Aalto University
MS-E2177 - Seminar on Case Studies in Operations Research

Final Report

Optimal investment strategy for nonlinear life insurance liabilities

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1 Introduction

1.1 Background

The Fennia Group consists of Fennia, Fennia Life, and Fennia-service Ltd. Together, the companies offer their clients competitive and high-quality non-life and life insurance services. Fennia's insurance activities date back to the 19th century, and the Fennia Group provides various risk management and insurance services for enterprises, their personnel, entrepreneurs, and private households, as well as flexible client financing solutions.

Fennia offers clients non-life property and casualty insurance products, and subsidiary Fennia Life offers both savings and risk life insurance products. Investments of both insurance companies are managed using a group-level asset and liability management strategy that separates assets into a hedging portfolio that is backing the long liabilities and an investment portfolio that consists of excess capital.

The hedging portfolio's goal is to generate returns required by liabilities as well as replicate liability market risks to minimize risk coming from covering liabilities. This way, the excess capital in the investment portfolio can use the majority of the group's risk capacity with the sole goal of accumulating the group's own capital.

While market sensitivities of the parent company's products are straightforward, Fennia Life has offered more complex voluntary savings and pension products with a guaranteed minimum rate of return for savings and an option to get extra benefits if the current interest rate level exceeds the guaranteed rate. To meet these promises, the assets of the company must generate returns that match the guaranteed rate and the option in all scenarios. In addition, liabilities are valued with market interest rates, and the value of assets must match or exceed the market consistent present value of future cash flows for the company to stay solvent in the regulatory framework.

Moves in market interest rates affect liability value significantly. Lower interest rates increase the present value of guarantees that essentially behave as fixed-rate bond instruments. On the other hand, higher interest rates, as well as expected volatility of rates, increase the expected value of extra benefits paid to policyholders and the option present value. The value of the guarantee and option combined form the market-consistent value liability that needs to be covered with suitable financial instruments. Due to the complexity of policy terms, no financial instrument can fully replicate the sensitivities of liabilities, and the hedging/investment strategy needs to be planned carefully to optimally balance the return expectations and risks for the company's own capital as well as transaction costs.

1.2 Objectives

The main goal of the project is to form an optimal investment or hedging strategy for Fennia Life for liability matching. As the behaviour of the liability cashflows is complex, finding the correct assets to replicate the cashflows is key. The excess return of assets compared to liabilities should be maximized while the risk to the own capital of the company (adverse mismatch between asset and liability values) should be minimized.

The project aims to build a framework of hedging against this type of nonlinear liabilities. To identify mismatches, we build a model to simulate the expected return and mismatch of a multitude of insurance policies and hedging instruments under an investment or hedging strategy. This model should take simulated yield curve paths as arguments for the insurance policies and hedging instruments, and be able to balance the assets and liabilities in the balance sheet for optimal results.

The goal for the model implementation is to be a clean and cohesive program, that can be easily run with different data and be expandable or adaptable to other similar problems with low effort. The code should be high quality including comments, and its development should be documented using version management.

The investment and hedging strategy should be based on literature reviews, client input and team analysis. Literature review is also used to analyse the reliability of the created framework. Validating the use of hedging products as well as commentating on the yield curve simulations is in scope.

2 Literature review

2.1 Simulating yield curves

Modelling the structure of interest rates is important in both theoretical and practical side of simulating the development of other financial instruments as it depicts the price of money. The difficulty with the task lies on the nonlinear and dynamic nature of interest rates. There is also a relationship between interest rates and economic cycles and other macroeconomic variables, mainly the government monetary policies to control inflation. [1]

The authorities typically use a Monetary policy rate (MPR) to set a baseline for all other local interest rates. They also issue "risk-free" debt in different denominations and maturities. The implicit zero-coupon rates for these risk-free rates form a *yield curve*, which is one of the most important economic indicator. [2] [1]

Both researchers and market participants are interested in developing an understanding of the relationship between economic cycle variables and interest rates. Theoretical approach includes studying the properties of the yield curve such as slopes at different maturities, whereas empirical approach includes econometric measures and knowledge-discovery techniques in order to model the structure and dynamics of the yield curve. The latter is important as econometric measures, such as the unemployment rate, are easier to estimate. However, the limitation of empirical techniques is that the results are difficult to interpret due to the abundance of non-linear patterns. [1]

Using stochastic modelling to study interest rate development has it pitfalls. Stochastic interest rate simulations might overweight the probabilities of the unrealistic interest rate scenarios, thus making analysis using them unreliable, or they might not capture all possible outcomes. [3] However, simulated interest rates have the benefit of being drawn from a distribution. They are consistent with the market and therefore they can thus be used to price assets over all simulations.

While there are many ways to model the behaviour of the yield curve, the results may be hard to interpret and they may not cover all possible outcomes. Thus when analyzing the development of portfolio with respect to interest rate changes, special care needs to be taken to be sure that the limitations of the simulations are understood.

2.2 Hedging strategies

Hedging a portfolio of liabilities has been studied extensively. The idea is to hedge the portfolio by mimicking the movement of liability value with the changing market environment, in our case interest rate regime [4]. The hedging can be done in continuous time as presented in [5], where the optimal investment strategies for funding ratio optimization portfolio and dynamic surplus optimization are stated at closed form solutions. As these results are however quite technical, we will analyze liability hedging strategies on a more general level.

