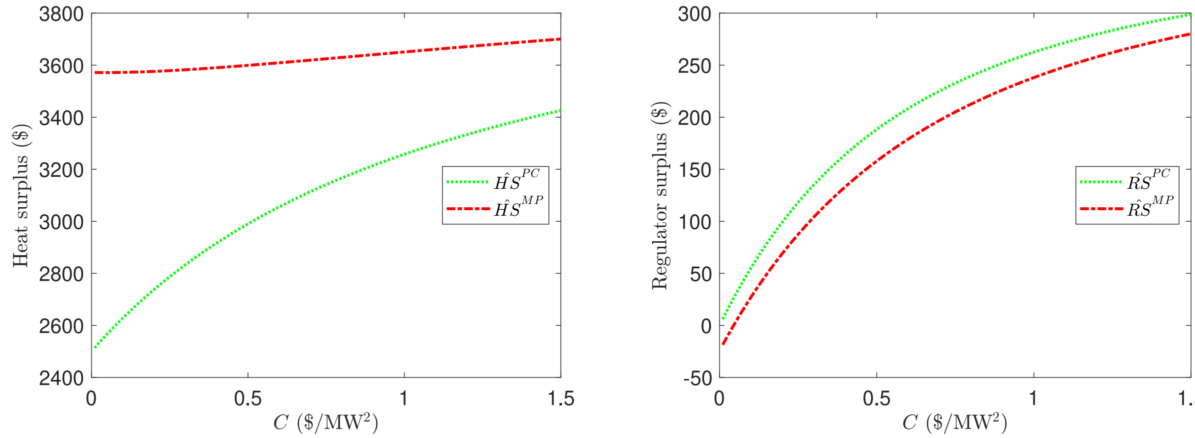
The comparison of this regulated CHP heat output under MP, $\hat{y}^{\mathrm{MP}}$, with strategic CHP heat output under MP, y^{MP}, is based on similar logic. Consequently, the regulatory mechanism under MP almost always decreases CHP heat output (Fig. 6(a)) vis-à-vis MP and generally leads to an anticipated increase in CHP power output (Fig. 6(b)), a decrease in the electricity price (Fig. 7(b)), and an increase in social welfare (Fig. 8(d)) compared to the corresponding values under MP. In terms of comparative statics, we have $\frac{\partial \hat{y}^{\mathrm{MP}}}{\partial C} < 0$, cf. Proposition P9, as welfare-maximising CHP heat output under MP monotonically decreases with C to compensate for the exertion of market power by power-only generation. The implementation of the regulatory mechanism reduces the heat surplus for $C \geq \hat{C}^{\mathrm{MP}}$, cf. Figs. 8(c) and 10(a). However, as evidenced by Proposition P12, it is possible for $\hat{y}^{\mathrm{MP}} > y^{\mathrm{MP}}$ if power-only generation is relatively cheap, i.e., $C < \hat{C}^{\mathrm{MP}}$. In that situation, the exertion of market power by power-only generation increases the electricity price without drastically eroding consumption. As a consequence, a strategic heat producer under MP is able to maintain a comparatively higher CHP power output (or, lower CHP heat output), i.e., above the welfare-maximising CHP power output under MP (Fig. 6(b)). Note that the critical threshold, $\hat{C}^{\mathrm{MP}}$, is 0.0446 and is indicated by the positive root of the characteristic cubic function $\hat{Q}^{\mathrm{MP}}(C)$ (Fig. 11(b)), which also marks the critical point below which the regulatory mechanism becomes a subsidy (Fig. 9).

5. Conclusions

Decarbonisation pathways proposed by legislation in most OECD countries are underpinned by a VRE-based power sector and the electrification of adjacent energy sectors. Such so-called sector coupling aims to avail of the flexibility of industrial processes as well as heat, natural-gas, and transport systems to buffer intermittencies in VRE output. While tighter integration between interdependent sectors can lead to better coordination of energy production and consumption, it may also create new opportunities for strategic agents to leverage their market power. The literature has, indeed, marshalled empirical evidence of price manipulation even in well-functioning electricity markets. Although district-heating sectors are subject to closer regulatory scrutiny than electricity markets, the greater prominence of CHP in coupling the two sectors due to electrification could facilitate the exercise of market power by large producers even if their overall heat output is fixed and sold at a regulated price.

