Minimization of mean-CVaR evacuation time of a crowd using rescue guides: a scenario-based approach

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Abstract

In case of a threat in a public space, the crowd in it should be moved to a shelter or evacuated without delays. Risk management and evacuation planning in public spaces should also take into account uncertainties in the traffic patterns of crowd flow. One way to account for the uncertainties is to make use of safety staff, or guides, that lead the crowd out of the building according to an evacuation plan. Nevertheless, solving the minimum time evacuation plan is a computationally demanding problem. In this paper, we model the evacuating crowd and guides as a multi-agent system with the social force model. To represent uncertainty, we construct probabilistic scenarios. The evacuation plan should work well both on average and also for the worst-performing scenarios. Thus, we formulate the problem as a bi-objective scenario optimization problem, where the mean and conditional value-at-risk (CVaR) of the evacuation time are objectives. A solution procedure combining numerical simulation and genetic algorithm is presented. We apply it to the evacuation of a fictional passenger terminal. In the mean-optimal solution, guides are assigned to lead the crowd to the nearest exits, whereas in the CVaR-optimal solution the focus is on solving the physical congestion occurring in the worst-case scenario. With one guide positioned behind each agent group near each exit, a plan that minimizes both objectives is obtained.

Keywords: Evacuation, rescue guides, multi-agent system, scenario-based approach, bi-objective optimization, numerical simulation, genetic algorithm

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1. Introduction

In case of a safety threat in a public place, the crowd has to be evacuated fast out of the building. There are many studies, where the optimal evacuation routes have been calculated Haghani (2020). However, these theoretical solutions are not accompanied by practical solutions to implement and enforce these routes. An extreme suggestion is to send the routes to crowd members, to their cellphones Wong et al. (2017). Yet, it is known that in emergency situations people tend to follow clear orders and guidance given by authorities, like security staff, and the use of security staff improves evacuation efficiency Gwynne et al. (2016); Proulx (2002). Thus, the coordination of security staff, or guides, is a more useful problem to solve.

The evacuation time of a crowd is sensitive to changes in conditions, even to deviations in individuals' positions Heliövaara et al. (2012); Helbing et al. (2003). A working evacuation plan should be robust against them. Related to this, we recently raised the question in von Schantz & Ehtamo (2020): how should a crowd be evacuated when there is the possibility that a large part of it deviates from its usual rules of motion? Here, our objective is to study how the guides should be coordinated so that the crowd is evacuated in minimum time considering different crowd flow traffic scenarios. We are also interested in how the evacuation plan changes when the number of guides varies; e.g. some of them can be needed at the same time in other operations, too. We will present our study in the context of a passenger terminal, even though other spaces containing a crowd could be used as well. Passenger terminals are characterized by high and fluctuating crowd flow, with a wide variety of people with different destinations to reach Schultz & Fricke (2011); Ali et al. (2019). Also, they typically have security staff that can be utilized to respond fast and guide the evacuation if needed.

When modeling the evacuation of a crowd using guides, it is appropriate to use the microscopic scale, as it allows focusing directly on individuals and on one-to-one interactions Bellomo et al. (2012). The social force model Helbing & Molnar (1995) and cellular automaton model Burstedde et al. (2001) are the most well-known microscopic models. Here, we use the social force model. In it, a mixture of socio-psychological and physical forces are assumed to influence an agent's motion in the crowd, which is modeled with Newtonian mechanics. When guide agents are added to the crowd, they also influence the motion of the other agents. In the original, stochastic version of the model, a Gaussian random force is added to an agent's equation of motion to describe intentional or unintentional deviations from the usual rules of motion.

Different aspects of the optimal use of guides in an evacuation have been studied. The main focus has been on the optimal proportion or number of guides Pelechano & Badler (2012); Wang & Cheng (2012); McCormack & Chen (2014). Choosing the number of guides alone does not guarantee a fast evacuation. In fact, if the routes of the guides are not simultaneously optimized, it can even make the evacuation slower Yang et al. (2016). Most of the research on using guides are comparative studies, where different configurations are numerically simulated and compared. There are only few studies where mathematical optimization has been used Albi et al. (2016); Zhou et al. (2019); von Schantz & Ehtamo (2020). On the other hand, the need for rigorous mathematical approaches have been recognized Haghani (2020).

In our recent study von Schantz & Ehtamo (2020), the number of guides, their initial positions and exit assignments needed to minimize the crowd evacuation time is solved in a single optimization problem. In it, the crowd is modeled with the stochastic social force model to which interaction rules between regular agents and guides are added. The minimum time crowd evacuation problem is first formulated as a stochastic control problem. Then it is reformulated as a scenario optimization problem and solved with a combined numerical simulation and genetic algorithm (GA) von Schantz & Ehtamo (2020).

A typical concern when modeling crowd evacuation is that in real life people can take part in various activities that do not immediately get them to the exit Gwynne et al. (2016). While these behavioral deviations could be implemented in the social force model framework, we assume them to be small compared to the deterministic part of a moving crowd that is controlled by guides, and to be approximated with the Gaussian random force term. This approach does not suffice if a larger part of the crowd behaves differently. Instead the different crowd flow traffic patterns could be modeled by changing some of the input parameters.

To deal with the uncertainty of the input parameters, robust optimization Beyer & Sendhoff (2007) or stochastic programming Birge (1982) can be used. In robust optimization, the input parameters are defined to belong to an uncertainty set, but no probabilities are assigned to them. Typically, a solution that optimizes the worst-case scenario is solved. This is practical when probabilities cannot be assigned to different scenarios. But, it might result in an overly conservative evacuation plan. On the other hand, in stochastic programming, different realizations of the input parameters are modeled as probabilistic scenarios. The scenario probabilities in passenger terminals could be obtained from passenger data or security cameras. Typically, the mean of the objective function is optimized, which in our case is the mean of the evacuation time over the scenarios. This is appropriate when the optimal evacuation plan works well across all scenarios. The slowest scenarios can be accounted for by minimizing a risk measure like the conditional value-at-risk (CVaR_α). It gives the conditional mean evacuation time for the $1-\alpha$ percentage of the slowest scenarios Rockafeller & Uryasev (2000). Note that in the special case that the worst-case scenario probability equals or exceeds $1-\alpha$, the worst-case scenario equals CVaR_α. This will be the situation in the toy example of our paper.

Instead of deciding whether the mean, or CVaR_α, is a more appropriate objective, the minimum time crowd evacuation problem can be formulated as a bi-objective problem similar to that in portfolio optimization Rockafeller & Uryasev (2000). Due to problem complexity, usual derivative-based methods cannot be applied for optimization. Instead combined numerical simulation and GA approaches are used. The GA iteratively searches for the nondominated solutions, while the values of the objective function on different scenarios are evaluated using numerical simulation. For bi-objective problems a special kind of GA is needed. In this paper we will use NSGA-II Deb et al. (2002).

In this paper, we model the evacuating crowd of passengers and guides with a deterministic version of the social force model from von Schantz & Ehtamo (2020). When a guide is within an interaction range from a passenger, it starts to follow the guide. Different crowd flow traffic patterns are modeled as probabilistic scenarios. Our main contribution is that we formulate a mean-CVaR$_\alpha$ crowd evacuation problem with rescue guides. The optimization variables are the initial positions and exit assignments of guides. Then, we present a solution procedure combining numerical simulation and NSGA-II.

Our paper is structured as follows. In Sec. 2 the mathematical details of the crowd movement model is presented. In Sec. 3 we define the optimization model. In Sec. 4 we present the solution procedure and its implementation details. In Sec. 5 we present the case study of an evacuation of a passenger terminal. The numerical results and their analysis are presented in Sec. 6. Finally, Sec. 7 is for discussion and conclusion.

2. Evacuation model with guides

Next, we will present the crowd movement model used for evacuation of a passenger terminal. It is the deterministic version of the social force model presented in von Schantz & Ehtamo (2020). It is essentially Helbing's original social force model, with a velocity-dependent social repulsion force. The social force model has been extensively discussed in many previous researches. We refer the reader to von Schantz & Ehtamo (2020) and its appendix for an exact mathematical description of the individual force terms and parameter values used.

Let us start by denoting the index set of passenger agents, or passengers, with $N = \{1, ..., n\}$, the index set of guide agents, or guides, with $G = \{n+1, ..., n+m\}$, their combined index set with $I = N \cup G$, the building space with $\Omega \subset \mathbb{R}^2$, the exits with $\mathcal{E} \subset \Omega$, and the walls with $W \subset \Omega$. The points associated with an exit we denote by $\varepsilon \subset \mathcal{E}$, so that $\mathcal{E} = \cup\varepsilon$. And, the points associated with a wall segment we denote by $w \subset W$, so that $W = \cup w$.

An agent $i \in I$ is circle-shaped with radius r_i and mass m_i. At time t, its center of mass is $\mathbf{x}_i(t)$ and its velocity vector is $\mathbf{v}_i(t)$. Its change of position is then:

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i. \tag{1}$$

Initially, all passengers $l \in N$ are heading towards their destination exit $\varepsilon_l^{des} \in \mathcal{E}$. The guides $g \in G$ have been instructed by the evacuation planner to move towards the exits $\varepsilon_g \in \mathcal{E}$. If at some point in time, a passenger comes within the interaction range r_{guide} of a guide, the passenger starts to follow the guide. If the passenger is within the r_{guide} range to two or more guides, it starts to follow the closest one. Once it has started to follow a guide, it will not start to follow another. Let us define a binary variable:

$$u_{lg}(t) = \begin{cases} 1, & \text{if passenger } l \text{ follows guide } g \text{ at time } t, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

for which it initially holds.

