Computational Management Science (2025) 22:6
https://doi.org/10.1007/s10287-025-00534-5

ORIGINAL PAPER

Understanding the role of technological complexity in sustainability transitions using stochastic, bi-level optimization

Nathan T. Boyd[1] · **Steven A. Gabriel**[1,2,3]

Received: 25 May 2024 / Accepted: 30 April 2025 / Published online: 28 May 2025

Abstract
Practically unlimited natural resources, such as solar energy and advanced seawater desalination, are potential solutions to sustainable resource consumption. However, accessing these natural resources depends on complex technologies that require further research and development. This technological complexity introduces significant uncertainty in the actions required to transition societies to more sustainable levels of resource consumption. Such uncertainty has important implications in terms of risk. The short-term depletion of limited natural resources, such as fossil fuels and freshwater, can pay off if these sustainable technologies mature in the long term. However, this short-term, resource-depletion policy carries the risk that these sustainable technologies will not materialize. In such a case, economic decline, population decline, or both are possible undesirable outcomes. To address this challenge, a stochastic, bi-level optimization problem is developed for sustainability transitions in natural-resource contexts. This model is formulated as a mathematical program with equilibrium constraints and is solved as a mixed-integer, non-linear program. This model is applied to an illustrative water-resources problem with two lower-level players where a policymaker manages freshwater in conjunction with a new water-treatment technology. Overall, this model demonstrates how policies for sustainable resource management can be quantified in terms of risk aversion to adopting new technologies.

Keywords Sustainability transitions · Mathematical programs with equilibrium constraints · Emerging technologies · Energy, natural resources, and the environment

Extended author information available on the last page of the article

1 Introduction

Policymakers face a challenging dichotomy when establishing limits to the sustainable consumption of energy and other natural resources: consumption is derived from both accessible, limited resources and less accessible, but nearly unlimited resources (e.g., renewable energy, desalinated seawater). Harnessing the full abundance of the less-accessible resources depends on complex, technological solutions with uncertain, future prospects. Policies that are too optimistic regarding these technologies could result in resource over-extraction, while overly pessimistic policies could encourage resource hoarding.

Such technological trade-offs can be understood in terms of risk. The short-term depletion of limited natural resources can pay off if technology emerges to harness alternative resources in the long term. However, this short-term, resource-depletion policy carries the risk that the technology will not materialize. In such a case, economic decline, population decline, or both are possible undesirable outcomes.

Given this challenge, our work seeks to identify sustainable energy and natural resource policies that also account for technological transitions. We concentrate on water-resource systems specifically to serve as a counterpoint to the already extensive literature on energy transitions (Saraji and Streimikiene 2023). However, our water model is also mathematically general to promote multi-disciplinary engagement among experts in energy and other natural-resource systems.

Technology plays an important role in water-resource systems because human engineering alters a natural environment's water quantity and quality. These technologies vary greatly in their complexity. A simple water-supply system for humans often features a shared well in a village. Simple technologies are typically used with freshwater sources such as streams, lakes, or groundwater. On the other hand, a complex water system often features reservoirs, water treatment facilities, and distribution networks of pipes and pumps. Even more complex technologies can also be used to treat seawater or wastewater.

Regardless of complexity, water-resource systems involve more than just their technical features. Socio-economic institutions construct and maintain these technologies. They also vary widely in complexity. Simple water institutions often rely on cultural norms for hygiene and sharing of resources. Water utilities represent complex water institutions, which often have separate departments for planning, maintenance, construction, and finance. Other societal institutions, such as government and industry, are stakeholders who benefit from and influence the decisions of water institutions.

In most water-resource systems, societal norms and technologies have catered to the use of freshwater sources such as rivers, lakes, streams, and groundwater. Alternative water sources, such as seawater and wastewater, require a lot of energy and advanced technology to remove impurities and pathogens. These technologies are broadly viewed as either too expensive, risky, or unsanitary (Marlow et al 2015). For example, desalination satisfies only 1% of global water demand (Voutchkov 2016), and only 11 % of wastewater is reused (Jones et al 2021).

Despite the dominance of freshwater sources, these alternative water sources play a central role in certain specialized contexts. Desalination has become commonplace in middle-eastern countries that have large oil reserves, few freshwater sources, and large coastal populations (Jones et al 2019). Additionally, recycling of wastewater into drinking water has become technically feasible and socially acceptable in certain places such as Windhoek, Namibia. This became desirable because the city is located far from the ocean and has very scarce freshwater resources (Lafforgue 2016).

These alternative water sources are likely to become more relevant in more water-resource contexts in the future. Climate change is expected to alter the distribution of precipitation such that mega-droughts are likely in many regions across the world (Cook et al 2022). In contrast, seawater is highly abundant and thus is not vulnerable to changes in precipitation. Similarly, wastewater recycling allows small quantities of water to be reused multiple times, which effectively increases the water supply.

Water-resource issues are related to broader issues of sustainability. Resource extraction and pollution are human activities that impact the earth's natural systems including climate, ecology, freshwater, and nutrient cycles. These systems will likely destabilize unless resource extraction and pollution concentration limits are established at a global scale. These limits are collectively referred to as earth's system boundaries (ESBs). Research indicates that humanity has already crossed many of the ESBs including the ones for surface water and groundwater. Crossing these ESBs carries consequences such as ecosystem degradation and loss of human life, livelihoods, and access to basic needs (Rockström et al 2023).

ESBs are expected to place limits on economic growth because resource availability constrains industrial output. For instance, the index of industrial production (IIP) quantifies the gross output of the resource extraction, manufacturing, and energy sectors. A theoretical model of global population, food, resources, and pollution explores several scenarios for IIP change over time (Herrington 2021). This model, supported by empirical data, suggests that IIP will peak in the mid-twentieth century due in part to resource depletion and pollution accumulation.

Complex “green" technologies (e.g., desalination and renewable energy) may help overcome the limits that ESBs (e.g., water supply and carbon concentrations) impose on industrial production. Put another way, industrial output can be decoupled from ESBs (Vadén et al 2020). Indeed, technology often becomes more complex over time to overcome previous performance limitations. An allegorical example is the complexity of the modern jet engine. It evolved from the incremental additions of subsystems over time to regulate temperature, airflow, and combustion. These innovations accommodated progressively higher air pressures in airplane engines (Arthur 1993).

The allegorical jet engine example extends conceptually to environmental systems. Wastewater-treatment technologies were developed to lessen the impacts of urban growth on water pollution (Burian et al 2000). Controls in the power and automotive sector reduced the emissions of non-methane, volatile organic compounds by half between 1990 and 2010 (Xing et al 2013). In both cases, environmental innovations introduced complexity in the form of new subsystems to water and air ESBs, respectively. These innovations helped to overcome the waste assimilation limits associated with human activity while allowing economic growth to continue.

Despite these historical arguments, unproven and complex green technologies will likely pose challenges. There is concern whether green technologies can be developed quickly enough to mitigate the ESB impacts (Vadén et al 2020). For instance, existing desalination technologies bolster water supplies reducing stress on the surface and groundwater ESBs. However, truly green desalination technologies need innovative solutions to reduce the discharge of saline waste products to the environment and to transition to renewable energy resources (Gude 2016). Otherwise, saline pollution and CO2 emissions will continue to undermine the ecological and climate ESBs, respectively.

Consequentially, the decoupling concept has come under scrutiny in recent years. Alternatively, recent thinking asserts that the growth mindset of economic institutions is fundamentally incompatible with the ESB limits. In response, researchers have begun to envision how global society could thrive in the absence of economic growth. These alternative conceptions have been aptly called “degrowth" mindsets. Degrowth generally emphasizes social change over technological ones including improvements to socio-economic equity and demand reduction of natural resources (Kallis et al 2018; Kerschner et al 2018).

Regardless of one’s opinion of degrowth or decoupling, transitions to technology, social norms, or both are required to re-stabilize the ESBs. Research into this imperative has become known as the field of sustainability transitions (ST) and has an impressive body of literature (Markard et al 2012). In ST, technology (e.g., desalination) and societal values (e.g., water-demand reduction) can change simultaneously. This simultaneous consideration provides a multi-faceted approach for addressing sustainability challenges.

For this reason, the ST field provides the conceptual basis for our model in this paper. In Sect. 2, we present a literature review of the ST field and then explain in Sect. 3 how we incorporate ST into our novel modeling approach. In Sect. 4, the mathematical formulation is developed. In Sect. 5, this formulation is applied to a stylized sustainable water-resource transition with two lower-level players and an upper-level policymaker. This water-resources example serves as a counterpoint to the already extensive literature on energy transitions (Saraji and Streimikiene 2023). It also builds on recent operations research (OR) work in water systems where multiple decision-makers are explicitly modeled (Boyd et al 2023; Britz et al 2013; Kuhn et al 2016). Lastly, Sect. 6 presents our final conclusions to the paper.

2 Literature review

The ST field has many qualitative concepts that need more mathematical rigor (Safarzyńska et al 2012). Economists and complexity theorists have contributed some important mathematical details to ST related to innovation, economic growth, and the scale of cities (Arthur 1994; Bettencourt et al 2007). The field of OR has contributed mathematical models to inform decisions regarding sustainable technology investment, infrastructure provision, and technology diffusion (Brozynski and

Leibowicz 2020, 2022). As will be substantiated, our work is unique in that it integrates many of these modeling features together to support ST decisions related to energy and natural resource management.

Economic theory plays an important role in ST because it addresses issues of economic growth, technology, and the price stability of scarce resources. Classical economic theory explains price stability while subsequent economic theories link economic growth to technological innovation. Innovators of new technologies (e.g., personal computers) often disrupt the economic status quo and can capitalize on this disruption to gain an advantage over competitors (Arthur 1994). Economic growth is possible in such a dynamic setting because market incentives favor technologies that progressively extract more value from commodities (Romer 1990).

Classical economics often assumes diminishing returns such that the marginal value of a resource diminishes with increasing utilization (i.e., negative feedback). This leads to the conditions for an economic equilibrium (Arthur 1990). Equilibrium programming and game theory have been very successful in explaining how markets allocate scarce resources in sectors relevant to sustainability transitions such as energy (Gabriel et al 2012; Zhuang and Gabriel 2008; Hobbs 1999) and water (Boyd et al 2023; Britz et al 2013; Safari et al 2023). These equilibrium models have been subsumed into mathematical programs with equilibrium constraints (MPECs) for infrastructure investment (U-tapao et al 2016) and market regulation (Allen 2022).

In contrast, technology can become more valuable the more it is utilized. This phenomenon is known as increasing returns to scale (Arthur 1994). Firms that innovate early in a niche are more likely to increase the number of adopters of their technology relative to their competitors. The presence of adopters is likely to further adoption in a positive feedback loop (Arthur 1990). For example, early electric vehicle (EV) manufacturers are more likely to have charging infrastructure and thus are more likely to continue increasing the number of consumers.

Consequentially, an objectively inferior technology can become “locked in" because of early adoption (Arthur et al 1987). This effect is called path dependence and is often associated with the difficulty of sustainability transitions with technological components (e.g., renewable energy). For instance, non-renewable energy is likely to persist over renewable energy in places where fossil fuels are widely adopted for use in the power grid.

Population modeling plays an important role in ST because it can be used to examine the relationship between innovation, per-capita wealth, and city size. In Bettencourt et al (2007), the authors proposed the following power law model:

$$y = aN^b$$

where y represents a metric for wealth (e.g., GDP) or innovation (e.g., number of patents), N is population size, and a and b are the model's parameters. If $b > 1$, then the metric exhibits super-linear scaling with city population size. Similarly, if $b < 1$, then the scaling is sub-linear. These scaling possibilities reveal the costs and benefits of cities with progressively larger populations.

The results from this power law model indicate that metrics for wealth and innovation exhibit super-linear scaling. Specifically, metrics for infrastructure (e.g., miles of utility lines, number of EV charging stations) exhibit sub-linear scaling. Many metrics for social and environmental problems (e.g., inequality, pollution) also exhibit super-linear scaling. Theoretical interpretations of these power law models (Bettencourt 2013) suggest that the social, environmental, and resource costs of city scaling will outpace wealth and innovation in the long run.