In contrast to the extant literature on coupling between the heat and power sectors, we explore the strategic incentives of a heat producer that participates in a deregulated electricity market via its CHP power output. Using a stylised game-theoretic model that permits analytical solutions and comparative statics, we first prove that such an entity would distort the composition of its heat output to restrict its CHP power output under PC relative to the first-best outcome (RQ 1). Next, we allow for a power-only producer that behaves à la Cournot to investigate how an imperfectly competitive electricity market would alter the strategic heat producer's leverage. We find that this additional layer of strategic behaviour would actually lead to more CHP power output compared to PC as the heat producer would aim to exploit the higher electricity price *ceteris paribus* from the power-only sector's withholding of generation as long as the electricity market were large compared to the heat sector (RQ 2). In order to limit welfare losses from such strategic behaviour, we devise a regulatory mechanism, i.e., either a subsidy to or a tax on CHP heat output, that could align incentives with either the first-best (in case of PC) or the second-best (in case of MP) outcome (RQ 3). Hence, by formalising how heat production would be reshuffled by a strategic producer and proposing countervailing incentive-based regulation, our work informs policy on sector coupling and future negotiations over CHP cost allocation as part of a sustainable, net-zero energy transition.

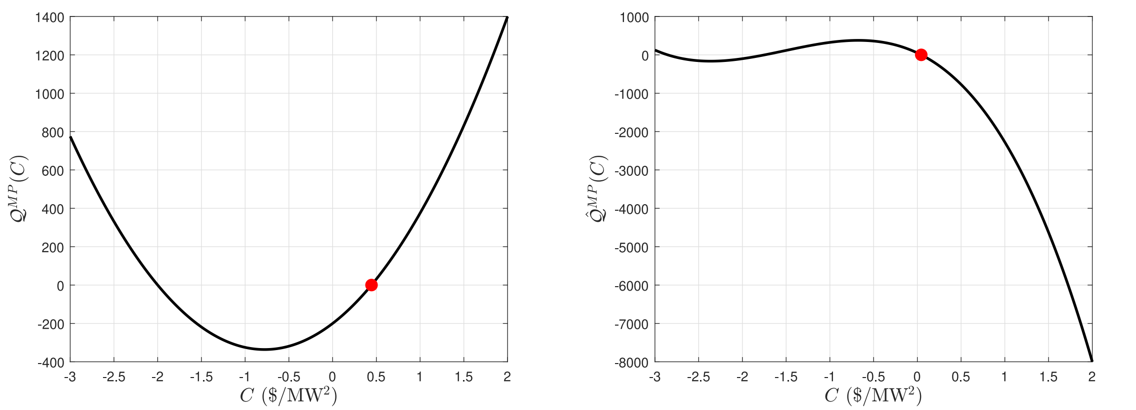
Directions for future work could include expanding our framework to more detailed bi-level programming problem instances that reflect heterogeneous firms and the spatio-temporal details of heat and power sectors (Mitridati and Pinson, 2016; Mitridati et al., 2020; Virasjoki et al., 2018). Such an approach would also require a scaleable computational implementation (Gabriel et al., 2013). Alternatively, staying within an analytical framework, we could explore incentives for such strategic behaviour in nascent commodity markets, e.g., hydrogen (Mégy and Massol, 2023), with emerging technologies, e.g., heat pumps (Siddique et al., 2024) and storage (Bjørndal et al., 2023), and under regulation that is intended to foster stable revenues for



(a) Heat surplus under regulatory mechanism with respect to C

(b) Regulator surplus with respect to C

Fig. 10. Impact of regulatory mechanism on heat surplus and regulator surplus.



(a) Characteristic quadratic, $\mathcal{Q}^{\mathrm{MP}}(C)$, indicating threshold, $C^{\mathrm{MP}} = 0.4444$, above which $y^{\mathrm{CP}} < y^{\mathrm{MP}}$

(b) Characteristic cubic, $\hat{\mathcal{Q}}^{\mathrm{MP}}(C)$, indicating threshold, $\hat{C}^{\mathrm{MP}} = 0.0446$, below which $\hat{y}^{\mathrm{MP}} > y^{\mathrm{MP}}$

Fig. 11. Characteristic functions.

producers (Chaiken et al., 2021). Finally, from a practical perspective, a phased approach to the proposed regulation would be worth exploring as part of a pilot study (Marnay et al., 2013).

CRediT authorship contribution statement

Afzal S. Siddiqui: Writing – review & editing, Writing – original draft, Validation, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Sebastian Maier:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Funding acquisition, Formal analysis.

Appendix A. Nomenclature

Parameters

A Consumers' maximum willingness to pay for electricity ($/MW).

B Factor by which CHP production cost is higher than that of the heat-only plant (–).[21]

C Quadratic cost coefficient for the power-only plant ($/MW^2).