The change of velocity at time t for passenger l is given by the equation of motion:

$$m_l \frac{d\mathbf{v}_l}{dt} = (1 - \sum_g u_{lg})\mathbf{f}_l^0 + \sum_g u_{lg}\mathbf{f}_{lg} + \sum_i \mathbf{f}_{li} + \sum_{w \subset W} \mathbf{f}_{lw}. \tag{3}$$

Here, the term $\mathbf{f}_l^0$ is the driving force of agent i to move towards its destination exit. It describes the attempt of agent l to change its actual velocity $\mathbf{v}_l$ to a desired velocity $v_l^0 \mathbf{e}_l^{des}$ with a certain characteristic reaction time τ:

$$\mathbf{f}_l^0 = m_l \frac{v_l^0 \mathbf{e}_l^{des}(\mathbf{x}_l) - \mathbf{v}_l}{\tau}. \tag{4}$$

Here, v_l^0 is the desired speed of agent l. For simplicity, we assume it to be constant throughout the evacuation. In position $\mathbf{x} \in \Omega \setminus \left\{\varepsilon_l^{des} \cup W\right\}$, the unit vector $\mathbf{e}_l^{des}$ gives the direction of the shortest path of agent l towards its destination exit ε_l^{des}:

$$\mathbf{e}_l^{des}(\mathbf{x}) = -\frac{\nabla D_l^{des}(\mathbf{x})}{\|\nabla D_l^{des}(\mathbf{x})\|}. \tag{5}$$

Here, D_l^{des} is the distance map to the destination exit, and it is obtained as a solution to the continuous shortest path problem:

$$\begin{cases} \|\nabla D_l^{des}(\mathbf{x})\| = 1, & \text{if } \mathbf{x} \in \Omega \setminus \left\{\varepsilon_l^{des} \cup W\right\}, \\ D_l^{des}(\mathbf{x}) = 0, & \text{if } \mathbf{x} \in \varepsilon_l^{des}, \\ D_l^{des}(\mathbf{x}) = \infty, & \text{if } \mathbf{x} \in W. \end{cases} \tag{6}$$

The problem is solved for all exits before the simulation is run. For detailed information on its numerical computation, see von Schantz & Ehtamo (2020); Sethian (1999).

If the passenger follows a guide, i.e., $\sum_g u_{lg} = 1$, it will instead move towards the guide's destination with the driving force,

$$\mathbf{f}_{lg} = m_l \frac{v_l^0 \mathbf{e}_g(\mathbf{x}_l) - \mathbf{v}_l}{\tau}. \tag{7}$$

Here, the unit vector $\mathbf{e}_g$ is the direction towards the destination exit of guide g, ε_g. It is calculated in the same way as $\mathbf{e}_l^{des}$ in Eqs. (5) and (6).

The term $\mathbf{f}_{li}$ includes the socio-psychological and physical forces between agents l and $i \in I$, and similarly, the term $\mathbf{f}_{lw}$ contains the physical forces between agent i and wall $w \subset W$. These have been extensively discussed in the article von Schantz & Ehtamo (2020); see its appendix for their exact mathematical expressions.

On the other hand, the change of velocity at time t for guide g is given by the equation of motion:

$$m_g \frac{d\mathbf{v}_g}{dt} = \mathbf{f}_g^0 + \sum_i \mathbf{f}_{gi} + \sum_{w \subset W} \mathbf{f}_{gw}. \tag{8}$$

The term $\mathbf{f}_{gi}$ includes the socio-psychological and physical forces between guide g and agent $i \in I$, and $\mathbf{f}_{lw}$ contains the physical forces between guide g and wall $w \subset W$. These forces are of the same form as the ones for passengers. Finally, the term $\mathbf{f}_g^0$ is the driving force of guide g to move towards its destination exit:

$$\mathbf{f}_g^0 = m_l \frac{v_g^0 \mathbf{e}_g(\mathbf{x}_g) - \mathbf{v}_g}{\tau}. \tag{9}$$

3. Optimization framework

We assume that the crowd flow traffic patterns are uncertain. More specifically, the uncertainty is related to the destination exits and desired speeds of the passengers, ε_l^{des} and v_l^0, respectively. The possible realizations, or scenarios, of the uncertain parameters are denoted by $\vartheta^k,\ k \in \{1, ..., K\}$. In this section, we define the optimization variables and objective functions. The notation for risk measures is based on the article Rockafeller & Uryasev (2000).

3.1. Probability definitions

Let us denote the evacuation times of the individual agents to get out of the building with $t_i,\ i \in I = N \cup G$. The maximal element of these evacuation times equals the evacuation time of the crowd, which we denote by T_{last}. It is a random variable that depends on the set of scenarios $\theta = \{\vartheta^1, ..., \vartheta^K\}$. The evacuation time associated with scenario ϑ^k is T_{last}^k. Furthermore, each scenario is associated with a probability $p^k,\ \sum_k p^k = 1$.

We are dealing with a finite number of scenarios; hence Eq. (10), the probability of T_{last} being less than or equal to a number $\zeta \in \mathbb{R}$, is calculated from the discrete cumulative distribution function, by summing all the probabilities of the scenarios for which T_{last}^k is less than ζ,

$$\begin{aligned} \mathrm{P}\big(T_{last} \le \zeta\big) &= \sum_{k,\ T_{last}^k \le \zeta} \mathrm{P}\Big(T_{last} = T_{last}^k\Big) \\ &= \sum_{k,\ T_{last}^k \le \zeta} p^k. \end{aligned} \tag{10}$$

One of the objectives for our optimization problem is the mean of T_{last}, w.r.t., θ. It is defined as a mean of a discrete random variable:

$$\mathbb{E}\left[T_{last}|\theta\right] = \sum_k p^k T_{last}^k. \tag{11}$$

From here on, we use the notation $\mathbb{E}\left[T_{last}\right]$ for $\mathbb{E}\left[T_{last}|\theta\right]$.

Next, we denote value-at-risk for probability level $\alpha \in (0,1)$ by VaR_α. For our problem, it is the smallest number ζ for which the cumulative probability $\mathrm{P}\big(T_{last} \le \zeta\big)$ exceeds α, and is defined as,

$$\mathrm{VaR}_\alpha\left[T_{last}\right] := \inf\left\{\zeta \in \mathbb{R} : \mathrm{P}\big(T_{last} \le \zeta\big) \ge \alpha\right\}. \tag{12}$$

We can rewrite Eq. (12) by using Eq. (10):

$$\mathrm{VaR}_{\alpha}\left[T_{last}\right] = \inf\left\{\zeta \in \mathbb{R} : \sum_{k,\ T^{k}_{last} \leq \zeta} p^{k} \geq \alpha\right\}. \tag{13}$$

Now, we can define our second objective, the conditional value-at-risk for probability level α, CVaR_{α}. It is the conditional mean of T_{last} exceeding VaR_{α}, and is defined as,

$$\mathrm{CVaR}_{\alpha}\left[T_{last}\right] = \mathbb{E}\left[T_{last} \middle| T_{last} \geq \mathrm{VaR}_{\alpha}\left[T_{last}\right]\right]. \tag{14}$$

Again, because we have a finite number of scenarios, we can rewrite Eq. (14) as a probability-weighted sum of those T^{k}_{last} exceeding $\mathrm{VaR}_{\alpha}\left[T_{last}\right]$:

$$\mathrm{CVaR}_{\alpha}\left[T_{last}\right] = \mathrm{VaR}_{\alpha}\left[T_{last}\right] + \frac{1}{1-\alpha} \sum_{k,\ T^{k}_{last} \geq \mathrm{VaR}_{\alpha}\left[T_{last}\right]} p^{k}\left[T^{k}_{last} - \mathrm{VaR}_{\alpha}\left[T_{last}\right]\right]. \tag{15}$$

Note that in some situations, especially when there are a small number of scenarios, the worst-case scenario probability might equal or exceed the tail probability $1-\alpha$. Let us denote the index of the worst-case scenario by β, and the evacuation time associated with it, or the worst-case scenario value, by WCSV. Thus, it holds

$$\beta = \arg\max_{k}\ T^{k}_{last}, \tag{16}$$

and

$$\mathrm{WCSV}\left[T_{last}\right] = T^{\beta}_{last}. \tag{17}$$

When the worst-case scenario probability $p^{\beta} \geq 1-\alpha$, from Eq. (13) we get,

$$\mathrm{VaR}_{\alpha}\left[T_{last}\right] = \mathrm{WCSV}\left[T_{last}\right], \tag{18}$$

and furthermore, by inserting the above into Eq. (15),

$$\mathrm{CVaR}_{\alpha}\left[T_{last}\right] = \mathrm{WCSV}\left[T_{last}\right]. \tag{19}$$

So, in this situation, we can use the simpler WCSV instead of CVaR_{α}. In the example problem studied later in this paper, this will be the case.

3.2. Optimization problem

Next, we discretize the space Ω into square grid cells ω, so that $\Omega \subset \cup\omega = \bar{\Omega}$. In our problem, the number m of guides is fixed. Each guide $g \in G = \{n+1, ..., n+m\}$ is associated with an origin grid cell $\omega_g \subset \bar{\Omega}$, and a destination exit $\varepsilon_g \subset \mathcal{E}$. The optimization variables are feasible origin-destination pairs of the guides, i.e., feasible evacuation plans $\pi := \{(\omega_{n+1}, \varepsilon_{n+1}), ..., (\omega_{n+m}, \varepsilon_{n+m})\}\,,\ \omega_g \subset \bar{\Omega},\ \varepsilon_g \subset \mathcal{E}, g \in G$. Let Π be the set of all such evacuation plans.

