Designing Carbon Policy with Profit-Maximising Energy Storage

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Abstract

We examine carbon-policy design for a power system with energy storage as well as renewable and fossil-fuelled generation. A central-planning solution internalises the environmental externality of CO_2 emissions and curbs fossil-fuelled generation in proportion to the marginal cost of damage. By contrast, a decentralised solution leads to a bi-level setup—an upper-level welfare-maximising policymaker sets a CO_2 tax to impose upon lower-level profit-maximising generators. For completely efficient storage, an optimal CO_2 tax in this bi-level setting renders the first-best outcome. However, with inefficient storage, an infinitesimal increase in the CO_2 tax induces prices to increase at the same rate. As a result, storage shifts energy to the off-peak period to offset the loss in the value of stored energy. Hence, relative to the marginal cost of damage from emissions under central planning, the optimal CO_2 tax for the decentralised case is lower and may be nonmonotonic in energy storage's inefficiency.

Keywords: Bi-level optimisation, carbon policy, game theory, renewable energy, energy storage.

1. Introduction

Several OECD countries have adopted initiatives to decarbonise their power sectors and to facilitate the electrification of their broader economies. Examples of such legislation include the EU's Green Deal[1] and the U.S. Inflation Reduction Act of 2022.[2] While they differ in scope, such packages typically aim to support renewable-energy technologies such as wind and solar power. Due to their intermittent output (Newbery, 2023), renewables require variability management, *e.g.*, flexible generation and demand response, to integrate them into the power system. In this context, energy storage can also serve as a means to balance renewable generation but faces market and regulatory barriers that affect its operations by private entities in decentralised electricity industries (Sioshansi et al., 2012).

Given that a decarbonising power sector based on renewable energy will rely upon energy storage and flexible generation for variability management, carbon policy will have to evolve to ensure that fossil-fuelled generation does not exacerbate environmental externalities. While the literature examines the impact of carbon taxation on market equilibria (Downward, 2010; Hassanzadeh Moghimi et al., 2023), it typically does not endogenise how policy is determined. For example, Debia et al. (2019) demonstrate that storage-enabled renewable output in a decentralised industry with fossil-fuelled generation has the incentive to oppose exogenous improvements to storage efficiency. Intuitively, more efficient storage enables greater transfer of energy from the off-peak to the peak period, which enhances social welfare and reduces CO_2 emissions. Yet, after storage efficiency reaches a critical threshold, the renewable generator's profits actually decrease in storage efficiency because there will be "too much" energy available in the peak period, thereby depressing the peak price and causing renewable output to be diverted to the off-peak period. Here, a higher exogenous CO_2 tax on fossil-fuelled generation may better align private and social incentives by limiting the shift of renewable generation to the off-peak period due to improved storage efficiency.

Ideally, a welfare-maximising policymaker anticipates industry's response when setting carbon policy to maximise social welfare inclusive of the cost of environmental externalities (Barnett, 1980). This marks our point of departure from the extant literature: we investigate how carbon-policy design in a deregulated

[1] https://www.consilium.europa.eu/en/policies/green-deal/
[2] https://www.congress.gov/bill/117th-congress/house-bill/5376

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electricity industry leads to market outcomes that are fundamentally different from those in a centrally planned industry, especially if storage is not completely efficient. In particular, we expand the game-theoretic framework of Debia et al. (2019) to allow for a policymaker that acts as a Stackelberg leader *vis-à-vis* industry in setting the CO_2 tax endogenously. We prove that the optimal CO_2 tax in a decentralised industry is always lower than the marginal cost of damage from emissions under central planning. Indeed, in contrast to central planning, the CO_2 tax is unable to internalise perfectly the temporal rate of environmental damage from fossil-fuelled generation. Consequently, CO_2 taxation's immediate impact is to shift storage-enabled renewable generation to the off-peak period, which necessitates lowering the CO_2 tax relative to the marginal cost of damage under central planning. Finally, while the marginal cost of damage under central planning monotonically increases with storage inefficiency, a CO_2 tax in a decentralised industry may actually decrease in storage inefficiency to avoid exacerbating temporal distortions to operations.

The remainder of this paper is structured as follows. Section 2 details the modelling framework and underpinning assumptions. Section 3 obtains the first-best solution for the ideal benchmark of a centrally planned electricity industry, while Section 4 derives the result using a bi-level approach for the decentralised case. Numerical examples in Section 5 illustrate the main insights, and Section 6 summarises the findings and offers new directions for research. All proofs of propositions are in the Appendix.

2. Framework for Analysis

Our ultimate goal is to understand the interplay between carbon policy and the decisions of profit-maximising energy storage. To this end, we examine a Stackelberg leader-follower, *i.e.*, bi-level, model (Gabriel et al., 2013) to determine the optimal CO_2 tax in a decentralised setting. We contrast this decentralised case to a first-best central-planning solution to distil how optimal carbon policy and resultant carbon emissions compare between the two cases. We take a stylised approach to address our research objective and to make the results amenable to comparative statics (Sioshansi, 2010).[3] As is common in the literature, we use two representative time periods, $j = 1, 2$, with $j = 1$ being a low-demand, low-price off-peak period, and assume no uncertainty in either demand or generation.

Electricity consumers are represented passively by a linear inverse-demand function, $P_j(q_j) = A_j - q_j$ [in \$/MWh], where $A_j > 0$ [in \$/MWh] and q_j [in MWh] is quantity consumed. The slope of the inverse-demand function [in \$/MWh^2] is normalised to 1. A_j is the consumers' maximum willingness to pay for electricity during period j. Market clearing is implicit, *i.e.*, q_j equals total electricity generation.

Electricity supply consists of (i) energy storage that is coupled with a renewable generator (ESR) and (ii) a fossil-fuelled generator. Both generators are assumed to be price-taking profit-maximising agents. ESR has no explicit cost but faces a limited given stock of resource, $D > 0$ [in MWh], contained in its energy storage that it must allocate between the two periods. ESR output during period j is x_j [in MWh], and $F > 1$ is a unitless constant that denotes the inefficiency of energy storage. Thus, the ESR's resource-allocation constraint is $x_1 + Fx_2 = D$, where we assume that the entire stock of the ESR resource must be utilised fully. The period-j cost of fossil-fuelled generation is $\frac{1}{2}Cy_j^2$, where $C > 0$ [in \$/MWh^2] is the marginal-cost rate of fossil-fuelled generation and y_j [in MWh] is period-j fossil-fuelled generation. This cost function reflects increasing marginal fuel consumption by the fossil-fuelled generator, *e.g.*, due to diminishing efficiency of a single plant or to varying efficiency of a portfolio of plants. The social cost of damage from CO_2 emissions is $\frac{1}{2}K\left(y_1^2 + y_2^2\right)$, where $K > 0$ [in \$/MWh^2] is the marginal-cost rate of damage. This damage function captures the increasing marginal fuel use of the fossil-fuelled plant as reflected by its cost function. We assume implicitly that fossil-fuelled generation has a CO_2-emission rate of 1, and ESR use is emission free.

