Aalto University

MS-E2177 - Seminar on Case Studies in Operations Research

Final Report:
Impacts of solvency requirements on optimal asset allocation

Project manager: Mikko Närhi
Team: Vesa Ranta-aho, Oskar Emelianov

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1 Introduction

1.1 Background

The Finnish pension system is financed through a combination of contributions from employers, employees, and the government, as well as investment returns. The level of contributions is determined by factors such as the age of the employee and the type of work they perform. The contributions are then used to pay for the pensions of current retirees. Any excess funds are invested in a variety of instruments to help ensure the long-term viability of the pension system. The Finnish pension system is designed to be sustainable and adaptable to changing demographic and economic conditions.

Technical provisions are an important aspect of the financing of pensions in Finland. These are the funds that pension providers set aside to ensure they can meet their future obligations to pay out pensions to their clients. Technical provisions are calculated based on actuarial assumptions, taking into account factors such as life expectancy, inflation, and investment returns. They are designed to ensure that pension providers have adequate reserves to cover future payouts and can maintain financial stability in the long term. The Financial Supervisory Authority monitors technical provisions to ensure that pension providers have sufficient reserves and can meet their obligations to their clients. Overall, technical provisions are a crucial component of the pension system in Finland and help to ensure the financial security of retirees.

The Solvency Regulation in Finland is a set of rules that govern the financial stability of pension providers. It ensures that pension providers have enough assets to cover their liabilities and can meet their obligations to pension recipients. The regulation requires pension providers to maintain a certain level of solvency, which is the difference between their assets and liabilities. If the solvency level falls below the required amount, the pension provider must take corrective measures to restore its financial stability. The Solvency Regulation is an important part of the Finnish pension system, which aims to provide secure and sustainable pensions for all citizens.

Portfolio optimization is a process of selecting a mix of assets that maximize the discounted value of future returns (Markowitz, 1952). It involves analyzing various investment options, their historical performance, and their correlations with each other to create a diversified portfolio that balances risk and return. This approach helps investors minimize their exposure to any single asset or market sector, while maximizing their overall returns. In our project, we use portfolio optimization to study the effects of solvency requirements on the optimal mix.

1.2 Objectives

The main objective of this project is to analyze the impact of the Finnish solvency requirements on optimal asset allocation and realized portfolio return. The investment options include two baskets, equity index and bonds. Asset return paths are simulated from historical returns data using geometric Brownian motion for equity and Cox-Ingersoll-Ross for bonds. An optimization model for simulating the return paths for different assets and optimizing the asset allocation under solvency requirement constraints is developed based on a literature study on existing solutions.

The goal for the model implementation is to be a clean and cohesive program that can easily be easily run with different data and be expandable or adaptable to other similar problems with low effort. The code should be high quality including comments, and the development should be documented using version management.

2 Literature Review

In this section, we provide a literature review on the key concepts of this study. The literature review consists of previous studies regarding asset-liability management models in different environments and scenario generation for financial modelling.

Asset-liability management (ALM) is a crucial aspect of financial management for firms that need to manage their liabilities and assets in order to meet their financial obligations. The aim of ALM is to ensure that a company can meet its financial obligations, even in adverse market conditions.

A common approach to ALM is the use of stochastic models to simulate future scenarios of asset returns and liabilities. These models enable firms to simulate a range of potential outcomes and evaluate the impact of different investment strategies on their financial position. Many studies have proposed different stochastic models for scenario generation, such as the use of Geometric Brownian Motion (GBM) for equity prices and the Cox-Ingersoll-Ross (CIR) model for fixed income prices, which have been employed in this study.

2.1 Scenario Generation for financial modelling

We consider two assets in the model: equity index and fixed income. The scenario generation for each follows the suggestions of Kouwenberg and Zenios (2001) which suggests to use Stochastic Differential Equations (SDEs) to simulate the asset prices. Equity index prices are generated with Geometric Brownian Motion (GBM) and the fixed income asset prices are simulated with the Cox-Ingersoll-Ross model (CIR) (Cox et al., 1985).

2.2 Stochastic Programming in asset-liability management

Asset-liability management models have been used in a variety of environments, ranging from banking, insurance companies, personal finance, to pension funds (Zenios and Ziemba, 2007). This study focuses on the case of Solvency Regulation in Finland and modeling this case using Stochastic Programming.

Stochastic programming models have been applied to asset-liability management for a long time. Bradley and Crane (1972) proposed a multistage model for bond portfolio management in the early seventies. De Oliveira et al. proposes a multistage stochastic programming approach for the asset-liability management in the case of Brazilian pension funds. In this paper, they generate asset price scenarios with stochastic differential equations. They use Geometric Brownian Motion for equities and Cox-Ingersoll-Ross model for fixed income securities. De Oliveira et al. also develop a stochastic programming model that takes into consideration the solvency regulatory rules for Brazilian pension funds. To achieve this, they implement a VaR probabilistic constraint to achieve a positive funding ratio at each time step with a high probability.