The end-point conditions for our problem are the initial and final positions of the agents. The initial position of a guide g, $\mathbf{x}_g(0) \in \omega_g$, is a prespecified point in its corresponding origin grid cell ω_g. The final position of a guide g, $\mathbf{x}_g(t_g) \in \varepsilon_g$, is any point in its corresponding destination exit ε_g. The initial positions of the agents are $\mathbf{x}_l(0) = \mathbf{x}_l^0,\ l \in N$. The agents can evacuate using any of the available exits; thus it holds for the final positions $\mathbf{x}_l(t_l) \in \mathcal{E}$.

We are interested in finding an evacuation plan that optimizes both the mean and CVaR_α of T_{last}. Let us first define the following function:

$$\phi(\pi, \theta) := T_{last}. \tag{20}$$

Then we can write our bi-objective optimization problem as

$$\begin{aligned}
&\min_{\pi \in \Pi} \left\{ \mathbb{E}\left[\phi(\pi,\theta)\right], \text{CVaR}_\alpha\left[\phi(\pi,\theta)\right] \right\}; \\
&\text{subject to Eqs. (1), (3), (8);} \\
&\mathbf{x}_l(0) = \mathbf{x}_l^0,\ \mathbf{x}_l(t_l) \in \mathcal{E},\ l \in N;\ \mathbf{x}_g(0) \in \omega_g,\ \mathbf{x}_g(t_g) \in \varepsilon_g,\ g \in G.
\end{aligned} \tag{21}$$

Here we are minimizing two objectives at the same time in the sense of Pareto-optimality. To compare two solutions we use the concept of dominance. Solution π^1 dominates solution π^2 iff:

$$\begin{aligned}
&\mathbb{E}\left[\phi(\pi^1,\theta)\right] \le \mathbb{E}\left[\phi(\pi^2,\theta)\right],\ \text{and } \text{CVaR}_\alpha\left[\phi(\pi^1,\theta)\right] < \text{CVaR}_\alpha\left[\phi(\pi^2,\theta)\right], \\
&\text{or} \\
&\mathbb{E}\left[\phi(\pi^1,\theta)\right] < \mathbb{E}\left[\phi(\pi^2,\theta)\right],\ \text{and } \text{CVaR}_\alpha\left[\phi(\pi^1,\theta)\right] \le \text{CVaR}_\alpha\left[\phi(\pi^2,\theta)\right].
\end{aligned} \tag{22}$$

The set of solutions not dominated by any other solutions is called the set of nondominated, or Pareto-optimal, solutions. By definition, they are the solutions of the problem.

4. Solution method

The bi-objective optimization problem of Eq. (21) is solved with a combined simulation and genetic algorithm (GA) procedure Goldberg (1989). In it, the GA searches iteratively for the nondominated solutions, while the evacuation simulation evaluates the objective function values, or fitnesses, of the found solutions and steers the randomized search process. The procedure ends when a convergence criterion is met. The procedure is summarized in the flowchart of Fig. 1, and it is explained next step-by-step.

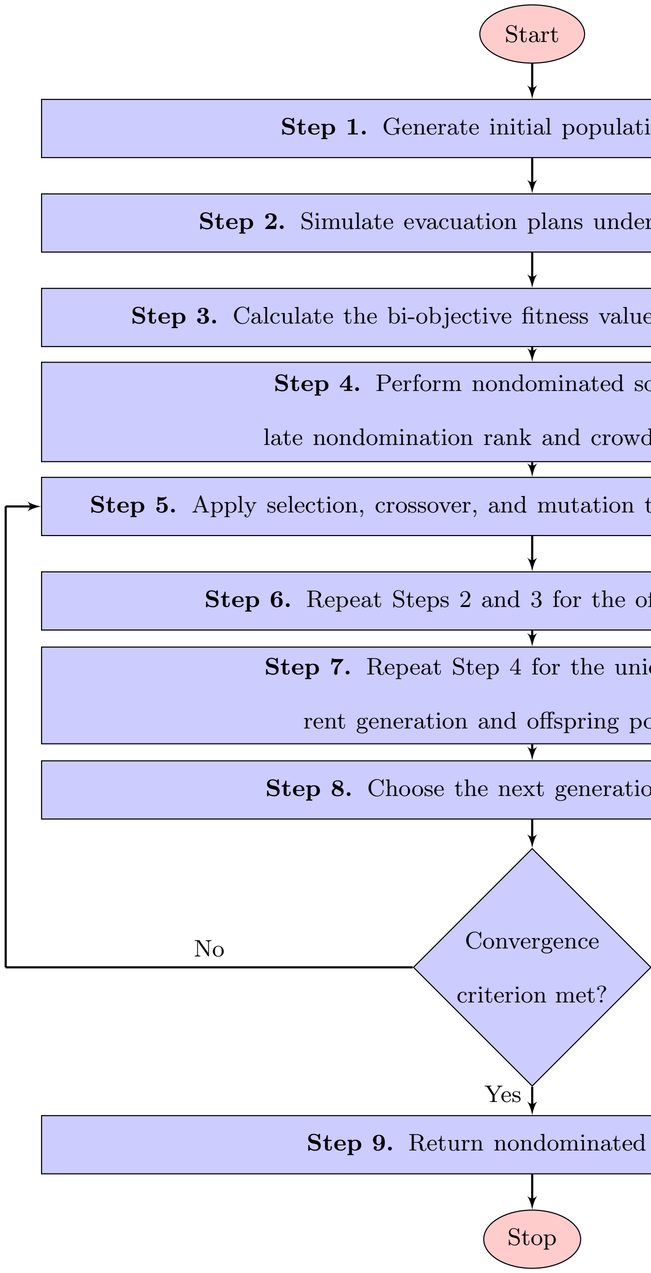


Figure 1: Flowchart of the combined simulation and GA procedure.

Step 1. In the first iteration, or generation, a random population with Q number of solutions is created. The solutions are evacuation plans $\pi^1, ...\pi^Q$. A gene in a solution contains the origin grid cell and destination exit of a single guide.

Step 2. The evacuation plans are simulated under different scenarios. More specifically, given an evacuation plan and a scenario ϑ^k, the system defined by the constraints in Eq. (21) is simulated with a numerical integration scheme to obtain T^k_{last} (see, e.g., appendix of von Schantz & Ehtamo (2019) for further details). This is done for all evacuation plans in the population and all scenarios.

Step 3. The bi-objective fitness value is calculated with the equations from Sec. 3.1.

Step 4. The solutions in the population are ranked with the Nondominated Sorting Genetic Algorithm

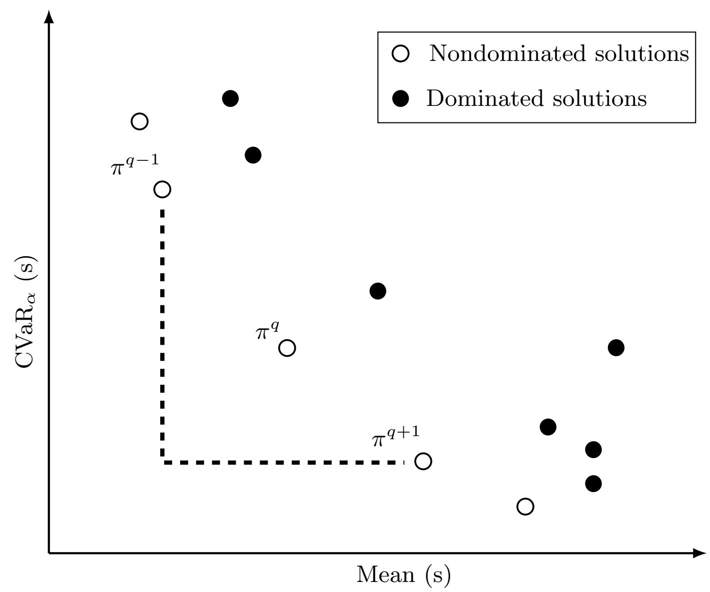


Figure 2: The crowding distance of solution π^q is the L^1 distance between its neighboring solutions π^{q-1} and π^{q+1} (the sum of the lengths of the two dashed lines).

II (NSGA-II) Deb et al. (2002). The solutions are sorted, or compared with each other, using the concept of dominance of Eq. (22) to find the set, or front, of nondominated solutions. The first front gets the nondomination rank 0. The solutions of the front are then set aside, and the sorting is repeated to find the next front. The solutions in the next front gets the nondomination rank 1. This procedure is repeated until all solutions have been ranked.

After nondomination ranks have been assigned, a crowding distance is calculated for each solution Fortin & Parizeau (2013). It estimates the density of solutions with the same rank surrounding a particular solution. For a given solution, in the objective space, it calculates the L^1 distance between its two neighboring solutions (see Fig. 2). Solutions that have a higher crowding distance, i.e., are located in a less crowded region, are preferred.

Step 5. The offspring solutions are obtained by applying selection, crossover and mutation operations on the current generation, or parent, solutions. In selection, we use the unique fitness tournament selection Fortin & Parizeau (2013). In it, a number of solutions with unique fitness values are sampled. Each sampled solution is paired against one other sampled solution. In the paired contest, the solution with lower nondomination rank wins, and is selected to undergo further operations. If the solutions have the same rank, the one with a higher crowding distance wins the paired contest. This process is repeated until Q solutions have been selected.