Under a benchmark central-planning (CP) setting, a single welfare-maximising entity determines generation directly, while taking account of the externality from CO_2 emissions. Thus, while there is no need to impose a CO_2 tax, the *implied* CO_2 tax, z^{CP}, can be calculated *ex post* as the marginal cost of damage (MCD) from total CO_2 emissions, *i.e.*, $\frac{K\left(y_1^{\text{CP}}+y_2^{\text{CP}}\right)}{2}$. By contrast, the decentralised setting with perfect competition (PC) leads to a policymaker at the upper level that sets a welfare-maximising CO_2 tax, z^{PC}, in anticipation of industry's output at the lower level.[4] At the lower level, each generator takes the CO_2 tax as given when maximising its profit, thereby giving a bi-level problem.[5]

Based on Debia et al. (2019), we make the following assumptions to ensure interior solutions under CP:

Assumption 1. $\frac{(F-1)A_2}{F^2+1} < A_1 < \frac{A_2}{F}$

[3] Comparative statics permit the decomposition of complex interactions among the model's attributes, whereas a more detailed model that is solved numerically provides only plausible explanations for such connections (Zhou et al., 2011).

[4] Under perfect competition, the ownership of assets would not affect the lower-level equilibrium outcome.

[5] In either the CP setting or the lower level of the PC setting, the resulting problem will be convex.

Assumption 2. $A_1 + A_2 \leq D < A_1 + FA_2$

Assumption 3. $A_j > 0,\ j = 1, 2$

Assumption 4. $C > 0$

Assumption 5. $D > 0$

Assumption 6. $F > 1$

Assumption 7. $K > 0$

The intuition that underlies each of the aforementioned assumptions is as follows. Assumption 1 formalises that period 2 is the peak one while ensuring that period 1's maximum willingness to pay is not "too low." It is coherent in the sense that $\frac{(F-1)}{F^2+1} < \frac{1}{F}$. Assumption 2 formalises that the stock of ESR is neither "too much" nor "too little." It ensures also that $D > F(A_2 - FA_1)$ because $A_1 > \frac{(F-1)A_2}{F^2+1}$ from Assumption 1. Assumptions 3–7 state that the maximum willingness to pay is positive, fossil-fuelled generation is costly, ESR stock is positive, energy storage is inefficient, and CO_2 emissions incur a social cost of damage.

3. First-Best Benchmark: Central Planning

Under CP, a single welfare maximiser makes all decisions via the following quadratic program (QP):

$$\max_{x_j \geq 0, y_j \geq 0} \quad \sum_{j=1}^{2} A_j (x_j + y_j) - \frac{1}{2} \sum_{j=1}^{2} (x_j + y_j)^2 - \frac{1}{2}(C+K) \sum_{j=1}^{2} y_j^2 \tag{1}$$

$$\text{s.t.} \qquad x_1 + Fx_2 = D : \ \mu \tag{2}$$

Eq. (2) enforces full ESR utilisation with a corresponding shadow price of μ. Eq. (1) comprises consumer surplus (CS), producer surplus (PS), and damage cost (DC).[6]

[6]The welfare components are defined as follows:

- CS is the gross benefit to consumers from electricity consumption, $\sum_{j=1}^{2} A_j (x_j + y_j) - \frac{1}{2}\sum_{j=1}^{2} (x_j + y_j)^2$, minus the cost of electricity purchased, $\sum_{j=1}^{2} p_j (x_j + y_j)$, where p_j is the equilibrium price in period j.
- PS is the revenue from electricity sales for the ESR producer, $\sum_{j=1}^{2} p_j x_j$, plus the revenue from electricity sales for the fossil-fuelled producer, $\sum_{j=1}^{2} p_j y_j$, less the cost of fossil-fuelled generation, $\frac{1}{2} C \sum_{j=1}^{2} y_j^2$.
- DC is the social cost of damage from CO_2 emissions, $\frac{1}{2} K \sum_{j=1}^{2} y_j^2$.

Because the cost of electricity purchased by the consumer, $\sum_{j=1}^{2} p_j (x_j + y_j)$, cancels with the revenue terms accruing to the generators, $\sum_{j=1}^{2} p_j x_j + \sum_{j=1}^{2} p_j y_j$, social welfare may be expressed as in (1).

Since (1)–(2) is a convex optimisation problem, it may be replaced by its Karush-Kuhn-Tucker (KKT) conditions for optimality:

$$0 \leq x_1 \perp \quad -A_1 + (x_1 + y_1) + \mu \geq 0 \tag{3}$$
$$0 \leq x_2 \perp \quad -A_2 + (x_2 + y_2) + F\mu \geq 0 \tag{4}$$
$$0 \leq y_1 \perp \quad -A_1 + (x_1 + y_1) + (C+K) y_1 \geq 0 \tag{5}$$
$$0 \leq y_2 \perp \quad -A_2 + (x_2 + y_2) + (C+K) y_2 \geq 0 \tag{6}$$
$$\mu \text{ free}, \quad D - x_1 - Fx_2 = 0 \tag{7}$$

Eqs. (3)–(7) are sufficient for a global optimum because we have a convex QP. We solve (3)–(7) analytically to yield interior solutions, *cf.* Assumptions 1–2:

$$x_1^{\text{CP}} = \frac{D + F(FA_1 - A_2)}{F^2 + 1} \tag{8}$$
$$x_2^{\text{CP}} = \frac{FD + A_2 - FA_1}{F^2 + 1} \tag{9}$$
$$y_1^{\text{CP}} = \frac{A_1 + FA_2 - D}{(C+K+1)(F^2+1)} \tag{10}$$
$$y_2^{\text{CP}} = \frac{F(A_1 + FA_2 - D)}{(C+K+1)(F^2+1)} \tag{11}$$
$$\mu^{\text{CP}} = \frac{(C+K)(A_1 + FA_2 - D)}{(C+K+1)(F^2+1)} \tag{12}$$
$$z^{\text{CP}} = \frac{K(F+1)(A_1 + FA_2 - D)}{2(C+K+1)(F^2+1)} \tag{13}$$

Eq. (13) is obtained implicitly by inserting y_1^{CP} and y_2^{CP} into the expression for $\text{MCD}^{\text{CP}} = \frac{K\left(y_1^{\text{CP}} + y_2^{\text{CP}}\right)}{2}$.