Understanding the liability behaviour is the most important step to selecting suitable hedging strategy and products. To match the liability behaviour in different interest rate regimes, the understanding of the liability structure is crucial. [6]

An important part of creating a hedging strategy is to select suitable hedging products to hedge a liability which has a non-linear dependency on interest rates. Fixed income products, such as bonds, seem reasonable choice, and do in fact work in an environment of parallel shifts in interest rates. The problem arises when there are non-parallel shifts in the yield curve (i.e. changes in slope). Then only fixed income products are not sufficient to hedge against the change of value of non-linear liabilities. [4]

Derivatives of fixed income are needed to create a sufficiently good hedge. Bond futures, floating rate bonds, interest rate swaps and swaptions provide customization and possible leverage to the hedging portfolio. [6]

No hedging strategy can usually provide a perfect match to the liabilites with nonlinear interest rate dependencies. There are real world challenges such as corporate bond issuance, transaction costs, and liquidity, which can be managed, but not completely eliminated. Thus focusing to bring the risk to suitable level instead of trying to create a perfect hedge should be the main focus. [6]

3 Data and methods

3.1 Liabilities

The liabilities to be matched are voluntary pension products with a minimum guaranteed rate of return. The products were sold at the start of 2000's when the prevailing interest rate environment was much different from that seen today. Thus the minimum guaranteed rates promised are quite high even compared to today's interest rates. As these products are no longer offered to customers due to the difficulty in pricing them, we know that no new sales are made.

The liabilities work as a savings account which accumulate interest based on the market conditions, but the interest rate must be at least the minimum guaranteed rate. Thus the rate paid to customer is

$$r = \max(r_f,\ r_g), \tag{1}$$

where r_f is the market rate to be used by the product and r_g the minimum guaranteed rate. At the maturity of the product (i.e. after 15 years), the savings will be paid to customer.

As the products are pension plans, there is a possibility of the policy holder dying before the maturity. In this case, the money accumulated including the original amount will be paid to the policy holders relatives. In our analysis, we assume that the yearly probability of death is 1%, which is an accurate estimate on the whole policy holders scale.

The liabilities can be thus modelled as yearly cash flows to be paid out. At start $t = 0$ an amount S is invested and it accumulates yearly interest using interest rate defined by Eq. 1. At each consequent time-step $t = 1, 2, \ldots, T$, the cash flow to be paid is the probability of death times the accumulated savings, i.e. $cf_t = 0.01 \cdot S_t$. The savings must then be multiplied by the probability of no death $\mathbf{P}(\text{no death}) = 1 - \mathbf{P}(\text{death}) = 0.99$. At time step T, the cash flow is the full accumulated savings.

The non linearity of these products can be illustrated by considering the present value of a cash flow of face value 100 which accumulates interest r to be paid in one year. The present value is

$$P = \frac{100(1+r)}{(1+r_1)}, \tag{2}$$

where r is the interest rate defined by Eq. 1 and r_1 is the one year spot rate. Assuming that the product is tied to the one year spot rate, if $r_f > r_g$, the present value of the product is just the face value $P = F = 100$. However, if $r_f < r_g$ then $\dfrac{(1+r)}{(1+r_f)} > 1$, and thus $P > 100$. This behaviour cannot be achieved with any fixed income product, which is why more complex hedging products must be considered.

3.2 Assets

We will consider the following assets to use for the hedging portfolio: zero-coupon bonds, variable rate bonds, swaps and swaptions. We will present the cash flow profile and valuation of each of these assets.

3.2.1 Zero-coupon bonds

A zero-coupon bond is a fixed-income security that can be bought at the present time $t = 0$ at price P and at maturity T will give a payoff of the face value $F > P$ [7]. Thus the cashflow vector for this instrument is

$$\bar{x} = [\underbrace{0, \ldots, 0}_{T-1 \text{ times}}, F]. \tag{3}$$

The yield-to-maturity (here on yield) of a zero-coupon bond is the value λ such that for interest compounded m times in n periods, the price of the bond is

$$P = \frac{F}{(1 + \frac{\lambda}{m})^n}. \tag{4}$$

Thus knowing the face value, yield and compounding method, the price of a zero-coupon bond can be calculated using Equation (4).

An important measure for interest rate sensitivity of bonds is duration, calculated as

$$D = \frac{PV(t_0)t_0 + PV(t_1)t_1 + \ldots + PV(t_n)t_n}{PV},$$

where $PV(t_i)$ is the present value of cashflow of time t_i and PV is the present value for the full cash flow. For a zero-coupon bond we have

$$D = \frac{PV(t_0)t_0 + PV(t_1)t_1 + \ldots + PV(t_n)t_n}{PV} = \frac{PV(t_n)t_n}{PV} = t_n,$$

as we have only one cashflow at t_n.

Bonds can also pay coupons, which are intermittent payments paid at each timestep from $t = 1, \ldots, T$. These bonds are called fixed coupon bonds and their price is

$$P = \frac{F}{(1 + \frac{\lambda}{m})^n} + \sum_{k=1}^{n} \frac{C/m}{(1 + \frac{\lambda}{m})^k},$$

where C is the coupon payment.