After selection, each selected solution is paired with one other selected solution for crossover. We use the single-point crossover operation, where the offspring solutions of the two parent solutions contain half of the genetic material of both parents. Thus, the offspring solutions contain half of the guides of each parent's evacuation plan. The crossover operator is applied with a certain probability. If it is not applied, the offspring solutions have the exactly same genetic information as their parents.

Finally, a mutation operator is applied on the offspring solutions. It is applied separately, with a certain

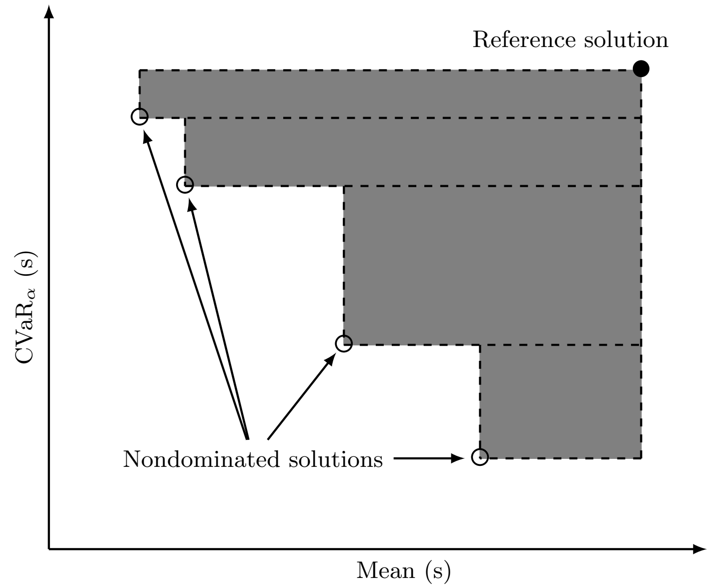


Figure 3: Hypervolume is the sum of the rectangular gray areas defined by the nondominated solutions and the reference solution.

probability on each gene. It can either change the origin grid cell of a guide, or its destination exit, or both of them.

Step 6. The bi-objective fitness of the offspring solutions is calculated by repeating Steps 2 and 3.

Step 7. Step 4 is repeated for the union of the current generation and offspring populations.

Step 8. For the next generation population, Q solutions are chosen from the union of the current generation and offspring populations. The solutions with lowest nondomination ranks are chosen. If all solutions of the same rank do not fit, those with highest crowding distance are prioritized.

Step 9. Return the first nondominated front calculated in Step 7.

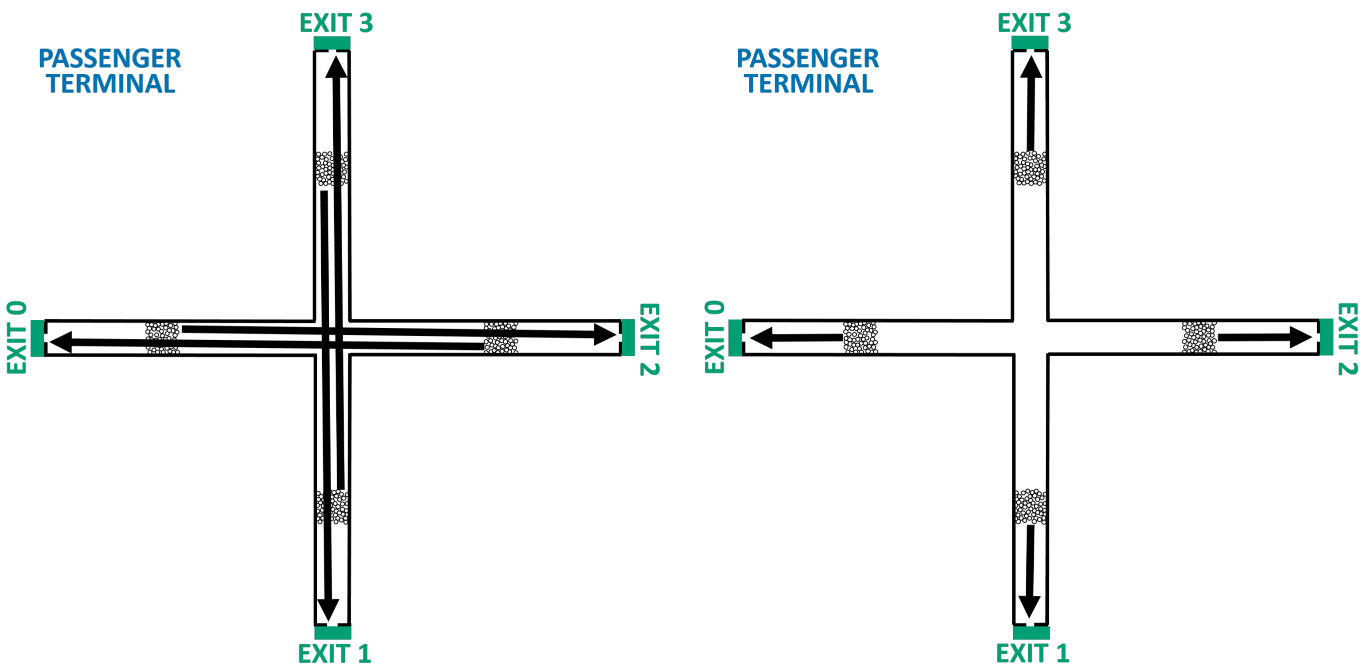
After Step 8, we check if a convergence criterion is met. More precisely, to measure convergence, we use a hypervolume indicator Bader & Zitzler (2011). To calculate its value, we first construct a reference solution. The reference solution is a point in the objective space that has larger CVaR_α and mean evacuation time than any feasible solution. The hypervolume indicator is calculated as the sum of the rectangular areas defined by the reference solution and the first nondominated front from Step 7 (see Fig. 3).

The hypervolume indicator is a measure of the quality of the set of nondominated solutions. The solutions of a bi-objective optimization problem give the largest hypervolume. When the hypervolume indicator has not increased for a predefined number of generations, we consider the algorithm to have converged. Note that the choice of the reference solution can affect the convergence of NSGA-II.

5. Case study: evacuation of a passenger terminal

Here, we have in mind a busy passenger interchange terminal. Passengers with different characteristics are arriving there through one exit and later departing from another. At such places, common walking areas can get very crowded at times Schultz & Fricke (2011); Ali et al. (2019). The changing crowd conditions make planning an evacuation particularly difficult. The question we ask is: how should the evacuation planner instruct the safety staff, or guides, to lead the passengers out of the terminal in case of emergency when accounting for different scenarios?

More specifically, if the passengers have just arrived, they move to the exits on the opposite end of a hallway. If they are departing, they move to their nearest exits. Passengers move in groups. They can either move with normal speed, or with slow speed. Slow-moving passengers are elderly or somehow disabled. The passengers are modeled as agents with our agent-based model from Sec. 2. We create four characteristic scenarios, which are depicted in the schematic diagram of the fictional interchange terminal in Fig. 4. We assign the probabilities 0.3, 0.2, 0.2 and 0.3 to the scenarios, respectively. Also, we set the probability level $\alpha = 0.95$ for the risk measure. Thus, in this example, minimizing CVaR_α equals to minimizing the evacuation time of the worst-case scenario.



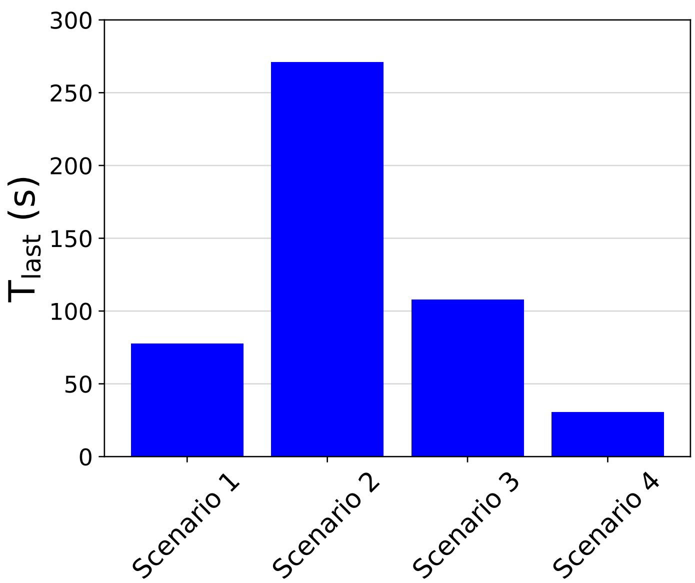
(a) Scenarios 2 and 3

(b) Scenarios 1 and 4

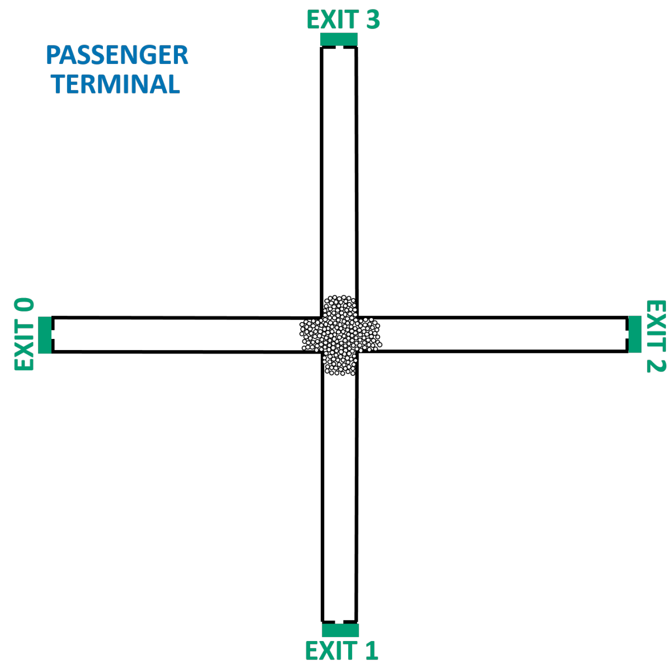
Figure 4: A schematic diagram of possible scenarios in a passenger terminal: (a) arriving agents in Scenario 2 (slow) and Scenario 3 (fast), (b) departing agents in Scenario 1 (slow) and Scenario 4 (fast).