E Quadratic cost coefficient for the heat-only plant ($/MW$_{\mathrm{th}}{}^2$).

H Total heat supply (MW$_{\mathrm{th}}$).

$H^{\mathbf{CM}}$ Cost-minimising CHP heat output (MW$_{\mathrm{th}}$).

R Fixed per-unit price for heat ($/MW$_{\mathrm{th}}$).

$\hat{S}^{\mathbf{MP}}$ Regulatory mechanism under MP ($/MW$_{\mathrm{th}}$).

$\hat{S}^{\mathbf{PC}}$ Regulatory mechanism under PC ($/MW$_{\mathrm{th}}$).

Variables

Primal variables

q Power quantity demanded (MW).

w Power output from the CHP plant (MW).

x Power output from the power-only plant (MW).

y Heat output from the CHP plant (MW$_{\mathrm{th}}$).

Dual variables

p Market-clearing price for electricity ($/MW).

λ Shadow price on the CHP plant's capacity ($/MW).

[21] "(–)" refers to a unitless item.

Appendix B. Proofs of Propositions

Proof of Proposition P1 Partially differentiating (9) with respect to C yields $\frac{E[BH-A(B+1)]}{[C+(B+1)(C+1)E]^2}$, which is negative from Assumptions A1–A3, viz., $E>0$ and $A>H$. $\square$

Proof of Proposition P2 By comparing (22) with (9), we note that $y^{\mathrm{PC}} > y^{\mathrm{CP}} \Leftrightarrow AC > -B(C+1)EH$, which follows from Assumptions A1–A3. $\square$

Proof of Proposition P3 From (21), it is evident that $\mathcal{F}^{\mathrm{PC}}(y)$ is a strictly decreasing linear function of y because $\frac{\partial \mathcal{F}^{\mathrm{PC}}}{\partial y} = -\frac{2C}{C+1} - (B+1)E < 0$. Total differentiation of $\mathcal{F}^{\mathrm{PC}}\left(y^{\mathrm{PC}}\right) = 0$ yields $\frac{\partial y^{\mathrm{PC}}}{\partial C} = -\left.\frac{\frac{\partial \mathcal{F}^{\mathrm{PC}}}{\partial C}}{\frac{\partial \mathcal{F}^{\mathrm{PC}}}{\partial y}}\right|_{y=y^{\mathrm{PC}}}$. Since $\left.\frac{\partial \mathcal{F}^{\mathrm{PC}}}{\partial C}\right|_{y=y^{\mathrm{PC}}} = \frac{\left(2H-A-2y^{\mathrm{PC}}\right)}{(C+1)^2} \geq 0$ only if $2BH \geq A(B+1)$ using the definition of y^{PC} in (22), it follows that $\frac{\partial y^{\mathrm{PC}}}{\partial C} \geq 0$ when the electricity market is "small" relative to the heat market. $\square$

Proof of Proposition P4 Under PC, the regulatory mechanism is devised by first solving a modified version of (20) that internalises the regulatory mechanism, i.e., $\max_{y\geq 0} p^{\mathrm{PC}}(y)\, w^{\mathrm{PC}}(y) + Sy - \frac{1}{2}BEy^2 - \frac{1}{2}E(H-y)^2 + RH$. The solution is similar to that obtained for y^{PC} from (22) except for the presence of the S term in the numerator, viz., $\frac{C(2H-A)+(C+1)(EH+S)}{2C+(B+1)(C+1)E}$. Next, the latter expression is equated with y^{CP} (or, $\hat{y}^{\mathrm{PC}}$) from (9). Solving for S, we find that $\hat{S}^{\mathrm{PC}} = -\left(\frac{BCEH+\frac{AC^2}{C+1}}{C+(B+1)(C+1)E}\right)$ and also verify that $\hat{S}^{\mathrm{PC}} < 0$ because all parameters inside the parentheses are positive. $\square$