Figure 1 is a "bathtub diagram" (Førsund, 2015) of the ensuing first-best solution under CP. From origin O, period-1 (period-2) decisions are indicated to the left (right). The dashed line that connects the points A_j on the vertical and horizontal axes indicates the period-j inverse-demand function. The utilisation of x_1^{CP} MWh of ESR during period 1 leads to a residual inverse-demand function faced by fossil-fuelled generation that has its vertical intercept, A_1, on the shifted vertical axis to the right of O. At an interior optimum, (5) reveals that the marginal benefit (MB) of consumption, $P_1\left(x_1^{\text{CP}} + y_1^{\text{CP}}\right)$, equals the marginal cost (MC) of generation, $(C+K)\, y_1^{\text{CP}}$, inclusive of the damage cost. In a similar vein, if x_2^{CP} MWh of ESR output is allocated to period 2, then the residual inverse-demand function faced by fossil-fuelled generation has a vertical intercept, A_2, that is on the shifted vertical axis to the left of O. For an interior solution, (6) equates the MB of consumption, $P_2\left(x_2^{\text{CP}} + y_2^{\text{CP}}\right)$, to the MC of generation, $(C+K)\, y_2^{\text{CP}}$. From the initial D-MWh stock, Fx_2^{CP} MWh of gross energy is allocated to period 2.

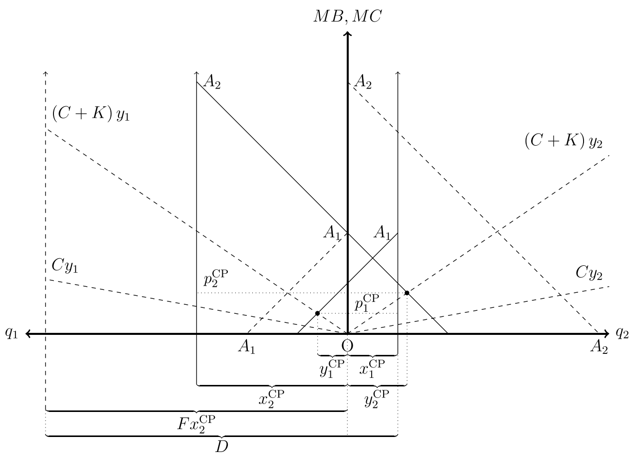

Figure 1. Bathtub diagram of ESR and fossil-fuelled operations under CP.

Given the solutions (8)–(13), Proposition 1 formalises the impact of the marginal-cost rate of damage, K.

Proposition 1. $\frac{\partial x_j^{CP}}{\partial K} = 0,\ j = 1, 2,\ \frac{\partial y_j^{CP}}{\partial K} < 0,\ j = 1, 2,$ $\frac{\partial \mu^{CP}}{\partial K} > 0$, *and* $\frac{\partial z^{CP}}{\partial K} > 0$.

In effect, an infinitesimal increase in the marginal-cost rate of damage has no impact on ESR operations because the immediate impact on the price is proportional to the periodic price. Since the period-2 price is F times as high as the one in period 1, *cf.* (3) and (4), there is no immediate gain from increasing period-2 ESR output by a unit, which would imply F fewer units of period-1 ESR output. Meanwhile, the signs of $\frac{\partial y_j^{CP}}{\partial K}$, $j = 1, 2$, $\frac{\partial \mu^{CP}}{\partial K}$, and $\frac{\partial z^{CP}}{\partial K}$ are all intuitive because a higher environmental impact of CO_2 emissions curbs fossil-fuelled production, increases the equilibrium price (or, equivalently, the marginal value of ESR-supplied energy), and boosts the implied CO_2 tax.

As for comparative statics with respect to device inefficiency, F, the findings follow straightforwardly from Debia et al. (2019) as summarised by Propostion 2.

Proposition 2. $\frac{\partial x_1^{CP}}{\partial F} < 0$ *if* $F < \hat{F}^{CP}$, $\frac{\partial x_2^{CP}}{\partial F} < 0$, $\frac{\partial y_1^{CP}}{\partial F} > 0$ *if* $F < \hat{F}^{CP}$, $\frac{\partial y_2^{CP}}{\partial F} > 0$, $\frac{\partial \mu^{CP}}{\partial F} > 0$ *if* $F < \hat{F}^{CP}$, *and* $\frac{\partial z^{CP}}{\partial F} > 0$.

Intuitively, as energy-storage efficiency worsens, *i.e.*, F increases, period-2 ESR operations decrease monotonically while period-2 fossil-fuelled generation increases monotonically. By contrast, period-1 ESR and fossil-fuelled production, as well as the marginal value of energy storage are decreasing, increasing, and increasing in F only for a relatively efficient storage device, *i.e.*, as long as $F < \hat{F}^{\text{CP}}$, where $\hat{F}^{\text{CP}} > 1$ is the positive root of the characteristic quadratic function $\hat{Q}^{\text{CP}}(F) = -A_2F^2 - 2(A_1 - D)F + A_2$. This result follows because the degrading of highly efficient energy storage incentivises the preservation of the ESR stock for use during period 2, which induces an increase in period-1 fossil-fuelled generation to fill the ensuing gap. Yet, the degradation of a relatively inefficient device makes it not worthwhile to preserve the ESR stock for use during period 2. As for the implied CO_2 tax, it is monotonically increasing in F because there is ample ESR stock to warrant curbing total fossil-fuelled generation.

Analogous expressions to (8)–(12) for a central planner that ignores the social cost of carbon (UI) are obtained by setting $K = 0$. Ignoring the cost of the externality does not affect ESR output, *i.e.*, $x_j^{\text{UI}} = x_j^{\text{CP}}$, but increases fossil-fuelled generation, *i.e.*, $y_j^{\text{UI}} > y_j^{\text{CP}}$, and the associated damage cost *vis-à-vis* CP with a concomitant social-welfare loss.

4. Decentralised Industry with Perfectly Competitive Profit-Maximising Agents

Under PC, both ESR and the fossil-fuelled generator act as price takers at the lower level when making their electricity-output decisions. Both generators take the CO_2 tax as given but are ignorant of the direct consequences of the social cost of damage from CO_2 emissions. At the upper level, the policymaker acts as a Stackelberg leader when deciding upon the welfare-maximising CO_2 tax in anticipation of the generators' responses. We solve this problem via backward induction by obtaining first a Nash equilibrium at the lower level and subsequently inserting the generators' response functions into the policymaker's upper-level objective function.

4.1. PC: Lower Level

Given an arbitrary CO_2 tax, z, each generator at the lower level selects period-j output to maximise its profit:

$$\max_{x_j \geq 0} \quad \sum_{j=1}^{2} \left[A_j - (x_j + y_j)\right] x_j \tag{14}$$

$$\text{s.t.} \quad x_1 + Fx_2 = D : \ \mu \tag{15}$$

$$\max_{y_j \geq 0} \quad \sum_{j=1}^{2} \left[A_j - (x_j + y_j)\right] y_j - \frac{1}{2} C \sum_{j=1}^{2} y_j^2 - z \sum_{j=1}^{2} y_j \tag{16}$$

Because each lower-level problem, *i.e.*, (14) s.t. (15) for the ESR and (16) for the fossil-fuelled generator, is convex and satisfies Slater conditions, it may be replaced by its necessary and sufficient KKT conditions, *viz.*:[7]

$$0 \le x_1 \perp \quad -A_1 + (x_1 + y_1) + \mu \ge 0 \quad (17)$$

$$0 \le x_2 \perp \quad -A_2 + (x_2 + y_2) + F\mu \ge 0 \quad (18)$$

$$0 \le y_1 \perp \quad -A_1 + (x_1 + y_1) + Cy_1 + z \ge 0 \quad (19)$$

$$0 \le y_2 \perp \quad -A_2 + (x_2 + y_2) + Cy_2 + z \ge 0 \quad (20)$$

$$\mu \text{ free}, \quad D - x_1 - Fx_2 = 0 \quad (21)$$

In contrast to CP, there is no direct impact of K on KKT conditions (19)–(20).