Coupon-paying bonds behave similarly to zero-coupon bonds with respect to changes in the yield curve. Their duration is less than the maturity $D < T$, as there are payments done before maturity. Coupon-paying bonds are much more common instruments, but as they are slightly more complicated as the reinvestment of the coupons would have to be taken into account, we only consider zero-coupon bonds without loss of generality. As we have multiple zero-coupon bonds with different maturities in our hedging portfolio, optimizing the portfolio will provide the optimal duration of the portfolio even with zero-coupon bonds.

3.2.2 Variable coupon bonds

Variable coupon bonds or floating rate bonds have a fixed face value and maturity, but the coupon payments are tied to the current short rate [7]. The coupon rate is reset at predefined points to match the spot rate. The present value of such an instrument is at any reset point the face value of the bond, in other words

$$P = F.$$

The proof of this can be found in [7], but the basic idea is that the coupon at each reset point is discounted with the corresponding discount rate $d_i = \frac{1}{1+r_i}$, and as the coupon payment of that reset point is tied to the same rate r_i, the present value stays fixed.

With variable rate bonds, the reinvestment of coupon payments have to be considered in the hedging framework. However, the yield tied to the spot rate is a versatile tool in our hedging portfolio, thus implementing this is important.

3.2.3 Interest Rate Swap

Interest rate swap is a financial contract in which two parties agree to exchange one stream of interest payments for another, based on a specified principal amount. These are typically used to manage exposure to fluctuations in interest rates or to obtain a marginally lower interest rate than would be possible otherwise. The most common type of interest rate swap is a fixed-for-floating rate swap, where one party agrees to pay a fixed rate, while the other pays a floating rate linked to an index such as LIBOR or EURIBOR. [8]

While a swap is inherently a linear product, we aim to match the non-linearity of the payouts introduced by the guaranteed rate with a fixed-for-floating interest rate swap. In our approach, we enter into a fixed-for-floating interest rate swap, receiving a fixed rate that matches the guaranteed rate while paying a floating rate that aligns with the rates discussed in the subsequent chapter. Essentially, the interest rate swap serves as a hedge against falling interest rates. By owning variable coupon bonds with coupon rates that correspond to the paid floating rate, we secure a fixed income from the swap's fixed-rate payments, which offsets the impact of the guaranteed rate and get to enjoy the higher floating rates.

In (3), the cashflow for paying a floating rate and receiving a fixed rate is the vector:

$$\bar{x} = [0, x_1, x_2, \ldots, x_T]$$

where each x_t for $t = 1, 2, \ldots, T$ is

$$x_t = N \cdot (r_f - r_{v,t})$$

Here, N represents the notional value, r_f is the fixed rate received, and $r_{v,t}$ is the variable (floating) rate paid at time t. The swap is then valued by taking the Net Present Value of the future cashflows which can be calculated as

$$\mathrm{NPV}_{\mathrm{swap}} = N \cdot \left(\sum_{t=1}^{T} \frac{x_t}{(1 + r_{d,t})^t} \right),$$

where

- N is the notional principal amount of the swap,
- x_t represents the net cashflow at time t, defined as $x_t = r_f - r_{v,t}$, where r_f is the fixed rate received and $r_{v,t}$ is the floating rate paid,
- $r_{d,t}$ is the discount rate applicable for the time t, and
- T is the total number of payment periods in the swap agreement.

3.3 Data from Fennia

To study the development of the cash flows of liabilities and the hedging products chosen to match the liabilities, a simulation of the development of interest rate curves is needed. We have simulations of the yield curve development over time provided by Fennia.

The simulations cover two different possible methods to estimate the development of the yield curve. Method 1 is to use the market-consistent, risk-neutral interest rates to create the simulations. We have 1000 trials of such simulations, each of which consists of a 60-period simulation of the yield curve. The interest rates provided for the yield curve are the short rate, 1, 5, 10, 30 and 60-year rates. There are also a different scenarios for market shocks ranging from -1.5% to 1.5% to be used for pricing the liabilities under different shocks.

Method 2 consists of real-world simulations calibrated with economist opinions of the possible developments of the interest rates. For these simulations we have 5000 trials with 10 periods each. For this method's yield curve, we have the interest rates for 1-30 years.

3.4 Hedging framework

The optimal hedging strategy is built with the hedging framework. This framework takes provided market data, liabilities portfolio and available asset types as an input and outputs the optimal hedging portfolio with respect to minimizing the target function.

For a given set of liabilities the resulting cashflow vector for each market simulation $i \in I$ is denoted by $cf_i^{[l]}$. For each type of asset $a \in A$, cashflow vector is denoted by $cf_i^{[a]}$ for each market simulation $i \in I$. Then the resulting cashflow can be calculated as

$$cf_i = x^T cf_i^{[a]} + cf_i^{[l]},$$

where $x = (x_a), a \in A$ is a vector describing how many units of each type of assets we buy. The target function can be expressed as

$$\tau(x) = \mathbf{T}(\{\theta(cf_i(x))\}_{i \in I}).$$

Here the function θ translates the space of cashflows to the real numbers. The most simple and meaningful function of this form is

$$\theta(cf) = cf^T \mathbf{1} = \sum_{t=0}^{T} cf[t],$$

which is the total absolute cash difference at the end of the period. The function $\mathbf{T}$ then calculate the value of the target function based on the evaluations of cashflows for each simulation path. See the discussion on which functions $\mathbf{T}$ were used in the next subsection. Hence, the purpose of the hedging framework is to find

$$x_* = \arg\min_{x \in X} \tau(x),$$

where is X is a set of possible vectors x.