In Scenarios 2 and 3 agents are arriving (see Fig. 4(a)), while in Scenarios 1 and 4 the agents are departing (see Fig. 4(b)). One can think of Scenarios 2 and 3 representing the movement of the same agents as in the other two scenarios, but earlier, when they have just arrived to the terminal. In Scenarios 1 and 2, the agents move slowly, while in Scenarios 3 and 4 they move fast.

In an emergency situation, for example a bomb threat, we assume that passengers will go about their business unless instructed otherwise. This could happen if they are uninformed of the threat, or if they aim to move to their destination exits regardless of the circumstances. Thus, in an unguided evacuation, the evacuation time of the crowd equals the time for them to get to their destination exits (see Fig. 5(a)). The evacuation times of Scenarios 2 and 3 are longest, since the arriving agents create a four-way counterflow at the intersection, which quickly causes a large congestion (see Fig. 5(b)). Also, the agents have a longer way to walk when they move to the exits on the opposite end of the hallway instead of the nearest exits. Moreover, in Scenario 2, the agents move slowly, which means that it is noticeably the slowest of all four scenarios.



(a)



(b) Scenario 2

Figure 5: A large congestion in the worst-case scenario, Scenario 2, results in a slow evacuation: (a) T_{last} (evacuation time) for different scenarios, (b) a snapshot of the congestion.

We acknowledge that in reality, for example, also loudspeakers could be used to give information to the passengers, but here our focus is only on how to use guides in an evacuation. We are interested in studying the tradeoff between number of guides used and evacuation time of the crowd. Furthermore, we assume guides are beforehand instructed for one evacuation plan by the evacuation planner, and they will use it for all scenarios.

The halls of the terminal are 5 m wide, 85 m long, and the distance from the exit to the intersection area is 40 m. The exit widths are 1.2 m. Initially, each passenger agent group is located in the middle of one leg of the terminal. In each of the four groups there are 50 agents. For passenger agents $l \in N$, the initial positions $\mathbf{x}_l^0$, radii r_l and masses m_l are fixed for all scenarios. Before fixing the values, the parameters m_l and r_l are drawn from a truncated normal distribution with a cutoff at three times the standard deviation. The mean and standard deviations are 73.5 kg and 8.0 kg, and 0.255 m and 0.035 m, respectively for m_l and r_l. The reaction time is set $\tau = 0.5$ s for all agents. These values are all taken from the FDS+Evac user manual Korhonen & Hostikka (2009). In Scenarios 1 and 2, the desired speeds of passengers are $v_l^0 = 0.5$ m/s, whereas for Scenarios

3 and 4 they are $v_l^0 = 1.55$ m/s. Also, as stated above, the destination exits ε_l^{des} of passengers vary between scenarios. On the other hand, for guide agents $g \in G$, we set the values of a typical male: $m_g = 80$ kg, $r_g = 0.27$ m, and $v_g^0 = 1.15$ m/s. Also, the interaction range of guide agents is set to $r_{guide} = 10$ m. The origin grid cells for guides, $\omega \in \bar{\Omega}$, are 2 m $\times$ 2 m.

6. Numerical results

6.1. Implementation details and performance

We solve the bi-objective optimization problem for the fictional passenger terminal for 1, 2, 3 and 4 guides, with the procedure presented in Fig. 1. Typically, to solve an optimization problem with a GA, the algorithm parameters are tuned manually in a problem-specific manner. Here, we tried extensively different algorithm parameters, to assure most accurate and efficient convergence. We set the GA population size to $Q = 40$, crossover probability to 0.85 and mutation probability to 0.10. We consider the procedure to have converged when the hypervolume has not increased for 15 consecutive generations. When calculating the hypervolume, as the objective values of the reference solution we use 271 s for both CVaR_α and mean evacuation time. The reason being that, in the unguided evacuation, in Scenario 2, the evacuation time is 271 s, and we assume the solutions cannot have worse objective function values than that.

The crowd simulation model is implemented in Python code. Some of the core parts of the code are written as Numba-decorated functions, which translates Python functions to optimized machine code at runtime. Numba-compiled numerical algorithms in Python can approach the speeds of C or FORTRAN Oliphant et al. (2020). The GA is implemented in Bash script that calls the crowd simulation code written in Python. For reproducibility, all codes are published von Schantz (2020). The procedure has been run on the Aalto University high-performance computing cluster Triton. A single generation of the GA has been run in parallel on Triton using its computing nodes that are Intel Xeon X5650 2.67 GHz with 48 GB or 96 GB memory, and Xeon E5 2680 v2 2.80 GHz with 64 GB or 256 GB memory.

Simulation of one generation takes a maximum of 10 min. The algorithm converges in 27, 34, 58 and 32 generations for 1, 2, 3 and 4 guides, respectively. In computation time, this is approximately 4 h 30 min, 5 h 40 min, 9 h 40 min and 5 h 20 min. It is not surprising that when we increase the amount of guides, i.e., optimization variables, the computation time increases. However, with 4 guides, the computation time decreases again. This could be related to the fact that there is only one Pareto-optimal solution with 4 guides, as we will see soon.

6.2. Near-optimal evacuation plans

Note that the GA is a heuristic rather than an exact solution algorithm, thus it is appropriate to call the solutions near-optimal instead of optimal. However, for readability, we drop the prefix *near-*. We can deduce from the problem setting that if there is no restriction on the number of guides, the solution would be to take the agent groups to their nearest exits, which can be done using 4 guides. This results in minimum evacuation time for all scenarios. The algorithm is able to find the 4 guide optimum; see Fig. 6. So, we are confident that the other obtained solutions are also optimal.

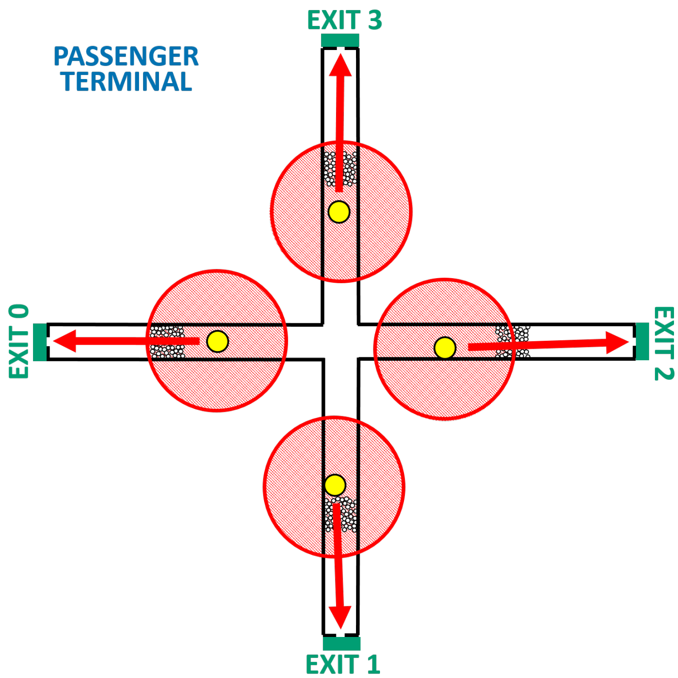


Figure 6: The solution that minimizes both CVaR_α and mean evacuation time is obtained with 4 guides. The red circles represent the interaction range of the guides, and the red arrows their paths to their destination exits.

The obtained Pareto-optimal solutions, or Pareto fronts, for different number of guides are all presented in Fig. 7. For 1, 2 and 3 guides, there are multiple solutions, which means that there is a tradeoff between minimizing CVaR_α and mean evacuation time. On each Pareto front, the solution down on the right minimizes CVaR_α, and is called CVaR_α-optimal solution. Whereas, the solution up on the left minimizes the mean evacuation time, and is thus called the mean-optimal solution. We can see that the GA only finds a few solutions per Pareto front. Actually, there exists more solutions, and if the algorithm was able to find them, the fronts would look more dense. By tuning algorithm parameters, we can obtain more solutions. However, we do not find it necessary, as we will see below, the CVaR_α- and mean-optimal solutions already characterize all solutions on a Pareto front.

Let us study the optimal evacuation plans for $1, 2$ and 3 guides more closely. Figs. 8, 9, 10 present schematic diagrams of them. In each figure, the left subfigure represents the CVaR_α-optimal evacuation plan, while the right figure represents the mean-optimal evacuation plan. The figures depict the initial situation, when using these plans. The red circles describe the interaction range of the guide, and the red arrow describes the path the guide takes to its destination exit. Along the path, the guide interacts with agents within the interaction range, and these agents are led to the guide's destination exit.

Note that the exit assignments are the same both in the CVaR_α- and mean-optimal solutions (see Figs. 8, 9, 10).