Assuming interior solutions, we solve (17)–(21) analytically to yield:

$$x_1^{\mathrm{PC}}(z) = x_1^{\mathrm{PC}}(0) + \tfrac{F(F-1)}{C(F^2+1)} z \quad (22)$$

$$x_2^{\mathrm{PC}}(z) = x_2^{\mathrm{PC}}(0) - \tfrac{(F-1)}{C(F^2+1)} z \quad (23)$$

$$y_1^{\mathrm{PC}}(z) = y_1^{\mathrm{PC}}(0) - \tfrac{\left[F(F-1)+C\left(F^2+1\right)\right]}{C(C+1)(F^2+1)} z \quad (24)$$

$$y_2^{\mathrm{PC}}(z) = y_2^{\mathrm{PC}}(0) + \tfrac{\left[(F-1)-C\left(F^2+1\right)\right]}{C(C+1)(F^2+1)} z \quad (25)$$

$$\mu^{\mathrm{PC}}(z) = \mu^{\mathrm{PC}}(0) + \tfrac{(F+1)}{(C+1)(F^2+1)} z \quad (26)$$

Eqs. (22)–(26), which characterise a lower-level solution, are parameterised on z and follow straightforwardly from the work of Debia et al. (2019). However, we express the solutions in terms of the no-tax levels, which are:

$$x_1^{\mathrm{PC}}(0) = \frac{D + F(FA_1 - A_2)}{F^2+1} \quad (27)$$

$$x_2^{\mathrm{PC}}(0) = \frac{FD + A_2 - FA_1}{F^2+1} \quad (28)$$

$$y_1^{\mathrm{PC}}(0) = \frac{A_1 + FA_2 - D}{(C+1)(F^2+1)} \quad (29)$$

$$y_2^{\mathrm{PC}}(0) = \frac{F(A_1 + FA_2 - D)}{(C+1)(F^2+1)} \quad (30)$$

$$\mu^{\mathrm{PC}}(0) = \frac{C(A_1 + FA_2 - D)}{(C+1)(F^2+1)} \quad (31)$$

Note that $x_1^{\mathrm{PC}}(0)$ and $x_2^{\mathrm{PC}}(0)$ are identical to the corresponding CP solutions, *cf.* (8)–(9). Conversely, $y_1^{\mathrm{PC}}(0)$ and $y_2^{\mathrm{PC}}(0)$ differ from the corresponding CP solutions, *cf.* (10)–(11), only by the factor, $\frac{C+K+1}{C+1}$. Likewise, $\mu^{\mathrm{PC}}(0)$ differs from its corresponding CP solution, (12), by the factor, $\frac{C(C+K+1)}{(C+1)(C+K)}$.

[7]Each producer acts as a price taker and ignores the impact of its own production on the equilibrium price.

Based on the lower-level solutions, (22)–(26), comparative statics with respect to an arbitrary z follow straightforwardly from Debia et al. (2019) as summarised by Proposition 3.

Proposition 3. $\frac{\partial x_1^{PC}}{\partial z} > 0$, $\frac{\partial x_2^{PC}}{\partial z} < 0$, $\frac{\partial y_1^{PC}}{\partial z} < 0$, $\frac{\partial y_2^{PC}}{\partial z} > 0$ *if* $C < \frac{F-1}{F^2+1}$, *and* $\frac{\partial \mu^{PC}}{\partial z} > 0$.

Intuitively, the immediate impact of an infinitesimal increase in the CO_2 tax, z, is to increase the electricity price by the same amount in each period. Thus, in contrast to the finding under CP, here, it is worthwhile for ESR output to shift towards period 1 because the reduction of period-2 output by one unit will lead to an extra F units of period-1 output. Otherwise, fossil-fuelled generation tends to decrease in both periods with the CO_2 tax unless the marginal-cost rate is low, which could lead to an increase in period-2 fossil-fuelled generation to compensate partially for the shifted ESR output. Finally, the marginal value of ESR stock increases with the CO_2 tax, which is in line with intuition.

The comparative statics of the lower-level solutions, (22)–(26), with respect to F are not amenable to straightforward analytical expressions as pointed out by Debia et al. (2019). Thus, Debia et al. (2019) attempt comparative statics only for the no-tax results, (27)–(31). Consequently, the findings mirror the results of Proposition 2, *viz.*, period-2 ESR (fossil-fuelled) generation decreases (increases) monotonically as energy-storage efficiency degrades, whereas the other results are nonmonotonic depending on whether $F < \hat{F}^{\mathrm{CP}}$ or not. Thus, it may be anticipated that if a higher K were to increase z and to reduce period-2 ESR supply, then degradations to energy-storage efficiency would further incentivise less period-2 ESR supply.

4.2. PC: Upper Level

The upper-level problem of the policymaker is to select the CO_2 tax, z, to maximise social welfare. It is constrained by industry's lower-level problems, (14) s.t. (15) for the ESR and (16) for the fossil-fuelled generator, which yields the bi-level formulation. Because each lower-level problem may be replaced by its necessary and sufficient KKT conditions, the bi-level problem may be reformulated as a mathematical program with equilibrium constraints (MPEC). By inserting the interior solutions from the lower level, (22)–(26), parameterised on an arbitrary z, we can subsequently tackle the policymaker's welfare-maximisation problem as the following single-level unconstrained QP:

$$\max_z \quad \sum_{j=1}^{2} A_j \left(x_j^{\text{PC}}(z) + y_j^{\text{PC}}(z)\right) - \frac{1}{2}\sum_{j=1}^{2}\left(x_j^{\text{PC}}(z) + y_j^{\text{PC}}(z)\right)^2 - \frac{1}{2}(C+K)\sum_{j=1}^{2} y_j^{\text{PC}}(z)^2 \tag{32}$$