Two qualitative but important ST concepts have been synthesized from historical case studies (Geels 2005; Markard et al 2012). First, a regime is defined as the dominant technologies, standards, and institutional structures for a particular society (e.g., freshwater supply). To give another example, non-renewable power generation is a regime that is preserved by physical infrastructure, profit motive, and political incentives alike. Second, a niche is defined as a unique context where novel approaches can be developed independent of the dominant regime (e.g., Windhoek, Namibia for wastewater reuse).

Another qualitative idea in ST explains how regimes, like freshwater treatment and delivery systems, can persist even when they become ineffective. For instance, over-extraction of freshwater may continue even in extreme droughts. One approach to remedying such a situation is known as strategic niche management (Kemp et al 1998). It seeks to transfer knowledge from a given niche and scale it up to redefine the capabilities of the regime. For instance, a desalination niche near the ocean could be scaled up to diversify freshwater supply regimes further inland.

The ST literature also calls for the use of several mathematical modeling frameworks to make these qualitative concepts more quantitative (Safarzyńska et al 2012). These frameworks include the following:

1. Mathematical models could clarify how sustainable technologies emerge and mature over time. For example, the US Military uses the technology readiness level (TRL) metric to track how a technology progresses from concept, to prototype, to final implementation (Eckhause et al 2009, 2012). Modeling these temporal aspects is called the *multi-phase* framework.
2. Mathematical models could clarify how infrastructure and new technologies evolve alongside one another (e.g., EVs and charging infrastructure). This modeling framework is called *co-evolution*.
3. *Social learning* could be modeled to characterize how agents learn from one another. For instance, how might a utility manager in a desalination niche transfer their knowledge to a utility manager in a freshwater supply regime who wants to augment this supply?
4. Lastly, mathematical models could clarify how niches fit within regimes, and how regimes fit into the surrounding landscape. For example, wastewater reuse innovations in Windhoek will not necessarily become commonplace elsewhere (e.g., surface water treatment regimes) unless the broader innovation landscape changes. Increased water scarcity due to climate change could be a potential driver for such changes. Modeling this nested structure is called the *multi-level* framework.

Recent work in OR model these ST frameworks to some degree. Brozynski and Leibowicz (2020) model the multi-phase dynamics of electric vehicle maturity (e.g., cost, vehicle range) using a Markov Decision Process (MDP). Boyd and Dumm (2019) modeled the co-evolution of sewer design and wastewater disposal practices. Lastly, Brozynski and Leibowicz (2022) create a hierarchy of policymakers, companies (e.g., EV developers), and consumers using a multi-level model. However, no OR work models either social learning or impacts to ESBs, such as the impact of withdrawals from surface water.

3 Contributions of this paper

Our paper uses these frameworks as inspiration for an OR model to answer the following research question: How does the uncertainty that is inherent to complex green technologies influence a policymaker's resource-management decisions? Organized by framework, we enumerate exactly how our work addresses this question:

1. The *multi-phase* framework is modeled using a two-stage, stochastic recourse problem (Birge and Louveaux 2011). In the first stage, research and development (R&D) is invested into a technology that offers the potential to unlock abundant resources (e.g., renewable energy, green desalination). However, the outcome of this R&D is uncertain. In the second stage, this uncertainty is revealed: the associated technology either matures or stagnates (i.e., in terms of cost).
2. Resources, especially water, are frequently shared among multiple players. Different players also have different R&D costs, but the investment of one player can lower the cost for others. For example, a desalination design for one region could be adapted for another at lower cost. This adaptive process is *social learning* and is modeled as a Generalized Nash Equilibrium Program (Harker 1991).
3. The *multi-level* framework is modeled using a bi-level optimization problem. This problem provides the main structure for resource management: Specifically, a policymaker oversees the management of an ESB, (e.g., climate, surface water, or ground water) and charges lower-level consumers for their resource consumption

4 Mathematical formulation

As shown in Fig. 1, we build on several sub-models to arrive at our final mathematical model. Subsection 4.1 introduces the first sub-model, which introduces multiple phases for which population and economic growth change over time. Subsection 4.2 introduces social learning to model how multiple decision-makers learn from each other's technology investment decisions. Lastly, Subsection 4.3 introduces the bi-level optimization problem to consider natural-resource management policies.

Different types of decision makers (i.e., players) are introduced in each sub-model. Each player is denoted as $p \in \mathcal{P}$, which represent land developers. These players could develop either urban or rural land and could be either municipal or private organizations. Sub-model 1 considers a single decision maker to start, thus

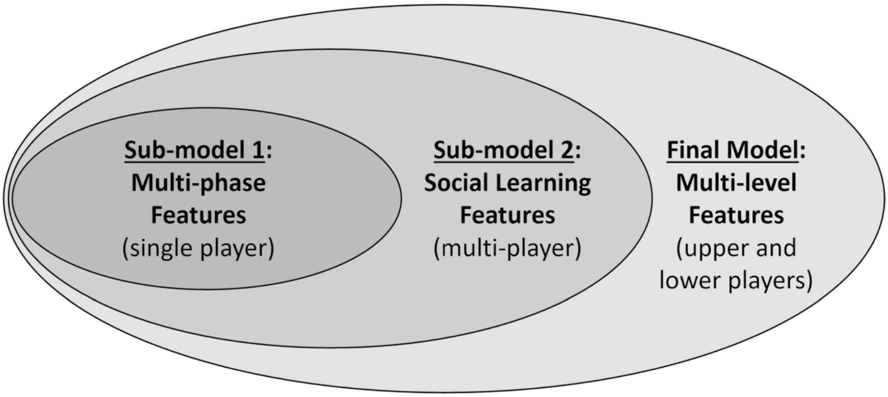


Fig. 1 Organization of the mathematical formulation by ST modeling features

$|\mathcal{P}| = 1$. Sub-model 2 considers multiple players, thus $|\mathcal{P}| > 1$. The Final Model refers to these multiple players collectively as lower-level players. The upper-level player is a policymaker (e.g., regional government), but is an implicit, un-indexed player.

The final, bi-level optimization model, which is a type of mixed-integer non-linear program (i.e., MPEC), is of the following general form:

$$\max_x f(x, y)$$

s.t.

$$(x, y) \in Z$$

$$y \in S(x)$$

where x represents resource-management decisions (e.g., taxes) and y represents how consumers decide to consume resources. The objective function f is a social welfare metric, such as the consumer surplus of water or energy. The first two sub-models can be understood as developing Z and $S(x)$, which models how consumers arrive at their decisions (e.g., water or energy usage). This nested structure makes the bi-level optimization problem difficult to solve (Jeroslow 1985; Hansen et al 1992).

The terms defined in this section, which include sets, variables, and parameters, are summarized in Appendix 7.1, 7.2 and 7.3, respectively. The formatting conventions applied to these terms distinguish their type. Upper-case Roman letters represent model variables, while lower-case Roman letters indicate model parameters and endogenous cost functions. Probability parameters are one exception to this lower-case convention; they are identified using the “$\mathbb{P}$" symbol. Lower-case λ and γ Greek letters represent Lagrange multipliers to their associated constraints. Superscripts distinguish similar terms (e.g., unit costs) from one another, while subscripts represent the indices of variables and parameters.

Further, variables that represent flow rates (e.g., fresh or desalinated water per unit time) take on representative average values within each planning period. This is done to make the model easier to interpret, without loss of generality. For instance, the amount of water that flows into a region over a five-year planning period would be large. Instead, monthly averages are assumed because this is a representative timescale for resource consumption (e.g., utility billing of energy or water).

4.1 Multi-phase model features

As mentioned in Sect. 3, each phase characterizes the research and development (R&D) status of a sustainability-related technology (e.g., desalination, clean energy). For instance, these phases could represent the prototype phase, the early adoption phase, the technological maturity phase, or the stagnation phase. Phase transitions are modeled with respect to discrete time periods t and $t + 1$ within a finite set $\mathcal{T}$. Each possible transition is assumed to be a discrete, uncertain scenario $s \in S$ to model the inherent complexities in the R&D process.

Technology-related risks are modeled using return on R&D investment (ROI). The R&D investor (i.e., Player p) is assumed to suffer a financial loss if the technology enters an undesirable phase (e.g., stagnation). These losses are assumed possible because R&D costs, c_p^{RD}, are incurred here and now (i.e., t), while the uncertain, ROI scenario s is resolved later in $t + 1$ with probability $\mathbb{P}(s)$. This limited time horizon is used because decision-makers in complex, engineering contexts often make preliminary decisions and refine them over time as new information becomes available (Devine et al 2016).

The R&D investment, c_p^{RD}, is assumed to be a fixed cost that is either forgone or paid according to the binary decision $X_p^{RD} \in \{0, 1\}$, respectively. For instance, this fixed cost could represent the cost to install a pilot wastewater reuse plant or a renewable-energy-powered desalination facility. Future research could make this R&D cost continuous where larger investments translate into a higher success probability $\mathbb{P}(s)$. Such decision-dependent probabilities are an example of endogenous uncertainty (Noyan et al 2022), which is a computationally challenging, emerging area of research.

The underlying probabilities $\mathbb{P}(s)$ of the scenarios are Bayesian in the sense that they are based on beliefs towards the technology. Subject matter experts are typically used to estimate the probability of a technology being successful. However, these probability estimates are still useful because they help decision-makers to hedge their technology investment decisions. Intuitively, we assume that these probabilities are related to the complexity of the technology.

With these model elements in mind, Sub-model 1 is formulated as a two-stage, stochastic optimization problem (Birge and Louveaux 2011) with a rolling horizon. The first and second stages represent t and $t + 1$, respectively. Information from investments in previous time periods become data for the current time period. The starting time period is called the “roll" and the collection of all $|\mathcal{T} - 1|$ rolls is the rolling horizon. Notationally, assume that the current roll r occurs at Time Period t where r is a shorthand for $t = r$.

Expression (1a) represents the objective function to be maximized in this two-stage optimization problem:

$$\max\left(1-\beta_p\right)\left(f_{p,r}^{FS}+d_{p,r+1}\sum_{s\in S}\mathbb{P}(s)f_{p,r+1,s}^{SS}\right)+\beta_p f_{p,s}^{rsk} \tag{1a}$$

where f^{FS} and f^{SS} represent generic first and second-stage payoff functions, respectively, and f_p^{rsk} is a risk measure across all s. The parameter $\beta_p \in [0,1]$ is a weighting term between payoff and risk. Additionally, the parameter $d_{p,r+1}$ is the discount rate between $t=r$ and $t=r+1$, The specific forms of f^{FS} and f^{SS} will be chosen to quantify the green technology's ROI across different s (i.e., phases). Similarly, the specific form of f^{rsk} will quantify the risk of technology stagnation across s.

The function f_p^{FS} includes the R&D investment cost $c_p^{RD}X_p^{RD}$ as well as several new terms:

$$f_{p,r}^{FS}=b_pN_{p,r}^{FS}-c_p^fF_{p,r}^{FS}-c_p^eE_{p,r}^{FS}-c_p^{RD}X_p^{RD} \tag{1b}$$

In the first stage (i.e. $t=r$), Player p's region has a population $N_{p,r}^{FS}$. Assume that each player derives a benefit b_p (e.g., tax revenues) for each member of the population. This population requires a flow of finite resource (e.g., surface, ground water ESBs), $F_{p,r}^{FS}$, which carries a cost c_p^f per unit (e.g., storage and pumping costs). Lastly, emigration from the region, $E_{p,t}^{FS}$, is possible, which carries a cost per capita of c_p^e.

This emigration cost represents the lost economies of scale from maintaining infrastructure such as roads and utilities that were designed for a larger population. For example, Detroit, Michigan, USA lost 25% of its population between 2000 and 2010 resulting in declining city services, higher infrastructure costs, and greater social inequality (Doucet and Smit 2016).

The second stage also has terms that have the same interpretation as the first stage as well as a new cost term:

$$f_{p,r+1,s}^{SS}=b_pN_{p,r+1,s}^{SS}-c_p^fF_{p,r+1,s}^{SS}-c_p^eE_{p,r+1,s}^{SS}-c_{p,s}^u(U_{p,s}) \tag{1c}$$

where $c_{p,s}^u(U_{p,s})$ represents the cost to provide a given flow of a practically unlimited resource, $U_{p,s}$, in scenario s. For example, $U_{p,s}$ could represent the flow rate of desalinated water while $c_{p,s}^u(U_{p,s})$ is the corresponding cost. This cost is integral to the ROI for the green technology, which results from the first-stage R&D investment.