2.3 Solvency requirements

Another key concept in this study is the impact of regulatory requirements on optimal asset allocation. Solvency requirements are regulations that mandate the minimum level of capital an insurer must hold to meet its financial obligations. These regulations can significantly impact investment decisions, as insurers must balance the need to meet regulatory requirements with the goal of achieving high investment returns. Many studies have examined the impact of solvency requirements on investment decisions and optimal asset allocation. De Oliveira et al. proposed an ALM model for the case of Brazilian pension funds. In this study, the solvency requirement was that the funding ratio, defined as the ratio of current assets to the present value of future liabilities, cannot be smaller than one in more than consecutive years. De Oliveira et al. found that if the initial funding ratio is high, the fund allocates 70 % and 30 % in fixed income and stocks, respectively. As the initial funding ratio goes slightly below 1, the fund concentrates invests more in fixed income to reduce its risks of not paying the liabilities.

The Finnish solvency requirements are defined according to the Finnish Pension Alliance TELA (2023), which states that a pension provider's solvency capital must exceed the solvency limit for the provider to be solvent. The solvency limit applied in this study is defined according to the Finnish Centre for Pensions Eläketurvakeskus (2023), which is the technical provisions considered as the liabilities of the ALM model. The technical provisions are of form:

$$RSV = b_{16} + i_0 + \lambda \cdot j, \tag{1}$$

where b_{16} is a supplementary factor, i_0 is a discount rate, λ is the degree of stock return dependence and j is an equity-linked provision. The supplementary factor b_{16} is subject to the solvency capital of each pension fund.

3 Data & Methods

3.1 Data

We received the dataset used in this study from Varma's contact person, Hamed Salehi. The dataset includes daily price data for the equity index and the fixed income from December 29, 2000 to January 26, 2023. The price data only includes trading days, meaning that weekends and holidays are not included in the data. The data contains 5737 rows and each row contains the date, equity index price and fixed income price. The timeseries for both are plotted in Figure 1.

Figure 1: Daily price for both the equity index and bond.

Returns	Equity (daily)	Bond (daily)	Equity (annualized)	Bond (annualized)
Mean	0.02%	0.01%	6.06%	2.90%
Std Dev	1.02%	0.39%	16.20%	6.21%
Minimum	-9.70%	-2.51%	-2443.95%	-631.87%
Maximum	8.67%	2.83%	2183.90%	712.40%

Table 1: Summary statistics of the dataset

From Table 1, we can see the summary statistics of the dataset. The minimum is smaller, the maximum and standard deviation are larger for the equity index which means that it is much more volatile compared to the bond timeseries.

3.2 Scenario Generation

The validity of the optimization model's results depends heavily on the quality of the scenarios generated to represent the stochasticity of the assets' prices Dupacová and Polívka (2009). This means that simulating the prices of the assets considered for this study is extremely important for the model's performance. In this study, the asset prices follow correlated Stochastic Differential Equations (SDEs) of form

$$d\xi_{it} = \mu(\xi_{it}, t)\, dt + \sigma(\xi_{it}, t)\, dW_{it}, \tag{2}$$

where W_{it} is a Wiener process normally distributed with mean zero and variance u, $W_{t+u} - W_t \sim \mathcal{N}(0, u)$.

When we simulate more than one asset, we have to take the returns' correlated errors into account (Dempster et al., 2003). The correlation coefficients ρ_{ij} between two assets i and j at time t are defined by

$$dW_i \cdot dW_j = \rho_{ij} dt, \rho_{ii} = 1, \forall i, j. \tag{3}$$

We use the Geometric Brownian Motion model for generating the prices for the equity index (Mitra, 2006)

$$d\xi_{1t} = \mu(\xi_{1t}, t)\, dt + \sigma(\xi_{1t}, t)\, dW_{1t}. \tag{4}$$

The GBM above also has the analytic solution for an arbitrary initial value S_0

$$\xi_{1t} = \xi_{1(t-1)} e^{(\mu - \frac{1}{2}\sigma^2)d_t + \sigma\epsilon\sqrt{d_t}} \tag{5}$$

with $\epsilon \sim \mathcal{N}(0,1)$. For the price of the fixed income asset, we use the Cox-Ingersoll-Ross model (Cox et al., 1985)

$$d\xi_{2t} = \alpha(\beta - \xi_{2t})dt + \sqrt{\xi_{2t}}\sigma dW_{2t}, \tag{6}$$

where α, β and σ are model parameters. The drift function $\mu(\xi_{2t}, \alpha, \beta) = \alpha(\beta - \xi_{2t})$ is linear and it ensures mean reversion of the interest rate in the long run. The diffusion function $\sigma^2(\xi_{2t}, \sigma) = \xi_{2t}\sigma^2$ ensures that the interest rate stays positive at all times t.

