

## Operational Research: methods and applications

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#### Abstract

Throughout its history, Operational Research has evolved to include methods, models and algorithms that have been applied to a wide range of contexts. This encyclopedic article consists of two main sections: methods and applications. The first summarises the up-todate knowledge and provides an overview of the state-of-the-art methods and key developments in the various subdomains of the field. The second offers a wide-ranging list of areas where Operational Research has been applied. The article is meant to be read in a nonlinear fashion and used as a point of reference by a diverse pool of readers: academics, researchers, students, and practitioners. The entries within the methods and applications sections are presented in alphabetical order. The authors dedicate this paper to the 2023 Turkey/Syria earthquake victims. We sincerely hope that advances in OR will play a role towards minimising the pain and suffering caused by this and future catastrophes.


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### 2.2. Behavioural $O R^{3}$

Behavioural OR (BOR) is concerned with the study of human behaviour in OR-supported settings. Specifically, BOR examines how the behaviour of individuals affects, or is affected by, an OR-supported intervention ${ }^{4}$. The individuals of interest are those who, acting in isolation or as part of a team, design, implement and engage with OR in practice. These individuals include OR practitioners playing specific intervention roles (e.g., modellers, facilitators, consultants), and other individuals with varying interests and stakes in the intervention (e.g., users, clients, domain experts, sponsors).

A concern with the behavioural aspects of the OR profession can be traced back to past debates in the 1960s, 1970s and 1980s (Churchman, 1970; Dutton \& Walton, 1964; Jackson et al., 1989). Although these debates dwindled down in subsequent years, the emergence of BOR as a field of study represents a return to these earlier concerns (Franco \& Hämäläinen, 2016; Hämäläinen et al., 2013). What motivates this resurgence is the recognition that the successful deployment of OR in practice relies heavily on our understanding of human behaviour. For example, overconfidence, competing interests, and the willingness to expend effort in
searching, sharing, and processing information are three behavioural issues that can negatively affect the success of OR activities. Attention to behavioural issues has been central in disciplines such as economics, psychology and sociology for decades, and BOR studies draw heavily from these reference disciplines (Franco et al., 2021).

It is important to distinguish between the specific focus of BOR and the broader focus of behavioural modelling. The creation of models that capture human behaviour has a long tradition within OR, but it is not necessarily concerned with the study of human behaviour in OR-supported settings. For example, in the last 20 years operational researchers have produced an increasing number of robust analytical models that describe behaviour in, and predict its impact on, operations management settings (Cui \& Wu, 2018; Donohue et al., 2020; Loch \& $\mathrm{Wu}, 2007$ ). Operational researchers also have produced simulation models that capture human behaviour within a system with different levels of complexity. For example, systems dynamics models incorporate high-level variables representing average behaviour (Morecroft, 2015; Sterman, 2000, \$2.22), and discrete event simulation models capture human processes controlled by simple behavioural rules (Brailsford \& Schmidt, 2003; Robinson, 2014, $\$ 2.19$ ). More complex agent-based simulation models represent behaviour as emergent from the interactions of agents with particular behavioural attributes (Sonnessa et al., 2017; Utomo et al., 2018, $\$ 2.19$ ). Overall, behavioural modelling within the OR field is concerned with examining human behaviour in a system of interest in order to improve that system ${ }^{5}$. In contrast, BOR takes an OR-supported intervention as the core system of interest where human behaviour is examined. The ultimate goal of BOR is to generate an improved understanding of the behavioural dimension of OR practice, and use this understanding to design and implement better OR-supported interventions.

Another important distinction worth stating is that between BOR and Soft OR. At first glance, this distinction may seem unnecessary as BOR is a field of study within OR, while Soft OR refers to a specific family of problem structuring approaches ( $\$ 2.20$ ). Soft OR approaches have been developed to help groups reach agreements on problem structure and, often, appropriate responses to a problem of concern (Franco \& Rouwette, 2022; Rosenhead \& Mingers, 2001). However, while Soft OR intervention design and implementation typically require the consideration of behavioural issues, this is not the same as choosing human behaviour in a Soft OR intervention context as the unit of analysis. Of course, a study with such a focus would certainly
fall within the BOR remit (e.g., Tavella et al., 2021). But note that BOR is also concerned with the study of human behaviour in other OR-supported settings, such as those involving the use of 'hard' and 'mixed-method' OR approaches.

Studies of behaviour in OR-supported settings assume implicitly or explicitly that human behaviour is either influenced by cognitive and external factors, or is in itself an influencing factor (Franco et al., 2021). In the first case, observed individual and collective action is taken to be guided by cognitive structures (e.g., personality traits, cognitive styles) manifested during OR-related activity behaviour is influenced. In contrast, the second case assumes that individuals and collectives are responsible for determining how OR-related activity will unfold - behaviour is influencing. This raises the practical possibility that the same OR methodology, technique, or model could be used in distinctive ways by various individuals or groups according to their cognitive orientations, goals and interests (Franco, 2013). Whilst behaviour in practice is likely to lie somewhere between the influenced and influencing assumptions, BOR studies tend to foreground one of the extremes as the focus, while backgrounding the other.