For this project, we have used two different definitions of X. First considered case is where cashflow at $t = 0$ must be positive, so $cf_i[0] > 0$. Note that cashflows at time $t = 0$ do not depend on market data, thus they do not depend on the $i \in I$. Second considered case where NPV of the resulting asset portfolio is matched to the given liabilities portfolio.

3.4.1 Target function

The vital part for evaluating the asset portfolio is the function $\mathbf{T}$ introduced in the previous subsection. We consider three different alternatives for the function $\mathbf{T}$:

1. The most simple case of the function $\mathbf{T}$ is the average of its arguments, that is,

$$\tau_{av}(x) = \mathbf{T}_{av}(\{\theta(cf_i(x))\}_{i\in I}) = -\frac{1}{|I|}\sum_{i\in I}\theta(cf_i).$$

   Here the value of $\mathbf{T}_{av}$ equals to the average loss among the simulation paths.

2. Another considered function is the 5th percentile of losses, that is

$$\tau_5(x) = \mathbf{T}_5(\{\theta(cf_i(x))\}_{i\in I}) = -\frac{1}{|I'|}\sum_{i\in I'}\theta(cf_i),$$

   where I' is the set of simulation paths on which the largest losses occur and $\frac{|I'|}{|I|} = 0.05$. This function is more sensible to lossesand applicable in order to develop a risk averse strategy.

3. Another risk averse strategy can be based on the function which takes average over losses, that is,

$$\theta_l(cf) = \sum_{t=0}^{T} cf[i]^2 \cdot \mathbb{1}_{cf[i]<0}, \quad \tau_l(x) = \mathbf{T}_l(\{\theta_l(cf_i(x))\}_{i\in I}) = -\frac{1}{|I|}\sum_{i\in I}\theta_l(cf_i).$$

3.5 Optimization algorithm

The optimization problem in Section 3.4 is solved using minimize-function from the Scipy-library for Python [9]. It solves the optimization problem using sequential least squares programming (SLSQP), which is a sequential quadratic programming method (SQP). In general, SQP methods are used on optimization problems in which the objective function and constraints are twice continuously differentiable but not necessarily convex.

The algorithm works by solving a sequence of subproblems, where the objective function is approximated by a quadratic model and the constraints are linearized. Given a nonlinear problem

$$\begin{aligned} \min . & f(x) \\ \text{s.t. } & h(x) \geq 0 \\ & g(x) = 0, \end{aligned}$$

and its Lagrangian

$$\mathscr{L} = f(x) + \lambda h(x) + \sigma g(x),$$

for an iterate $(x_k, \lambda_k, \sigma_k)$ the quadratic subproblem is

$$\begin{aligned}\min_d . \ & f(x_k) + \nabla f(x_k)^\top d + \frac{1}{2} d^\top H d \\ \text{s.t. } & h(x_k) + \nabla h(x_k)^\top d \geq 0 \\ & g(x_k) + \nabla g(x_k)^\top d = 0.\end{aligned}$$

Here H is the Hessian of the Lagrangian. Solving each subproblem for a direction d_k, such that $[x_{k+1}, \lambda_{k+1}, \sigma_{k+1}]^\top = [x_k, \lambda_k, \sigma_k]^\top + d_k$, an optimum can be achieved such that the parent problem passes a convergence test. Direction d_k is the such that the subproblem is minimized within the bounds of the constraint.

4 Results

The following results of our cashflow optimization have been computed with a Python script built for this purpose. It can be found on GitHub [10].

Section 4.1 presents the cashflow profiles of single-type asset portfolios. It also highlights the ineffectiveness of only using one type of asset for hedging purposes. Section 4.2 presents the optimization results of using a different mix of assets.

Table 1 presents the calculated mean, standard deviation and 5th percentile outcome for total cashflows out of all simulations. The results are shown in thousands. We use 2 different optimization objectives because for the third option (maximizing mean gain) the optimization algorithm malfunctioned. This is done to show their difference and to analyze the goodness of the hedge in these cases, i.e., how well the risks are minimized and payouts maximized. The results are also shown for different interest rate shocks from $-1,5\%$ to $1,5\%$. The interest rate shock is a shock affecting the risk-neutral scenarios just before buying the assets. Therefore, the interest rate shock impacts the NPV of the liabilities and thus the hedging result.

However, achieving a global optimum in optimizing for the 5th percentile of losses turned out to be challenging possibly due to the non-convexity of the objective function. This percentile measure is highly sensitive to outliers and extreme values, leading to an optimization space with multiple local minima. The sequential quadratic programming method used may struggle with this complexity, as it approximates the function by a quadratic model, which may not effectively capture the overall dynamics within the distribution's tails. Consequently, the algorithm may converge to local minima, as observed.

Optimization objective	Measure of total cashflows from all simulations	Interest rate shock						
		-1,5 %	-1,0 %	-0,5 %	0,0 %	0,5 %	1,0 %	1,5 %
Minimizing the sum of squared yearly losses	Mean	217	141	72	12	-33	-67	-90
	Standard deviation	75	81	89	99	108	112	110
	5th percentile	93	-1	-93	-180	-244	-288	-306
Maximizing 5th percentile gain	Mean	422	339	287	27	-9	-59	37
	Standard deviation	63	65	64	67	64	62	62
	5th percentile	317	242	189	-95	-103	-158	-69

Table 1: Measures of the results for different optimization objectives and interest rate shocks.

