2015

Seminar on Case Studies in Operations Research (Mat-2.4177)

CLIENT: NORDEA

RISK ANALYSIS OF A DERIVATIVES PORTFOLIO

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1. Introduction

This student project was made for the Aalto University School of Science as the assignment of the course Seminar on Case Studies in Operations Research. It was executed in collaboration with Nordea Markets which acted as our client and set the topic for the project. We explored counterparty risk of a derivative portfolio by developing a mathematical method for analyzing changes in the risk. As the result, we are providing our client with an analysis tool which is used for explaining, why a derivative portfolio's risk level has changed during a selected period. The tool can be used on a regular basis, supporting our client's reporting tasks that relate to their credit risk management.

When talking about derivatives in banking business, management of counterparty risk is an important issue to be taken into account. Counterparty risk, or credit risk, refers to the risk that a customer (i.e. counterparty) will default on debt by failing to make the required payments. Within derivatives trading, counterparty risk may vary significantly even in the short term, causing unexpected impacts to customers' financial ratios, solvency, or liquidity. Especially in turbulent economic environments the capability of risk management to produce different kinds of analyses and reports for internal and external stakeholders is emphasized. Effective management of credit risk has recently become even more essential, because authorities have considerably tightened regulations.

As a concrete output of this project, we have built an Excel-based tool for analyzing realised changes in a portfolio's credit risk. After feeding the tool with a portfolio of derivative contracts and historical data of related market variables, such as interest rates and exchange rates, it calculates for each contract to what extent every underlying market variable has changed the risk level of the particular contract. The analysis can be run between any two selected days whenever the required historical data is available. The tool thus explains how much each market variable has affected the risk level of each involved contract during the selected period. Further, these results can be aggregated to a sub-portfolio (e.g. particular customer) and the whole portfolio level, to explain how much each involved contract (or set of contracts) has affected to the change of risk level of the selected portfolio during the particular period.

As the Excel-tool enables our client to produce quickly and easily information about changes in their derivative portfolios, it is a useful aid when executing weekly reporting for internal stakeholders. Similarly, the tool can be utilized when communicating to customers when changes in their credit risks occur.

2. Literature review

2.1 Financial derivative securities

A financial derivative contract is a financial instrument that is linked to another specific financial instrument, indicator or commodity and through which specific financial risks such as interest or foreign exchange rate risks can be traded in their own right in financial markets. The value of financial derivative security derives from the price of the underlying asset, but transactions and positions in financial derivatives are traded separately from the values of the underlying assets to which they are linked.

Financial derivatives are mainly used for risk management and hedging. The risk associated with the potential future price of an asset can be hedged against by using financial derivatives. For example interest rate risks can be hedged against by using interest rate swaps or forwards.

Financial derivatives are not debt instruments. Generally, no principal amount is exchanged in advance that is required to be repaid, and no investment income accrues on any financial derivative. An overdue obligation on a financial derivative is, however classified as an account receivable or payable. In that case, the claim becomes a debt instrument. The most common instrument types are foreign exchange rate swaps and forwards, interest rate swaps, cross currency swaps, options of various types and commodity derivatives such as oil swaps or soy bean futures (Valdivia-Velarde, 2012). In this project, the examination is limited to foreign exchange rate swaps and forwards, interest rate swaps and cross currency swaps.

2.2 Useage and valuation

Financial derivatives enable parties to trade specific financial risks to other parties more willing to take and manage these risks. The risk embodied in a financial derivative can be traded either by trading the contract itself or by creating a new contract offsetting the risks of the existing contract. Offsetability means that it is often possible to eliminate the risk associated with a financial derivative by creating a new but reverse contract having charecteristics that countervail the risk underlying the firs derivative. Since hedging and risk management often include multiple derivative instruments, the derivative market amounts to be more than four times larger than the combined equity and bond markets overall. However, the estimate is biased, because the notional amounts cumulate even though the instruments cancel out each other. The estimated gross market values of all derivatives combined is approximately a quarter of the combined equity and bond markets size (Deutsche Börse Group, 2008).

The value of financial derivative contract derives from the price of an underlying asset (reference price). Because the future reference price is not known with certainty, the value of financial derivative at maturity can only be estimated. The current value can however, be calculated with certainty. To calculate the price of a financial derivative, it is important that a prevailing market price for the underlying item is observable or can be estimated. Exchange traded derivatives have an observable price underlying them. The value of over-the-counter (OTC) derivatives is often established in markets with use of models. Valuation models for the instruments used in this project, are known.

2.3 Derivative types

A forward-type contract is an unconditional contract by which two parties agree to exchange a specified quantity of an underlying financial item at an agreed price on a specified date. Forward-type contracts include forwards, futures and swaps. At the inception of a forward-type contract, risk exposures of equal market value are exchanged so the contract typically has zero market value at that time, and no transactions are recorded. As the price of the underlying item changes, the market value of the derivative will change. Therefore, the classification of a forward-type contract may change between asset and liability positions. Many forward-type contracts involve net cash settlement payments, based on the difference between the agreed contract price and the prevailing market price of the spread between two reference prices, times quantity, for the underlying item. Typically, the underlying item is foreign exchange rate or interest rate (Valdivia-Velarde, 2012).

A foreign exchange (FX) forward is used by market participants to lock in an exchange rate on a specific date. It is a binding obligation for a physical exchange of specified amount of funds at a future date. There is no payment upfront (MFX Currency Risk Solutions, 2008 (1)).

A foreign exchange swap is a contract under which two counterparties agree to change two currencies at a set rate and then to re-exchange those currencies at an agreed upon rate at a fixed date in the future. These are used to exchange currency risks between two parties. Since each forward and swap contract carries a specific delivery or fixing date, they are more suited to hedging the foreign exchange risk on a bullet principal repayment as opposed to a stream of interest and principal payments. The latter is more often covered with a cross currency swap (Foreign Exchange Committee, 2010).

A cross currency swap (CCS) is the best way to fully hedge a loan transaction as the terms can be structured exactly mirror the underlying loan. It is also flexible in that it can be structured to fully hedge a fixed rate loan with a combined currency and interest rate hedge via a fixed-floating cross currency swap. The contract is a binding agreement between two counterparties where one stream of future interest payments is exchanged for another. The two streams are paid in different currencies. When the CCS is made, also notional amounts of different currencies are changed between counterparties. In the maturity the notional amounts are changed back to the opposite direction. The streams paid by the counterparties are called float legs or fixed legs according to the underlying interest type. Contract is called fixed-float-CCS, if one leg is fixed and the other one is float. Float-float-CCS and fixed-fixed-CCS are similarly defined (MFX Currency Risk Solutions, 2010 (2)).

CCS can be used to decrease risk associated to known or unknown, or to change currency of, future cash flows. For instance, a company that has fixed interest debt in US dollars, but gets its revenues in euros, may want to do a fixed-fixed-CCS, in which it pays euros and receives dollars. Then it will pay fixed interest to the counterparty and get cash flows in USD, which it uses to amortize its debt.

Interest rate swaps (IRS) are like cross currency swaps, except that the exchanged principals are of the same currency and no initial transactions are made. Interest rate swap is a contract between two parties who agree to exchange payments based on a defined principal amount, for a fixed period of time. In an interest rate swap, the principal amount is not actually changed between the counterparties even at the maturity date but the payments are based on a “notional” amount. Typically, payments made by one counterparty are based on a floating rate of interest, such as the London Interbank Offered Rate (LIBOR) while the payments made by the other counterparty are fixed. Interest rate swaps can be used to hedge against changes in floating interest rates (California Debt and Investment Advisory Commission, 2007).