Proof of Proposition P5 By comparing (33) with (9), we note that $y^{\mathrm{MP}} < y^{\mathrm{CP}} \Leftrightarrow (A+BEH)C^2 + (A+2BEH)C + E[2BH - A(B+1)] < 0$. In other words, $y^{\mathrm{MP}} < y^{\mathrm{CP}}$ requires $Q^{\mathrm{MP}}(C) < 0$, where $Q^{\mathrm{MP}}(C)$ is an upward-facing parabola. If $2BH - A(B+1) < 0$, then $Q^{\mathrm{MP}}(C)$ has one positive root, C^{MP}, and one negative root, which means that it is possible to have $Q^{\mathrm{MP}}(C) < 0$ only for $C < C^{\mathrm{MP}}$. Therefore, $y^{\mathrm{MP}} < y^{\mathrm{CP}}$ as long as (i) the electricity market is "large" relative to the heat market, i.e., where constraining CHP power output has little leverage, and (ii) power-only generation is relatively inexpensive. Otherwise, $y^{\mathrm{MP}} \geq y^{\mathrm{CP}}$ may still hold for a "large" electricity market as long as $C \geq C^{\mathrm{MP}}$, i.e., constraining CHP power output is effective because power-only output is relatively expensive. Finally, if $2BH - A(B+1) \geq 0$, i.e., the electricity market is relatively "small", then $y^{\mathrm{MP}} \geq y^{\mathrm{CP}}$ because $Q^{\mathrm{MP}}(C)$ has only negative roots. $\square$

Proof of Proposition P6 By comparing (33) with (22), we note that $y^{\mathrm{MP}} < y^{\mathrm{PC}} \Leftrightarrow 2BH - A(B+1) < 0$, i.e., the electricity market is "large" relative to the heat market. $\square$

Proof of Proposition P7 From (32), it is evident that $\mathcal{F}^{\mathrm{MP}}(y)$ is a strictly decreasing linear function of y because $\frac{\partial \mathcal{F}^{\mathrm{MP}}}{\partial y} = -\frac{2(C+1)}{C+2} - (B+1)E < 0$. Total differentiation of $\mathcal{F}^{\mathrm{MP}}\left(y^{\mathrm{MP}}\right) = 0$ yields $\frac{\partial y^{\mathrm{MP}}}{\partial C} = -\left.\frac{\frac{\partial \mathcal{F}^{\mathrm{MP}}}{\partial C}}{\frac{\partial \mathcal{F}^{\mathrm{MP}}}{\partial y}}\right|_{y=y^{\mathrm{MP}}}$. Since $\left.\frac{\partial \mathcal{F}^{\mathrm{MP}}}{\partial C}\right|_{y=y^{\mathrm{MP}}} = \frac{\left(2H-A-2y^{\mathrm{MP}}\right)}{(C+2)^2} \geq 0$ only if $2BH \geq A(B+1)$ using the definition of y^{MP} in (33), it follows that $\frac{\partial y^{\mathrm{MP}}}{\partial C} \geq 0$ when the electricity market is "small" relative to the heat market. $\square$

Proof of Proposition P8 From (36), $\hat{y}^{\mathrm{MP}} > 0$ holds as long as $A < H + EH\frac{(C+2)^2}{C+(C+1)^2}$. Based on Assumption A4, we have $A < H + EH\frac{(C+2)}{C+1}$. Thus, $\frac{(C+2)^2}{C+(C+1)^2} > \frac{(C+2)}{C+1}$ will ensure $\hat{y}^{\mathrm{MP}} > 0$. Note that $\frac{(C+2)^2}{C+(C+1)^2} > \frac{(C+2)}{C+1} \Leftrightarrow C^2+3C+2 > C^2+3C+1$, which is always true and guarantees $\hat{y}^{\mathrm{MP}} > 0$. Next, we can determine that $\hat{y}^{\mathrm{MP}} < H$ is equivalent to $-A\left[C+(C+1)^2\right] < B(C+2)^2 EH$, which holds due to Assumptions A1–A3. $\square$

Proof of Proposition P9 From (35), it is evident that $\hat{\mathcal{F}}^{\mathrm{MP}}(y)$ is a strictly decreasing linear function of y because $\frac{\partial \hat{\mathcal{F}}^{\mathrm{MP}}}{\partial y} = -\frac{(C^2+3C+1)}{(C+2)^2} - (B+1)E < 0$. Total differentiation of $\hat{\mathcal{F}}^{\mathrm{MP}}\left(\hat{y}^{\mathrm{MP}}\right) = 0$ yields $\frac{\partial \hat{y}^{\mathrm{MP}}}{\partial C} = -\left.\frac{\frac{\partial \hat{\mathcal{F}}^{\mathrm{MP}}}{\partial C}}{\frac{\partial \hat{\mathcal{F}}^{\mathrm{MP}}}{\partial y}}\right|_{y=\hat{y}^{\mathrm{MP}}}$. Since $\left.\frac{\partial \hat{\mathcal{F}}^{\mathrm{MP}}}{\partial C}\right|_{y=\hat{y}^{\mathrm{MP}}} = \frac{\left(H-A-\hat{y}^{\mathrm{MP}}\right)(C+4)}{(C+2)^3} < 0$ and $\frac{\partial \hat{\mathcal{F}}^{\mathrm{MP}}}{\partial y} < 0$, it follows that $\frac{\partial \hat{y}^{\mathrm{MP}}}{\partial C} < 0$. $\square$