In Table 1, the interest shock has an expected impact on the hedging result. When the interest rate shock is negative, the liabilities tend to perform better against the market because of the guaranteed rate and thus the hedging is better. A good hedge is defined here as the total cashflows having a high mean and 5th percentile and a low standard deviation. The explanation for this result is that on low risk-neutral interest rates, the NPV of the liabilities is higher and we are able to buy more assets as we buy them according to the NPV of the liabilities. The real rates stay the same and the outcome is therefore better.

Optimization objective	Measure of total cashflows from all simulations	Guaranteed rate		
		2,0 %	3,5 %	6,0 %
Minimizing the sum of squared yearly losses	Mean	17	13	13
	Standard deviation	121	99	68
	5th percentile	-220	-179	-86
Maximizing mean gain	Mean	1597	5343	459
	Standard deviation	2218	6991	82
	5th percentile	-2561	-7263	330

Table 2: Measures of the results for different optimization objectives and guaranteed rates.

In Table 2, we present the same measures as in Table 1 but for different guaranteed rates from $2,0\%$ to $6,0\%$. In this instance, the optimization objectives "minimizing the sum of squared yearly losses" and "maximizing mean gain" are the ones where the optimization algorithm worked properly. We can again see anticipated results. The variation of the guaranteed rate does not produce a strong linear response in the results. However, even if in the second optimization objective the results do not have a linear correlation, we can see that in the first one, an increase in the guaranteed rate leads to slightly worse hedging results. This behaviour could be explained for example by the fact that with higher guaranteed rates the payoff of liabilities is determined by the guaranteed rate more often which increases non-linearity in the cashflows which further would lead to a more difficult hedge.

4.1 Linear asset hedging power

We first present the hedging results of only using linear assets, i.e. zero-coupon and variable coupon bonds. We use a guaranteed rate of 3.5% and try to minimize the sum of squared yearly losses.

In Figure 1 the optimal asset distribution without nonlinear assets is presented when optimizing for the sum of squared yearly losses. Mostly fixed coupon bonds are chosen for most of the maturities and variable coupon bonds are chosen for only 1 year maturity and 15 year maturity.

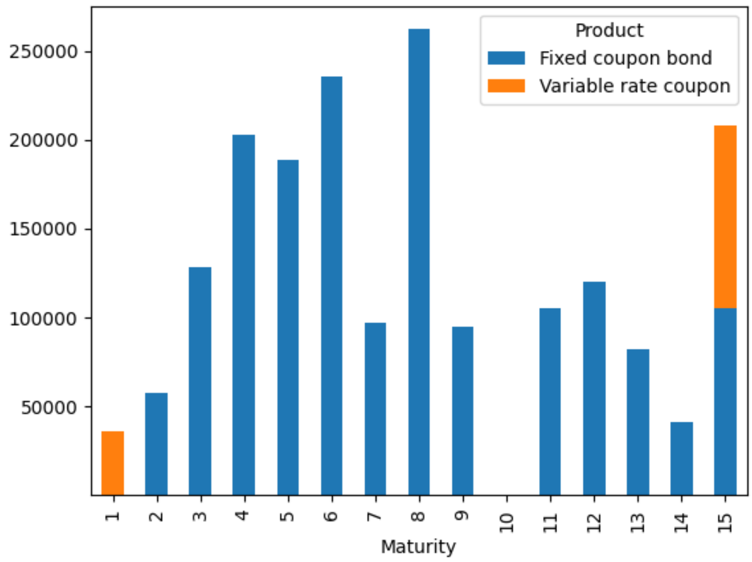


Figure 1: The distribution of assets when optimizing the sum of squared yearly losses.

Figure 2 shows the average cashflows across all simulations from the liabilities and the team's optimal asset portfolio. The payoff from the assets is mostly just slightly below the liability cashflow, but in the final year, the payoff is large enough to make the total net cashflow positive.