Both the interest rate swap and cross currency swap can be modified to hedge risks of a loan which is amortized during the time. This means that OTC-contracts such as interest rate swaps can be modified such that the underlying nominal amount changes over time. The modification is called "Amortizing Swap" and it should be taken into account when choosing the valuation models for these instruments (Fernald, 1993).

2.4 Credit risk of a derivatives portfolio

The credit crisis has highlighted the need for transparent and robust methods for valuing, hedging and measuring the risk of credit portfolios. Besides effective valuation and management techniques, the knowledge of risk allocation is vital. The risk of a portfolio can be allocated to various components. Most commonly, risk managers calculate risk contributions of positions, such as individual instruments or obligors (Rosen and Sanders, 2010).

The credit risk associated to an individual counterparty (counterparty risk) consists of individual contracts or trades made with that specific counterparty. These contracts compose a sub-portfolio. The problem of allocating the counterparty-level credit risk to the individual contracts composing the portfolio can be reduced to calculating contributions of the contracts to the counterparty-level exposure conditional on the counterparty's default. Counterparty exposure can be measured using for example current exposure (CE), which is the current value of the exposure to a counterparty. Current exposure can be defined as the amount at risk should the counterparty default now and it is normally assumed to be the market value or the mark-to-market (MtM) value (le Roux, 2008) of the sub-portfolio. The exposure can be reduced greatly by means of netting agreements. A netting agreement is a legally binding contract between two counterparties that, in the event of default, allows using transactions with negative value to offset the positive ones. Only the net positive value of the sub-portfolio represents the current exposure to a specific counterparty (Zhu and Pykhtin, 2007).

From the bank's point of view, a derivatives portfolio is a credit portfolio of over-the-counter (OTC) derivative instruments which are traded between two parties, one of them being the bank. The counterparty risk associated to the portfolio consists of risks associated to single counterparties. If counterparty risk is not mitigated in any way, the maximum loss that the non-defaulting party can suffer equals the net positive sum of the contract level exposures (i.e. market values). Thus, the total risk of the derivatives portfolio is the sum of current exposures at the counterparty level (Zhu and Pykhtin, 2007).

The measurement of contributions of risk factors to portfolio risk is also important. These risk factors include for example financial variables (market factors) such as interest and foreign exchange rates, volatilities and equity prices. When each position depends only on one independent risk factor, the problem can be addressed effectively as the position contributions represent the factor contributions. However, mathematical difficulties arise when the portfolio consists of for example interest rate derivatives which depend on multiple factors (Rosen and Sanders, 2010). In this project, the objective is to explain changes in total counterparty risk by allocating the changes in risk to factor level. Counterparty risk is measured using current exposure.

Since the total risk of the derivatives portfolio can be relatively easily divided to counterparty level and contract level, the changes of total risk can be straightforwardly divided to counterparty level and contract level as well. The problem is to divide the changes of contract-level risk (i.e. market value) to factor level. This can be done using greeks: delta, gamma, vega, etc. which are commonly used to calculate options' price sensitivities respect to their underlying assets. More generally, for any type of instrument, the market value's sensitivities respect to market variables can be calculated by the instrument value's derivatives respect to the market variables. Instrument prices change all the time due to changes in market variables. This change between two distinct time points can be approximated by constructing a Taylor series expansion for the price at starting point. By using the derivatives and changes in market variables, the total change of market value can be divided into factor level components (Carr, 2000), (Tuckman 2002).

3. Data

For this project we obtained two data sets from Nordea Markets. Firstly, we got a data set of a fictional derivative portfolio and secondly, data on selected market variables. All the data is dated between the first of September and the 31st of December, 2014.

__Table 1__. List of the columns in the data set of the derivative portfolio. The columns with an asterisk are only for IR swap and cross currency swap

Netting set ID
Deal ID
Product type
Trade date
Date of maturity
Pay currency
Pay principal outstanding
Pay principal original
Pay rate type
Pay rate
Rec currency
Rec principal outstanding
Rec principal original
Rec rate type
Rec rate
Market value
Next interest pay date*
Accrued interest*
Next pay leg pay date*
Next rec leg pay date*

When defining the scope of the project, we decided to focus on a static portfolio, which include deals only in the most common currencies in Nordea and all the deals are one of the following four types of derivatives: FX swap, FX forward, IR swap or cross currency swap. Our example portfolio includes approximately 1000 such deals divided into 52 netting sets. In the portfolio data set, for each deal and date there are 16 or 20 important numbers, which are listed in Table 1. The amount of the required information depends on the type of the deal.

The data set includes some basic information about the deals, such as deal type and maturity day. Information related to pay leg and received (rec) leg is shown separately. Rate type is shown for IRS and CCS and it is either fixed or float. The original and current (outstanding) principal or nominal amounts are also shown separately. Next interest leg pay dates represent the fixing days of accrued interest payments for pay and received legs of the contract.

In order to explain the changes in the credit risk, we need data on the underlying market variables, i.e. interest rates and exchange rates. As the project scope is limited to most common currencies in Nordea, we got the interest rates of EUR, SEK, NOK, DKK and USD. For each currency and date, we got one month, three month, six month, one year, five year and ten year interest rates. The exchange rate data consist of time series of the exchange rates of all the relevant currencies against the euro.

Due to the close collaboration between the project team and the client, it was possible to update the data sets whenever it was deemed necessary. For example, the information on the next pay dates of IR swap and cross currency swap deals was not included in the initial data set, but was added to it after it turned out to be essential when estimating the changes in market values.

4. Methodology

When explaining the changes in portfolio's credit risk, we focus on explaining the changes in current exposure. The current exposure (CE) risk measure for a portfolio with k netting sets at time t is

$$C_{total}(t) = \sum_{j=1}^{k} C(S_j, t),$$

where S_j is the j:th netting set and $C(S_j,t)$ is the current exposure of the netting set S_j at time t, defined as

$$C(S_j, t) = \max\{0, V(S_j, t)\},$$

where $V(S,t)$ is the market value of the netting set S at time t. Market value of a netting set S_j, that consists of l deals D_{ji} is

$$V(S_j, t) = \sum_{i=1}^{l} V(D_{ji}, t).$$

Changes in market value can be obtained using the Taylor approximation. Generally, first order Taylor approximation for a two-variable function f(x,y) is:

$$f(x, y) \approx f(a, b) + \frac{\delta f(a, b)}{\delta a}(x - a) + \frac{\delta f(a, b)}{\delta b}(y - b),$$

when values f(a,b), x and y are known. In this project the analogy is that the market value f(a,b) is known at t=T, market variables x and y are known at t=T (a and b) and t=T+1 (x and y) and the new market value f(x,y) can be calculated using Taylor approximation. The changes in the market value can be divided into components which represent the changes in the underlying market variables. The changes driven by variables x and y are $\frac{\delta f(a,b)}{\delta a}(x-a)$ and $\frac{\delta f(a,b)}{\delta b}(y-b)$, respectively. Function f represents the valuation formula for this example.