Proof of Proposition P10 Comparing (36) with (9), we have $\hat{y}^{\mathrm{MP}} > y^{\mathrm{CP}} \Leftrightarrow A(B+1) < BH$, which never holds from Assumptions A1 and A3. $\square$

Proof of Proposition P11 In order for $y^{\mathrm{PC}} < \hat{y}^{\mathrm{MP}}$, we compare (22) and (36) to conclude that the result is possible as long as $-BEH\left(C^3+4C^2+4C-1\right) - AC\left(C^2+3C+1\right) - A(B+1)E > 0 \Leftrightarrow \hat{Q}^{\mathrm{PC,MP}}(C) > 0$, where $\hat{Q}^{\mathrm{PC,MP}}(C) = -(A+BEH)C^3 - (A+BEH)C^2 - (A+4BH)C - (-BEH + A(B+1)E)$ is a cubic function with no real positive roots, $\hat{C}^{\mathrm{PC,MP}}$, thereby ensuring that $y^{\mathrm{PC}} > \hat{y}^{\mathrm{MP}}$. $\square$

Proof of Proposition P12 By comparing (33) with (36), we note that $\hat{y}^{\mathrm{MP}} > y^{\mathrm{MP}} \Leftrightarrow -A(C+1)\left(C^2+3C+1\right) - B(C+2)EH\left(C^2+3C+3\right) + A(B+1)(C+2)E > 0 \Leftrightarrow \hat{Q}^{\mathrm{MP}}(C) > 0$, where we define $\hat{Q}^{\mathrm{MP}}(C) = -(A+BEH)C^3 - (4A+5BEH)C^2 - (4A+9BEH-A(B+1)E)C - (A+6BEH-2A(B+1)E)$ as a cubic function with positive real root $\hat{C}^{\mathrm{MP}}$ (if it exists). Thus, $\hat{y}^{\mathrm{MP}} > y^{\mathrm{MP}} \Leftrightarrow C < \hat{C}^{\mathrm{MP}}$. $\square$

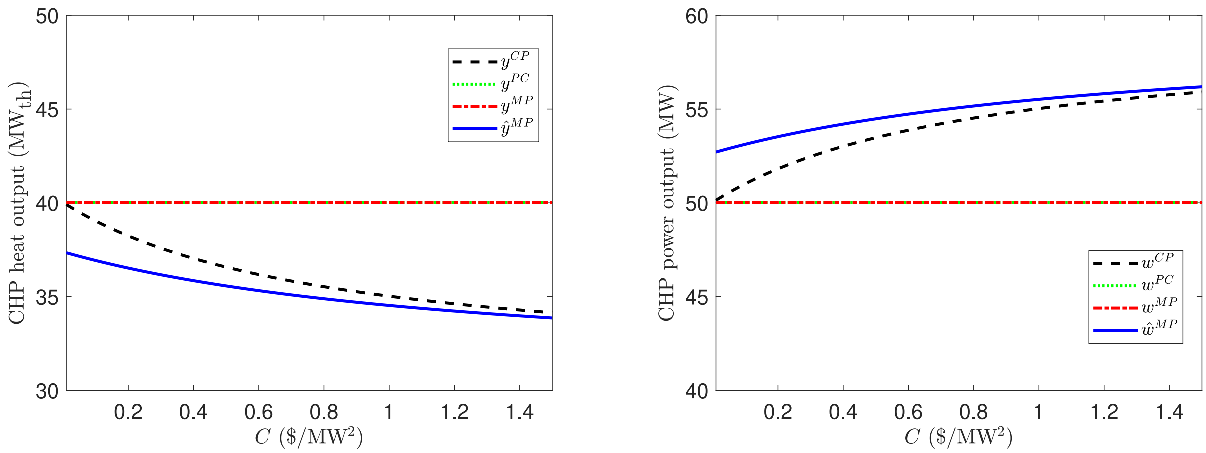
Proof of Proposition P13 Under MP, the regulatory mechanism is devised by first solving a modified version of (31), i.e., $\max_{y\geq 0} p^{\mathrm{MP}}(y)\, w^{\mathrm{MP}}(y) + Sy - \frac{1}{2}BEy^2 - \frac{1}{2}E(H-y)^2 + RH$. The solution is similar to that obtained for y^{MP} from (33) except for the presence of the S term in the numerator, viz., $\frac{(C+1)(2H-A)+(C+2)(EH+S)}{2(C+1)+(B+1)(C+2)E}$. Next, the latter expression is equated with $\hat{y}^{\mathrm{MP}}$ from (36). Solving for S, we find that $\hat{S}^{\mathrm{MP}} = -\left(\frac{B(C^2+3C+3)EH+A\left(\frac{C+1}{C+2}(C^2+3C+1)-(B+1)E\right)}{C^2+3C+1+(B+1)(C+2)^2E}\right)$ and also verify that $\hat{S}^{\mathrm{MP}} > 0$ as long as $-BEH\left(C^2+3C+3\right)(C+2) - A\left(C^2+3C+1\right)(C+1) + A(B+1)(C+2)E > 0$, which corresponds to $\hat{Q}^{\mathrm{MP}}(C) > 0$ or $\hat{y}^{\mathrm{MP}} > y^{\mathrm{MP}}$ from Proposition P12. $\square$