We use the maximum likelihood estimation method for estimating the model parameters α and β of the Cox-Ingersoll-Ross model (Kladivko, 2007) with N observations $r_{t_i}, i = 1, ..., N$

$$\mathcal{L}(\theta) = \prod_{i=1}^{N-1} p(r_{t_{t+1}} | r_{t_i}; \theta, \Delta t). \tag{7}$$

However, it is computationally more efficient to work with the log-likelihood-function

$$\ln \mathcal{L}(\theta) = \prod_{i=1}^{N-1} \ln p(r_{t_{t+1}} | r_{t_i}; \theta, \Delta t). \tag{8}$$

After formulating the likelihood function, the goal is to find the values of the model parameters a and b that make the observed data most probable. This is achieved by maximizing the likelihood

$$(\hat{\alpha}, \hat{\beta}) = \arg\max_{\theta} \ln \mathcal{L}(\theta). \tag{9}$$

Because the logarithmic function is monotonically increasing, maximizing the log-likelihood function also maximizes the likelihood function.

3.3 Optimization model

The objective of asset-liability management is to allocate wealth in financial assets $i = 1, ..., N$ to maximize profits while liabilities L_{ts} are always covered. This is a stochastic dynamic allocation problem due to the randomness of the asset prices and the time-dependency of the investment and rebalancing decisions. To solve the problem, an optimization model was formulated following loosely de Oliveira et al. (2017). The indices and sets, decision variables and parameters are summarized in Table 2. The basic profit maximizing allocation problem is

$$\max \quad \sum_{s=1}^{S}\sum_{i=1}^{N} P_s X_{iTs} \tag{10}$$

$$\text{subject to} \quad \sum_{i=1}^{N} X_{i0s} = k_0 L_0, s = 1, ..., S \tag{11}$$

$$X_{i0s} = a_i \sum_{j=1}^{N} X_{j0s}, i = 1, ..., N, s = 1, ..., S \tag{12}$$

$$X_{i1s} = (1 + R_{i1s})X_{i0s}, i = 1, ..., N, s = 1, ..., S \tag{13}$$

$$\sum_{j=1}^{N} X_{jts} = \sum_{j=1}^{N}(1 + R_{jts})X_{j(t-1)s}, t = 2, ..., T, s = 1, ..., S \tag{14}$$

$$X_{its} \geq 0, i = 1, ..., N, t = 1, ..., T, s = 1, ..., S \tag{15}$$

$$a_i \geq 0, i = 1, ..., N. \tag{16}$$

The objective function (10) is the expected value of the portfolio at the final time-step. Constraint (11) sets the initial value of the portfolio to be equal to the initial solvency ratio k_0 times initial liabilities L_0. To obtain a single best initial allocation that considers all scenarios, constraint (12) imposes a common initial asset allocation a_i for all scenarios. To increase the weight of the initial allocation decision, in the first time-step rebalancing the assets is not allowed. Therefore, the new amount invested in each asset is simply the old amount after returns, according to constraint (13). After the first time-step, however, assets can be rebalanced at every time-step. The rebalancing is ruled by constraint (14), which ensures that the new value of the portfolio is equal to the sum of the old amounts invested in each asset after returns.

The solvency ratio, assets to liabilities, of Finnish pension insurance companies is never allowed to go below one. We enforce this rule softly using a stochastic

Indices	
t	Time index $t = 1, ..., T$
i	Index of asset classes $i = 1, ..., N$
s	Index of scenarios $s = 1, ..., S$
Decision variables	
X_{its}	Amount of asset i to hold at time t at scenario s
X_{i0s}	Amount of asset i to hold initially at scenario s
a_i	Initial allocation of asset i
C_{ts}	Binary variable with value 1 if solvency requirement not fulfilled in time t and scenario s and 0 otherwise
L_{ts}	Liability at time t at scenario s
k_{ts}	Solvency ratio at time t at scenario s
j_{ts}	Stock return provision at time t at scenario s
i_{ts}	Interest provisions at time t at scenario s
p_{ts}	Supplementary basis at time t at scenario s
b^*_{ts}	Supplementary factor at time t at scenario s
b_{ts}	Supplementary factor liability addition at time t at scenario s
Random variables	
R_{its}	Random return of asset i at time t at scenario s
c_{its}	Cumulative return of asset i at time t at scenario s
Deterministic parameters	
P_s	Probability of scenario s
i_0	Discount rate
λ	Degree of stock return dependence
K	Required solvency ratio
k_0	Initial solvency ratio
L_0	Initial liability
α	Risk level
w	Fund weight
M	Large positive coefficient