The valuation formula of deal D_{ji} depends on the type of the deal. In general, the market value of a deal is the difference between the present values of the cash flows of the receiving leg and the pay leg. All deals are evaluated using euro as the basis currency. FX swap and FX forward are so similar products, that they are valuated using the same function V_{FX}

$$V_{FX}(N_P, N_R, c_P, c_R, s_P^T, s_R^T, T, t) = \frac{N_P c_P(t)}{\left(1+s_P^T(t)\right)^T} - \frac{N_R c_R(t)}{\left(1+s_R^T(t)\right)^T},$$

where N_P and N_R are the principal amounts of the deal, $c_P(t)$ and $c_R(t)$ are the exchange rates at time t, $s_P^T(t)$ and $s_R^T(t)$ are the T-year spot rates at time t in pay currency and receiving currency, respectively. T is the time to maturity in years. From now on, the subscripts P and R refer to pay leg and receiving leg, respectively.

To begin the valuation of IR swap, consider two bonds where the first bond has a fixed rate coupon while the second bond features a floating rate coupon. Values of the fixed rate bond B^{Fix} and the floating rate bond B^{Flt} are determined as follows:

$$B^{Fix}(N, r_{Fix}, d, n, s, T, t) = \sum_{i=1}^{n} \frac{N(t+t_i) r_{Fix} d}{(1+s^{t_i}(t))^{t_i}} + \frac{N(t+T)}{(1+s^T(t))^T}$$

$$B^{Flt}(N, r_{Flt}, d, s, t) = N(t) \frac{1 + r_{Flt}(t) d}{(1+s^{t_1}(t))^{t_1}}$$

In IR swaps the principal may change, so in the above expressions $N(t)$ denotes the principal at time t, n is the number of the remaining fixing dates, t_i is the time to the i:th fixing date in years, r_{Fix} is the rate of the fixed rate coupon, d is the length of the fixing interval in years, $s^i(t)$ is the i-year spot rate at time t and $r_{Flt}(t)$ is the rate of the floating rate coupon at time t. As n can be really large, the spot rates must be known for various different times. Therefore the entire spot rate curve must be estimated and the function input s denotes the estimated spot rate curve. In the data there are only six data points from the curve each day. For the sake of simplicity, linear approximation is used to estimate the spot rates between the given data points, as the accuracy of that approximation turned out to be sufficient.

An example of an interest rate curve is presented in figure 1.

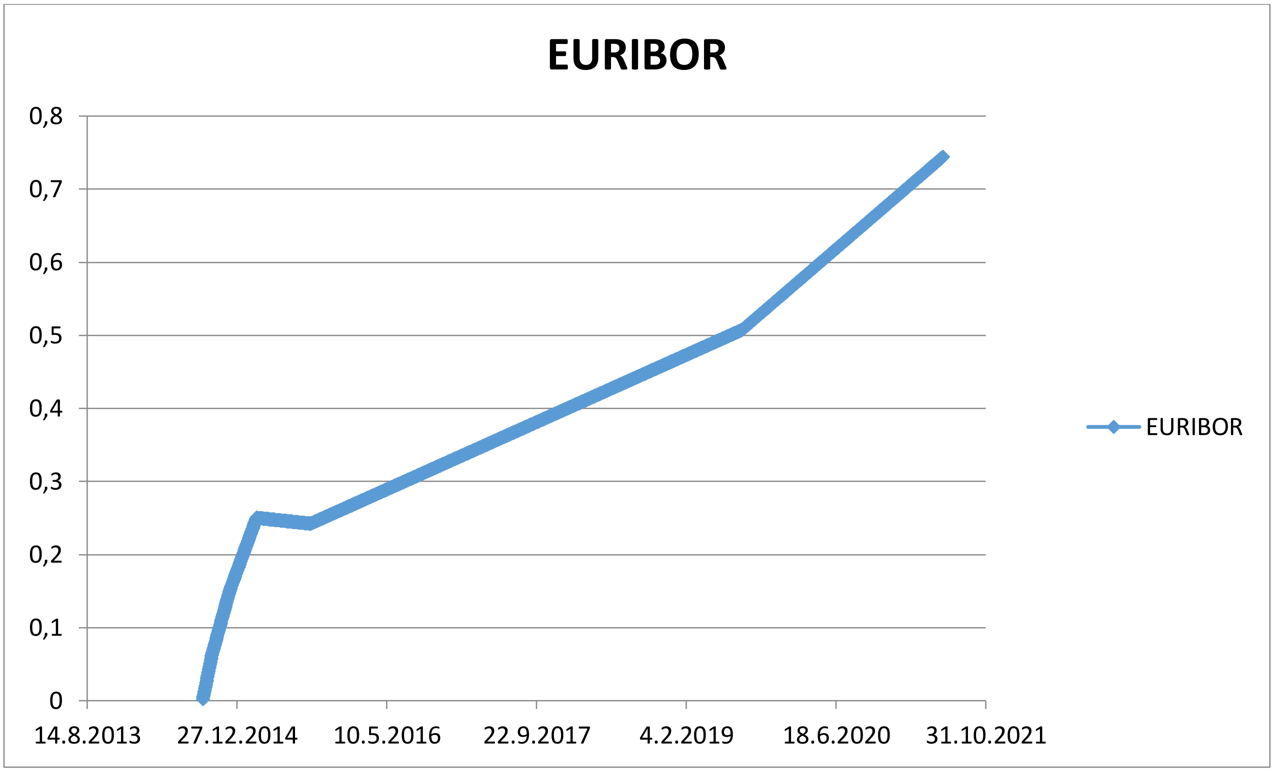

Figure 1: Euro area interest rates

The curve represents interest rates from 3.9.2014. The values in the y-axis are percentages.

The market value of the deal is the difference between the values of two different bonds, which represent the pay leg and receiving leg.

$$V_{IRS}(N_P, N_R, r_P, r_R, d_P, d_R, n_P, n_R, s, c, T, t) = (B^{pay\ type} - B^{rec\ type})c(t),$$

where *pay type* and *rec type* can be *Fix* or *Flt* and *c* is the exchange rate between the deal currency and euro at time *t*.

The valuation of a cross currency swap is similar to the valuation of IR swap. Only difference is that the currencies are different in each leg. The valuation formula for cross currency swap is therefore

$$V_{CCS}(N_P, N_R, r_P, r_R, d_P, d_R, n_P, n_R, s_P, s_R, c_P, c_R, T, t) = B^{pay\ type}c_P(t) - B^{rec\ type}c_R(t)$$

Because the current exposure of the portfolio is essentially determined by the market values of the deals within the portfolio, the changes in the risk level of the portfolio can be better analyzed by starting from the bottom, i.e. how the market values of the individual deals have changed due to changes in market variables. The problem is now to estimate how much and for what reasons a market value of a given deal has changed between time *t* and *t+1*, when the data on the portfolio and market variables is available for the whole period. For the estimation, Taylor series approximations are used.