Appendix C. Numerical example for a relatively "small" electricity market

All parameters here are the same as those in Section 4 except that we set $H = 90.05$, i.e., $A(B+1) = 225 < 225.125 = 2BH$, which leads to $H^{\mathrm{CM}} = 40.02$ as the cost-minimising CHP heat output. As alluded to earlier, some of the results under PC and especially under MP are sensitive to the relative sizes of the electricity market and the heat contract. For example, Proposition P3 states that the optimal CHP heat output by a strategic producer under PC is nondecreasing in the rate of power-only generation's marginal cost as long as the electricity market is relatively "small" (Fig. C.1(a)). In effect, more costly power-only generation leads to less CHP power output in order to avoid crashing the electricity price (Fig. C.1(b)).[22] Likewise, in terms of Proposition P5, a "small" electricity market ensures that CHP heat output under MP is greater than that under CP, which also leads to lower CHP power output under MP. Intuitively, the low sales volumes in the electricity market discourage the strategic heat producer from selling more power in order to capitalise on the higher electricity price that stems from power-only generation's withholding. Instead, it is more profitable simply to restrict CHP power output to yield higher margins on the residual electricity sales. On a related note, comparing y^{PC} and y^{MP} based on Proposition P6 in Fig. C.1(a), a "small" electricity market boosts CHP heat output under MP to depress CHP power output (Fig. C.1(b)).[23] In contrast to the results from Section 4, here, a "small" electricity market also causes CHP heat output under MP to be nondecreasing in C, cf. Proposition P7, thereby yielding monotonically less CHP power output to take advantage of the relatively high electricity price.

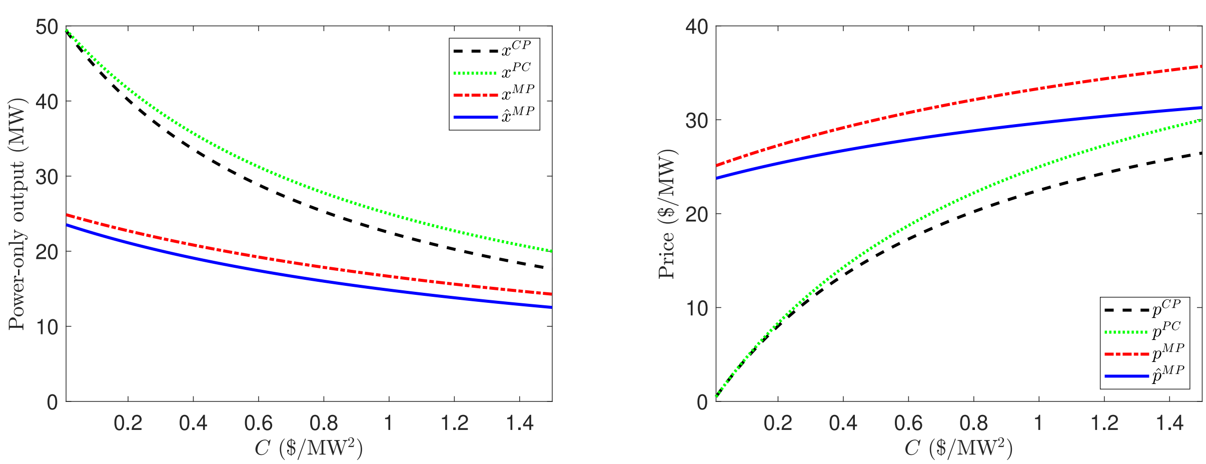
[22] Here, y^{PC} and w^{PC} are very close to y^{MP} and w^{MP}, respectively, which makes it difficult to discern their trends. For sake of clarity, y^{PC} and w^{PC} have ranges of $[40.0223, 40.0281]$ and $[50.0277, 50.0219]$, respectively, in Fig. C.1.