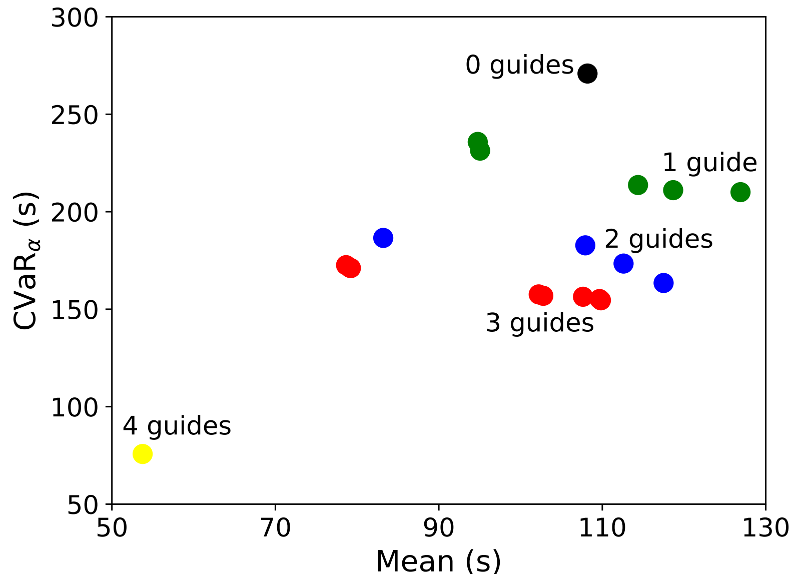
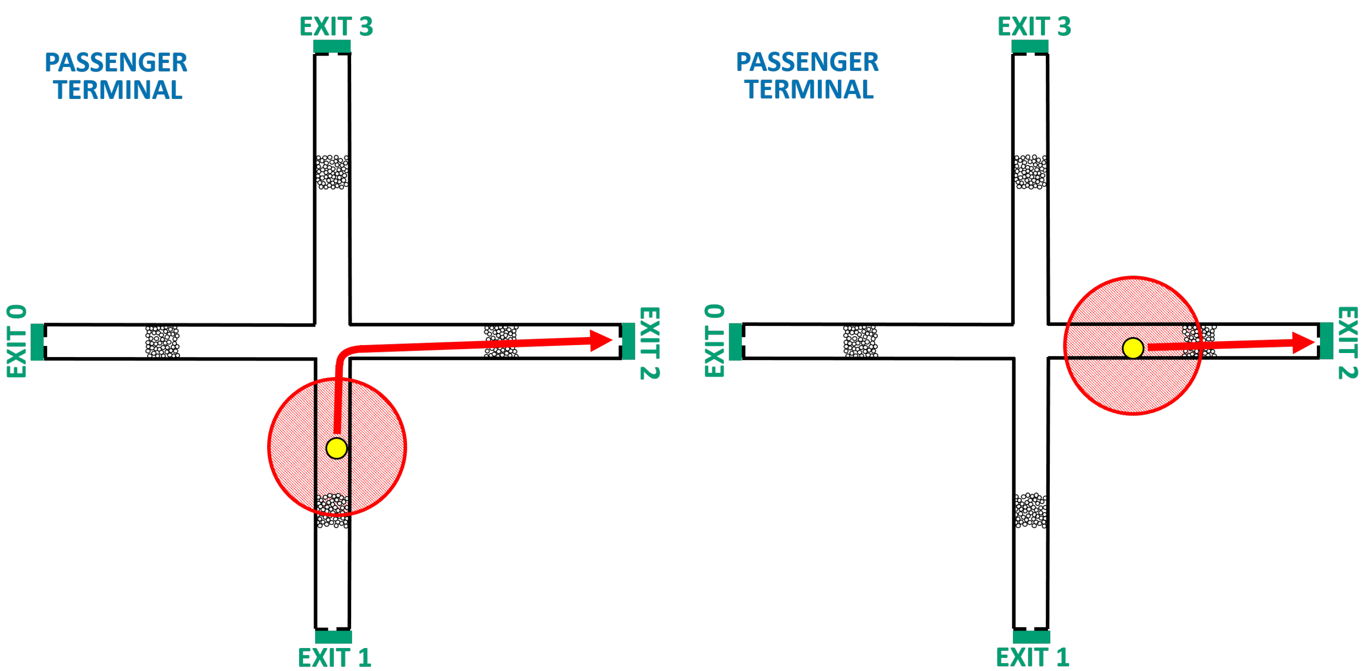


Figure 7: Pareto fronts for different number of guides.

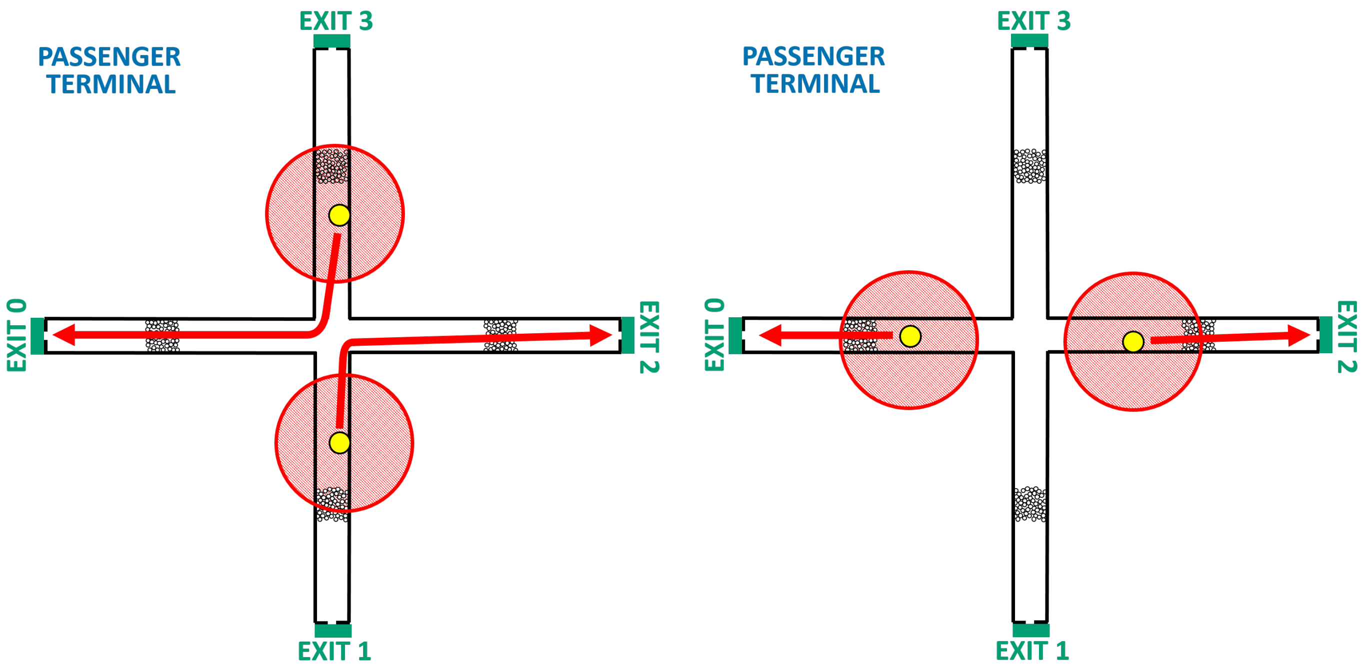


(a)

(b)

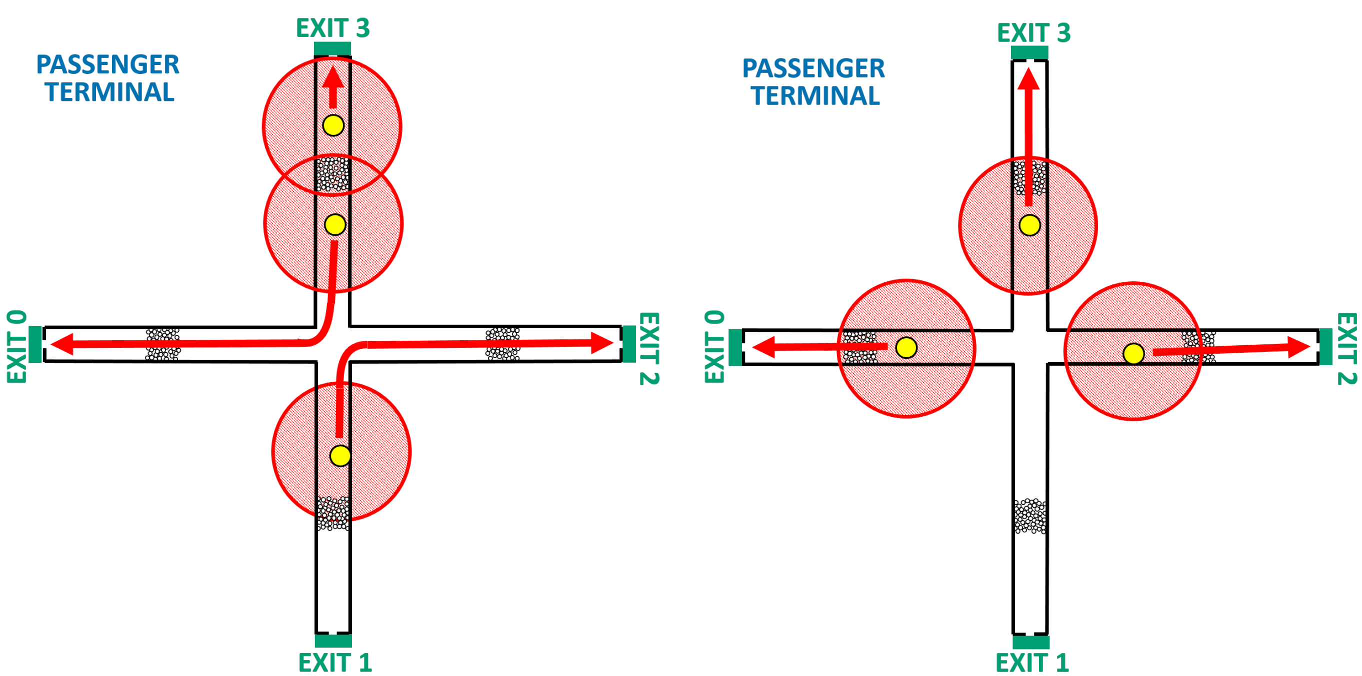
Figure 8: (a) CVaR$_\alpha$-, and (b) mean-optimal evacuation plan with 1 guide.