In FX swap and FX forward, there are four underlying market variables: two exchange rates and two spot rates. For those three types of deals, the first order Taylor approximation at time t_0 is

$$V(D_{ij},t) \approx V(D_{ij},t_0) + \frac{\delta V(D_{ij},t_0)}{\delta c_P}(c_P(t) - c_P(t_0)) + \frac{\delta V(D_{ij},t_0)}{\delta c_R}(c_R(t) - c_R(t_0)) + \frac{\delta V(D_{ij},t_0)}{\delta s_P^T}(s_P^T(t) - s_P^T(t_0)) + \frac{\delta V(D_{ij},t_0)}{\delta s_R^T}(s_R^T(t) - s_R^T(t_0))$$

For FX swap and FX forward deals, the change between *t* and *t+1* is therefore approximately

$$\Delta V(D_{ij},t) \approx \frac{\delta V(D_{ij},t)}{\delta c_P}(c_P(t+1) - c_P(t)) + \frac{\delta V(D_{ij},t)}{\delta c_R}(c_R(t+1) - c_R(t)) + \frac{\delta V(D_{ij},t)}{\delta s_P^T}(s_P^T(t+1) - s_P^T(t)) + \frac{\delta V(D_{ij},t)}{\delta s_R^T}(s_R^T(t+1) - s_R^T(t))$$

In IR swaps there is only one exchange rate in the underlying market variables, but the amount of different spot rates might be big. Similarly as above, the change between *t* and *t+1* for IR swap is

$$\Delta V(D_{ij},t) \approx \frac{\delta V(D_{ij},t)}{\delta c}(c(t+1) - c(t)) + \frac{\delta V(D_{ij},t)}{\delta s^T}(s^T(t+1) - s^T(t)) + \sum_{i=1}^{n} \frac{\delta V(D_{ij},t)}{\delta s^{t_i}}(s^{t_i}(t+1) - s^{t_i}(t))$$

Cross currency swap is similar to the IR swap, but instead of only one exchange rate and spot rate curve, there are two exchange rates and spot rate curves involved. For cross currency swap, the change between *t* and *t+1* is

$$\begin{aligned}\Delta V(D_{ij},t) \approx{}& \frac{\delta V(D_{ij},t)}{\delta c_P}\big(c_P(t+1) - c_P(t)\big) + \frac{\delta V(D_{ij},t)}{\delta c_R}\big(c_R(t+1) - c_R(t)\big) \\ &+ \frac{\delta V(D_{ij},t)}{\delta s_P^T}\Big(s_P^T(t+1) - s_P^T(t)\Big) + \frac{\delta V(D_{ij},t)}{\delta s_R^T}\Big(s_R^T(t+1) - s_R^T(t)\Big) \\ &+ \sum_{i=1}^{n_P} \frac{\delta V(D_{ij},t)}{\delta s_P^{t_i}}\Big(s_P^{t_i}(t+1) - s_P^{t_i}(t)\Big) + \sum_{i=1}^{n_R} \frac{\delta V(D_{ij},t)}{\delta s_R^{t_i}}(s_R^{t_i}(t+1) - s_R^{t_i}(t))\end{aligned}$$

The differentials for each deal type and each underlying market variable are listed in

Table 2.

Table 2. *The differentials for each deal type and each underlying market variable. *In IR swap columns, it is assumed that the examined bond belongs to pay leg. For receiving leg, the signs of the differentials of B^{Fix} and B^{Flt} must be changed. ** In cross currency swap the differentials are always similar to the differentials of IR swap. The correct formula is chosen according to the type of the interest rate and whether it is a pay leg or received leg. In the formulas, spot rates and exchange rates are replaced by the spot rates and exchange rate of that leg's currency.*

Deal type / Market variable	FX swap/forward	IR swap*		Cross currency swap
		B^{Fix}	B^{Flt}	
c_P	$\frac{N_P}{1+s_P^T(t)T}$	-		$B^{pay\ type}$
c_R	$-\frac{N_R}{1+s_R^T(t)T}$	-		$-B^{rec\ type}$
c	-	$B^{pay\ type} - B^{rec\ type}$		-
s_P^T	$-\frac{N_P c_P(t)T}{(1+s_P^T(t)T)^2}$	-		IRS**
s_R^T	$\frac{N_R c_R(t)T}{(1+s_R^T(t)T)^2}$	-		IRS**
s^T	-	$-\frac{N(t+T)c(t)}{(1+s^T(t))^{T+1}}$	-	-
$s_P^{t_i}$	-	-		IRS**
$s_R^{t_i}$	-	-		IRS**
s^{t_i}	-	$-\frac{N(t+t_i)r_{Fix}dc(t)}{(1+s^{t_i}(t))^{t_i+1}}$	- $N(t)\frac{(1+r_{Flt}(t)d)c(t)}{(1+s^{t_1}(t))^{t_1+1}}$ only for *i=1*	-

When the changes in the market values of each of the deals are known, the change in the netting set's S_j market value is obtained by

$$\Delta V(S_j, t) = \sum_{i=1}^{l} \Delta V(D_{ij}, t).$$

The change in the current exposure of the netting set S_j is therefore

$$\Delta C(S_j, t) = max\{0, V(S_j, t) + \Delta V(S_j, t)\} - max\{0, V(S_j, t)\}.$$

The contribution Y of the change in the market value of one deal D_{ij} to the change in the current exposure of the netting set S_j is defined as

$$Y(D_{ij}, t) = \begin{cases} 0 & , \quad if\ \Delta V(S_j, t) = 0 \\ \dfrac{\Delta V(D_{ij}, t)}{\Delta V(S_j, t)} \Delta C(S_j, t) & , \quad otherwise \end{cases}$$

From $Y(D_{ij},t)$, the change in the exchange rate of the pay leg currency has caused a change of X euros, where X is defined as

$$X(c_P, D_{ij}, t) = \begin{cases} 0 & , \quad if\ \Delta V(D_{ij}, t) = 0 \\ \dfrac{\dfrac{\delta V(D_{ij}, t)}{\delta c_P}(c_P(t+1) - c_P(t))}{\Delta V(D_{ij}, t)} Y(D_{ij}, t) & , \quad otherwise \end{cases}$$

The contributions of every other underlying market variable can be calculated similarly. By adding up all *X*s over all *i*, *j* and market variables, the change of the current exposure of the whole portfolio is obtained. If the analysis is required at the netting set level, the *X*s can be added up over all *i* and all market variables separately for all *j*. By doing so, the contributions of each netting set to the change of the portfolio's current exposure are obtained. On the other hand, if the analysis is required at the market variable level, the *X*s can be added up over all *i* and *j* separately for all market variables.

5. Results

The model for explaining changes in CE of a portfolio was written as an Excel program. It can be used to calculate and visualize the CE change and individual components' contributions to it on different levels (portfolio, client and deal level) between two chosen dates. A few illustrative screenshots of the program's output infographics can be found below. Here are some example pictures of the tool. Figure 2 shows the control buttons of the Excel program.