Table 2: Notation summary

constraint - the probability of the solvency ratio k_t being below 1 must be less than the risk level α at each time-step:

$$\mathbb{P}(k_t < 1) \leq \alpha, t = 1, ..., T. \tag{17}$$

We implement this stochastic constraint in steps with the constraints

$$k_{ts} L_{ts} = \sum_{i=1}^{N} X_{its}, t = 1, ..., T, s = 1, ..., S \tag{18}$$

$$K - k_{ts} \leq M C_{ts}, t = 1, ..., T, s = 1, ..., S \tag{19}$$

$$\sum_{s=1}^{S} C_{ts} \leq \alpha S, t = 1, ..., T \tag{20}$$

$$k_{ts} \geq 0, t = 1, ..., T, s = 1, ..., S \tag{21}$$

$$C_{ts} \in \{0, 1\}, t = 1, ..., T, s = 1, ..., S. \tag{22}$$

Constraint (18) calculates the solvency ratio. C_{ts} is a binary variable which indicates whether or not the solvency ratio is below the required limit. If it is, the indicator is forced to one by the big-M constraint (19). Finally, (20) constrains the share of insolvent scenarios below the risk level at each time-step. The formulation used is quite lenient - it allows every scenario to become insolvent during its lifetime, just not simultaneously. A stricter and more realistic constraint would limit the insolvency probability for the whole duration of the scenario instead of separately for each time-step. Such constraint would, however, introduce additional inter-time dependencies to the model and affect solving speeds, so we settle for the simpler version.

A key part of the model is the calculation of liabilities. We assume that short-term liabilities are covered by short-term contributions, so the liabilities to be covered by the portfolio are the technical provisions as defined in Eläketurvakeskus (2023). The piecewise defined supplementary factor

$$b_{16} = \begin{cases} (1-\lambda) \cdot 0.36 \cdot p - 0.057 & p < 0.198 \\ 0 & 0.198 \leq p < 0.218 \\ (1-\lambda) \cdot 0.15 \cdot p - 0.026 & p \geq 0.218, \end{cases} \tag{23}$$

defined in Eläketurvakeskus (2023), is formulated using Special Ordered Set of Type

2 (SOS2) constraints of the form

$$p_{ts} = \sum_k \overline{p}_k \gamma_{kts}, t = 1, ..., T, s = 1, ..., S \tag{24}$$

$$b^*_{ts} = \sum_k \overline{b}_k \gamma_{kts}, t = 1, ..., T, s = 1, ..., S \tag{25}$$

$$\sum_k \gamma_{kts} = 1, t = 1, ..., T, s = 1, ..., S \tag{26}$$

$$\gamma_{kts} \geq 0, k = 1, ..., 4, t = 1, ..., T, s = 1, ..., S \tag{27}$$

$$SOS2(\gamma_1, ..., \gamma_4). \tag{28}$$

Here γ_k are members of a Special Ordered Set of Type 2, meaning that only two of them can be nonzero and they have to be neighbors. Therefore they act as weights which are used to interpolate between breakpoints $(\overline{p}_k, \overline{b}_k)$ and $(\overline{p}_{k+1}, \overline{b}_{k+1})$. We achieve the wanted piecewise definition of b_{16} by using four breakpoints, (-2, -0.633), (0.198, 0) (0.218, 0) and (2, 0.214). The quarterly supplementary factor liability addition b_{ts} is then calculated as

$$b_{1s} \cdot 4 = b^*_{1s} L_0, s = 1, ..., S \tag{29}$$

$$(b_{ts} - b_{t-1,s}) \cdot 4 = b^*_{ts} L_0, t = 2, ..., T, s = 1, ..., S. \tag{30}$$

Since we simulate only a single pension fund, we calculate the average supplementary basis p as a weighted average of our own supplementary basis

$$p^*_{ts} = k_{t-1,s} - 1 \tag{31}$$

and the average supplementary basis from years 2011 to 2022 (0.31) is calculated from Työeläkelakipalvelu (2023)

$$p_{ts} = (1 - w) \cdot 0.31 + w \cdot p^*_{ts}, t = 1, ..., T, s = 1, ..., S. \tag{32}$$

In reality, the weight of each pension fund in the average calculation is capped at 0.15 to limit the effect of large funds on the average. However, we assume that the performance of different funds is correlated through macroeconomic factors, so we set our weight to 0.5 to simulate the effect of the performance of multiple funds moving in the same direction simultaneously.

Due to the exact calculations of the stock return factor j being nonlinear, we approximate stock return provisions linearly using cumulative returns:

$$j_{ts} = \lambda(c_{1ts} - 0.01 \cdot t/4)L_0, t = 1, ..., T, s = 1, ..., S. \tag{33}$$

Similarly, interest provisions are linearized as

$$i_{ts} = i_0 L_0 t/4, t = 1, ..., T, s = 1, ..., S. \tag{34}$$

Finally, liabilities are

$$L_{ts} = j_{ts} + i_{ts} + b_{ts} + L_0, t = 1, ..., T, s = 1, ..., S. \tag{35}$$

4 Results

4.1 Simulation

Figures 2 and 3 show simulated equity and bond prices in 300 5-year scenarios generated using geometric Brownian motion for equity and Cox-Ingersoll-Ross for bonds. These are used as the scenarios in optimization. The mean and standard deviations of the annualized returns are 6.4% and 16.2% for equity and 0.01% and 0.6% for bonds, respectively. For equity, the statistics match well to the observed values in table 1. For bonds, however, both the mean and standard deviation are too small. The Cox-Ingeroll-Ross assumes that the underlying process is mean reverting, while the input data has a clear trend. The model therefore seems to have fit poorly to the data. Despite the ill-fitted model, we accept the bond simulations as they are, because the main factors separating them from equity still hold - both the drift and volatility are smaller.