The cost $c_{p,s}^u$ is assumed to be concave and increasing for all s. This reflects an intuitive assumption of endogenous technological learning: the marginal cost of new technologies often becomes cheaper with increasing utilization. Thus, the slope of the total cost decreases as $U_{p,s}$ increases. As will be shown, this technology's cost is modeled as a piece-wise linear function. Alternative conceptions of technological learning (Nemet 2006) are left for future research. We assume that U is implemented in $t=r+1$. For notational simplicity, the time index is dropped such that $U_{p,s,r+1}\equiv U_{p,s}$.

The realized R&D scenario carries risk if $c_{p,s}^u$ is too high to be economically viable. The decision-maker could overestimate resource availability and lack enough to cost-effectively maintain a locality's population. Because of its convexity, Conditional Value at Risk (CVaR) (Conejo et al 2010) is used to model this risk:

$$f_p^{rsk} = Z_p - \frac{1}{1-\alpha_p} \sum_{s \in S} \mathbb{P}(s) V_{p,s} \tag{1d}$$

where Z_p helps calculate the payoff at the (1 - α_p)-quantile of the net-benefit distribution and $V_{p,s}$ is net-benefit shortfall in scenario s with respect to this quantile. The term $\alpha_p \in [0, 1]$ is an arbitrary model parameter.

Sub-model 1 is non-linear due to the binary variable X_p^{RD} and the assumed piecewise structure of $c_{p,s}^u$. As will be shown, these non-linear terms make Sub-model 1 a mixed-integer linear program (MIP). This model structure has implications for most of the constraints that follow, (1e)–(1t). Specifically, each constraint has a Greek letter in parenthesis, which represent a shadow price. These prices are only valid for continuous-valued linear programs. However, in Section 4.2 onward, the non-linear, integer variables are either treated as exogenous parameters or represent the decisions of an upper-level policymaker.

Constraint (1e) calculates Z_p and $V_{p,s}$, for each Scenario s:

$$\begin{aligned} & Z_p - \Big(b_p N_{p,r}^{FS} - c_p^f F_{p,r}^{FS} - c_p^e E_{p,r}^{FS} - c_p^{RD} X_p^{RD} + \\ & d_{p,r+1} \Big(b_p N_{p,r+1,s}^{SS} - c_p^f F_{p,r+1,s}^{SS} - c_p^e E_{p,r+1,s}^{SS} - c_{p,s}^u (U_{p,s}) \Big) \Big) \le V_{p,s} \quad (\gamma_{p,s}^{rsk}) \quad \forall s \in S \end{aligned} \tag{1e}$$

This is Z_p minus the payoff (i.e., R&D ROI) in a given scenario. $V_{p,s}$ is also constrained to be non-negative for all scenarios s. Further, the $V_{p,s}$ variables are minimized in (1a). Therefore, $V_{p,s}$ is intuitively zero whenever a scenario's payoff is greater than the threshold payoff Z_p.

Objective Function (1a) is also subject to the constraints of population growth. We assume that an individual in $N_{p,r}^{FS}$ (or $N_{p,r+1,s}^{SS}$) nets on average g_p additional individuals via reproduction or invitation to the region during each Time Period t. The population thus grows exponentially from an initial population n_p^i via the difference equations $N_{p,r}^{FS} = g_p n_p^i$ and $N_{p,r+1,s}^{SS} = g_p N_{p,r}^{FS}$.

Unless resources are unlimited, exponential growth is not possible indefinitely. Thus, Player p may need to cap the accommodations made for future population growth. Although this capping mechanism is not explicitly modeled, it could represent the reduction of land zoned for new residents. We assume that prospective residents above this cap will locate elsewhere instead. Thus, (1f) and (1g) modify the difference equations in the previous paragraph reflecting the possibility of population caps:

$$N_{p,r}^{FS} \le g_p n_p^i \quad (\gamma_{p,r}^{max1}) \tag{1f}$$

$$N_{p,r+1,s}^{SS} \le g_p N_{p,r}^{FS} \quad (\gamma_{p,r+1,s}^{max2}) \quad \forall s \in S \tag{1g}$$

The population cap could be less than the current population if insufficient resources exist to sustain it. Such a case implies emigration, $E_{p,t}$, out of the region. The possibility of population loss can be expressed mathematically for both the first stage and second stage as follows:

$$N^{FS}_{p,r} \geq n^{i}_{p} - E^{FS}_{p,r} \quad (\gamma^{min1}_{p,r}) \tag{1h}$$

$$N^{SS}_{p,r+1,s} \geq N^{FS}_{p,r} - E^{SS}_{p,r+1,s} \quad (\gamma^{min2}_{p,r+1,s}) \quad \forall s \in S \tag{1i}$$

To quantify the notion of sufficient resources, the population in Player p's region demands a quantity q_p of resources (e.g., water) per capita. Supply of this finite resource in the first stage, $F^{FS}_{p,r}$ (e.g., groundwater), must be greater than or equal to the quantity demanded by the current population:

$$F^{FS}_{p,r} \geq q_p N^{FS}_{p,r} \quad (\gamma^{dem1}_{p,r}) \tag{1j}$$

Empirical research has validated the linear relationship in (1j) and (1k) between resource consumption and population levels for both household electrical and water consumption (Bettencourt et al 2007).

Resource consumption in the second stage is the same except that the resource pool is expanded to include the practically unlimited resource, $U_{p,s}$:

$$F^{SS}_{p,r+1,s} + U_{p,s} \geq q_p N^{SS}_{p,r+1,s} \quad (\gamma^{dem2}_{p,r+1,s}) \quad \forall s \in S \tag{1k}$$

The inclusion of $U_{p,s}$ in principle allows exponential growth to continue per (1g). However, the degree to which Player p uses $U_{p,s}$ is dependent on the cost $c^{u}_{p,s}$.

To quantify finite resources, a resource limit l constrains the supply of $F^{FS}_{p,r}$ unless a withdrawal $W^{FS}_{p,t}$ is made from a depletable reserve at time $t = r$. This limit is associated with an ESB (e.g., air, water, land, etc.). For instance, l could represent the amount of groundwater recharged by rainfall, while W could represent the extraction of groundwater reserves. For simplicity, we assume that withdrawal from the reserve does not occur any additional unit costs beyond c^{f}. This accounting of resource limits, supply, and withdrawals is stated mathematically as follows:

$$W^{FS}_{p,t} \geq F^{FS}_{p,r} - l \quad (\gamma^{w}_{p,r}) \tag{1l}$$

The variable $W^{FS}_{p,t}$ is not summed across multiple players because this sub-model assumes the players do not share resources.

Withdrawals can be used for supply until the reserve is completely depleted. We do not assume the reserve can be recharged because this paper is primarily focused on resource-depletion issues. The reserve available for supply is mathematically expressed as follows:

$$F^{FS}_{p,r} \leq l + a \quad (\gamma^{lim1}_{p,r}) \tag{1m}$$

where a is the initial accumulation at the beginning of the current roll.

In the second stage for each scenario, $F^{SS}_{p,r+1,s}$ can be as high as the limit l plus any reserves left over after the first stage withdrawal:

$$F^{SS}_{p,r+1,s} \leq l + a - W^{FS}_{p,t} \quad (\gamma^{lim2}_{p,r+1,s}) \quad \forall s \in S \tag{1n}$$

We do not explicitly represent the second stage withdrawals in the model for parsimony. Instead, we assume that a is updated in the two-stage problem for the next roll (i.e., $r = t + 1$) based on the slack in (1n) for the realized scenario.

Additional constraints are needed to define the piece-wise cost function c^u_p and its argument $U_{p,s}$. A combination of linear and binary variables are used to activate the piece-wise segments of c^u_p. These are modeled as two-segment functions as described in Winston (2004). Examples are plotted in the results section (Section 5.2) as red lines on Figures 3b and 4b. The three break-points are denoted as A, B, and C. The domain values at these break-points are 0, i^B_p, and i^C_p, respectively, while the function values are c^A_p, $c^B_{p,s}$, and $c^C_{p,s}$, respectively. The weighting terms $Y^A_{p,s}$, $Y^B_{p,s}$, $Y^C_{p,s} \in [0, 1]$ quantify how close the domain value $U_{p,s}$ is to the corresponding break-point.

Mathematically, (1o) - (1x) model these piece-wise aspects. Expressions (1o) and (1p) model the function $c^u_{p,s}$ and its argument $U_{p,s}$, respectively. These expressions are defined in terms of the breakpoints A, B, C and the weighting terms between each breakpoint. Expressions (1q) - (1v) control the activation of each piece-wise segment. In these expressions, the binary terms $X^A_{p,s}$ and $X^B_{p,s}$ activate the piece-wise segments between A and B and B and C, respectively (Winston 2004).

$$c^u_{p,s}(U_{p,s}) = c^A_p Y^A_{p,s} + c^B_{p,s} Y^B_{p,s} + c^C_{p,s} Y^C_{p,s} - c^{RD}_p X^{RD}_p \quad (\lambda^{pw}_{p,s}) \quad \forall s \in S \tag{1o}$$

$$U_{p,s} = i^A_p Y^A_{p,s} + i^B_p Y^B_{p,s} + i^C_p Y^C_{p,s} \quad (\lambda^u_{p,s}) \quad \forall s \in S \tag{1p}$$

$$Y^A_{p,s} \leq X^A_{p,s} \quad (\gamma^A_{p,s}) \quad \forall s \in S \tag{1q}$$

$$Y^B_{p,s} \leq X^A_{p,s} + X^B_{p,s} \quad (\gamma^B_{p,s}) \quad \forall s \in S \tag{1r}$$

$$Y^C_{p,s} \leq X^B_{p,s} \quad (\lambda^C_{p,s}) \quad \forall s \in S \tag{1s}$$

$$Y^A_{p,s} + Y^B_{p,s} + Y^C_{p,s} = X^{RD}_p \quad (\lambda^{rd}_{p,s}) \quad \forall s \in S \tag{1t}$$

$$X^{RD}_p \in \{0, 1\} \tag{1u}$$

$$X^A_{p,s}, X^B_{p,s} \in \{0, 1\} \quad , \quad X^A_{p,s} + X^B_{p,s} = X^{RD}_p \quad \forall s \in S \tag{1v}$$

$$Y^A_{p,s}, Y^B_{p,s}, Y^C_{p,s} \geq 0 \quad \forall s \in S \tag{1w}$$

$$c^{u}_{p,s}(U_{p,s}) \quad free \quad \forall s \in S \tag{1x}$$

The last two constraints establish which of the model variables are nonnegative:

$$N^{FS}_{p,t}, E^{FS}_{p,t}, F^{FS}_{p,t}, W^{FS}_{p,t} \geq 0 \quad , \quad t = r \tag{1y}$$

$$N^{SS}_{p,t,s}, E^{SS}_{p,t,s}, F^{SS}_{p,t,s}, U_{p,s}, V_{p,s} \geq 0 \quad \forall s \in S \quad , \quad t = r + 1 \tag{1z}$$

4.2 Social learning model features

Sub-model 2 considers multiple players, P, who make decisions that are consistent with Sub-model 1. Each player $p \in P$ is still a local developer or a local government within a broader region. The region could represent a watershed with a shared surface or groundwater system. Each locality could have a geographical niche for alternative water technologies. For example, a coastal city might be more likely to implement desalination compared to an inland city. Similarly, a farming community on an urban fringe might use recycled wastewater for irrigation.

As will be explained, some adjustments and extensions are made to Sub-model 1 to account for the interactions among the players. Only these changes are stated in this section to avoid repeating many unchanged expressions. However, the full formulation for Sub-model 2 is provided in the supplementary information associated with this paper. Players share finite resources with each other as one form of interaction. However, social learning is the primary interaction considered per the ST frameworks. This framework accounts for how players learn from the R&D investments of others.

Players can also learn from their own R&D investments over multiple rolls of a rolling-horizon model (e.g., Sub-model 1). Mathematically, the Bayesian probabilities $\mathbb{P}(s)$ can be updated between each roll using a learning model (e.g., Bayes' Theorem). This approach is called endogenous learning and has been applied to decision-making in energy markets (Devine et al 2016). The details of such an algorithm are excluded from the current study and are left to future research.