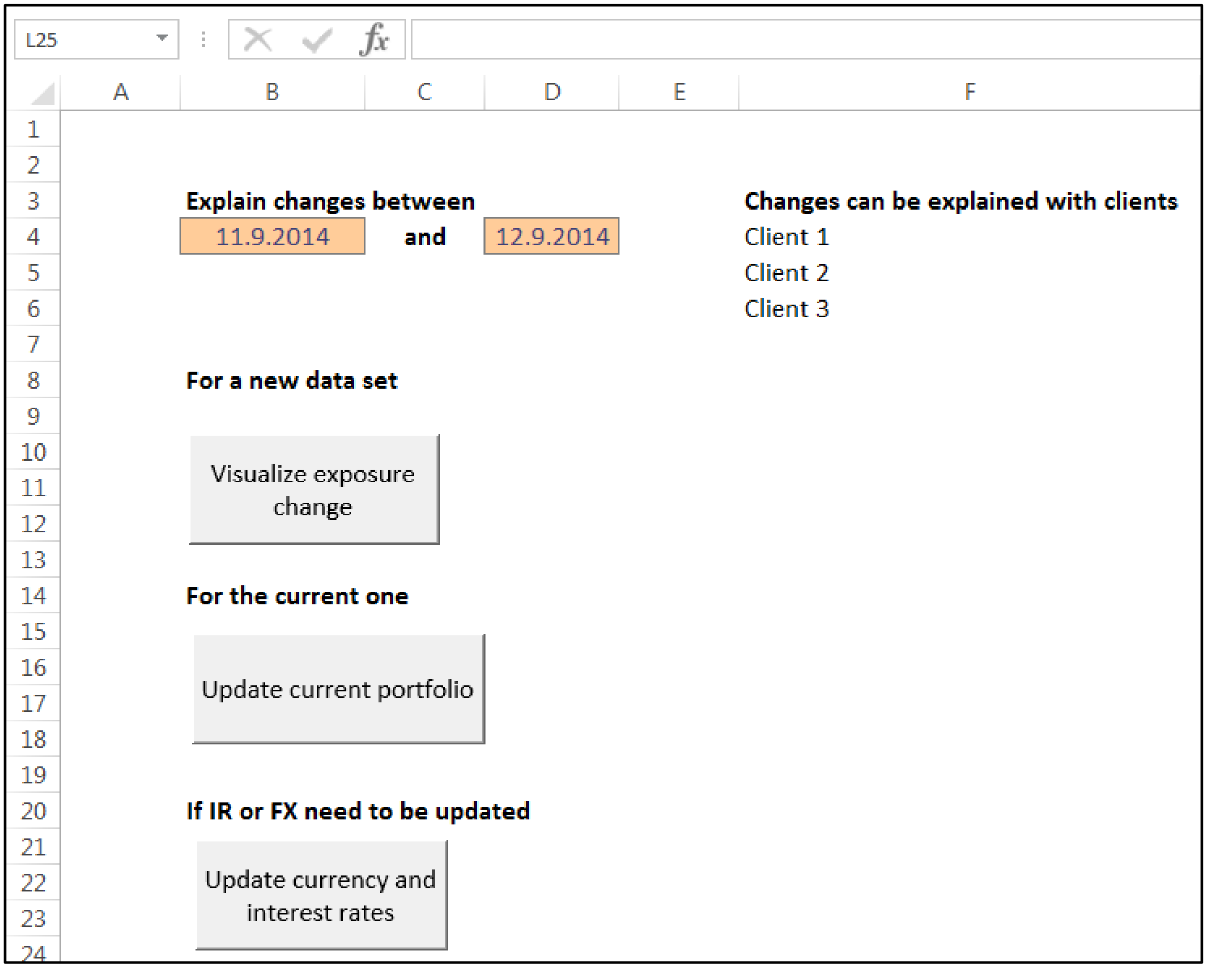

Figure 2: Control buttons

Figure 3 shows the portfolio's CE change and each market variable's individual contribution to it between two arbitrary consecutive dates as a bar graph. It can be seen from the graph that EURNOK and EURSEK currency rates have increased the CE, whereas EURUSD and EURIBOR have decreased it. The CE is measures in euros. The net effect of all changes, i.e. CE change of the portfolio, is shown on the red "Total" bar.

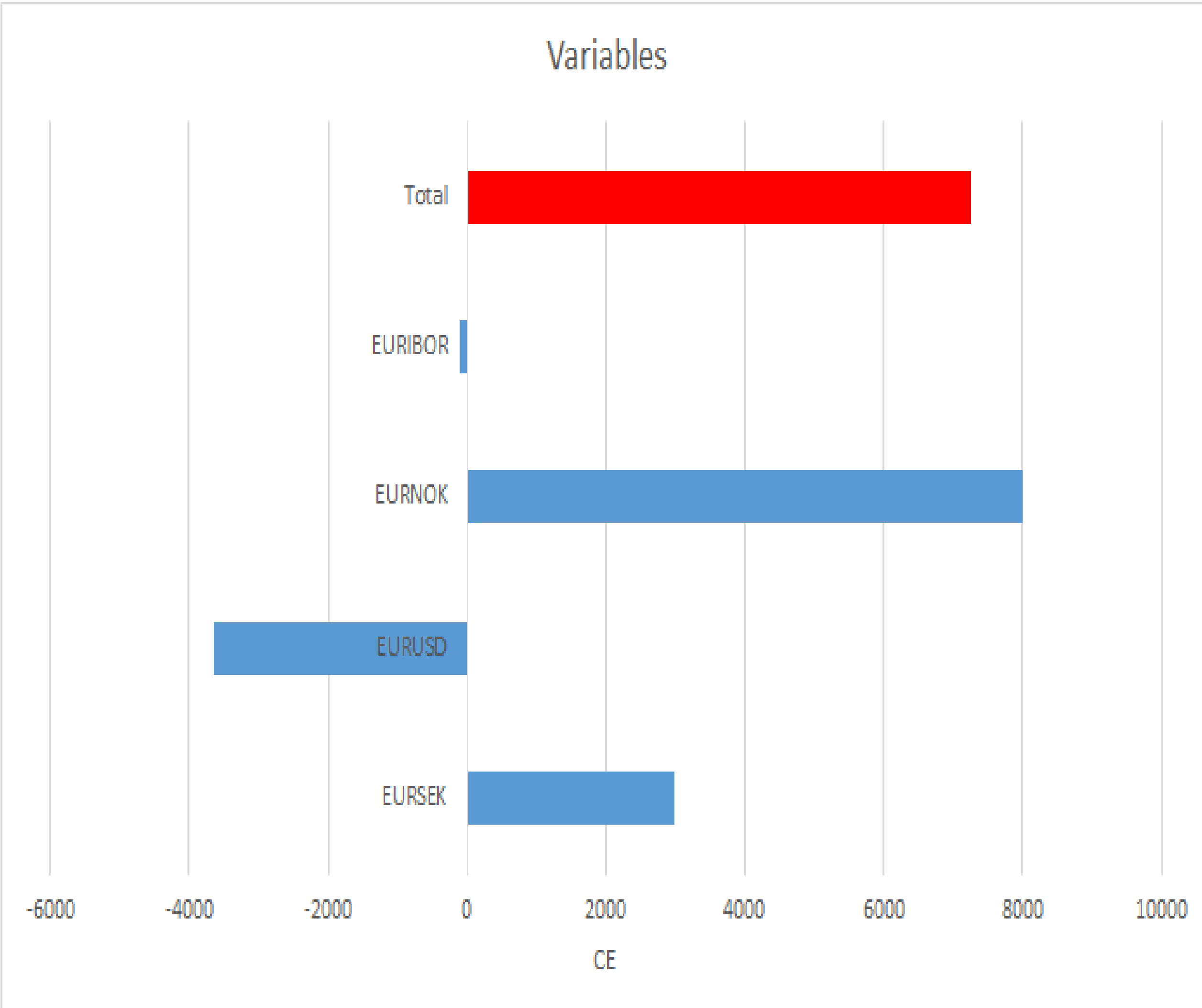

Figure 3: Changes in current exposure divided into factor level

In figure 4, the portfolio's CE change and each client's individual contribution to it between the same dates as above are shown as a bar graph. It can be seen that clients 1, 3, and 5 have contributed positively to the CE change, whereas clients 2 and 4 have contributed negatively to it. The CE change of the portfolio is shown on the red "Total" bar.