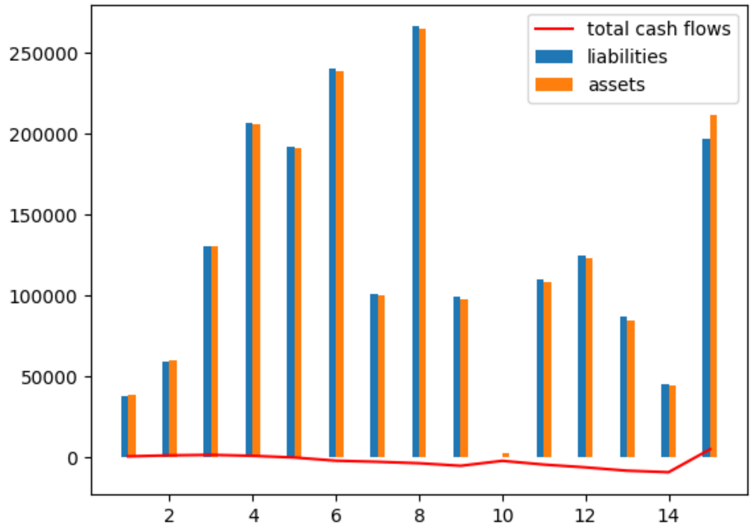
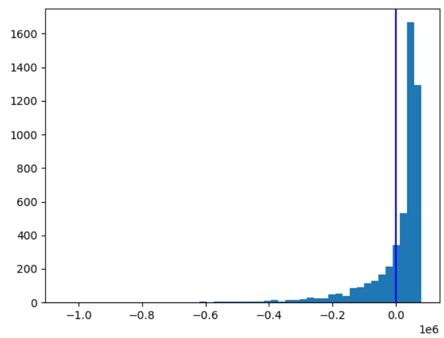
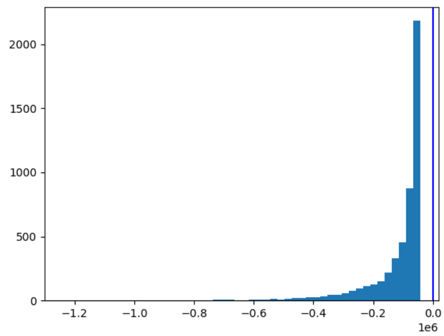


Figure 2: The average cashflows from liabilities and assets for each simulated year and the net total cashflow.

When assessing the goodness of the hedging portfolio, analysing the total cumulative cashflow distribution across all simulations is instructive. Figure 3 presents the distributions of the no-shock simulations and the corresponding 1.5% positive interest rate shock simulations are presented. Without an interest rate shock, optimizing for the sum of squared yearly losses yields relatively good results, but when a shock is present, the linear assets fail to provide sufficient hedging.



(a) No interest rate shock.



(b) 1.5% interest rate shock.

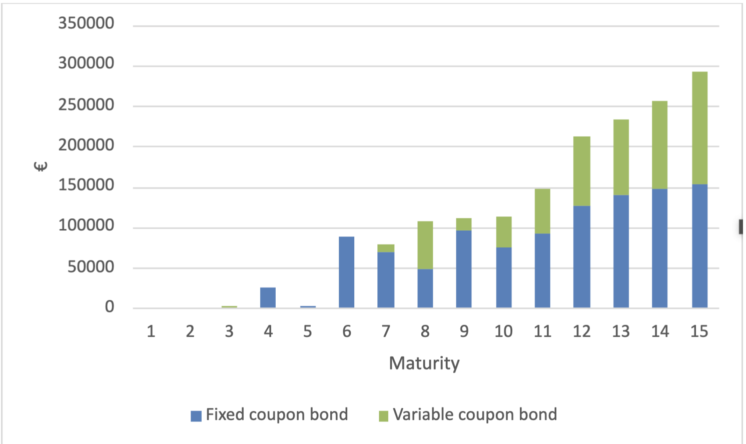
Figure 3: The distributions of the total cashflows for the simulations with and without interest rate shock. The shock highlights the ineffectiveness of using only linear assets for hedging.

4.2 Hedging with a combination of products

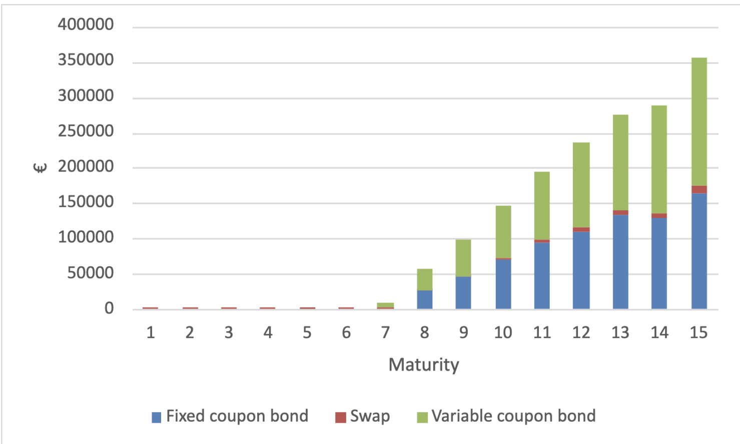
To examine the multi-asset portfolio with non-linear products available we added swaps with all possible maturities to the set of available assets. We considered cases for the guaranteed rate of the liability contracts, interest rate shocks and target functions.

In all considered combinations of parameters swaps do not become the most significant part of the portfolio, however introducing swaps helped to improve the resulting target function.

Firstly the non-linear assets (swaps) effect the optimal asset portfolio as follows



(a) Asset distribution without swaps



(b) Asset distribution with swaps

Figure 4: The optimal asset distributions with and without swaps. Swaps are the smallest part of the portfolio and do not change the shape of the asset distribution.

However, swaps are able to improve the value of a target function. This improvement depends on the choice of parameters such as guaranteed rate, interest rate shock and the target function. Below Table 3 shows the improvements that could be obtained by adding swaps to the asset portfolio.