But, initially, in the mean-optimal solutions, the guides are closer to their assigned exits. As it is shown above in Fig. 7, there are multiple Pareto-optimal solutions for a fixed number of guides. When we monitored the solutions more closely, we noticed that moving on the Pareto front from the CVaR$_\alpha$- to the mean-optimal solution, the guides' initial positions are set closer to their destination exit. The only exception is the mean-optimal solution in Fig. 10(b), where the upper guide is positioned a little farther away in the mean-optimal solution.



(a) (b)

Figure 9: (a) CVaR_α-, and (b) mean-optimal evacuation plan with 2 guides.



(a) (b)

Figure 10: (a) CVaR_α-, and (b) mean-optimal evacuation plan with 3 guides.

But, this does not matter, since it still influences the upper group of agents.

If we think about the unguided situation, the slowest evacuation occurs in Scenario 2, when agents are just arriving, and are heading to the exit gates on the opposite end of the hallway. This results in a four-way counterflow and large congestion at the intersection. Hence, the main feature of CVaR_α-optimal solutions is to decrease congestion effects at the intersection. In the most probable Scenarios 1 and 4, the agents head to the nearest exits even without guides. Hence, when minimizing the mean evacuation time, each added guide is positioned close to an exit, until there is one guide close to each exit.

6.3. Congestion at the intersection

Let us analyze the CVaR_α-optimal solutions in the worst-case scenario, Scenario 2. With 1 guide, the guide starts from the lower hallway and takes a large part of the lower group to Exit 2. On the way, it encounters the right group and reroutes it to Exit 2. Still a small part of the lower group, the left and the upper group create a congestion at the intersection (see Fig. 11(a)).

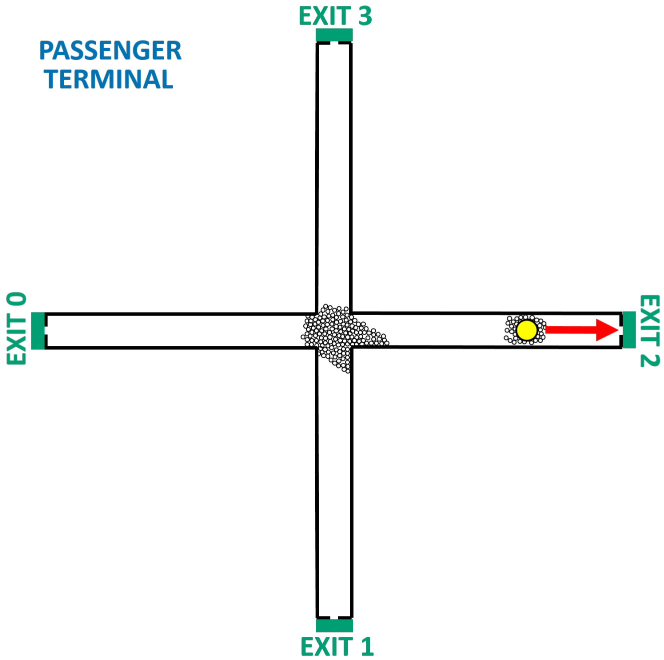
With 2 guides, the congestion at the intersection is completely cleared. As with 1 guide, one of the guides starts from the lower hallway and takes a large part of the lower group to Exit 2. The guide also reroutes the right group to Exit 2. The second guide, starts from the upper hallway, and takes the upper group to Exit 0. The guide also reroutes the right group to Exit 0. The upper and lower groups do not collide since they move along the walls at the intersection (see Fig. 11(b)). With 3 guides, we notice the same features as with 2 guides. In addition there is the third guide positioned in the upper hall that influences part of the upper group members, and takes them to Exit 3. This results in fewer agents going to Exit 0, which improves the evacuation slightly (see Fig.11(c)). With 4 guides, the agents are all taken to their nearest exits (see Fig.11(d)).

6.4. Comparison of evacuation times

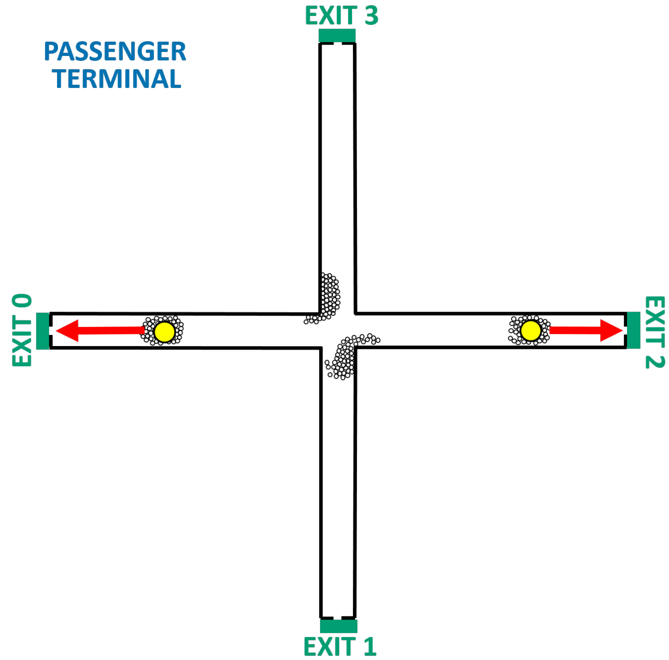
In Fig. 12 we see the evacuation times of different scenarios for CVaR_α- and mean-optimal solutions. First, notice that adding guides always decreases evacuation time for Scenario 2. It is the worst-case scenario, so decreasing its evacuation time improves both objective function values. For mean-optimal solutions, Scenarios 1 and 4 are unaffected by an increase in the number of guides. But, for mean-optimal solutions, the evacuation time of Scenario 3 is decreased for each added guide.

For CVaR_α-optimal solutions, the results are not so straightforward. Using only 1 guide actually worsens the evacuation time of Scenario 3. The reason being that the guide is slower than the passengers, and it reaches the intersection when all agents are jammed there, and influences them all to go to Exit 2, and creates a further jam there. Also, the evacuation times of Scenarios 1 and 4 get worse from adding guides, and are the slowest with 2 guides. With 4 guides, the evacuation times are again the same as with 0 guides. The reason for this is quite clear: the agents evacuate optimally in Scenarios 1 and 4 already without any guides, so adding guides elsewhere than close to exits, will worsen the evacuation time.

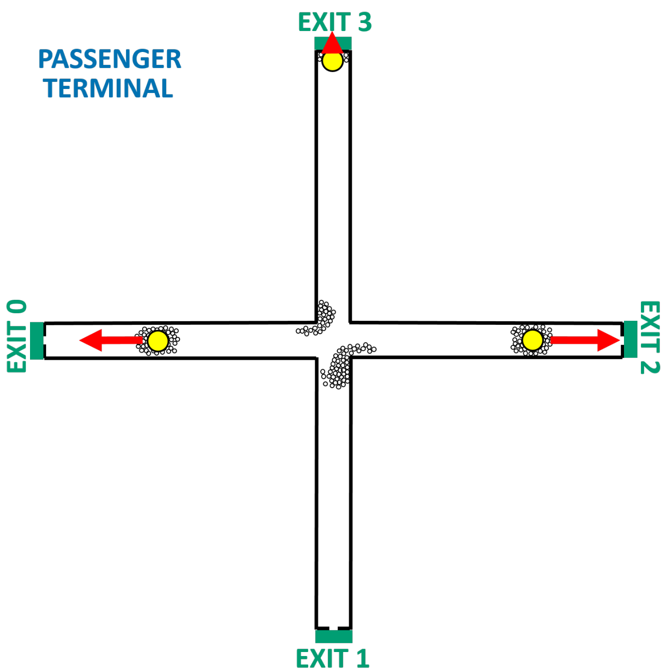
When we monitored the simulations closely, we noticed that on a Pareto front, when moving from the CVaR_α- to the mean-optimal solution, the evacuation time of Scenarios 1 and 4 is improved, and the evacuation time