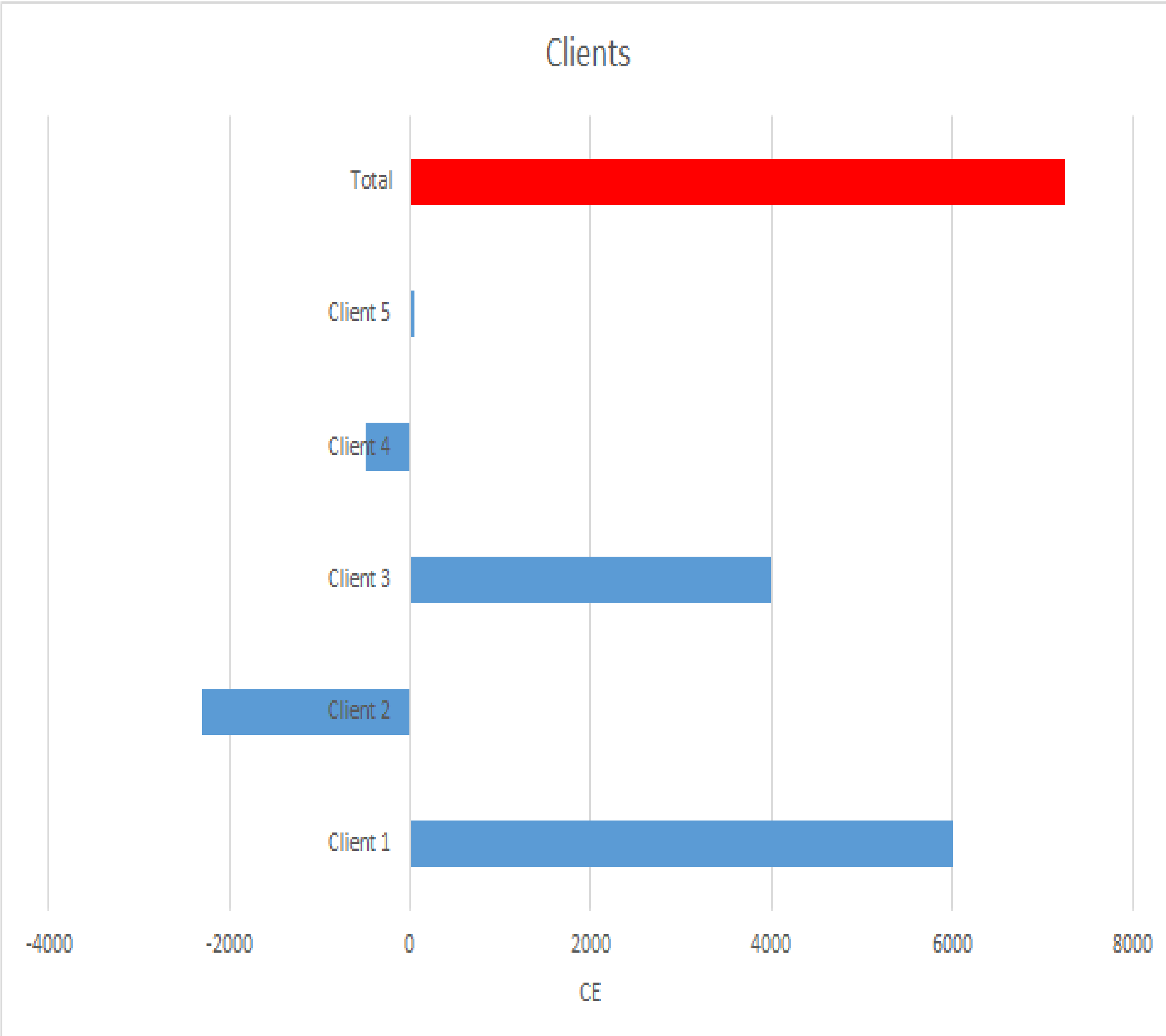

Figure 4: Changes in current exposure divided to netting set (client) level contributions

The contract level contributions are shown in figure 5. It can be seen that deals 1-3 have contributed positively to the CE, whereas deals 4-7 have contributed negatively to it. Also in this graph, the CE change of the portfolio is shown on the red "Total" bar.

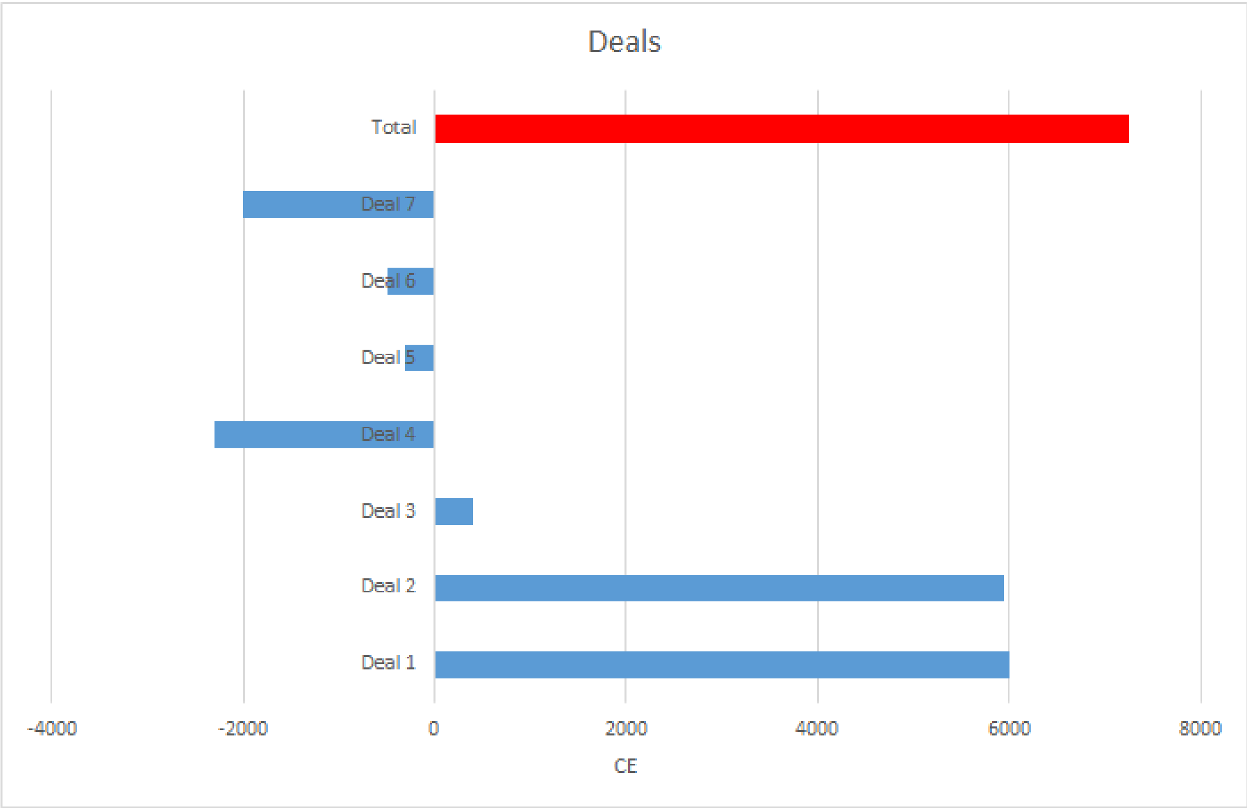

Figure 5: Changes in current exposure divided into contract level

6. Validation

The validation of results was done by comparing the changes in CE calculated by the Excel program (x) to the known real CE changes (y). R-squared statistic a statistical measure of how close the data are to the fitted regression line. In ideal situation the relationship between x and y is linear: x=y, and then the R-squared statistic is R^2 =1, whereas R^2 =0 would mean that the variation in y is not at all explained by values of x. The accuracy of the tool was examined by calculating R^2-values for all the deals and dates for which data was available. The results were good (R^2>0.9), and one scatter plot of CE changes for one example portfolio is shown in figure 6. It can be seen that the points are close to the orange line x=y, but not exactly on it.