[23] Again, for sake of clarity, y^{MP} and w^{MP} have ranges of $[40.0273, 40.0289]$ and $[50.0227, 50.0211]$, respectively, in Fig. C.1.



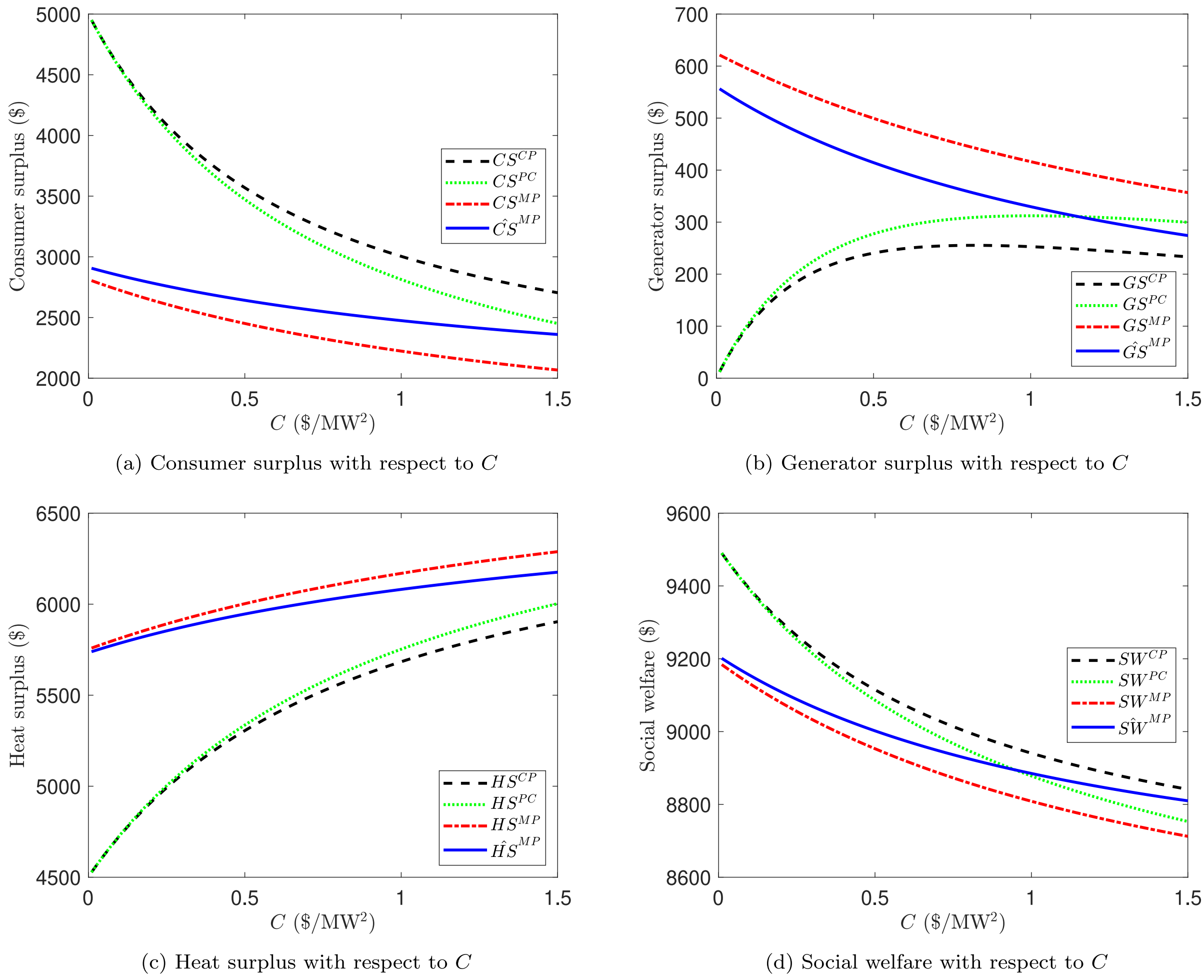
(a) Optimal CHP heat output with respect to C (b) Optimal CHP power output with respect to C

Fig. C.1. CHP operations with a relatively "small" electricity market.



(a) Optimal power-only output with respect to C (b) Equilibrium electricity price with respect to C

Fig. C.2. Power-only operations and equilibrium electricity price with a relatively "small" electricity market.