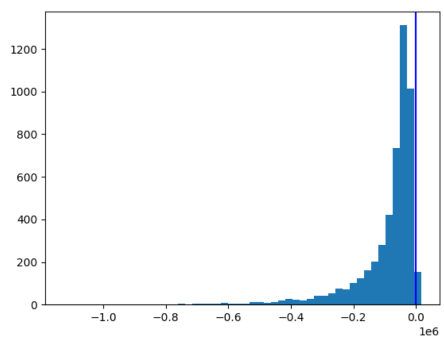
Interest Rate Shock	-1.5	-1	-0.5	0	0.5	1	1.5
Target Value (no swaps)	204	130	64	5	-45	-85	-117
Target Value	217	141	72	12	-33	-67	-90
Share of swaps	0.019	0.020	0.019	0.023	0.017	0.023	0.019

Table 3: Improvements achieved by introducing swaps to the asset portfolio.

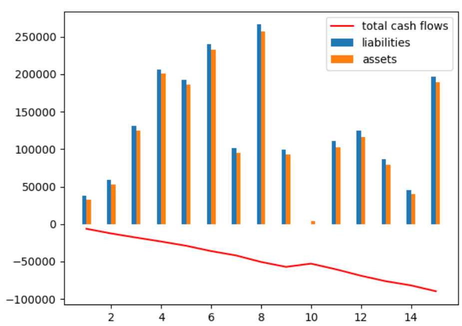
In the table above we considered the case of the target function being the mean average cashflow (first example of a target function in 3.4.1).

The swaps constitute a small share of the asset portfolio. However, one can see that we obtain a significant improvement in the value of the target functions.

Adding swaps slightly alters the distribution of total cashflows as the squared yearly losses method is applied. Figure 6b illustrates how a positive 1.5% interest rate shock impacts the distributions of the total cashflows of a portfolio made up of fixed and variable rate coupon bonds. Figure 5 shows a plot that includes swaps, along with the associated average cashflows.



(a) Total Cashflows Distribution



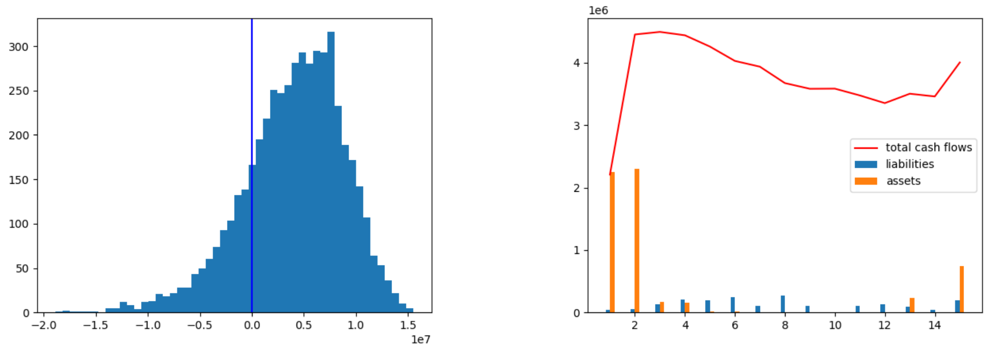
(b) Average Cashflows, Swap Inclusive

Figure 5: Swap inclusive sum of squared yearly losses method

The subpar performance persists, but the mean is marginally higher. The result was achieved by applying the same squared yearly losses method, which also yielded the results displayed in figure 6b.

As this optimization method performs poorly across all scenarios the more effective ones must be considered. For example, maximizing the mean cashflows appeared to provide an adequate hedge towards interest shocks to both directions.

For comparison, the figure 6 displays the total distribution and average cashflows for this method in a positive 1.5% interest rate shock environment.



(a) Total Cashflows Distribution

(b) Average Cashflows

Figure 6: Swap inclusive mean cashflow optimization

As one can notice, the hedging is much more efficient. This behaviour suggests that particularly under stressed conditions such as interest rate shocks, mean cashflow maximization provides better results than squared yearly losses minimization.

5 Discussion and conclusion

5.1 Optimal mix of assets

The strategy that yields the lowest returns is the squared yearly losses minimization strategy. However, if one seeks to strictly match liability and asset cash flows across time steps, the squared yearly losses minimization strategy yields overwhelmingly the best result.

Therefore, Fennia should consider their needs and solvency requirements in their hedging strategy. The squared yearly losses minimization strategy could provide with a safe baseline, with tactical bets to find better results. Fennia could also consider combining multiple strategies to yield a return profile that makes the most sense to them. However, it should be noted that there is not a single strategy that is consistently superior to another.

5.2 Potential issues with simulating yield curves

The results of this work are based on the simulated interest rate data provided to us by Fennia. Therefore, when analysing the results, one should be very mindful of the impact of assumptions behind the yield curve predictions.

The yield curves simulated using the risk-neutral data assume that the markets are complete, thus no arbitrage opportunities exist, and that the prices reflect all available information in the market [11]. These are the assumptions behind many pricing formulas for financial derivatives, as the price is formed as the expected value of the future payoff with this probability measure [12]. The markets, however, are never truly complete, as a shortage of traded assets can lead to prices that differ from the expected payoff [13]. The yield market is less affected by the shortage of assets due to its size, but it too is not completely shielded from market constraints.

The market-expectation-calibrated data is heavily affected by economists' analysis and expectations of future market developments. Even though many of the parameters used to build the yield curves can be estimated quite accurately, errors could occur in both the estimation and the modelling of the relationship of the parameters to the yield curve.