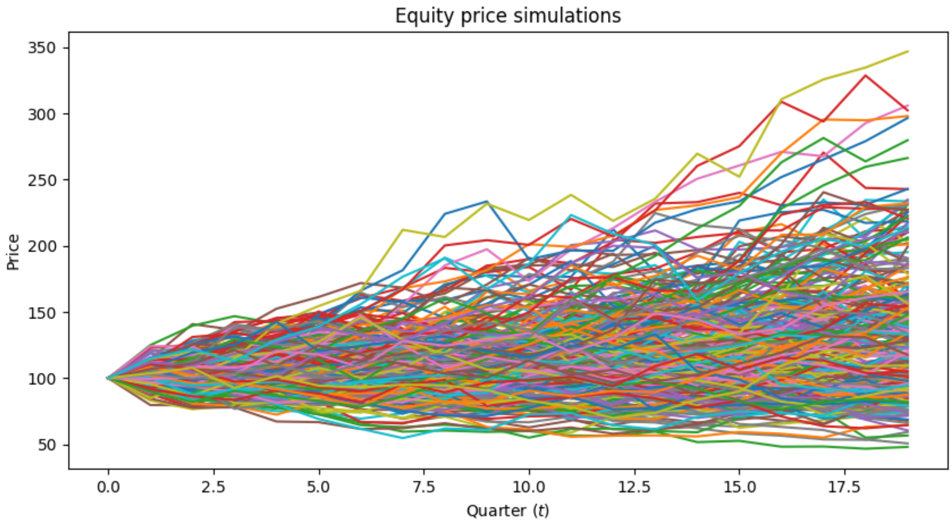

Figure 2: 300 scenarios generated using Geometric Brownian Motion for the equity index.

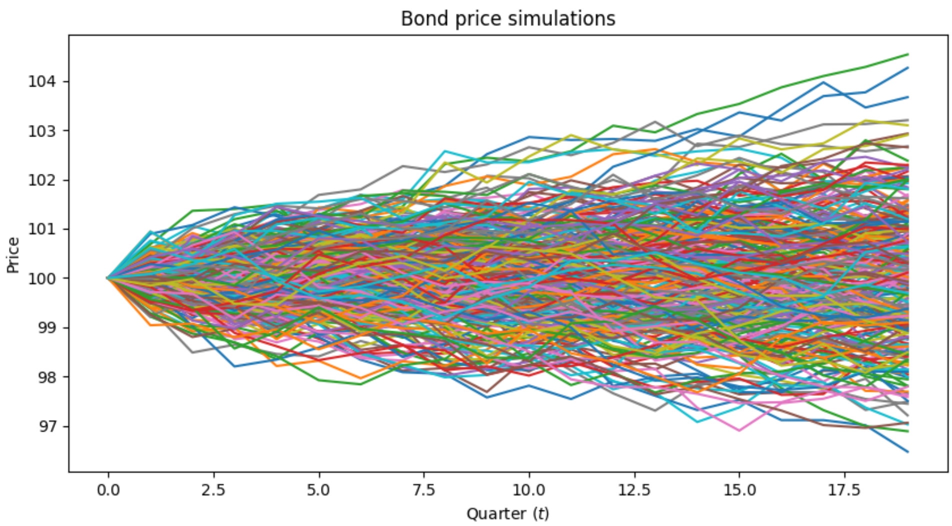

Figure 3: 300 scenarios generated using Cox-Ingersoll-Ross for bonds.

4.2 Effect of time span

Due to the complexity of the model, one has to choose between a high number of scenarios or a long time span to be able to solve the model in a reasonable time.

Long time spans allow the scenarios to develop properly and can offer insight into good long term allocations, but on the other hand a large number of scenarios is needed to make full use of the probabilistic solvency constraint. We therefore study how the time span affects the results to determine a sufficient time span for further tests. The test is performed using an initial solvency ratio of 1.3 and solvency requirements of 1.15 and 1.2, because when the initial solvency ratio is high compared to the requirement, the initial allocation is 100% stocks independent of the time span.

Table 3 shows the optimal allocation with time spans from 1 to 4 years when the solvency requirement is 1.15 and 1.2. We see that the optimal allocation is mostly not affected by the time span. Figure 4, containing the solvency ratios in the 4 year simulation with solvency requirement 1.2, explains why this is the case. Due to the high equity returns, stable bond prices and optimization's ability to allocate assets perfectly, all of the scenarios are likely to maintain good solvency in the long run. This may not be the case in reality, where the price movements can not be predicted perfectly, and where trading costs prevent as aggressive allocation changes as we allow. The most demanding moment for the portfolios is the beginning, where the initial allocation has not yet been reallocated to the scenario-specific optimum. Due to these findings, we can choose a short 1-year time span for the following tests in order to be able to maximize the number of scenarios used.