(a) Consumer surplus with respect to C

(b) Generator surplus with respect to C

(c) Heat surplus with respect to C

(d) Social welfare with respect to C

Fig. C.3. Social welfare and its components with a relatively "small" electricity market.

The other numerical results in Figs. C.2–C.5 generally mirror those in Section 4, viz., they retain the same ordering and monotonicity. However, due to a "small" electricity market, there are differences in the generator surplus, cf. Figs. 8(b) and C.3(b), and the regulatory mechanism, cf. Figs. 9 and C.4. First, in a "small" electricity market, the generator surplus under PC is more likely to exceed that under MP with a welfare-maximising heat producer. This is because the latter entity endeavours to boost CHP power output in order to prevent the twin distortions of withholding by power-only generation and CHP power output. Thus, in contrast to a "large" electricity market, which could exhibit a countervailing increase in CHP power output by a strategic heat producer, here, both facets of market power, viz., power-only withholding and CHP power restriction, are exerted in the same direction. Consequently, welfare-maximising CHP power output requires substantial bolstering, thereby dampening the increase in the electricity price to the benefit of consumers. Since the electricity market is "small" and power-only generation is withholding output, generator surplus can, therefore, drop below that under PC with a strategic heat producer and perfectly competitive power-only generation. Second, the magnitude of the regulatory intervention required under MP is greater than that under PC with a "small" electricity market due to the withholding by both power-only and CHP producers. By contrast, a "large" electricity market necessitates a higher tax on CHP heat output under PC (Fig. 9).

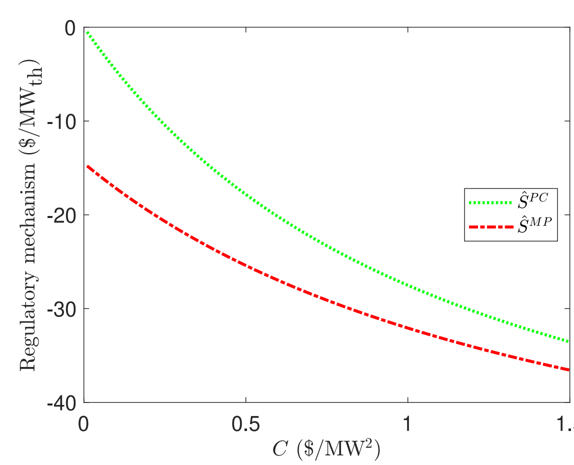
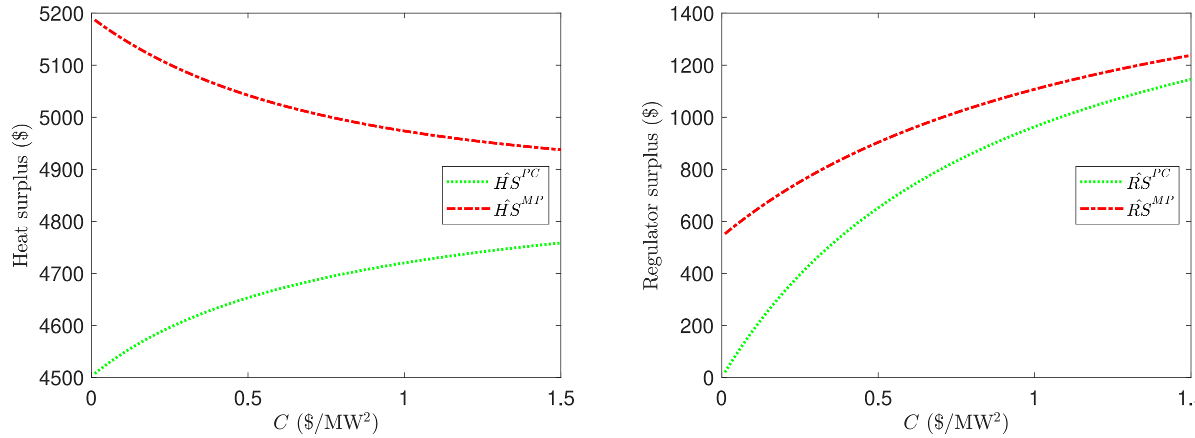


Fig. C.4. Regulatory mechanism with respect to C with a relatively "small" electricity market.



(a) Heat surplus under regulatory mechanism with respect to C (b) Regulator surplus with respect to C

Fig. C.5. Impact of regulatory mechanism on heat surplus and regulator surplus with a relatively "small" electricity market.

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