4.2.1 Formulation for each player

The formulation for each player in Sub-model 2 uses all the expressions from Sub-model 1 with two changes, which account for resource sharing among the players. Throughout this subsection, a specific player, denoted p, is considered. Firstly, Expressions (2a) and (2b) replace Expressions (1m) and (1n), respectively:

$$\sum_{p' \in P} F^{FS}_{p',r} \leq l + a \quad (\gamma^{lim1}_{p,r}) \tag{2a}$$

$$\sum_{p' \in P} F^{SS}_{p',r+1,s} \leq l + a - W^{FS}_{r} \quad (\gamma^{lim2}_{p,r+1,s}) \quad \forall s \in S \tag{2b}$$

Secondly, Expression (11) is removed because a specific player can no longer completely control the first-stage withdrawal, W_r^{FS}. As will be shown, this withdrawal is now a consequence of all players consuming beyond the sustainable limit, l. These two adjustments make Sub-model 2 a type of Generalized Nash Equilibrium Model because the player's can now affect the constraint set (i.e., resource availability) of others (Harker 1991).

The players' optimization problems, which incorporate these two adjustments into Sub-model 1, are solved simultaneously. To achieve this, each player's optimization problem is re-expressed as the Karush-Kuhn Tucker (KKT) conditions (Gabriel et al 2012). The KKT conditions are necessary and sufficient for linear programs (LPs), but Sub-model 1 is an example of a MIP. To represent Sub-model 2 as an LP, the binary "X" variables are considered exogenous decisions eventually to be determined by an upper-level player. These binary variables could be interpreted as government funded research.

Within this LP context, the KKT conditions represent the three characteristics of optimal LP solutions: those that satisfy primal feasibility, dual feasibility, and complementary slackness Winston (2004). Henceforth, the KKT conditions are sometimes described in terms of these characteristics as KKT^{prim}, KKT^{dual}, and KKT^{comp}, respectively. The KKT conditions for Sub-model 2 are provided in the supplementary material of this paper, and a description is added to relate each KKT condition to these LP characteristics.

4.2.2 System constraints

In addition to the KKT conditions, other system constraints are included outside of each player's optimization problem. These constraints provide additional details into how the players interact. In this sub-model, the system constraints specify (1) withdrawals beyond the sustainable limit, (2) the distribution of finite resources among the players, and (3) the dynamics of social learning. In related work, system constraints are often related to market-clearing conditions (Gabriel et al 2012; Boyd et al 2023).

The first system constraint determines the first-stage withdrawals, W_r^{FS}, as a consequence of the joint, finite-resource consumption of all the players.

$$0 \leq W_r^{FS} + l - \sum_{p \in P} F_{p,r}^{FS} \perp W_r^{FS} \geq 0 \tag{2c}$$

In words, we assume that W_r^{FS} is zero unless the finite resource consumed across all players exceeds the sustainable limit l. The "$\perp$" operator is shorthand for the product of two variables (e.g., a,b) to be equal to zero (i.e., $a \perp b$). This is also called a complementarity relationship. Thus, in (2c), $W_r^{FS}(W_r^{FS} + l - \sum_{p \in P} F_{p,r}^{FS}) = 0$.

The second set of system constraints characterize how the players access the shared resources as described in (2a) and (2b). While the players share these same constraints, their shadow prices (i.e., "γ" terms) for these constraints could be different. To see this, (2a) and (2b) are summed over the index p', while the shadow prices are indexed by p. In our water example, different shadow prices can result from differing costs to pump groundwater.

To model this mathematically, each shadow price is assumed to be a weighted sum of the other players' shadow prices:

$$\gamma_{p,r}^{lim1} = \sum_{p' \neq p} v_{p,p'}^{FS} \gamma_{p',r}^{lim1} \quad , \quad p < |P| \tag{2d}$$

$$\gamma_{p,r+1,s}^{lim2} = \sum_{p' \neq p} v_{p,p',s}^{SS} \gamma_{p'r+1,s}^{lim2} \quad , \quad p < |P| \quad , \quad s \in S \tag{2e}$$

where the "v" terms are non-negative parameters representing these weights. Both (2d) and (2e) require only $|P| - 1$ equations each to make Sub-model 2 a square system of equations. Additionally, (2d) and (2e) only influence the output of Sub-model 2 if at least one expression in (2a) are (2b) are binding, respectively. Otherwise, the shadow prices associated with (2a) and (2b) will equal zero per the complementary slackness condition in linear programming.

The third set of system constraints model social learning. In a qualitative sense, social learning reflects how the one player can learn and/or benefit from the technological investments of other players. For example, a desalination design approach from one location could be repurposed for use in another location saving design costs. Players could also pay to connect to an existing desalination network instead of building a separate facility. Another conception of social learning could be correcting the failures of another player's previous R&D efforts.

These social learning possibilities are relevant because the players may have heterogeneous costs for sustainable technologies. For instance, solar power technologies are likely to be initially most cost-effective in niches that are very sunny and dry. Similarly, wastewater recycling is likely to first emerge near sites with high treatment standards. However innovation in these niches can transfer into the prevailing regimes in most other settings, such as power generation derived from fossil fuels and surface-water depletion.

Social learning is modeled as systematic reductions to the costs at the piece-wise linear break-points. The costs without any social learning at Break-points A and B are $c_p^{A'}$ and $c_{p,s}^{B'}$, respectively. This reflects no technological investment from the other players. In contrast, Δc_p^A and $\Delta c_{p,s}^B$ represent the maximum decrease of the break-point function values if other players (i.e., $p' \in \{\mathcal{P} : p^{'} \neq p\}$) invest. The impact of one player's investment on that of another is given as $w_{p',p}$. As in Sub-model 1, the technology is considered mature at a constant cost $c_{p,1}^m$ beyond break-point B.

Social learning represents a band of possible costs, which are shown as the dark gray region on Figures 3b and 4b of Sect. 5.2. The top of this dark gray region represents the case when no other players invest. The bottom of the region represents the case when all other players invest as much as possible. The intermediate region represents all other cases where some players invest at different levels.

These social learning relationships are summarized mathematically as the following system constraints:

$$c_p^{RD} = c_p^A = c_p^{A'} - \Delta c_p^A \sum_{p' \in \{\mathcal{P}:p' \neq p\}} w_{p,p'} X_{p'}^{RD} \tag{2f}$$

$$c_{p,s}^B = c_{p,s}^{B'} - \Delta c_{p,s}^B \sum_{p' \in \{\mathcal{P}:p' \neq p\}} w_{p,p'} (Y_{p',s}^B + Y_{p',s}^C) \quad \forall s \in S \tag{2g}$$

$$c_{p,s}^C = c_{p,s}^B + c_{p,s}^m (i_p^C - i_p^B) \quad \forall s \in S \tag{2h}$$

These system constraints specify how c_p^A, $c_{p,s}^B$, and $c_{p,s}^C$ show up as exogenous data for each player. The piece-wise curves start at c_p^A, which also represents the first-stage cost c_p^{RD}. In (2f), the *fixed* R&D decisions of other players, $X_{p'}^{RD}$, decrease the *fixed* costs, c_p^A, for Player p. Similarly, other players' *continuous* investment decisions (i.e., $U_{p',s}$) decrease the *continuous* investment costs of Player p. The sum $Y_{p',s}^B + Y_{p',s}^C$ is a normalized indicator of this continuous investment level in (2g). This sum increases from zero to one as $U_{p',s}$ approaches $i_{p'}^B$.

In summary, Sub-model 2 consists of the KKT conditions from multiple players' optimization problems and system constraints. These constraints govern the interactions among the players. The KKT conditions for each player are derived from Sub-model 1 except that (1) Expressions (1m) and (1n) are replaced with (2a) and (2b), respectively. (2) Expression (1l) is omitted, and lastly, (3) the binary decisions are taken as data. The system constraints consist of (2c)–(2h). Sub-model 2 is solved as a mixed-complementary problem (MCP) (Gabriel et al 2012). For completeness, the full mathematical details of Sub-model 2, including the KKT conditions, are provided in the supplementary information associated with this paper.

4.3 Multi-level model features

Multi-level features are added to Sub-model 2 to form the full bi-level optimization model. Our bi-level optimization problem adds a single, upper-level, regional policymaker to manage resources for the lower-level players. This upper-level player could be a state or national government who is responsible for balancing economic benefits in Sub-models 1–2 with environmental damage.

In this bi-level optimization problem, environmental damage is a function of the withdrawals beyond the sustainable limit. In the water-resources context, saltwater intrusion from the over-extraction of groundwater could cause environmental damage. In the energy context, this damage could be climate-induced, biodiversity loss from the over-extraction and burning of fossil fuels. One approach to internalizing such damages could involve estimating the costs to restore damaged ecosystems.

In mathematical terms, let $f_s^{dmg}(W_r^{FS} + W_{r+1,s}^{SS})$ represent the function to estimate environmental damage in Scenario s. This damage is a function of the sum of withdrawals across Time Period $t = r$ and $t = r + 1$. The withdrawal in Time Period $t = r$ is the same as W_r^{FS} in Sub-model 2. The withdrawal in Time Period $t = r + 1$ is in the second-stage and thus is dependent on Scenario s. Accordingly, the second-stage withdrawal is denoted as $W^{SS}_{r+1,s}$.

One policy instrument for the upper-level player is a set of tariffs that are levied against Player p's consumption of the finite resource. These tariffs are indexed in the usual way for first and second stage variables: $T^{FS}_{p,r}$ and $T^{SS}_{p,r+1,s}$. The lower-level players perceive these tariffs as increased unit costs for the consumption of the finite resource. Thus, this perception discourages low-value uses of the finite resource. Additionally, these tariffs also serve as a potential revenue source for funding R&D investments.

Leveraging revenue from tariffs, the second instrument is a set of subsidies that are used to reimburse the lower-level players for investments in the unlimited resource. These reimbursements are player and scenario dependent and are modeled as cost reductions at point B and C of the piecewise cost curves. These reimbursements are denoted as: $R^{B}_{p,s}$ and $R^{C}_{p,s}$, accordingly.

As alluded to in Sub-model 1, the policymaker also decides the binary variables related to technology investment. These decisions can be interpreted as government-funded research for the green technology. Mathematically, having $X^{RD}_{p} = 1$ enables Player p to continue to invest and commercialize this technology. Furthermore, for a given Scenario s, having $X^{B}_{p,s} = 1$ indicates that the policymaker develops capacity for i^{B}_{p} units of $U_{p,s}$. This capacity installation could represent the government building a desalination facility and then transferring the facility's operation to a municipal government.