Taking the first-order necessary condition for (32) and assuming an interior solution, we obtain:

$$\begin{aligned}
&- \frac{A_1(F+1)}{(C+1)(F^2+1)} - \frac{(F+1)^2}{(C+1)^2(F^2+1)^2}z \\
&+ \frac{\left(x_1^{\text{PC}}(0) + y_1^{\text{PC}}(0)\right)(F+1)}{(C+1)(F^2+1)} \\
&- \frac{(C+K)\left[F(F-1) + C\left(F^2+1\right)\right]^2}{C^2(C+1)^2(F^2+1)^2}z \\
&+ \frac{(C+K)\,y_1^{\text{PC}}(0)\left[F(F-1) + C\left(F^2+1\right)\right]}{C(C+1)(F^2+1)} \\
&- \frac{A_2F(F+1)}{(C+1)(F^2+1)} - \frac{F^2(F+1)^2}{(C+1)^2(F^2+1)^2}z \\
&+ \frac{\left(x_2^{\text{PC}}(0) + y_2^{\text{PC}}(0)\right)F(F+1)}{(C+1)(F^2+1)} \\
&- \frac{(C+K)\left[(F-1) - C\left(F^2+1\right)\right]^2}{C^2(C+1)^2(F^2+1)^2}z \\
&- \frac{(C+K)\,y_2^{\text{PC}}(0)\left[(F-1) - C\left(F^2+1\right)\right]}{C(C+1)(F^2+1)} \\
&= 0
\end{aligned} \tag{33}$$

The second-order sufficient condition is satisfied by checking the sign of the second derivative of the objective function, which is:

$$-\frac{(F+1)^2}{(C+1)^2(F^2+1)^2} - \frac{(C+K)\left[F(F-1)+C\left(F^2+1\right)\right]^2}{C^2(C+1)^2(F^2+1)^2} - \frac{F^2(F+1)^2}{(C+1)^2(F^2+1)^2} - \frac{(C+K)\left[(F-1)-C\left(F^2+1\right)\right]^2}{C^2(C+1)^2(F^2+1)^2} < 0 \tag{34}$$

Hence, by solving (33), we obtain the optimal CO_2 tax under the decentralised setting, which is:

$$z^{\text{PC}} = \frac{C^2K(F+1)(A_1+FA_2-D)}{C^2(F+1)^2+(C+K)\left[(2C+1)(F-1)^2+2C^2(F^2+1)\right]} \tag{35}$$

Given z^{PC}, Figure 2 provides a bathtub diagram, which can facilitate understanding of the solution under PC. The concept is similar to that in Figure 1 except that now the social cost of damage from CO_2 emissions is not internalised directly. Instead, the tax on fossil-fuelled generation merely shifts up the private MC during each period, *cf.* (19)–(20).

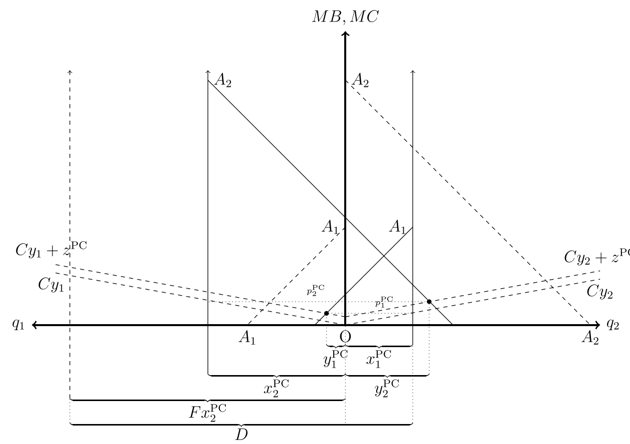

Figure 2. Bathtub diagram of ESR and fossil-fuelled operations under PC.

Based on the CP and PC solutions, (13) and (35), it is possible to prove that (i) the optimal CO_2 tax under PC is lower than that under CP, (ii) the marginal value of ESR stock under PC is lower than that under CP, and (iii) the MCD under PC is higher than that under CP. Propositions 4–6 formalise these results.

Proposition 4. $z^{PC} < z^{CP}$.

The result $z^{\text{PC}} < z^{\text{CP}}$ in Proposition 4 indicates that the implied CO_2 tax under CP is always higher than that in a deregulated industry. Intuitively, this occurs because an infinitesimal increase in the CO_2 tax under PC incentivises ESR production to shift to the off-peak period, thereby potentially exacerbating the damage from fossil-fuelled generation, *cf.* Proposition 3. Thus, the policymaker under PC has a quandary and optimally reduces the CO_2 tax *vis-à-vis* CP. However, with a perfectly efficient storage device, the same outcome is rendered under both CP and PC, *i.e.*, $\lim_{F\to 1^+} z^{\text{CP}} = \frac{K(A_1+A_2-D)}{2(C+K+1)} = \lim_{F\to 1^+} z^{\text{PC}}$.

Proposition 5. $\mu^{PC}\left(z^{PC}\right) < \mu^{CP}$.

In effect, Proposition 5 demonstrates that the propensity of the ESR generator to shift its production to the off-peak period under PC for any positive CO_2 tax obviates the value of storage.

Proposition 6. $MCD^{PC} > MCD^{CP} \equiv z^{CP}$.

Proposition 6 shows that a decentralised industry under PC always has a higher MCD than one under CP.

This outcome is intuitive, given the need to curb the CO_2 tax under PC to avoid exacerbating the misallocation of ESR energy storage.

Analogous to Proposition 1 under CP, we can demonstrate in the following proposition that the optimal CO_2 tax under PC (35) monotonically increases with respect to K.

Proposition 7. $\frac{\partial z^{PC}}{\partial K} > 0$.

Given a higher social-cost rate of damage from CO_2 emissions, the policymaker increases the CO_2 tax on the fossil-fuelled generator.

In a similar vein to Proposition 2 under CP, the following proposition demonstrates how the optimal CO_2 tax under PC (35) behaves with respect to F.

Proposition 8. $\frac{\partial z^{PC}}{\partial F} < 0$ *if* $F > \hat{F}^{PC}$.

Note that the CO_2 tax under PC is increasing in energy-storage efficiency loss for a perfectly efficient device, *i.e.*, $lim_{F \to 1^+} \frac{\partial z^{\mathrm{PC}}}{\partial F} = \frac{K(A_2 - A_1 + D)}{4(C+K+1)} > 0$ from (A-20). However, a higher CO_2 tax incentivises greater period-1 ESR output unconditionally as well as more period-2 fossil-fuelled output (for sufficiently degraded energy-storage efficiency), *cf.* Proposition 3. Thus, it is plausible for the CO_2 tax under PC to actually decrease in F once storage efficiency has degraded sufficiently, which is in contrast to the monotonic result under CP, *cf.* Proposition 2.

5. Numerical Examples

We use the following parameter values: $A_1 = 100$, $A_2 = 250$, $C = 0.18$, $D = 350$, $K = 0.5$, and $F \in (1, 2.5]$. These are consistent with Assumptions 1–7 in yielding interior solutions as F is varied.