Neither risk-neutral simulations nor market-expectation-calibrated simulations take into account the possibility of the severe market shock that would lead to never-before-seen yield curves. From recent history, negative real rates are examples of situations that were thought to be impossible. Thus the simulations presented in this analysis should be understood as being the result of the best estimates of future interest rate development.

As the interest rates are drawn from a distribution, it is also important to keep in mind, that the number of simulations selected for the analysis heavily impacts the results. The law of large numbers guarantees that with a sufficiently large number of simulations, the simulations converge to the attributes of the selected distribution, but the individual simulations may still exhibit unusual behaviour.

5.3 Conclusion

The main purpose of the analysis presented in this report was to find an optimal asset mix to hedge a nonlinear liability portfolio. The approach taken included analysing the cashflows of the liabilities and assets under different yield curve regimes, which allowed for optimization of asset mix with respect to wanted attributes, such as minimizing negative cashflows or maximizing positive discounted cashflows.

The initial results showed, that in an optimal mix of assets, the inclusion of nonlinear assets improves the hedging power of a portfolio with nonlinear liabilities. Even though more analysis could be done with a wider range of even more complex products, these results alone help understand the requirements of a hedging portfolio. In future studies, one could strive to implement swaptions in the analysis, since they should offer a more cost-efficient alternative for hedging against extreme market eventualities.

One could also make the simulation more robust and faster to decrease simulation times and allow for more trials. The Sequential Least Squares Programming Approach was the best one our team had available, but one could aim to build or acquire an optimization algorithm with better convergence, allowing more reliable results to be computed.

As a large market player, Fennia has certain regulatory and strategic limitations which also can play a role in forming the actual hedging portfolio and are not considered in this project.

The most viable hedging portfolios for nonlinear liabilities for Fennia are include swaps, which outperform portfolios containing only fixed- and variable-rate bonds. Fennia should either consider a hedging strategy that minimizes the sum of squared yearly losses of negative asset cashflows, or alternatively a hedging strategy that maximizes the mean of the cashflows. For achieving minimum risk of mismatch between the cashflows, the squared yearly losses minimization strategy should be chosen.

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6 Self assessment

6.1 How closely did the project follow the project plan?

The project followed the project plan very closely from beginning to the end, with no major deviations. The scope was followed for the entire project and was not changed in any way during the project. At some points, the team contemplated expanding the project scope, but decided against it as the project drew to an end. For example, the ideas of adding swaptions or performing interest rate simulations ourselves was rejected as the team felt like these either didn't add much value to the project or were too hard to implement properly in the time frame.

While the timeline was followed quite closely for most of the project, in the final stages the model took slightly longer to validate and finalize than the team initially expected, and thus the team had to rush towards the end. This caused minor changes in the scheduling of the project towards the end. However, overall the project stayed well within the confines of the schedule provided, and the scope was kept unchanged for the entire project.

6.2 In what regard was the project successful?

The entire team concurs that the scope as it was initially defined was well-conceived. The team also found it as a major positive that the project scope remained unchanged throughout the entire project.

Furthermore, the team felt like another positive was in how the scope given by the client was transformed into a project plan. The idea of creating a framework for simulations was also seen as a positive.

Another factor contributing to the success of the project was the division of tasks among the group members, in which each team member got a chance to shine. The team had quite differing backgrounds ranging from management consulting to derivatives trading and research-oriented mathematics, and each team member felt like their strengths were considered well when dividing tasks. Every member thus felt like they got to bring their own expertise to the project while learning from other members'. Learning client interaction, presentation and team working skills also felt valuable to the members, especially to those whose past professional experiences were limited to academia.

6.3 In what regard was the project less successful?

In hindsight, the implementation of the algorithm could have been approached differently. A long time was spent working on the minimum viable product, while the algorithm could have been to be more robust or include more assets. Swaptions had to be left out, and the algorithm often failed to converge to a global maximum. We focused excessively on minor details, while the "alpha" version of our simulation should have been capable of handling more complex products beyond bonds. This issue is closely linked to our failure to achieve the desired outcomes in exploring the hedging with a wider range of products.

6.4 What could have been done better?

6.4.1 Team

In hindsight, there are multiple things the team could have done better to achieve better results. For example, the team could have allocated more time to project from the beginning. The team should have been more careful when conducting analyses, since many times the project was postponed because the model was implemented incorrectly or the team misunderstood how the hedging portfolio actually worked. A more active approach towards the client would have probably expedited the process.

Before starting building the Python model, the team could have built the minimum-viable product in Excel and then build the Python model. However, it was good that the Python model was begun early, as it could not have been able to complete in time otherwise.

6.4.2 Client

The communication and scoping of the project from Fennia was commendable from start to finish. The data was provided in a clear and concise manner, and Fennia was very forthcoming with all of our questions.

However, we also felt like the communication regarding the topic was sometimes slightly confusing. The problem at hand is very complex and thus explaining it can be very difficult, especially in the span of a few hours, but many times the team did not realize how the modelling tasks should be done and how, for example, the risk-neutral and real-world data should be utilized and how the are used in different situations.

6.4.3 Teaching staff

Looking back, we believe there would have been value of offering more explicit guidance for each phase of the project. This could have included clarifying the specific components expected in project deliverables, presentations, and written feedback for the opposing team, along with clear instructions on the e-mails. Also, instead of writing an intermittent report, a spoken report on progress taken could have been as beneficial as a written one, and these could have been more frequent.