Program (3) formalizes the bi-level model. The terms without subscripts are vectors that contain all the related set-indexed variables. In (3a), the policymaker seeks to maximize the sum of the lower-level objective functions (i.e., f^{ll}_{p} as Expression (3b)) minus environmental damage. This objective function captures social and environmental trade-offs. The lower-level players' KKT conditions are Expressions (3c), (3d), and (3e), which enforce primal feasibility, dual feasibility, and complementary slackness for LPs, respectively. Expressions (3f)–(3q) represent constraints with variables that the lower-level players do not directly control.

$$\max \sum_{p\in P} f^{ll}_{p} - \phi \mathbb{E}_{s\in S}\{f^{dmg}_{s}(W^{FS}_{r} + W^{SS}_{r+1,s})\} \tag{3a}$$

s.t.

$$\begin{aligned} f^{ll}_{p} = \Big(1-\beta_p\Big)\Big(& b_p N^{FS}_{p,r} - (c^{f}_{p} + T^{FS}_{p,r})F^{FS}_{p,r} - c^{e}_{p}E^{FS}_{p,r} - c^{RD}_{p}X^{RD}_{p} \\ &+ d_{p,r+1}\sum_{s\in S}\mathbb{P}(s)\Big(b_p N^{SS}_{p,r+1,s} - (c^{f}_{p} + T^{SS}_{p,r+1,s})F^{SS}_{p,r+1,s} - c^{e}_{p}E^{SS}_{p,r+1,s} - c^{u}_{p,s}(U_{p,s})\Big)\Big) \\ &+ \beta_p\left(Z_p - \frac{1}{1-\alpha_p}\sum_{s\in S}\mathbb{P}(s)V_{p,s}\right) \end{aligned} \tag{3b}$$

$$\begin{aligned}&\{N^{FS}, N^{SS}, E^{FS}, E^{SS}, F^{FS}, F^{SS},\\&U, V, Z, Y^{A}, Y^{B}, Y^{C}, c^{u}\} \in KKT^{prim}(T^{FS}, T^{SS}, X^{RD}, X^{A}, X^{B})\end{aligned} \tag{3c}$$

$$\begin{aligned}&\{\gamma^{rsk}, \gamma^{dem1}, \gamma^{dem2}, \gamma^{lim1}, \gamma^{lim2}, \gamma^{max1}, \gamma^{max2},\\&\gamma^{min1}, \gamma^{min2}, \lambda^{pw}, \lambda^{u}, \gamma^{A}, \gamma^{B}, \gamma^{C}, \lambda^{rd}\} \in KKT^{dual}(T^{FS}, T^{SS})\end{aligned} \tag{3d}$$

$$\begin{aligned}&\{N^{FS}, N^{SS}, E^{FS}, E^{SS}, F^{FS}, F^{SS}, U, V, Y^{A}, Y^{B}, Y^{C},\\&\gamma^{rsk}, \gamma^{dem1}, \gamma^{dem2}, \gamma^{lim1}, \gamma^{lim2}, \gamma^{max1}, \gamma^{max2},\\&\gamma^{min1}, \gamma^{min2}, \gamma^{A}, \gamma^{B}, \gamma^{C}\} \in KKT^{comp}\end{aligned} \tag{3e}$$

$$R^{B}_{p,s}(Y^{B}_{p,s} + Y^{C}_{p,s}) + R^{C}_{p,s}Y^{C}_{p,s} \leq T^{FS}_{p,r}F^{FS}_{p,r} + T^{SS}_{p,r+1,s}F^{SS}_{p,r+1,s} \quad \forall s \in S \tag{3f}$$

$$\sum_{p \in P} F^{SS}_{p,r+1,s} - l \leq W^{SS}_{r+1,s} \quad \forall s \in S \tag{3g}$$

$$0 \leq W^{FS}_{r} + l - \sum_{p \in P} F^{FS}_{p,r} \perp W^{FS}_{r} \geq 0 \tag{3h}$$

$$\gamma^{lim1}_{p,r} = \sum_{p' \neq p} v^{FS}_{p,p'}\gamma^{lim1}_{p',r} \quad , \quad p < |P| \tag{3i}$$

$$\gamma^{lim2}_{p,r+1,s} = \sum_{p' \neq p} v^{SS}_{p,p',s}\gamma^{lim2}_{p' r+1,s} \quad , \quad p < |P| \quad , \quad s \in S \tag{3j}$$

$$X^{RD}_{p} \in \{0, 1\} \tag{3k}$$

$$X^{A}_{p,s}, X^{B}_{p,s} \in \{0, 1\} \quad \forall s \in S \tag{3l}$$

$$X^{A}_{p,s} + X^{B}_{p,s} = X^{RD}_{p} \quad \forall (p, s) \in P \times S \tag{3m}$$

$$c^{RD}_{p} = c^{A}_{p} = c^{A'}_{p} - \Delta c^{A}_{p} \sum_{p' \in \{\mathcal{P}: p' \neq p\}} w_{p,p'} X^{RD}_{p'} \tag{3n}$$

$$c^{B}_{p,s} = c^{B'}_{p,s} - \Delta c^{B}_{p,s} \sum_{p' \in \{\mathcal{P}: p' \neq p\}} w_{p,p'}(Y^{B}_{p',s} + Y^{C}_{p',s}) - R^{B}_{p,s} \tag{3o}$$

$$c^{C}_{p,s} = c^{B}_{p,s} + c^{m}_{p,s}(i^{C}_{p} - i^{B}_{p}) - R^{C}_{p,s} \tag{3p}$$

$$T^{FS}, T^{SS}, R^{B}, R^{C} \geq 0 \tag{3q}$$

The upper-level objective function (3a) relies on a weighted sum involving both the social benefits and the environmental damage. The social benefits are simply the sum across all the lower-level players' objective functions. These objective functions are the same as those in Sub-models 1 and 2 except for the introduction of tariffs, T^{FS} and T^{SS}. Environmental damage is calculated as an expected value across scenarios, and its weight relative to social benefits in the upper level objective function is specified with a new parameter, ϕ.

The KKT conditions for this final model are identical to those used in Sub-model 2 with one exception. Tariffs (i.e., T^{FS}, T^{SS}) enter into the primal and dual feasibility KKT conditions (3c) and (3d) because they are part of (3b). These additions do not change the linear nature of the problem. Thus, these KKT conditions are still necessary and sufficient conditions for optimal solutions to the lower-level players' optimization problems. These updated KKT conditions are included in the supplementary material for this paper.

Constraints (3f) and (3g) are new expressions relative to the previous sub-models. Constraint (3f) states that the reimbursement given to the lower-level players is less than or equal to the revenue collected from tariffs. Using the same mathematical approach as Expression (1o), the variables R^B and R^C are multiplied by the weighting variables Y^B and Y^C. These products calculate the actual value of the reimbursement along the piecewise curve. Constraint (3g) states that second stage withdrawals must occur whenever the second-stage resource consumption exceeds the sustainable limit.

Lastly, Constraints (3h) - (3p) are the remaining system constraints carried over from Sub-model 2. These constraints are unchanged except for an additional term added to both Constraints (3o) and (3p). Specifically, the unlimited-resource costs at breakpoints B and C are adjusted to incorporate the reimbursements from the upper-level player.

5 Water resources application

5.1 Overview

Water-Resource transitions parallel energy systems, the latter having received a lot of attention in the literature (Chang et al 2021). For instance, both systems have a limited quantity of economically accessible resources (e.g., fossil fuels, freshwater). Furthermore, these systems have additional resources that are very abundant but are difficult to access. In energy systems, these are renewable energy sources such as wind and solar. In water systems, the equivalent involves water treatment of alternative water sources such as saltwater or wastewater.

The illustrative results in this section focus on desalinated seawater because the scale of its future implementation is a major unknown in water policy. Desalination is an example of a large, capital improvement that often generates a large quantity of water supply. In this sense, desalination is similar to other physical infrastructure developed widely in the twentieth century such as dams, reservoirs, and large water treatment facilities. However, the continuation of water extraction is not likely to be sustainable in the long term (Gleick 2003).

Other water systems place less emphasis on physical capital and more emphasis on behavioral changes (i.e., social capital). For example, home retrofits can increase the efficiency of water fixtures or enable wastewater to be recycled for gardening. Similarly, market-based approaches like dynamic water pricing can be used to regulate water demand. Solutions that emphasize these behavioral changes over “hard" infrastructure investment are called “soft path" solutions (Gleick 2003).

Both hard and soft infrastructure need to be considered together in water policies. Two case studies investigated how the premature adoption of water desalination hindered the adoption of alternatives like wastewater reuse (Fuenfschilling and Truffer 2016; Miörner et al 2022). On the other hand, a degree of abundance is necessary to create the conditions for intermediate scarcity, which is necessary for soft solutions to actually work (Kuhn et al 2016). Desalination can provide this water abundance in some settings where it would otherwise be lacking.

Desalination occupies a water-supply niche in high-income, water-scarce countries, but obstacles still exist for larger-scale deployment (Jones et al 2019). First, removing salt from water is very energy-intensive. Second, the desalination process yields hyper-concentrated “brine", which requires environmentally responsible disposal. The management of desalination brine is part of a broader set of challenges with wastewater resource recovery in general (Palmeros Parada et al 2022). Thus, in most cases, freshwater treatment and distribution comprises the prevailing regime for water supply.

Within this context, these illustrative results use the program of Sect. 4 to model the role of desalination in sustainability transitions. Table 1 provides an interpretation of the abstract model terminology as applied to these results. The subscripts for these terms are dropped for parsimony. In general, the model terms that are related to the finite resource pertain to traditional freshwater sources. These could include rivers, lakes, and groundwater. In contrast, the terms that are related to the alternative, unlimited resource pertain to desalination.

The stylized input data used to produce these illustrative results is provided in the supplementary information associated with this paper. This data specifies the environmental damage function and the characteristics of the players. A general overview of the data is described presently.

Environmental damage from the over-extraction of water is assumed to grow rapidly as groundwater and surface water reserves are depleted (i.e., W^{FS}, W^{SS}). For simplicity, this assumption is modeled using $f_s^{dmg} = (W_r^{FS} + W_{r+1,s}^{SS})^2$. For the weighting value, $\phi = 0.1$ is used. As will be shown in subsequent computational experiments, this value for ϕ illustrates plausible trade-offs between environmental damage and social benefits.

The players are identical in terms of the input data with several exceptions. First, Player 2 has lower technology investment costs than Player 1. In the desalination example, this could mean that Player 2 is closer to the ocean. Second,

Table 1 Interpretation of our ST model for water-resource systems

Term (s)[a]	General description	Water-resources interpretation
$t = r$, $t = r + 1$ (SE)	time periods within the rolling-horizon	Interval between developments of water supply master plans (e.g., 5–10 years)
$s = 1$ (SE)	Transition scenario where the technology improves	R&D yields promising technology to decrease brine management cost
$s = 2$ (SE)	Scenario where technology stagnates (i.e., no transition)	R&D does not yield promising brine management technology
l (PR)	Quantity of the finite resource	Replinishable supply of fresh water sources (e.g., rivers, lakes, groundwater)
a (PR)	Initial accumulation of the finite resource at the start of the rolling-horizon	Non-replinishable supply of fresh water sources (e.g., deep groundwater reserves)
q (PR)	Resource required per unit of population supported	Water-use per capita
c^f (PR)	Cost of accessing the finite resource	Marginal cost to treat and distribute fresh water sources
F^{FS}, F^{SS} (MV)	Finite resources consumed across all players in the first and second stages, respectively	Average monthly fresh water withdrawals in the current and next planning period, respectively
U (MV)	The alternative, unlimited resource that is harnessed using technology	Desalinated seawater
c^A (MV)	Cost at the first piece-wise point	Fixed cost to install desalination facilities and R&D for brine management
c^B (MV)	Cost at the second piece-wise point	Minimum investment required to discover cheaper brine management alternatives
c^C (MV)	Cost at the third piece-wise point	Maximum desalination investment possible given limitations to brine discharge
w (PR)	Weight relating the impact of one player's technology investment to another	Making minor adjustments to existing desalination designs, inland communities connecting pipelines to a desalination network
c^m (PR)	Marginal cost of the technology after reaching maturity	Marginal cost of mature desalination technology

[a] The letters in parenthesis indicate the term's type. SE: set element, PR: parameter, MV: model variable

Player 1's desalination costs can be decreased via social learning from Player 2's investments, but the reverse is not true. Lastly, Player 1 is assumed to have a slightly smaller shadow price for the finite resource in the first stage (i.e., $v^{FS}_{1,2} = 0.5$). In the water context, this could mean that Player 1's distance from the ocean could place it upstream on a river relative to Player 2.

For instance, suppose that Player 2's desalination investments lower the cost of membrane filtration technology to treat sea water. Player 1 is located farther inland and can't directly apply the technology in the same way. However, suppose Player 1 can adapt the cheaper membrane filtration technology for another purpose. For example, cheaper membrane-treatment-technologies could be used to treat brackish water (i.e., less salty) water to drinking-water standards.

Table 2 Summary of computational experiments

	Adjustments to program 3	Upper bound, T^{FS}, T^{SS}	Upper bound, R^{B}, R^{C}	Purpose
Centralized	Dual feasibility (3d) and complementary slackness (3e) constraints removed	N/A	N/A	Justify bi-level structure of (3)
Bi-level, Full	None	M	M	Basis for comparison across experiments
Bi-level, No Subsidy	None	M	0	Illustrate the role of subsidies
Bi-level, No Tariff	None	0	0	Illustrate tariffs as scarcity pricing

This illustrative model was solved as a specific instance of the bi-level optimization problem (Program 3). It was compiled using Pyomo: an open source, Python-based modeling language.[1] Within Pyomo, this model is solved to global optimality as a non-convex, mixed integer quadratically constrained program (MIQCP) using the Gurobi solver.[2] Two separate model runs were performed for $\beta_1 = \beta_2 \in \{0.1, 0.5\}$ to see how the results were sensitive to varying degrees of risk aversion by the two players.