The main finding about the optimal CO_2 tax under CP and PC is illustrated in Figure 3: while z^{CP} increases monotonically with F, z^{PC} encounters a turning point at $\hat{F}^{\mathrm{PC}} = 1.6308$, *cf.* Propositions 2 and 8. The characteristic quadratic, $\hat{Q}^{\mathrm{PC}}(F)$, that identifies $\hat{F}^{\mathrm{PC}}$ is illustrated in Figure 4. Note that exactly a single $\hat{F}^{\mathrm{PC}} > 1$ exists due to the concavity of the characteristic quadratic. Moreover, following from Propositions 4 and 6, it is also evident that the CO_2 tax is lower under PC while its MCD is higher. For reference, the higher MCD under UI is also provided in Figure 3.

As for the marginal value of ESR stock, Figure 5 illustrates Proposition 5's finding that it is always higher under CP than under PC. Both $\mu^{\mathrm{PC}}\left(z^{\mathrm{PC}}\right)$ and μ^{CP} also exhibit turning points with respect to F. However, only the latter's turning point, $\hat{F}^{\mathrm{CP}} = 2.4142$ may be found analytically, as formalised by Proposition 2, via the root of the characteristic quadratic, $\hat{Q}^{\mathrm{CP}}(F)$ (see Figure 6).

Not surprisingly, social welfare decreases monotonically with F under all settings (see Figure 7). The ordering of the maximised welfare is also in line with intuition as the outcome under the (second-best) decentralised PC setting is sandwiched by those under first-best CP and the UI settings.

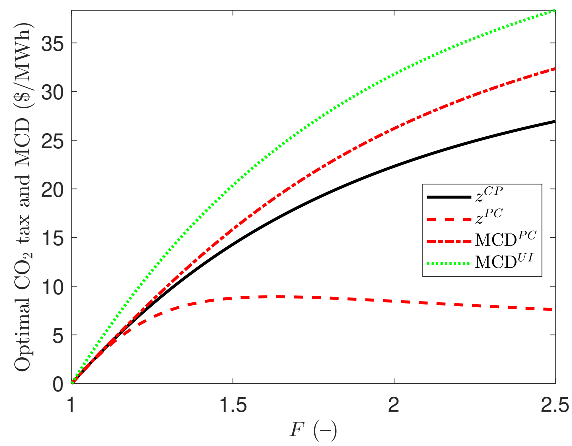

Figure 3. Optimal CO_2 tax and MCD with respect to F.

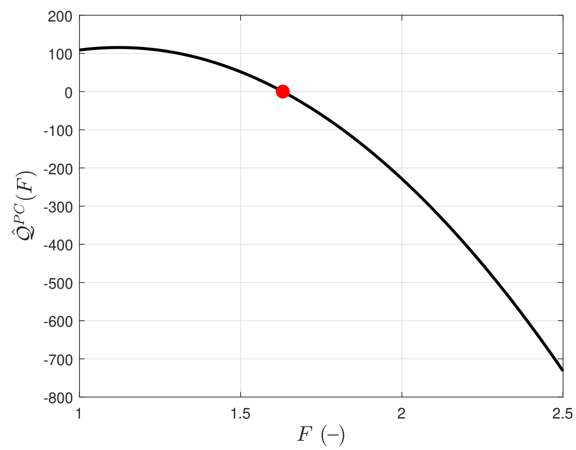

Figure 4. Characteristic quadratic, $\hat{Q}^{\mathrm{PC}}(F)$, that identifies $\hat{F}^{\mathrm{PC}}$, above which z^{PC} decreases with F.

6. Conclusions

Renewable-energy integration will necessitate an unprecedented transformation of the power system with increasing reliance on energy storage and other forms of flexibility. In this respect, fossil-fuelled generation may still play a role and require appropriate regulation in line with climate goals. While the extant literature has analysed storage operations in a deregulated industry with emission constraints, the design of carbon policy in a future power system has received less attention.

This paper examines optimal carbon pricing in an

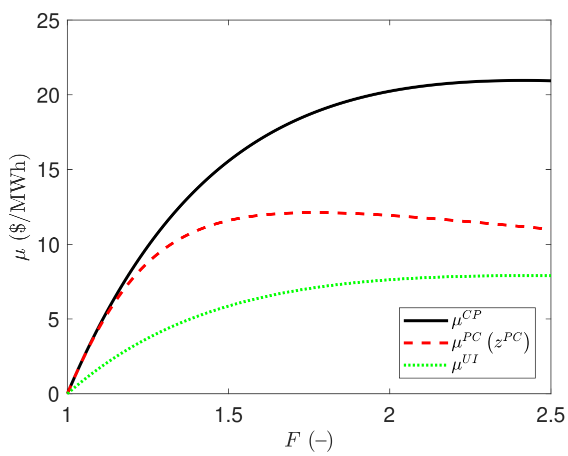

Figure 5. Marginal value of ESR stock with respect to F.

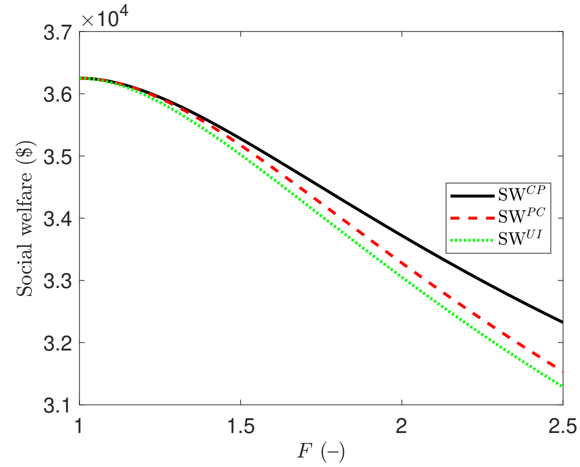

Figure 7. Social welfare with respect to F.

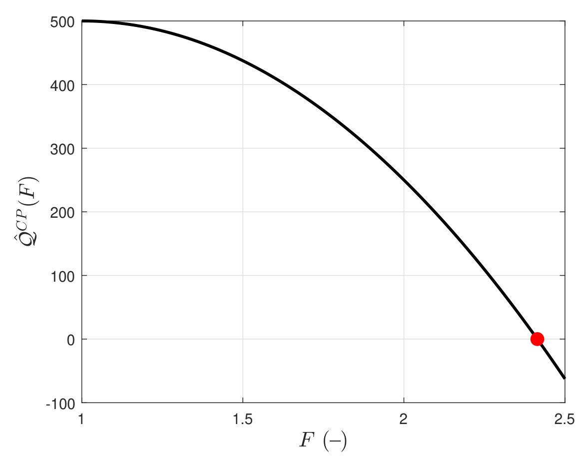

Figure 6. Characteristic quadratic, $\hat{Q}^{\mathbf{CP}}(F)$, that identifies $\hat{F}^{\mathbf{CP}}$, above which $\mu^{\mathbf{CP}}$ decreases with F.

electricity sector with carbon-intensive fossil-fuelled generation, carbon-free renewable generation, and energy storage. We do this by contrasting an ideal central-planning benchmark, wherein a welfare maximiser makes all production and energy-storage decisions while internalising the social cost of carbon, to a decentralised setting. One of our key findings is that with perfect competition, a decentralised setting yields social-welfare losses and socially suboptimal use of generation and energy-storage resources compared to the central-planning benchmark. This result is counterintuitive because conventional wisdom holds that perfect competition with properly priced and internalised externalities will yield a social-welfare-optimising outcome. Moreover, the contrast between the two settings becomes starker as storage efficiency decreases. Indeed, for a sufficiently degraded storage device, carbon policy in a decentralised setting behaves fundamentally differently *vis-à-vis* central planning as it necessitates a *decrease* in the CO_2 tax as efficiency drops.