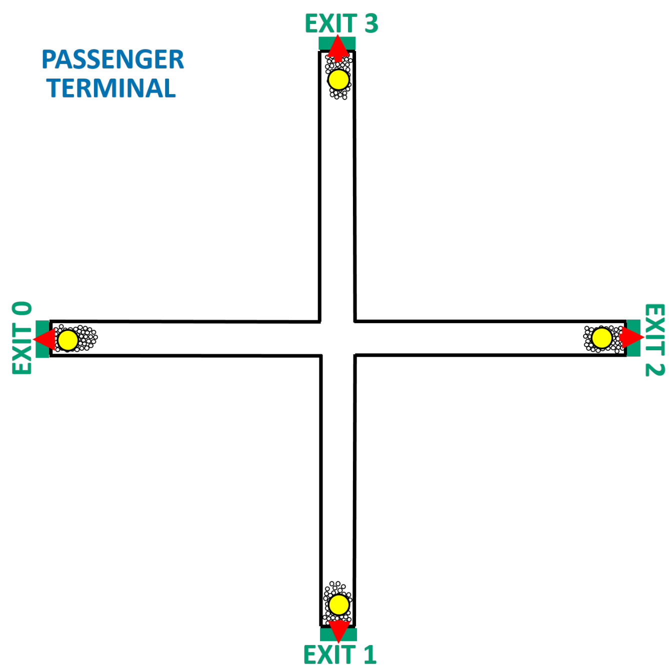
(a) Scenario 2



(b) Scenario 2



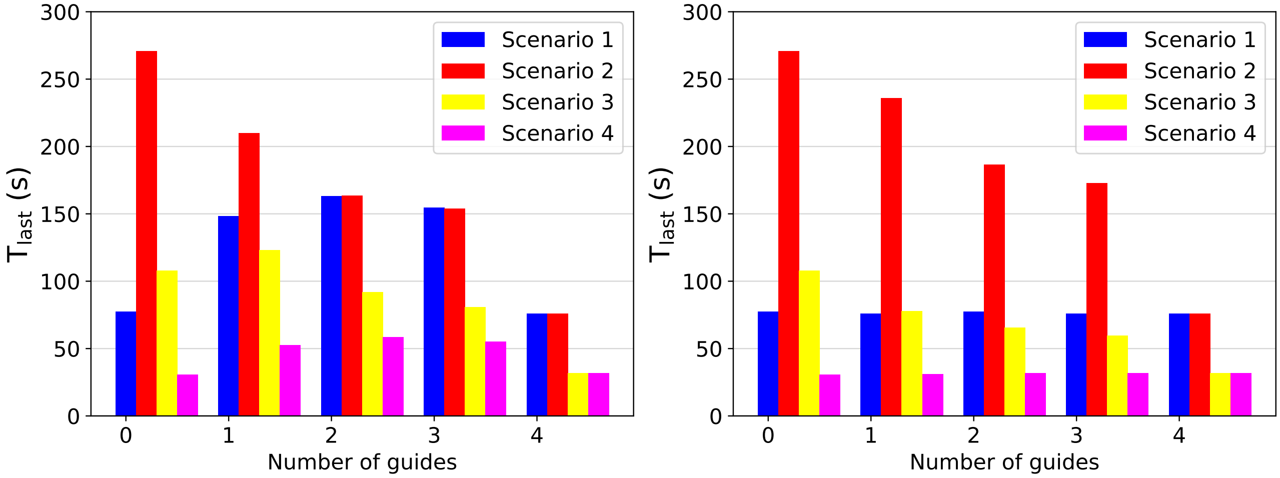
(c) Scenario 2



(d) Scenario 2

Figure 11: Snapshots of the evacuation of the worst-case scenario, Scenario 2, with CVaR_α-optimal evacuation plans: (a) 1 guide, (b) 2 guides, (c) 3 guides, and (d) 4 guides. The interaction ranges are not drawn in this scheme.

of Scenario 2 is worsened. For Scenario 3, the optimum would be a solution that is between the CVaR_α-, and mean-optimal solution on the Pareto front.



(a) (b)

Figure 12: T_{last} (evacuation time) under different scenarios: (a) CVaR$_{\alpha}$-optimal solutions, (b) mean-optimal solutions.

6.5. *Effect of model parameters*

For a comprehensive numerical and experimental analysis on the effect of physical model parameters on the congestion at the intersection we refer to our study Heliövaara et al. (2012a).

Because the geometry and crowd distribution is symmetric, in all four directions from the intersection, we can rotate the near-optimal solutions 90, 180, or 270 degrees clockwise or counter-clockwise. They can also be rotated 180 degrees around the horizontal, vertical, diagonal and cross-diagonal axes. Performing such operations can be thought to be part of the sensitivity analysis of the exits used. Otherwise, the initial positions of the guides can be changed as long as they still influence the same agents. However, if the initial positions or exit assignments are changed otherwise, it worsens the solution.

By afterwards running simulations with changed guide parameters, we found that CVaR$_{\alpha}$-optimal solutions could be further optimized, by moving the guides closer to the intersection, increasing their interaction range, and increasing their speed slightly. For 1 guide, the guide would take a small part of the upper and bottom group members and lead them to Exit 0. The guide would start closer to the right group, thus it would be able to lead its members faster to Exit 2. For 2 and 3 guides, we would get the same benefit, the guides would be able to faster lead the right and left group members to their near exits.

Also, the faster the guides are, the more agents they are able to inform about the optimal evacuation routes. However, it is not the absolute speed that matters, but the relative speed with respect to that of passengers. If the guide's speed is set higher, the CVaR$_{\alpha}$-optimal evacuation plans would be optimal for both Scenarios 2 and 3, when they now are only optimal for Scenario 2. In a more sophisticated model, the speed of the guides could be an optimization variable.

Even though our aim is not to optimize building design, let us analyze how the building dimensions and the crowd distribution affect the evacuation times. The larger the area the agents are distributed on, the more guides are needed to control them. Longer halls result in longer walking times, and shorter halls have the opposite effect. Narrower halls and exits increase queuing time at the intersection and exits and wider decrease it, respectively. The crowd spread over a larger area, or smaller number of agents, reduces physical contact forces, and thus reduces congestion and queuing time. A more densely packed crowd, or larger amount of agents has the opposite effect. If we change asymmetrically the dimensions of some hall, or initial positions of some agent group, the groups of agents will not meet at the same time at the intersection. Thus, it reduces the counterflow at the intersection. To conclude, if congestions are reduced with proper building design, the CVaR_α- and mean- optimal solutions will start to coincide.

7. Discussion and Conclusion

In this paper, we model the movement of a crowd consisting of passengers and guides with a modified social force model. A guide follows routes instructed by the evacuation planner. Whereas, a passenger goes about its business in the terminal unless a guide comes within an interaction range from it and leads it to an exit. The uncertainty of the crowd conditions are described as probabilistic scenarios. They are modeled by altering some of the model parameters. We formulate the problem of minimizing crowd evacuation time under different scenarios using rescue guides as a bi-objective scenario optimization problem. The two objectives are the mean and CVaR_α of evacuation time over the scenarios.

Then, a solution procedure combining numerical simulation and the NSGA-II algorithm is presented. It returns the set of nondominated evacuation plans. The procedure is applied on an evacuation of a fictional passenger terminal. If the number of guides is not restricted, we know intuitively that assigning one guide per exit, and positioning them behind the agent groups near those exits, optimizes the evacuation for all scenarios. The algorithm is able to efficiently converge to this solution, which assures us about its adequacy.

The research question of this paper is what happens to the optimal evacuation plan if there is a possibility for a large part of a crowd to deviate from its usual behavior. In our example case, for a fixed number of guides less than four, there is a tradeoff between minimizing the mean evacuation time or CVaR_α. With four guides, we obtain a single solution that is optimal for all scenarios. A similar result was found in our recent paper von Schantz & Ehtamo (2020). Although, there the uncertainty is not in the model input parameters, but in the equations of motion.

Often rules about the optimal proportion of guides are being presented in studies and in federal guidelines Cao et al. (2016); Hou et al. (2014); Ma et al. (2016); Wang et al. (2015); McCormack & Chen (2014); Ma et al. (2017). We do not think such rules by themselves to guarantee an efficient evacuation other than in very simple situations. Rather, the evacuation is a function of the building geometry Still (2000), the initial crowd distribution and behavioral conditions.

In Wang & Cheng (2012) it was found that near-exit positions are good, and in Hou et al. (2014) that for building geometries with multiple exits, the guides slow down the evacuation, unless all exits are utilized. These findings coincide with our results, given that average performance over scenarios is optimized. Our mean-optimal solutions use a near-exit strategy, where guides are positioned near their destination exits. Maybe a more useful quantity than the initial position is the proximity of a guide to the crowd. In Aubé & Shield (2004) it was found that simultaneously positioning guides inside the crowd, on its periphery and at a distance from it improved evacuation time. However, in the study, the aim was to evacuate a large crowd using the same route. In our study, since the agents groups are relatively small, it is enough to locate one guide per group within the interaction range.

If there is a possibility for a large congestion, and we decide to optimize worst-case performance, a different strategy is needed, as our CVaR_α-optimal solutions show. The guides focus should be on solving the congestion by moving parts of the crowd in it elsewhere. Most preferably, they should be moved before the congestion occurs.

Even though, in our toy example, CVaR_α equals the evacuation time of the worst-case scenario, this does not generally happen. In fact, optimizing the worst-case scenario instead of CVaR_α, when there is a large, or infinite, number of scenarios, is an ill-defined problem. Even if it could be precisely defined, it can lead to an overly conservative evacuation plan.

In our study, we minimize the evacuation time. It could be interesting to see how the optimal evacuation plans change if there is a time limit within which the crowd should be evacuated. For example, in fire safety literature, a distinction is made between required safe egress time (RSET) and available safe egress time (ASET). RSET defines the time it takes to evacuate the crowd, and ASET the time before the conditions become lethal Heliövaara et al. (2013); von Schantz & Ehtamo (2019). If RSET is less than ASET, the evacuation is efficient. This could be simply implemented by changing the objective function.

In future research, it would equally well be interesting to try our model on a real-world case with real data. Our solutions are qualitatively fairly insensitive to variations in model parameters, especially in the guide agent parameters. The building geometry we use can be found in many buildings, hence not only our mathematical framework, but the crowd management strategies can be used for more general cases. Also, since many buildings and evacuations share similar features, the solution process can be automated, by solving optimal evacuation plans for typical situations and storing them, and then training a neural network to give fast approximate optimal evacuation plans for previously unsolved problems.

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