In addition to the two Program 3 model runs, three additional computational experiments are performed for both of the two risk-parameter levels (for 2+6=8 experiments total). These experiments are variations of Program 3, which are designed to justify the upper-level player's use of tariffs and subsidies to mediate the lower-level player's water-usage decisions. The remainder of this section is dedicated to describing these experiments, and a high-level summary is provided in Table 2.

The values of M were chosen in an iterative manner. Each experiment was rerun with progressively larger values of M until the upper bounds were no longer binding constraints. The specific values of M obtained for each experiment are provided as supplementary information associated with this paper.

The Centralized Computational Experiments identify conflicts between the upper-level and lower-level players objectives to justify the complexity of the bi-level structure of Program (3). These experiments relax the optimality requirement for the lower-level players (i.e., the high-point relaxation), which place both the upper and lower variables under "centralized" control of the policymaker. In mathematical terms, the constraints for dual feasibility (3d) and complementary slackness (3e) are removed to achieve this control.

The Bi-level, Full Computational Experiments simply use Program (3) without any adjustments or bounds on the upper-level variables (i.e., the full bi-level model).

1 www.pyomo.org/
2 www.gurobi.com.

Table 3 Metrics used to analyze illustrative model results

Metric	Mathematical definition
Objective function value	Expression (3a)
Expected Net Economic Growth	$d_{p,r+1} \sum_{s\in S} \mathbb{P}(s)\Big(b_p N^{SS}_{p,r+1,s} - (c^f_p + T^{SS}_{p,r+1,s})F^{SS}_{p,r+1,s} - c^e_p E^{SS}_{p,r+1,s} - c^u_{p,s}(U_{p,s}) \Big)$ $-(b_p - c^f_p q_p) n^i_{r-1,p}$
Expected inequality	$\max_{(p,p')\in P\times P} \lvert f^{ll}_p - f^{ll}_{p'} \rvert$
Expected environmental damage	$\phi \sum_{s\in S} \mathbb{P}(s)(W^{FS}_r + W^{SS}_{r+1,s})^2$
Expected tariff charges	$\sum_{s\in S} \mathbb{P}(s)(T^{FS}_{p,r} F^{FS}_{p,r} + T^{SS}_{p,r+1,s} F^{SS}_{p,r+1,s})$

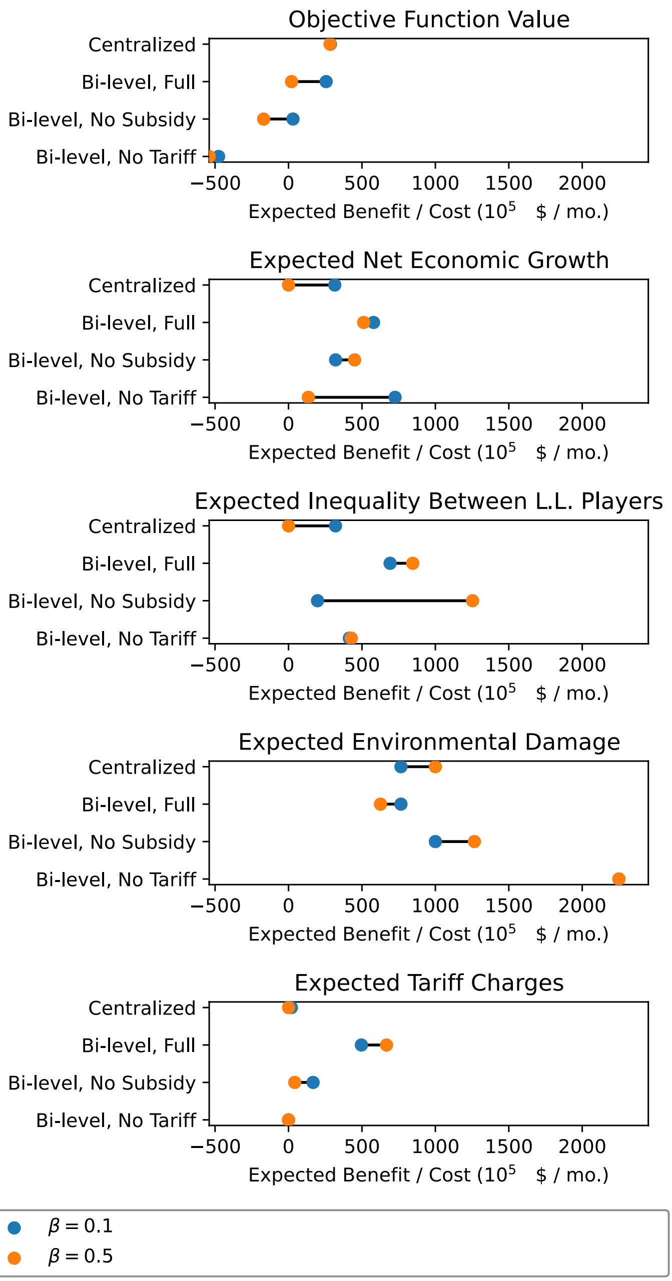
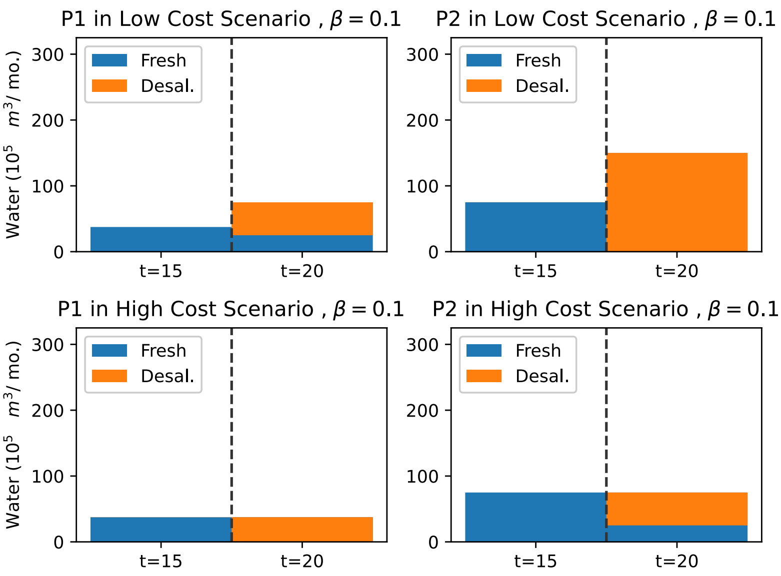
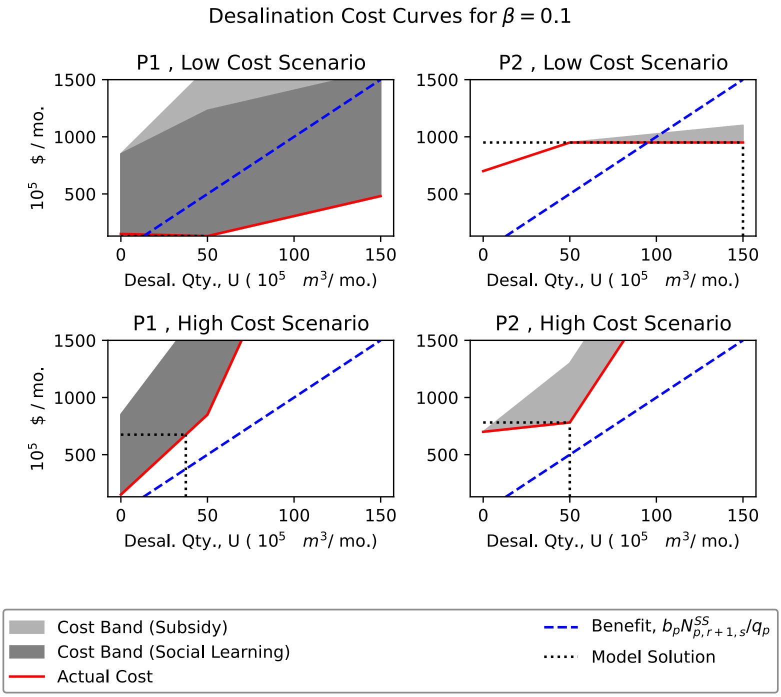


Fig. 2 Comparison of computational experiments

They serve as the primary contribution of this paper in terms of the ST frameworks. Additionally, it serves as the point of comparison for the other computational experiments.

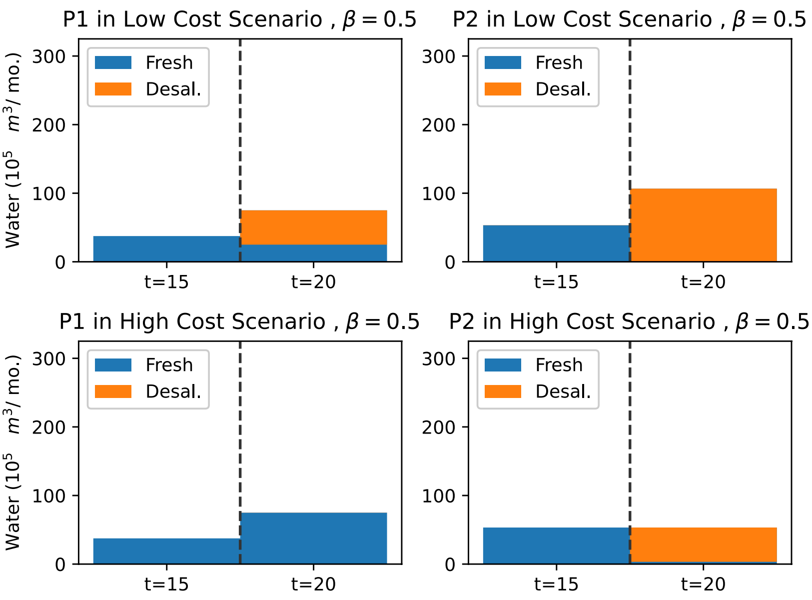


(a) Resource usage in the final model



(b) Social learning and cost curves for the final model

Fig. 3 **a** Resource usage in the final model, **b** Social learning and cost curves for the final model



(a) Resource usage in the final model

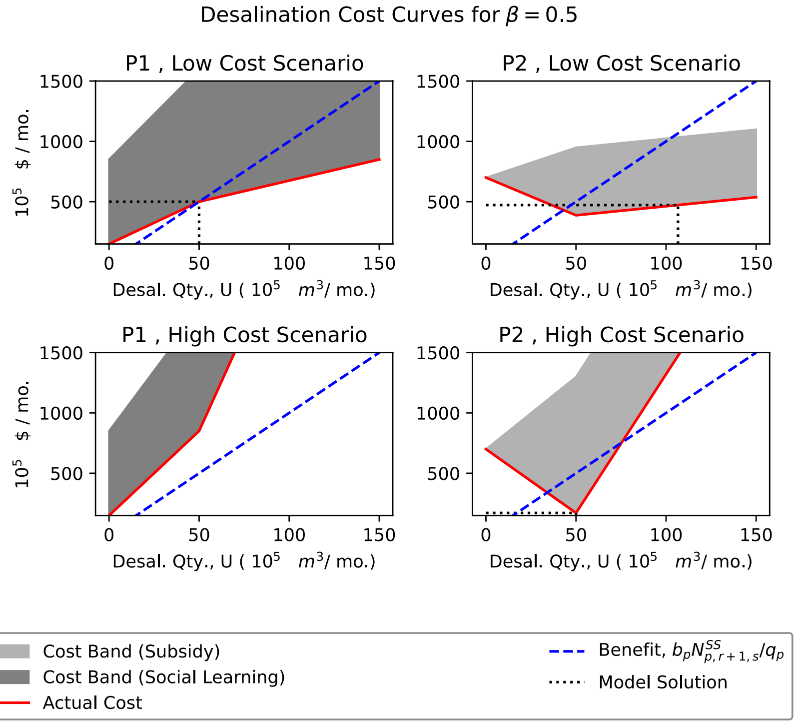


Fig. 4 **a** Resource usage in the final model, **b** Social learning and cost curves for the final model

The Bi-level, No Subsidy and No Tariff experiments illustrate how removing these upper-level variables from Program (3) impacts the upper-level objective function (3a). The No Subsidy Experiments show how the upper-level player use tariffs for the purpose regulating freshwater consumption. The No-Tariff Experiments also imply no subsidies because of the revenue constraint (3f). Thus, these experiments demonstrate the impact of a laissez-faire approach to water management.