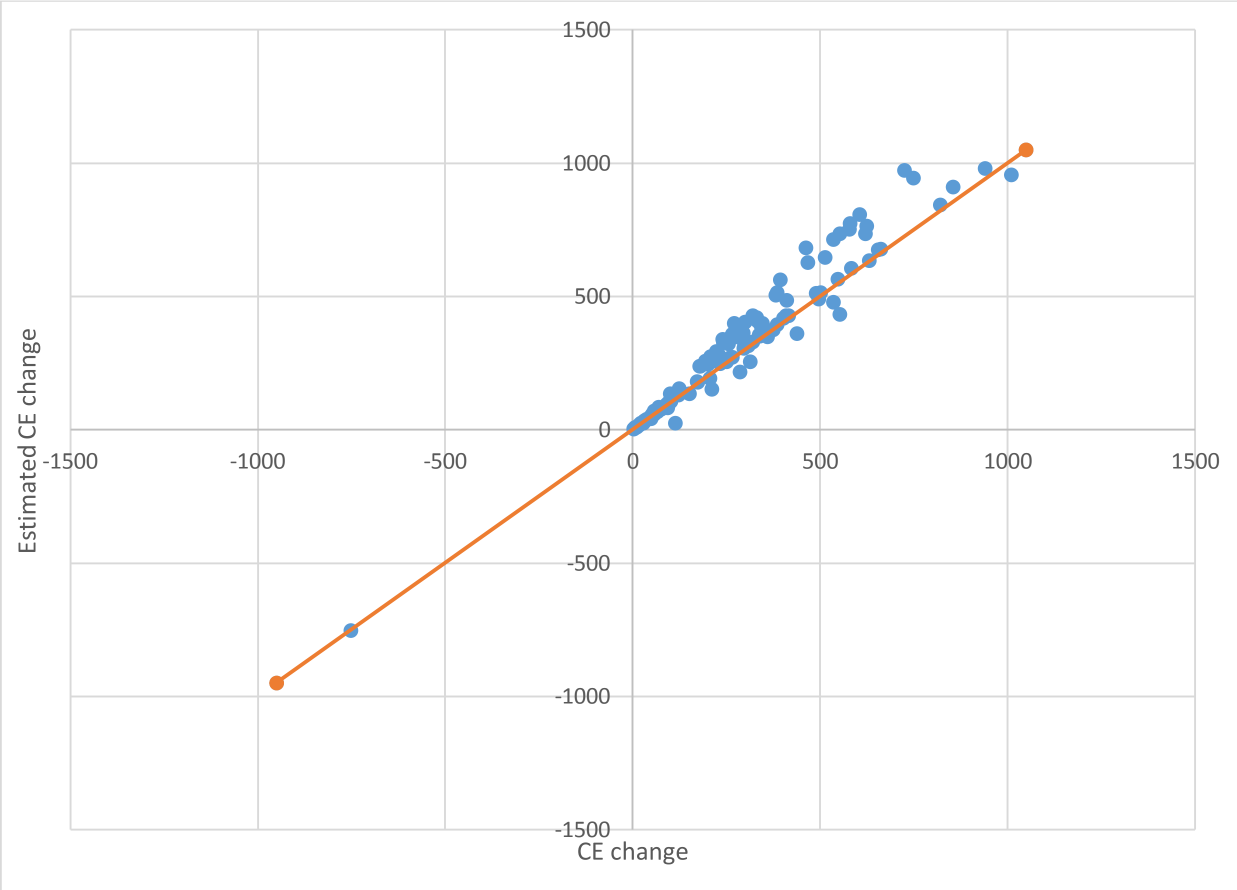

Figure 6: Estimated CE change vs real change. The observations are scattered close to the orange line (Estimated CE change = CE change), which means that the estimation accuracy is good. Also R-squared statistic is good (R^2=0.91).

For deal types that are not very dependent on interest rates, i.e. FX forward and FX swap, the R-squared value was practically 1 (>0.99). Good results were found for other deal types (IR: R^2 >0.7 and CC swap: R^2 >0.8), too, even though there were some inaccuracies, possibly as a result of differences between the approximated and real interest rate curves. There are several ways to improve the accuracy of the tool, explained in more detail in paragraph "Limitations". A few examples are using the real underlying interest rates of each deal for calculations, taking into account the impact of the collaterals, and using more interest rate-related terms of different lengths in the Taylor polynomial.

7. Limitations

To keep the workload of the project group within the limitations set by the 5 credits of the course on which this project was done, some issues of interest have been deliberately excluded from the project. The purpose, however, was to build a model that can be later developed to include more deal types and more changes in market variables, in order to extend the usage of the tool to examining the client's entire derivative portfolio. It is vital that the end user known the limitations. The following assumptions and exclusions are made in the model:

- Portfolio is assumed to be static in the sense that no contract begins or matures during the period of analysis. The impact of the new and matured deals could, however, be easily added to the tool by subtracting the CE of all matured contracts, and adding the CE of all new contracts. New contracts, however, do not contribute to large errors in CE calculation, as the initial CE of a derivative contract is typically zero.
- All interest rates in one currency for contracts of different lengths are treated as one explaining factor, even though it is possible that changes of rates are not parallel. It is even possible that short-term and long-term rates move to the opposite directions at the same time.
- Impact of fixing dates, i.e. dates of interest payments, is not taken into account, since there was not enough information about the payment schedules of the contracts. The fixing dates' impact could be easily added to the model, if precise information about them was available.
- Collaterals are not taken into account when calculating CE. The impact of a client's collateral is easy to add to the model, as it can be simply subtracted from any positive CE. In some cases this no-collaterals-assumption may cause even large errors in CE calculation.
- The impact of elapsed time on market values of contracts is not taken into account. This exclusion causes errors, which are relatively small for the chosen contract types, when the time elapsed is short (~a few days). The impact of the time elapsed should be considered, if the model was to be used over long periods of time, or for contracts whose values change rapidly as a function of time. The impact of time would be easy to add to the model using the Taylor polynomial approach, once the valuation formulas for each contract type are known.
- The interest payments related to float leg of interest rate and cross currency swaps are predicted using the spot and forward rates of the same interest rate curve that is used to discount these future payments. This assumption does not always hold and may cause (even large) errors in the present values of the calculated exposures. It would be possible to add separate rates for predicting and discounting cash flows, once the underlying rates of the legs are determined and appropriate discount rate is identified. In fact, the underlying rate of a contract might be for example 6-month Euribor + 1% interest rate margin, and appropriate discount rate could be dependent on the credit rating of the counterparty that pays the float leg.