BOR studies can adopt three different research methodologies to examine behaviour: variance, process, and modelling. A variance methodology uses variables that represent the important aspects or attributes of the OR-supported activity being examined. Variance explanations of behavioural-related phenomena take the form of causal statements captured in a theoretically-informed research model that incorporates these variables (e.g., A causes B, which causes C). The research model is then tested with data generated by the activity, and the research findings are assessed in terms of their generality (Poole, 2004). Adopting a variance research methodology typically requires the implementation of experimental, quasi-experimental, or survey research designs ${ }^{6}$. This involves careful selection of independent variables, which might be either manipulated or left untreated, and of dependent variables that act as surrogates for specific behaviours. Once information about all variables is collected, data is quantitatively analysed using a wide range of vari-ance-based methods (e.g., analysis of variance, regression, structural equation modelling).

Behavioural studies that use a variance research methodology can produce a good picture of the gencrative mechanisms underpinning behavioural processes if they test hypotheses about those mechanisms. For example, variance studies in BOR have examined the impact of individual differences in cognitive motivation and cognitive style on the
conduct of OR-supported activity (Fasolo \& Bana e Costa, 2014; Franco et al., 2016b; Lu et al., 2001). There is also a long tradition of testing the behavioural effects of reconfiguring different aspects of OR-supported activity such as varying model or information displays (Bell \& O'Keefe, 1995; Gettinger et al., 2013), and preference elicitation procedures (Cavallo et al., 2019; Hämäläinen \& Lahtinen, 2016; Pöyhönen et al., 2001; von Nitzsch \& Weber, 1993).

A process methodology is used to examine ORsupported activity as a series of events that bring about or lead to some behaviour-related outcome. Specifically, it considers as the unit of analysis an evolving individual or group whose behaviour is led by, or leading, the occurrence of events (Poole, 2004). Process explanations take the form of theoretical narratives that account for how event dynamics lead to a final outcome (Poole, 2007). These narratives are often derived from observation, but it is also possible to use an established narrative (e.g., a theory) to guide observation that further specifies the narrative.

Diverse and eclectic research designs are used to implement a process research methodology. Central to these designs is the task of identifying or reconstructing the process through the analysis of events taking place over time. For example, there is an important stream of BOR studies that examines the process of building models by experts and novices (Tako, 2015; Tako \& Robinson, 2010; Waisel et al., 2008; Willemain, 1995; Willemain \& Powell, 2007). There is also an increasing interest to use process methodologies to take a closer look at actual behaviour in OR-supported settings both, before, during and after OR-related activity is undertaken (Franco \& Greiffenhagen, 2018; Kaki et al., 2019; VelezCastiblanco et al., 2016; White et al., 2016).

The variance and process approaches may seem opposite to each other, but instead they should be seen as complementary (Franco et al., 2021; Van de Ven \& Poole, 2005). BOR studies using a variance research methodology can explore and test the mechanisms that drive process explanations of behaviour, while BOR studies adopting a process research methodology can explore and test the narratives that ground variance explanations of behaviour. One way of combining a variance and process approach within a single BOR study is by adopting modelling as a research methodology. A modelling approach would create models that capture the mechanisms that generate a process of interest such as, for example, trust on an OR-derived solution, and the model can be run to generate the characteristics of that process. Model parameters and structure can then be varied systematically to enable variance-based comparisons
of trust levels. Furthermore, the trajectory of trust levels over time can be used to gain insights into the nature of the trust development process. As already mentioned, there is a long behavioural modelling tradition within OR but, as far as we know, its potential as a research methodology tool to specifically examine behaviour in OR-supported settings is yet to be realised.

In sum, the variance, process and modelling methodologies offer rich possibilities for the study of human behaviour in OR-supported settings. Which is best for a particular study will depend on the types of question being addressed by BOR researchers, their assumptions about human behaviour, and the data they have access to. Ultimately, a thorough understanding of behaviour in OR-supported settings is likely to require all three research methodologies.

For a detailed review of BOR studies the reader is referred to Franco et al. (2021). A review of behavioural studies in the context of OR in health has been written by Kunc et al. (2020). There are also two collections edited by Kunc et al. (2016) and White et al. (2020). The European Journal of Operational Research published a feature cluster on BOR edited by Franco and Hämäläinen (2016a). Finally, BOR-related news and events can be found on the sites of the European Working Group on Behavioural $\mathrm{OR}^{7}$, and the UK BOR Special Interest Group ${ }^{8}$.