5.2 Results

Figure 2 summarizes the computational experiments and their similarities and differences in terms of several key metrics: overall objective function value, expected net economic growth, expected inequality, expected environmental damage, and expected tariff charges. These metrics are described qualitatively in this section and are also defined mathematically in Table 3.

The relationship between the experiment's risk-aversion level and each metric are shown using blue and orange points for $\beta_1 = \beta_2 = 0.1$ and $\beta_1 = \beta_2 = 0.5$, respectively. If only an orange is visible, then the metric for both risk-aversion levels are equal.

Per Fig. 2, the upper-level objective function (3a) gradually decreases as the upper-level player's control decreases across the computational experiments. A conflict between the upper and lower-level objectives is present because the centralized computational experiments and the full bi-level experiments have different objective function values. This conflict is indicative of the upper-level player's inability to control all the lower-level player's decisions in the full bi-level experiments (e.g., population growth).

Furthermore, risk aversion makes the most difference in the objective function value for the computational experiments featuring tariffs (i.e., Bi-Level, Full and Bi-level, No Subsidy). This observation provides two insights. Firstly, this paper's bi-level model enables tariffs to be quantified in terms of risk preferences. Secondly, tariffs provide benefits but also induce costs. Balancing these costs and benefits provide a potential explanation for the computational difficulty of this bi-level model.

The expected net economic growth is higher in the full bi-level experiments than the centralized experiments. This finding suggests that economic growth is an incentive associated with the upper-level player's tariffs and subsidies. However, this incentive contributes less to the upper-level player's objectives. Furthermore, the centralized experiments are associated with the least inequality between the lower-level players.

Tariffs and subsidies control environmental damage very effectively despite their limited socio-economic value. The computational experiments with no tariffs and subsidies (i.e., Bi-level, No Tariff) have the highest level of environmental damage. This damage is progressively reduced for the Bi-level, No Subsidy and Bi-level, Full experiments. However, environmental damage is not minimized in the centralized experiments relative to the others. Thus, minimizing environmental damage is not the sole factor in the upper-level objective function (3a).

Risk aversion influences these metrics differently depending on whether subsidies are used in conjunction with tariffs or not. Higher tariffs are used in the Bi-level, Full experiment for $\beta_1 = \beta_2 = 0.5$ than for $\beta_1 = \beta_2 = 0.1$. However, the impact of risk aversion on tariffs is opposite for the Bi-level, No Subsidy experiments. In these experiments, the upper-level player is unable to fund subsidies with tariff revenues as described in Sect. 4.3. Consequently, the upper-level player levies less tariffs for $\beta_1 = \beta_2 = 0.5$ than $\beta_1 = \beta_2 = 0.1$, which allows more economic-growth and subsequent environmental damage.

Figures 3 and 4 illustrate detailed resource-consumption quantities and costs for $\beta_1 = \beta_2 = 0.1$ and $\beta_1 = \beta_2 = 0.5$, respectively. Cost bands for subsidies are added to Figs. 3b and 4b, which are shown as light gray shading. The policy-maker's interventions enable Player 2 to have a greater access to freshwater than Player 1 in $t = 15$ as shown in Figs. 3a and 4a. For the low cost scenario, this greater access enables Player 2's population to grow and consequently use more desalination in $t = 20$. In the High Cost Scenario, both players are able to maintain their populations.

Player 2's greater freshwater access relative to Player 1 ultimately creates incentives for social learning. In the Low Cost Scenario, the net benefits of desalination increase the more it is utilized, as shown by the differences between the dashed blue and red lines in Figs. 3b and 4b. These benefits justify Player 2's desalination investment, and in turn enable Player 1's desalination costs to come down via social learning (dark gray shading in Figs. 3b and 4b).

Subsidies are still needed to control for factors that undermine desalination investment. In Fig. 3b, subsidies are used to decrease Player 2's costs in the High Cost Scenario. Additionally, subsidies are used for Player 1 in the Low Cost Scenario. Their desalination quantity is too low to generate a high net payoff otherwise. In Fig. 4b, risk aversion decreases desalination utilization overall relative to Fig. 3b. Thus, more subsidies are needed to compensate for the lower net benefits.

Interestingly, risk aversion ($\beta_1 = \beta_2 = 0.5$) decreases the differences in resource consumption between Players 1 and 2 relative to $\beta_1 = \beta_2 = 0.1$. In Fig. 3a, Player 2's resource consumption increases substantially in the Low Cost Scenario relative to Player 1. In Fig. 4a, this effect is comparatively reduced. Additionally, Player 1 is able to grow in the High Cost Scenario for $\beta_1 = \beta_2 = 0.5$, which is not possible otherwise.

6 Conclusions

In conclusion, the bi-level model with two lower-level players in this paper enables tariffs, subsidies, and other benefits and costs to be quantified in terms of risk aversion to adopting new technologies. Such quantification introduces nuance into diametrically opposed policies such as degrowth and decoupling. Furthermore, the ST frameworks in our model illustrate the complex interactions involving economic growth, technology, socio-economic interactions, and various policy mechanisms over time.

The utility of this bi-level model also depends on how well the water resource example generalizes trends with resource usage at large. In energy systems, the externalities of greenhouse gas emissions may create ambiguity between resource availability and sustainability transitions. In other systems, such as supply chains, technological complexity may not be easily represented as latent variables.

One future research direction could be to simulate the bi-level optimization problem over multiple rolls of the rolling-horizon. Subsequent rolls would require the development of updating rules to carry over population levels, resource reserves, and R&D outcomes from one roll to the next. A potential outcome of such research could be to understand if limited foresight likely results in long-run population growth or decline. Another outcome is to understand how a policy-maker might equitably manage such long-term population changes.

Another future research direction could consider multiple forms of interaction among the players and earth system boundaries. For example, the players could also interact in a market to buy or sell rights to natural resources. Secondly, multiple earth system boundaries and natural resources could be considered in the same problem. For instance, the high energy consumption levels of desalination could be related to both the climate and water ESBs.

A third research direction could consider different piece-wise cost and benefit curves. For instance, a three-segment cost curve for technology could model more nuanced resource-consumption levels. Additionally, piece-wise benefit curves could be added to show increasing or diminishing returns to population growth. Adding these benefit and cost features would likely increase the complexity of the model but could also make it more realistic.

Appendix

This appendix summarizes the sets, variables, and parameters of the final model for the reader's convenience.

Sets

- $p \in P$: Players utilizing natural resources for economic growth
- $t \in \mathcal{T}$: Planning period for which players make or adjust planning decisions related to population growth and resource utilization
- $s \in S$: Discrete scenarios representing technology ROI outcomes

Variables

Upper-Level Variables

- $T^{FS}_{p,t}$: The upper-level player's unit tariff levied on Player p for the consumption of finite resources in the current Planning Period t (\$/mass/month)
- $T^{SS}_{p,t+1,s}$: The upper-level player's unit tariff levied on Player p for the consumption of finite resources in Scenario s of the next Planning Period $t + 1$ (\$/mass/month)
- $R^{B}_{p,s}$: Reimbursement (i.e., subsidy) for Player p in Scenario s at point B of the piecewise cost curve (\$).
- $R^{C}_{p,s}$: Reimbursement (i.e., subsidy) for Player p in Scenario s at point C of the piecewise cost curve (\$)
- W^{FS}_{t}: Withdrawal of the finite resource from reserve in Planning Period t (mass/month)
- W^{SS}_{t+1}: Withdrawal of the finite resource from reserve in Planning Period $t + 1$ and Scenario s (mass/month)
- X^{RD}_{p}: Binary investment decision indicating if Player p pays the R&D fixed cost (dimensionless)
- $X^{A}_{p,s}$: Activation term for the first point on Player p's piece-wise cost curve in Scenario s (dimensionless)
- $X^{B}_{p,s}$: Activation term for the second point on Player p's piece-wise cost curve in transition Scenario s (dimensionless)

Lower-Level Variables

- $N^{FS}_{p,t}$: Population served by Player p during Planning Period t (Thousands)
- $N^{SS}_{p,t+1,s}$: Population served by Player p in Scenario s during Planning Period $t + 1$ (Thousands)
- $E^{FS}_{p,t}$: Population lost for Player p during Planning Period t (Thousands)
- $E^{SS}_{p,t+1,s}$: Population lost for Player p in Scenario s during Planning Period $t + 1$ (Thousands)
- $F^{FS}_{p,t}$: Player p's average monthly consumption of the finite resource during Planning Period t (mass/month)
- $F^{SS}_{p,t+1,s}$: Player p's average monthly consumption of the finite resource in Scenario s during Planning Period $t + 1$ (mass/month)
- $U_{p,s}$: Player p's average monthly consumption of the practically unlimited resource in Scenario s (mass/month)
- $V_{p,s}$: Player p's benefit shortfall in Scenario s (\$)
- Z_{p}: Player p's threshold payoff for experiencing shortfall (\$)
- $Y^{A}_{p,s}$: First domain weighting term for Player p's piece-wise technology cost curve in Scenario s (dimensionless)
- $Y^{B}_{p,s}$: Second domain weighting term for Player p's piece-wise technology cost curve in Scenario s (dimensionless)

- $Y^{C}_{p,s}$: Third domain weighting term for Player p's piece-wise technology cost curve in Scenario s (dimensionless)
- $c^{u}_{p,s}$: Player p's cost to access the practically unlimited resource in Scenario s ($)
- $\gamma^{rsk}_{p,s}$: Shadow price associated with the net-benefit-shortfall constraint for Player p in Scenario s (dimensionless)
- $\gamma^{dem1}_{p,t}$: Shadow price associated with the natural-resource-demand constraint for Player p in Planning Period t ($/mass/mo.)
- $\gamma^{dem2}_{p,t+1,s}$: Shadow price associated with the natural-resource-demand constraint for Player p in Planning Period $t + 1$ and Scenario s ($/mass/mo.)
- $\gamma^{lim1}_{p,t}$: Shadow price associated with the limit to the finite-resource for Player p in Planning Period t ($/mass/mo.)
- $\gamma^{lim2}_{p,t+1,s}$: Shadow price associated with the limit to the finite-resource for Player p in Planning Period $t + 1$ and Scenario s ($/mass/mo.)
- $\gamma^{max1}_{p,t}$: Shadow price associated with the constraint on the maximum population growth for Player p in Planning Period t ($/thousand)
- $\gamma^{max2}_{p,t+1,s}$: Shadow price associated with the constraint on the maximum population growth for Player p in Planning Period $t + 1$ and Scenario s ($/thousand)
- $\gamma^{min1}_{p,t}$: Shadow price associated with the constraint on maintaining the minimum population level for Player p in Planning Period t ($/thousand)
- $\gamma^{min2}_{p,t+1,s}$: Shadow price associated with the constraint on maintaining the minimum population level for Player p in Planning Period $t + 1$ and Scenario s ($/thousand)
- $\lambda^{pw}_{p,s}$: Shadow price associated with the definitional constraint for Player p's piece-wise cost function in Scenario s (dimensionless)
- $\lambda^{u}_{p,s}$: Shadow price associated with the definitional constraint for Player p's utilization of the practically unlimited resource in Scenario s ($/mass/month)
- $\gamma^{A}_{p,s}$: Shadow price associated with the constraint on the first domain weighting term for Player p in Scenario s ($)
- $\gamma^{B}_{p,s}$: Shadow price associated with the constraint on the second domain weighting term for Player p in Scenario s ($)
- $\gamma^{C}_{p,s}$: Shadow price associated with the constraint on the third domain weighting term for Player p in Scenario s ($)
- $\lambda^{rd}_{p,s}$: Shadow price associated with the constraint of $R[NONSPACE]\&D$ investment on the domain weighting terms ($)
- c^{A}_{p}: Cost at Player p's first piece-wise point ($)
- $c^{B}_{p,s}$: Cost at Player p's second piece-wise point in Scenario s ($)
- $c^{C}_{p,s}$: Cost at Player p's third piece-wise point in Scenario s ($)

Parameters

- b_p: Benefit obtained per unit of population maintained by Player p ($ / thousand / month)

- c_p^f: Player p's unit cost for the finite resource ($ / unit mass / month)
- c_p^e: Player p's cost per unit of lost population ($ / thousand)
- $d_{p,t}$: Player p's discount rate in Planning Period t (dimensionless)
- g_p: Population growth rate for Player p (dimensionless)
- q_p: Player p's resource demand per unit of population (mass / month / thousand)
- n_p^i: Player p's starting population (thousands)
- l: Extraction limit of the finite resource (mass / month)
- a: initial accumulation of finite resources expressed as the quantity that can be removed on average each month throughout one planning period (mass / month)
- $\mathbb{P}(s)$: Probability of ROI Scenario s (dimensionless)
- α_p: Complement of the quantile used in the CVaR calculation for Player p (dimensionless)
- β_p: Risk weighting parameter for Player p (dimensionless)
- ϕ: Environmental damage weighting term ($ / mass squared)
- i_p^B: Second break point for Player p's piece-wise cost curve (mass / month)
- i_p^C: Third break point for Player p's piece-wise cost curve (mass / month)
- $c_p^{A'}$: Technology cost at first piece-wise point for Player p without any social learning contributions from other players ($ / month)
- $c_{p,s}^{B'}$: Technology cost at second piece-wise point for Player p in Scenario s without any social learning contributions from other players ($ / month)
- $c_{p,s}^m$: Player p's marginal technology cost after reaching maturity in Scenario s ($ / mass / month)
- Δc_p^A: Player p's maximum possible decrease of c_p^A given social learning contributions from other players ($)
- $\Delta c_{p,s}^B$: Player p's maximum possible decrease of $c_{p,s}^B$ given technological learning contributions from other players in Scenario s ($)
- $v_{p,p'}^{FS}$: First-stage shadow price weight between Player p and Player p' (dimensionless)
- $v_{p,p',s}^{SS}$: Second-stage shadow price weight between Player p and Player p' in Scenario s (dimensionless)
- $w_{p,p'}$: Weight relating the impact of player p' on the social learning of Player p (dimensionless)

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s10287-025-00534-5.