Despite the counterintuitive nature of our finding, it is not without precedent. Downward (2010) demonstrates cases in which imposing a carbon price or tax can increase total carbon emissions. His finding can be explained as being due to a third-best outcome. His base case has three market failures—transmission congestion, market power, and an unpriced carbon externality. Addressing only one of the market failures (the unpriced externality), exacerbates the effect of the other two, which yields a less socially desirable outcome. Our finding differs, insomuch as we have neither transmission congestion nor market power. However, Sioshansi (2014) unveils cases in which energy storage that is owned by a generator, which is the case for the ESR supplier, can yield social-welfare losses compared to a no-energy-storage case. As such, our finding could be an extension of this phenomenon.

Future work in this area could proceed in two directions. First, market power by generators plagues even well-functioning electricity markets (Tangerås & Mauritzen, 2018) and could potentially be exploited by flexible producers in a future power system with higher renewable penetration (Hassanzadeh Moghimi et al., 2023). In this vein, departures from perfect competition at the lower level to allow for Cournot behaviour (Crampes & Moreaux, 2001) would enable a richer examination of carbon-policy design. Second, besides a carbon tax, renewable portfolio standards (Siddiqui et al., 2016) and rate-based measures (Tanaka et al., 2022) are part of the regulatory toolkit. By providing greater flexibility in meeting environmental constraints, such policies could address a simple carbon tax's limitations in curbing fossil-fuelled generation adequately and would also be amenable to rigorous analysis via our bi-level framework.

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Appendix: Proofs of Propositions

Proof of Proposition 1 From (8)–(9), it is evident that $\frac{\partial x_j^{\mathrm{CP}}}{\partial K} = 0,\ j = 1, 2$. Moreover, from (10)–(13) and Assumption 2, we obtain:

$$\frac{\partial y_1^{\mathrm{CP}}}{\partial K} = -\frac{(A_1+FA_2-D)}{(C+K+1)^2(F^2+1)} < 0 \tag{A-1}$$

$$\frac{\partial y_2^{\mathrm{CP}}}{\partial K} = -\frac{F(A_1+FA_2-D)}{(C+K+1)^2(F^2+1)} < 0 \tag{A-2}$$

$$\frac{\partial \mu^{\mathrm{CP}}}{\partial K} = \frac{\left(F^2+1\right)(A_1+FA_2-D)}{(C+K+1)^2(F^2+1)^2} > 0 \tag{A-3}$$

$$\frac{\partial z^{\mathrm{CP}}}{\partial K} = \frac{(C+1)(F+1)(A_1+FA_2-D)}{2(C+K+1)^2(F^2+1)} > 0 \tag{A-4}$$

□

Proof of Proposition 2 The following results are unconditional using (9), (11), and (13) based on Assumptions 1, 2, and 5:

$$\frac{\partial x_2^{\mathrm{CP}}}{\partial F} = \frac{D\left(1-F^2\right)-(FA_2+A_1)-F(A_2-FA_1)}{(F^2+1)^2} < 0 \tag{A-5}$$

$$\frac{\partial y_2^{\mathrm{CP}}}{\partial F} = \frac{F(A_2-FA_1)+\left(F^2-1\right)D+F(FA_2+A_1)}{(C+K+1)(F^2+1)^2} > 0 \tag{A-6}$$

$$\frac{\partial z^{\mathrm{CP}}}{\partial F} = \frac{K\left[(A_2+A_1-D)\left(1-F^2\right)+2(A_2-A_1+D)F\right]}{2(C+K+1)(F^2+1)^2} > 0 \tag{A-7}$$

Meanwhile, the remaining results using (8), (10), and (12) hold if $F < \hat{F}^{\mathrm{CP}}$, where $\hat{F}^{\mathrm{CP}} > 1$ is the positive root of the characteristic quadratic function $\hat{Q}^{\mathrm{CP}}(F) = -A_2F^2 - 2(A_1 - D)F + A_2$:

$$\frac{\partial x_1^{\mathrm{CP}}}{\partial F} = \frac{F(FA_2+A_1)-2FD-(A_2-FA_1)}{(F^2+1)^2} < 0 \tag{A-8}$$

$$\frac{\partial y_1^{\mathrm{CP}}}{\partial F} = \frac{(A_2-FA_1)+2FD-F(FA_2+A_1)}{(C+K+1)(F^2+1)^2} > 0 \tag{A-9}$$

$$\frac{\partial \mu^{\mathrm{CP}}}{\partial F} = \frac{C[(A_2-FA_1)+2FD-F(FA_2+A_1)]}{(C+K+1)(F^2+1)^2} > 0 \tag{A-10}$$

□

Proof of Proposition 3 From (22)–(26), we have:

$$\frac{\partial x_1^{\mathrm{PC}}}{\partial z} = \frac{F(F-1)}{C(F^2+1)} > 0 \tag{A-11}$$

$$\frac{\partial x_2^{\mathrm{PC}}}{\partial z} = -\frac{(F-1)}{C(F^2+1)} < 0 \tag{A-12}$$

$$\frac{\partial y_1^{\mathrm{PC}}}{\partial z} = -\frac{1}{(C+1)}\left[1 + \frac{F(F-1)}{C(F^2+1)}\right] < 0 \tag{A-13}$$

$$\frac{\partial y_2^{\mathrm{PC}}}{\partial z} = -\frac{\left[C\left(F^2+1\right)-(F-1)\right]}{C(C+1)(F^2+1)} > 0 \text{ if } C < \frac{F-1}{F^2+1} \tag{A-14}$$

$$\frac{\partial \mu^{\mathrm{PC}}}{\partial z} = \frac{(F+1)}{(C+1)(F^2+1)} > 0 \tag{A-15}$$