T	Equity (%) (K=1.15)	Fixed-income bond (%) (K=1.15)	Equity (%) (K=1.2)	Fixed-income bond (%) (K=1.2)
1	96.3	3.7	66.3	33.7
2	94.7	5.3	66.3	33.7
3	96.0	4.0	65.8	34.2
4	96.3	3.7	66.3	33.7

Table 3: Initial portfolio allocation with different total time spans.

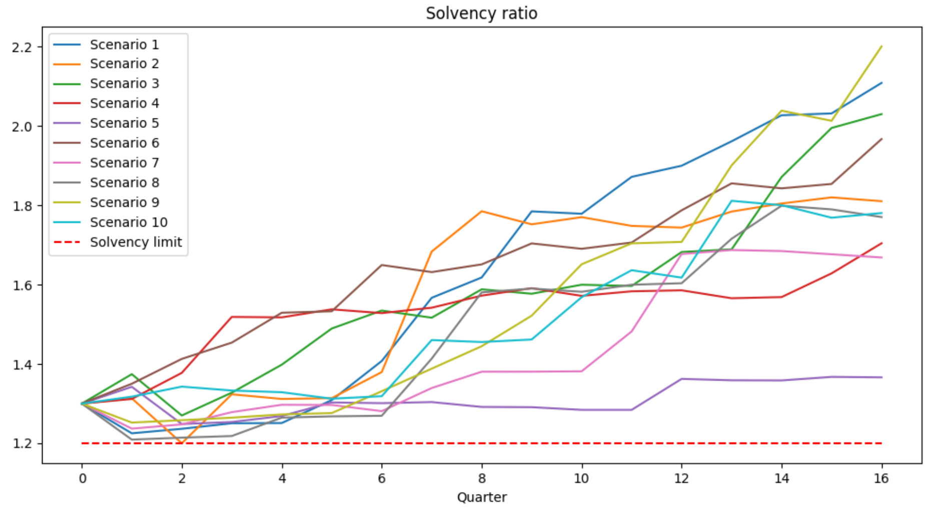

Figure 4: Solvency ratios in a 4-years simulation, when the solvency requirement is 1.2.

4.3 Effect of initial solvency ratio

Initial solvency ratio	Equity (%)	Fixed-income bond (%)	Obj. Value (corrected)	Insolvency probability (%)
1.5	100	0	165.6	0
1.4	100	0	165.5	0
1.3	100	0	165.6	0
1.25	100	0	165.5	1
1.2	95.3	4.7	165.4	2
1.15	78.2	21.8	164.9	2
1.1	59.4	40.6	164.2	2
1.05	39.4	60.6	163.6	2
1.02	23.9	76.1	163.1	2
1.01	NA	NA	NA	> 2

Table 4: Initial portfolio allocation and insolvency probability under different initial solvency ratios. The solvency requirement is fixed at 1.0.

In table 4, we present how the initial portfolio allocation, objective function value (expected portfolio value) and the insolvency probability vary with the initial solvency ratio. The insolvency probability is calculated as the share of scenarios that go insolvent at any point, and the effect of differing initial portfolio values is corrected. The effect was tested by optimizing the allocation for sets of 100 scenarios using a 1-year time span and iteratively lowering the initial solvency ratio from 1.5 to 1.01, where no feasible solutions were found in optimization due to the solvency constraint being violated. The required solvency ratio K was kept fixed at 1.0.

Table 4 highlights that with higher initial solvency ratios, we are able to withstand higher volatility in our constructed portfolio, which is reflected in the portfolio allocations. With initial solvency ratios ranging from 1.25 to 1.5, the portfolio is constructed entirely of equities and the allocation to fixed income securities is 0%.

The insolvency probability is stable and very low until the initial solvency ratio reaches 1.01, according to Table 4. When the initial solvency ratio gets this close to the required solvency ratio 1.0, some scenarios inevitably become insolvent, because the volatility of both investment options is too high for such a small buffer. However, initial solvency ratios of 1.2 to 1.05 seem to have stable insolvency probabilities, meaning that increasing the allocation of fixed-income securities when close to the required solvency ratio has a high probability of keeping the fund solvent. Increasing the share of fixed-income, however, reduces the expected value of the portfolio slightly.

Figure 5 displays the changing percentages of equity and fixed income allocations in the portfolio as a function of the initial solvency ratio. When the initial solvency ratio goes below 1.2, the allocation of equity in the portfolio starts to decline steeply and the allocation of fixed income securities increases rapidly. The portfolio allocation is 100 % in equity for higher initial solvency ratios.