Acknowledgements This research was funded by the National Science Foundation's Civil Infrastructure Systems Program (NSF Award # 2113891). Aside from funding, the agency did not play any other role in the study design, data collection, analysis, interpretation, writing of the report, or publication decisions. Special thanks to John Miller, Scott Page, and the participants of the Santa Fe Institute's 2023 Graduate Workshop in Computational Social Science for helping inspire the core themes featured in this paper.

Author contributions N.B. conceptualized and drafted the work, and S.G. analyzed and revised it critically. Both authors approve the manuscript and agree to be accountable for all aspects of the work.

Data availability The stylized input data is not uploaded to a public repository per University of Maryland policy. However, this data is provided as supplementary information associated with this paper.

Declarations

Conflict of interest The authors declare no Conflict of interest that pertain to the material in this article.

References

Allen SA (2022) Working in reverse: Advancing inverse optimization in the fields of equilibrium and infrastructure modeling. PhD thesis, University of Maryland, College Park
Arthur WB (1990) Positive feedbacks in the economy. Sci Am 262(2):92–99
Arthur WB (1993) Why do things become more complex. Sci Am 268(5):144
Arthur WB (1994) Increasing returns and path dependence in the economy. University of michigan Press
Arthur WB, Ermoliev YM, Kaniovski YM (1987) Path-dependent processes and the emergence of macro-structure. Eur J Operat Res 30(3):294–303
Bettencourt LM, Lobo J, Helbing D et al (2007) Growth, innovation, scaling, and the pace of life in cities. Proceed Nat Acad Sci 104(17):7301–7306
Bettencourt LMA (2013) The origins of scaling in cities. Science 340(6139):1438–1441. https://doi.org/10.1126/science.1235823
Birge JR, Louveaux F (2011) Introduction to stochastic programming. Springer
Boyd N, Dumm T (2019) Balancing present and future needs. Pipelines 2019. American Society of Civil Engineers Reston, VA, pp 187–195
Boyd NT, Gabriel SA, Rest G, et al (2023) Generalized nash equilibrium models for asymmetric, non-cooperative games on line graphs: Application to water resource systems. Computers & Operations Research 154:106194. https://doi.org/10.1016/j.cor.2023.106194, https://www.sciencedirect.com/science/article/pii/S0305054823000588
Britz W, Ferris M, Kuhn A (2013) Modeling water allocating institutions based on multiple optimization problems with equilibrium constraints. Environ Model Software 46:196–207
Brozynski MT, Leibowicz BD (2020) Markov models of policy support for technology transitions. Eur J Operat Res 286(3):1052–1069
Brozynski MT, Leibowicz BD (2022) A multi-level optimization model of infrastructure-dependent technology adoption: overcoming the chicken-and-egg problem. Eur J Operat Res 300(2):755–770
Burian SJ, Nix SJ, Pitt RE et al (2000) Urban wastewater management in the united states: past, present, and future. J Urban Technol 7(3):33–62
Chang M, Thellufsen JZ, Zakeri B et al (2021) Trends in tools and approaches for modelling the energy transition. Appl Energy 290:116731
Conejo AJ, Carrión M, Morales JM et al (2010) Decision making under uncertainty in electricity markets, vol 1. Springer
Cook BI, Smerdon JE, Cook ER et al (2022) Megadroughts in the common era and the anthropocene. Nat Rev Earth Environ 3(11):741–757

Devine MT, Gabriel SA, Moryadee S (2016) A rolling horizon approach for stochastic mixed complementarity problems with endogenous learning: application to natural gas markets. Comput Operat Res 68:1–15
Doucet B, Smit E (2016) Building an urban 'renaissance': fragmented services and the production of inequality in greater downtown detroit. J Housing and the Built Environ 31:635–657
Eckhause JM, Hughes DR, Gabriel SA (2009) Evaluating real options for mitigating technical risk in public sector r &d acquisitions. Int J Project Manag 27(4):365–377
Eckhause JM, Gabriel SA, Hughes DR (2012) An integer programming approach for evaluating r &d funding decisions with optimal budget allocations. IEEE Trans Eng Manag 59(4):679–691. https://doi.org/10.1109/TEM.2012.2183132
Fuenfschilling L, Truffer B (2016) The interplay of institutions, actors and technologies in socio-technical systems - an analysis of transformations in the australian urban water sector. Technol Forecast Social Change 103:298–312
Gabriel SA, Conejo AJ, Fuller JD et al (2012) Complementarity modeling in energy markets, vol 180. Springer
Geels F (2005) Co-evolution of technology and society: the transition in water supply and personal hygiene in the netherlands (1850–1930)-a case study in multi-level perspective. Technol Soc 27(3):363–397
Gleick PH (2003) Global freshwater resources: soft-path solutions for the 21st century. Science 302(5650):1524–1528.
Gude VG (2016) Desalination and sustainability - an appraisal and current perspective. Water Res 89:87–106.
Hansen P, Jaumard B, Savard G (1992) New branch-and-bound rules for linear bilevel programming. SIAM J Sci Stat Comput 13(5):1194–1217. https://doi.org/10.1137/0913069
Harker PT (1991) Generalized nash games and quasi-variational inequalities. Eur J Operat Res 54(1):81–94. https://doi.org/10.1016/0377-2217(91)90325-P
Herrington G (2021) Update to limits to growth: comparing the world3 model with empirical data. J Indust Ecol 25(3):614–626
Hobbs BF (1999) Lcp models of nash-cournot competition in bilateral and poolco-based power markets. In: IEEE Power Engineering Society. 1999 Winter Meeting (Cat. No. 99CH36233), IEEE, pp 303–308
Jeroslow RG (1985) The polynomial hierarchy and a simple model for competitive analysis. Math Program 32(2):146–164
Jones E, Qadir M, van Vliet MT et al (2019) The state of desalination and brine production: a global outlook. Sci Total Environ 657:1343–1356. https://doi.org/10.1016/j.scitotenv.2018.12.076
Jones ER, Van Vliet MT, Qadir M et al (2021) Country-level and gridded estimates of wastewater production, collection, treatment and reuse. Earth Syst Sci Data 13(2):237–254
Kallis G, Kostakis V, Lange S et al (2018) Research on degrowth. Ann Rev Environ and Resour 43:291–316
Kemp R, Schot J, Hoogma R (1998) Regime shifts to sustainability through processes of niche formation: the approach of strategic niche management. Technol Anal Strat Manag 10(2):175–198. https://doi.org/10.1080/09537329808524310
Kerschner C, Wächter P, Nierling L et al (2018) Degrowth and technology: towards feasible, viable, appropriate and convivial imaginaries. J Clean Product 197:1619–1636
Kuhn A, Britz W, Willy DK et al (2016) Simulating the viability of water institutions under volatile rainfall conditions-the case of the lake naivasha basin. Environ Model Software 75:373–387
Lafforgue M (2016) Supplying water to a water-stressed city: lessons from windhoek. La Houille Blanche 4:40–47. https://doi.org/10.1051/lhb/2016038
Markard J, Raven R, Truffer B (2012) Sustainability transitions: an emerging field of research and its prospects. Res Policy 41(6):955–967. https://doi.org/10.1016/j.respol.2012.02.013
Marlow DR, Beale DJ, Cook S et al (2015) Inves Water Infrastruct Tomorrow. Springer, Dordrecht, pp 217–236
Miörner J, Heiberg J, Binz C (2022) How global regimes diffuse in space - explaining a missed transition in san diego's water sector. Environ Innov Soc Trans 44:29–47. https://doi.org/10.1016/j.eist.2022.05.005
Nemet GF (2006) Beyond the learning curve: factors influencing cost reductions in photovoltaics. Energy Policy 34(17):3218–3232. https://doi.org/10.1016/j.enpol.2005.06.020

Noyan N, Rudolf G, Lejeune M (2022) Distributionally robust optimization under a decision-dependent ambiguity set with applications to machine scheduling and humanitarian logistics. INFORMS J Comput 34(2):729–751

Palmeros Parada M, Kehrein P, Xevgenos D et al (2022) Societal values, tensions and uncertainties in resource recovery from wastewaters. J Environ Manag 319:115759. https://doi.org/10.1016/j.jenvman.2022.115759

Rockström J, Gupta J, Qin D, et al (2023) Safe and just earth system boundaries. Nature pp 1–10

Romer PM (1990) Endogenous technological change. J Polit Econ 98(52):71–102

Safari S, Sharghi S, Kerachian R et al (2023) A market-based mechanism for long-term groundwater management using remotely sensed data. J Environ Manag 332:117409. https://doi.org/10.1016/j.jenvman.2023.117409

Safarzyńska K, Frenken K, van den Bergh JC (2012) Evolutionary theorizing and modeling of sustainability transitions. Res Policy 41(6):1011–1024. https://doi.org/10.1016/j.respol.2011.10.014

Saraji MK, Streimikiene D (2023) Challenges to the low carbon energy transition: a systematic literature review and research agenda. Energy Strat Rev 49:101163. https://doi.org/10.1016/j.esr.2023.101163

U-tapao C, Moryadee S, Gabriel SA et al (2016) A stochastic, two-level optimization model for compressed natural gas infrastructure investments in wastewater management. J Nat Gas Sci Eng 28:226–240

Vadén T, Lähde V, Majava A et al (2020) Decoupling for ecological sustainability: a categorisation and review of research literature. Environ Sci Policy 112:236–244. https://doi.org/10.1016/j.envsci.2020.06.016

Voutchkov N (2016) Desalination - past, present and future. https://iwa-network.org/desalination-past-present-future/, accessed 2/7/2024

Winston WL (2004) Operations Research APPLICATIONS AND ALGORITHMS. Thomson

Xing J, Pleim J, Mathur R et al (2013) Historical gaseous and primary aerosol emissions in the united states from 1990 to 2010. Atmos Chem Phys 13(15):7531–7549

Zhuang J, Gabriel SA (2008) A complementarity model for solving stochastic natural gas market equilibria. Energy Econ 30(1):113–147

Authors and Affiliations

Nathan T. Boyd[1] · Steven A. Gabriel[1,2,3]

✉ Nathan T. Boyd
hello@nathantboyd.com

Steven A. Gabriel
sgabriel@umd.edu

1 Department of Mechanical Engineering, University of Maryland, College Park, 2181 Glenn L. Martin Hall, Building 088, College Park, MD 20742, USA

2 Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Postboks 8900, NO-7491 Trondheim, Trondelag, Norway

3 Department of Mathematics and Systems Analysis, Aalto University, P.O. Box 11100, FI-00076 Espoo, Finland