□

Proof of Proposition 4 Note that $z^{\text{PC}} < z^{\text{CP}} \Leftrightarrow \frac{C^2K(F+1)(A_1+FA_2-D)}{C^2(F+1)^2+(C+K)\left[(2C+1)(F-1)^2+2C^2(F^2+1)\right]} < \frac{K(F+1)(A_1+FA_2-D)}{2(C+K+1)(F^2+1)}$, *cf.* (35) and (13). Simplifying, we obtain:

$$\begin{aligned}
&z^{\text{PC}} < z^{\text{CP}} \\
&\Leftrightarrow 2C^2\left(F^2+1\right) \\
&< C^2(F+1)^2 + (C+K)(2C+1)(F-1)^2 \\
&\Leftrightarrow C^2(F-1)^2 < (C+K)(2C+1)(F-1)^2 \\
&\Leftrightarrow C^2 < (C+K)(2C+1) \\
&\Leftrightarrow 0 < C^2 + 2CK + C + K
\end{aligned} \tag{A-16}$$

The latter holds from Assumptions 4 and 7, where in the next-to-last line of (A-16), we have used the fact that $(F-1)^2 > 0$. □

Proof of Proposition 5 Note that $\mu^{\text{PC}}\left(z^{\text{PC}}\right) < \mu^{\text{CP}} \Leftrightarrow \frac{C(A_1+FA_2-D)}{(C+1)(F^2+1)} + \frac{(F+1)}{(C+1)(F^2+1)}z^{\text{PC}} < \frac{(C+K)(A_1+FA_2-D)}{(C+K+1)(F^2+1)}$, *cf.* (12) and (26). Simplifying, we obtain:

$$\begin{aligned}
&\mu^{\text{PC}}\left(z^{\text{PC}}\right) < \mu^{\text{CP}} \\
&\Leftrightarrow C(C+K+1)(A_1+FA_2-D) \\
&+ (C+K+1)(F+1)z^{\text{PC}} \\
&< (C+K)(C+1)(A_1+FA_2-D) \\
&\Leftrightarrow (C+K+1)C^2(F+1)^2 \\
&< (C+K)\left[(2C+1)(F-1)^2 + 2C^2\left(F^2+1\right)\right] \\
&+ C^2(F+1)^2 \\
&\Leftrightarrow 0 < (2C+1)(F-1)^2 + C^2(F-1)^2
\end{aligned} \tag{A-17}$$

The latter holds from Assumption 4. □

Proof of Proposition 6 The MCD under PC is obtained by substitution of the total fossil-fuelled-generation level, $y_1^{\text{PC}}\left(z^{\text{PC}}\right) + y_2^{\text{PC}}\left(z^{\text{PC}}\right)$, into $MCD^{\text{PC}} = \frac{K\left(y_1^{\text{PC}}\left(z^{\text{PC}}\right)+y_2^{\text{PC}}\left(z^{\text{PC}}\right)\right)}{2} = \frac{K(F+1)(A_1+FA_2-D)}{2C(C+1)(F^2+1)} \times \frac{\left[C^3(F+1)^2+2C^4\left(F^2+1\right)+C(F-1)^2\left(2C^2+C+CK+K\right)\right]}{C^2(F+1)^2+(C+K)\left[(2C+1)(F-1)^2+2C^2(F^2+1)\right]}$. We prove now that $MCD^{\text{PC}} > MCD^{\text{CP}} \equiv z^{\text{CP}}$ by comparing the latter expression with (13):

$$\begin{aligned}
&MCD^{\text{PC}} > MCD^{\text{CP}} \\
&\Leftrightarrow C^2K(F+1)^2 \\
&+ 2C^2\left(F^2+1\right)\left[C(C+K+1) - (C+1)(C+K)\right] \\
&+ (F-1)^2(C+K+1)\left(2C^2+C+CK+K\right) \\
&- (F-1)^2(C+1)\left(2C^2+C+2CK+K\right) > 0 \\
&\Leftrightarrow C^2K(F+1)^2 - 2C^2K\left(F^2+1\right) \\
&+ (F-1)^2\left(C^2K+CK^2+K^2\right) > 0 \\
&\Leftrightarrow (F-1)^2(C+1)K^2 > 0
\end{aligned} \tag{A-18}$$

The latter holds from Assumptions 4 and 7. □

Proof of Proposition 7 Via (35), we have:

$$\begin{aligned}
&\frac{\partial z^{\text{PC}}}{\partial K} = C^2(F+1)(A_1+FA_2-D) \\
&\times \frac{\left[C^2(F+1)^2+C\left[(2C+1)(F-1)^2+2C^2\left(F^2+1\right)\right]\right]}{\left\{C^2(F+1)^2+(C+K)\left[(2C+1)(F-1)^2+2C^2\left(F^2+1\right)\right]\right\}^2} \\
&> 0
\end{aligned} \tag{A-19}$$

The latter holds from Assumptions 1, 4, 6, and 7. □

Proof of Proposition 8 Using (35), we obtain:

$$\begin{aligned}
\frac{\partial z^{\text{PC}}}{\partial F} &= \hat{Q}^{\text{PC}}(F)\,C^2K\left\{C^2(F+1)^2 + (C+K)\right. \\
&\times \left.\left[(2C+1)(F-1)^2 + 2C^2\left(F^2+1\right)\right]\right\}^{-2}
\end{aligned} \tag{A-20}$$

where $\hat{Q}^{\text{PC}}(F)$ is the quadratic function with F^2 coefficient $C^2(A_2-A_1+D) + (2C+1)(C+K)(D-A_1-3A_2) + 2C^2(C+K)(D-A_1-A_2)$, F coefficient $2C^2(A_2-A_1+D) + 2(2C+1)(C+K)(D-A_1+A_2) + 4C^2(C+K)(D-A_1+A_2)$, and constant term $C^2(A_2-A_1+D) + (2C+1)(C+K)(A_2+3A_1-3D) + 2C^2(C+K)(A_2+A_1-D)$. The necessary condition for $\frac{\partial z^{\text{PC}}}{\partial F} < 0$ corresponds to finding the root, $\hat{F}^{\text{PC}} > 1$, of $\hat{Q}^{\text{PC}}(F)$. Note that $\hat{Q}^{\text{PC}}(1) = 4C^2(D-A_1+A_2) > 0$ from Assumption 1 and $\hat{Q}^{\text{PC}\prime}(1) = 4C^2(A_2-A_1+D) + 4(2C+1)(C+K)(D-A_1-A_2) + 8C^2(C+K)(D-A_1) > 0$ from Assumptions 1–2. Thus, $\hat{Q}^{\text{PC}}(F)$ is positive and increasing at $F = 1$. Whether $\hat{Q}^{\text{PC}}(F)$ is convex or concave depends on the sign of $\hat{Q}^{\text{PC}\prime\prime}(F)$, which is $2C^2(A_2-A_1+D) + 2(2C+1)(C+K)(D-A_1-3A_2) + 4C^2(C+K)(D-A_1-A_2)$. If $\hat{Q}^{\text{PC}\prime\prime}(F) \geq 0$, then no $\hat{F}^{\text{PC}} > 1$ exists, but exactly one $\hat{F}^{\text{PC}} > 1$ exists if $\hat{Q}^{\text{PC}\prime\prime}(F) < 0$, *i.e.*, if A_2 is large enough to entice more period-2 fossil-fuelled output. □