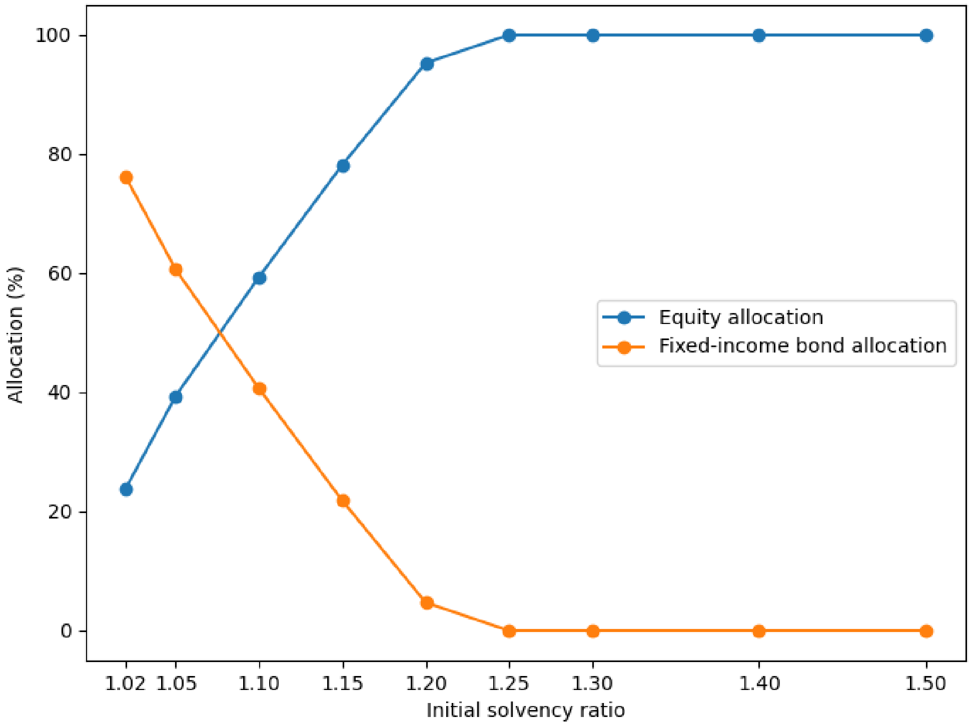

Figure 5: Portfolio allocation for different initial solvency ratios.

4.4 Effect of solvency requirement

Solvency requirement	Equity (%)	Fixed-income bond (%)	Obj. Value	Insolvency probability (%)
1.0	100	0	143.5	0
1.1	88.5	11.5	143.1	2
1.15	70.0	30.0	142.6	2
1.2	51.6	48.4	142.1	3
1.25	27.2	72.8	141.4	1
1.26	21.0	79.0	141.2	1
1.27	NA	NA	NA	> 1

Table 5: Initial portfolio allocation and insolvency probability under different solvency requirements. The initial solvency ratio is fixed at 1.3.

In Table 5, we present how the solvency requirement, the required solvency ratio K, affects the portfolio allocation, expected value and the insolvency probability. The solvency requirement ranges from 1.0 to 1.27 and the initial solvency ratio k_{0s} and the number of scenarios are fixed to 1.3 and 100, respectively.

While the setup is similar to the one with differing initial solvency ratios, the situations are not completely identical. This is because the liabilities grow with the solvency ratio, not with the distance between the ratio and the requirement. Nevertheless, the results in table 5 show that, as was the case with differing initial solvency ratios, the larger the distance from the initial solvency ratio to the solvency requirement is, the more we will favor equity in the portfolio over fixed income securities. When the requirement is raised, the equity allocation decline is similar in speed compared to how the portfolio allocation changes with the initial solvency ratio k_0 as variable (see Figure 5) when compared by the relative distance between the solvency ratio and the requirement. When the solvency requirement ranges from 1.0 to 1.2, the portfolio will have a majority allocation of equities. After this, majority will be fixed-security. The profit losses related to increased fixed-income allocation are also similar.

The portfolio is constructed mainly of fixed income securities as the solvency requirement K gets closer to the initial solvency ratio. When the solvency requirement goes below 1.27 with an initial solvency ratio of 1.3, the risk of insolvency grows too large. The apparent non-monotonicity of the insolvency probability is caused by the low number of scenarios (100). By chance, there are suitable price movements that allow several scenarios to become insolvent at different times. The effect should disappear if the number of scenarios was raised significantly.

5 Discussion and Conclusion

The aim of this project was to examine the impact of Finnish solvency requirements on optimal asset allocation, realized portfolio return, and insolvency probability. Two investment options were considered in the study, equity index and fixed income, and the results were analyzed for different time horizons, required solvency ratios and initial solvency ratios.

The asset return paths were simulated using historical returns data. Equity index prices were generated using geometric Brownian motion (GBM), and fixed income asset prices were simulated using the Cox-Ingersoll-Ross model (CIR). While the geometric Brownian motion model was able to create similar output scenarios as the input data, the Cox-Ingersoll-Ross model fit the data poorly. This was most likely due to the mean-reverting property not being fulfilled by the input data. Other models should therefore be investigated in place of CIR if the aim is to model fixed-income prices with a clear trend. Due to the ill-fitted model, the simulated fixed-income prices were too stable and contained no trend. The effect of this on the optimization results are significant, because this offers the high-volatility equity a relatively risk free alternative where assets can be allocated when equity prices drop. This meant that the portfolios rarely made significant loss. With higher bond volatility, the portfolios would be forced to make loss more often and could face more difficulty in maintaining a good solvency rate.