- Individual interest margin of the counterparty paying float payments (1% in the above example) is not taken into account, since there was no data about such margins. This may cause large errors in some cases, but can be fixed easily, if the margins are known.

- Spot rates (rate for discounting future cash flows to today) are approximated through linear interpolation. When the shape of the interest rate curve is "normal" (not too peculiar), it can be considered to be piece-wise linear, and thus this assumption does not usually contribute to very large errors. Adding other types of interest rate curves would be possible, too, once the appropriate method for estimation is determined. For an introduction to stochastic methods for interest curve estimation and prediction see e.g. (Beletski, 2003)

- The interest rate related to a float leg payment of IR or CC swap is assumed to be fixed on the same date that the previous interest payments are made. This date is called fixing date.

- Portfolio is assumed to be static in the sense that the nominal amounts of the legs do not change during the examining period. This means, that the nominal amount is paid in maturity, and only accrued interest is paid on fixing dates.

- The period between fixing dates is assumed to have always the same length, for example 3 months. In vast majority of contracts this is true.

- The analysis is only intended to explain changes in CE. It might, however, be interesting for practitioners to find information about changes that do not directly affect CE, but make it more likely to increase in future. Such changes happen when the market value of a contract or portfolio is more negative (CE=0) and increases to near-zero level.

- For the calculation of individual contribution of a contract to the CE change of a portfolio, it is assumed that the CE of the portfolio has changed. If CE did not change, it would not make sense to try to explain the change in it.

- For changes in IR and CC swap exposures explained by interest rates, the rates are approximated by mean of the known spot rates, i.e. duration-based CE change approximation is used (Luenberger, 1998). This assumption causes errors in CE, and more robust way would be to calculate the sensitivities to all spot rates separately, and to use them all in the Taylor polynomial.

- First-order Taylor polynomial is used to calculate the changes in CE. If a change in market variable is large, there may be inaccuracies in the calculation of that variables contribution to CE change. Also, the calculation becomes more inaccurate as the examining period becomes longer. Adding second-order terms to the Taylor polynomial calculation would make the calculation more accurate in such situations.

8. Simulated portfolio

In addition to the analysis of the past changes in the derivative portfolio's current exposure, the scope of this project contains also the examination of the risk allocation in the case of portfolio simulations. In portfolio simulations, the possible future risk levels of a selected portfolio are estimated using Monte Carlo simulation, i.e. large amount of market scenarios are generated and the current exposure of the portfolio is calculated in each scenario. The simulations are done to find out how much and for what reasons the risk level of the portfolio might rise in the future. The aim of this chapter is to introduce some methods, which can be used to explain to what extent an individual market variable or deal contributes to the change in the portfolio's risk level in a specific simulated scenario.

One possibility is to use the same method, which was used in this project work for analyzing the past changes in the portfolio's current exposure. The only difference would be that instead of the historical market data, the simulated market data would be used for calculating the changes in the deals' market values. Basically, this could be done by simulating set of market variables from for example multi lognormal distribution and then calculating the changes in market variables using Taylor's approximation. Parameters to the distribution would be historical averages and covariance matrix. Two of the main issues with that approach are the accuracy and required computational capacity.

For simple derivatives, such as FX swap, FX forward, IR swap and cross currency swap, the accuracy could be sufficient, but when more exotic and non-linear derivatives are included in the portfolio, the accuracy of the Taylor series expansion suffers significantly. One way to improve the accuracy is to use higher order Taylor series. In the case of historical data, the real market values are known for each day, so it is possible to correct the errors of the estimation each day by comparing the calculated change in market value to the real one and then use the corrected, i.e. real, market value for determining the next Taylor series. When the simulated data is used, this kind of corrections cannot be made and that affects the accuracy as well.

Monte Carlo simulation is computationally demanding because the number of simulated scenarios must be big enough. In a portfolio there can be thousands of deals, so constructing a Taylor series for each deal each day for every simulated market scenario might not be feasible. However, if only the worst case scenarios are of interest, it is not necessary to perform the analysis during the simulation. Instead of using the method for all scenarios, the market data of the worst scenarios could be saved during the simulation and the analysis done on them after the simulation run. One other problem is that the method requires differentiable valuation formulas for each deal, so if in the portfolio there is a derivative, which cannot be valuated by a differentiable function, the method will not work as it is.

9. Conclusion

The main objective of this project work was to provide a tool for analyzing the changes in the counterparty credit risk of a derivative portfolio. The analysis tool presented in this report is based on Taylor-series approximation of the derivatives' valuation functions. The tool is implemented in Excel and it currently works only for four types of derivatives: FX swap, FX forward, IR swap and cross currency swap. In principle, the same method works also for other types of derivatives as well, so the tool can be expanded to include more derivatives and market variables.

The test runs and validation show that the tool works reasonably well for the example portfolio. Especially in the case of FX swap and FX forward deals, the accuracy of the tool is really good. For some IR swap and cross currency swap deals, the estimated change in the risk differs significantly from the real change. The main reason for that is assumed to be the lack of accurate information on the interest rate curves. There are several simplifications used in the example portfolio, e.g. the portfolio is static, collaterals are not included etc., so the tool needs to be developed further for it to work smoothly in everyday use. The simplifications and limitations are listed in this report, but the accuracy of the tool might be sufficient even if not all of the limitations are fixed.

10. Self assessment

Altogether, our team got through the task well, and the project was successful. Almost at the outset, we had a clear idea of what we were expected to do and how to achieve it. Without any major problems, we were continuously able to progress and keep to the schedule. As the result, we managed to yield concrete value for our client, as they can utilize the final product in their daily working. Moreover, we gained experience in applying our theoretical knowledge on operations research to practice by working on a real case.

Workload of the project was quite large and we spent a lot of time on it. However, this was expected, so right from the beginning we took this into account when scheduling and planning our work. By having meetings every week, we ensured that the project progressed and the necessary tasks were done in time. In addition to the meetings, where we did mostly ideation and development of the model, we did quite a lot of individual work, especially on the pricing models. Furthermore, we had a few meetings with our client at Nordea's office, where we discussed some details of the model and had chances to ask specific questions, mostly related to the data.

We think we did particularly well in general team working and project management. We scheduled our work realistically, and were able to recognize all relevant tasks when planning the project. Also, we had sufficient amount and frequency of meetings in terms of workload and schedule, so we were not behind the schedule at any phase. Additionally, we succeeded in terms of communication to both our client and among ourselves. We were actively in connection with our client, resulting throughout the project a mutual image of what the project will eventually look like. In the same way, our active communication with each other ensured that we all had a clear view of what to do. On the whole, we succeeded to complete the project without significant modifications to the scope or the content of the project.

As always, there is also room for improvements. Probably the biggest challenge, in terms of project management, was the task of building the Excel-model. Since we did not make sufficient technical planning for the model, equal separation of the workload was not possible, resulting that most of the programming and technical work were done by a single person.

A big thanks belongs to our contacts from Nordea, who made a lot of effort to the project and were willing to help us whenever any issues occurred.

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