The optimal asset allocation was found by developing and applying a multi-stage stochastic programming ALM model for a Finnish pension fund that takes into account the Finnish solvency regulation. The results show that the optimal asset allocation is affected greatly by the Finnish solvency regulation and the constraints it imposes on the optimizaton model. There is a link between the optimal portfolio allocation and the difference between the initial solvency ratio and the required solvency ratio. Equity is preferred as long as solvency is on a good level, but the allocation of fixed income securities generally increases when the difference between the initial solvency ratio and the required solvency ratio decreases. The initial solvency ratio has to get close to the solvency requirement before the programming model formulation becomes infeasible, giving insights to fund managers on the required buffer to keep the fund solvent with a high probability. The results differed from other studies, such as de Oliveira et al. (2017), where fixed-income was preferred. The reason for this difference comes back to the unprofitable bond prices caused by the ill-fitted model.

Another key factor making it too easy for the portfolios to stay solvent is the absence of trading costs. With no limits on the allocation changes, the optimal allocation strategy uses bang-bang control, where assets are always fully allocated to the asset type which profits the most during the next period. In reality, such aggressive allocation changes are not possible. A large improvement for the model would therefore be to include trading costs, for example as a percentage of traded assets, or simply as a constraint for maximum allowed allocation change.

The optimization model turned out to be surprisingly heavy. The solving scales badly especially with the number of time-steps to solve, but also with the number of scenarios. There seems to be a threshold on the number of time-steps after which the model struggles to find a feasible solution at all. This is most likely due to the large number of equality constraints and dependencies to previous time steps in the liability calculations. This and the solving time could probably be improved by reformulating parts of the model and by turning some of the equality constraints into inequalities or by otherwise relaxing them.

The results and implementation of this project provide valuable insights into the impact of Finnish solvency requirements on optimal asset allocation, realized portfolio return, and realized solvency ratio. The developed models for simulating the return paths for different assets and optimizing the asset allocation under solvency requirement constraints could be adapted to other similar problems. The results of the project can assist funds in making informed investment decisions and help pension funds optimize their asset allocation under solvency constraints.

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7 Self Assessment

7.1 Project plan with respect to the project

7.1.1 Scope

The project was finished according to the initial project plan and there were no deviations from it. The planned sections were finished as discussed with the client and the main objectives were completed. Expansion of the scope to additional investment baskets was not done due to lack of time.

7.1.2 Risks

The risks, their likelihood, effect and the way to mitigate them in the initial project plan proved to rather accurate and the project plan included most risks associated with this kind of group study. The risks that realized at the end of the project were scheduling risks. We finalized the code and therefore the results way too close to the deadline of this project. We should have completed the coding tasks at least one week in advance. We should have also started writing the final report earlier. These risks were mainly a derivative of the members lack of time to work on the project. Gladly, the objectives of the study were met but the we were quite close to delivering the project late.

7.1.3 Schedule

We already discussed this briefly in the previous section. The schedule in the project plan was sufficient but the final validation of the model and verifying the results was done much later than the original schedule suggested. The main problem here was that every other course of every team member ended in April, meaning that our schedules were packed with other tasks. The number of team members also realized here. With only three people working on the project, unforeseen changes increase the work needed to finish the project significantly for each team member.

7.2 In what regard was the project successful?

Every main objective of the project set at the beginning of this course were completed and relevant insights and conclusions to the main research questions were found. We were able to simulate return paths for equities and fixed income and develop an optimization model that finds the optimal asset allocation under the Finnish solvency constraints. We were also able to develop the code that is capable of solving the aforementioned tasks. The scenario simulation and optimization model work seamlessly together, and it is easy to run the model with different parameters.

7.3 In what regard was the project less successful?

Too much work was left to do at the end of the course. Validating the model, verifying the results and writing the final report proved to be a lot of work. Since there are only three team members, the amount of work at the end of the course was high, and an additional team member would have helped.

This of course leads to a multitude of avenues that were not as successful as we would have wanted them to be. For example, sensitivity analysis was completely left out. Ideally, we would have also wanted to increase the number of assets considered in the portfolio from the two options which are now equity index and fixed income.

7.4 What could have been done better?

7.4.1 Team

We should have had frequent meetings, for example once a week to improve the communication and pace the project better. During the project, the team members mainly did tasks independently and then discussed the results or difficulties in our group chat. This proved to be sufficient but scheduled meetings would have improved the project with a high probability.

7.5 Client

Varma and their contact person, Hamed Salehi was very flexible and available for the whole duration of this project. This was appreciated by our team. Mr. Salehi responded to every question we had within a short interval which made getting over speedbumps easy. There was no point in the project where we were stuck on anything for an extended amount of time due to successful communication with Mr. Salehi.

7.5.1 Teaching staff

The teaching staff was clear in the communication concerning deadlines and the timeline of project deliverables, although the first excursion dates would have been useful to know a bit earlier. The course was well organized and structured, and the meetings were pleasant and had a great atmosphere. The project topics were also very interesting and the client organizations were diverse, which is excellent in a seminar case course.