Seminar on Case Studies in Operations Research (Mat-2.4177)

The pricing of Asian commodity options

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Client: Danske Bank

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Contents

1 Introduction

This student project was carried out for the Aalto University School of Science course Mat-2.4177 – A Seminar on Case Studies in Operations Research. It was executed in collaboration with Danske Bank Markets Finland which acted as a client for the project and set the topic for it.

We explore pricing methods for Asian options in commodity markets. Asian options are path-dependent in that they are settled against the predetermined strike price and the arithmetic average of spot prices calculated over a given time interval. They have several advantages. In thinly traded markets, average price options are not as vulnerable to price manipulation as standard European options whose payoff depends only on the price on maturity. Moreover, the averaging smoothens the high volatility observed in commodity markets. Thus, Asian options are often cheaper than European options. To hedge periodic cash-flows, Asian options can be used as an alternative to entering into multiple option contracts.

However, as a closed-form solution to the pricing of Asian options does not exist, approximative solutions must be used. Based on the extensive literature on the subject and expert advice from the project client, we focus on stochastic models with fundamental factors such as seasonality in weather. Also, we are particularly interested in using Monte Carlo simulation methods for the generation of large number of random price paths. These models are flexible but they can be hard and time-consuming to calibrate.

Commodities have some special characteristics that distinguish them from interest rates and foreign exchange market, for example. These include non-storability (e.g. electricity), seasonality (e.g. agricultural products), jumps and periods of high volatility, among others. Because of the special characteristics of commodity price process, it is necessary to study them independently.

Danske Bank set the following project objectives which remained unchanged through it. The high-level objective was to produce pricing models for Asian commodity options. There were three asset classes that were of main interest to our client: electricity, oil and agricultural products. Modelling of all these asset classes was not, however, required. Furthermore, the client instructed to prefer quality over quantity in developing the models. Therefore, we decided to build models for two asset classes – electricity and oil – so that the client is satisfied with them. In particular, we were asked to focus on the argumentation of the fundamental structure of the models and parameter selection in the modelling process. To accomplish this, we reviewed the literature for cases similar to ours.

The model chosen for electricity combines features from existing literature (especially Geman and Roncoroni (2006), Weron (2008)). For oil the best model was not as clear as explained in Section 3.2.2. Especially in the case of electricity more data would be needed to completely validate the results, as there were no relevant market prices available for comparison.

The final report is organized as follows: Section 1.1 presents necessary background definitions for understanding the work. Section 2 discusses relevant literature. Section 3 presents the pricing models and their results. In the Section 4, we validate the results based on two distinct criteria. Limitations of our produced models are discussed in the Section 5. Section 6 concludes the project work. Self-assessment is attached to the appendix.

1.1 Terminology

A few key concepts used in the project are shortly explained in this section.

Spot price is defined as the current price of a commodity at which it can be bought from or sold to markets. In this project, the data for spot prices consist of daily prices – electricity prices are quoted everyday and oil prices only on weekdays.

Futures price refers to a present buy or sell price of a specific standardized asset which has a certain future date for its delivery and payment. Hedging against volatile future spot price can be put into practise by locking the price with a futures price in a contract i.e. futures contract.

Implied volatility is a volatility value which is equivalent to the current market price of an option via Black-Scholes equation. It is also a convenient way to express the price of an option contract.

2 Literature review

Here, we present a review of existing solution methods and models for commodities option pricing with the special interest in stochastic models. There are various kinds of stochastic price processes for describing the spot price movements of commodities and thus we determined early on to study the processes broadly. The review begins from the very basics of Black and Scholes (1973) model and extends with models which provide amendments to it. Also, other methods such as approximate analytical solutions are discussed in general. Because we decided to research models widely, it was natural to add more models to the review as the project advanced through modelling process. We decided on the models in this literature review by exploring through – in part – the books of Geman (2010) and Glasserman (2003), searching for the scientific publications of commodity price processes, receiving advice from our client's experts and screening quantitative finance forums.

The original Black-Scholes model in Black and Scholes (1973) describes the evolution of a stock price through geometric Brownian motion.

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{1}$$

were S_t is the stock price, μ is the mean rate of return, σ the volatility of the stock price and W_t a standard Brownian motion. Under this framework, spot prices S_t are log-normally distributed and a closed-form solution exists for an arbitrary initial value S_0. As Black and Scholes (1973) show, an analytic formula also exists for pricing European options.

However, no closed-form solution exists for the pricing of arithmetic average options. The arithmetic average of n samples of spot prices is given by Equation 2.

$$A(T) = \frac{1}{n+1}\sum_{i=0}^{n} S(T_i), \tag{2}$$

where $T_i = T_0 + i(T - T_0)/n$. According to Turnbull and Wakeman (1991), the value of the arithmetic average option is given by Equation 3.

$$C(S(t), A(t), t) = e^{-r(T-t)} E^{S(t),A(t),t}\,\mathrm{Max}[A(T) - K, 0], \tag{3}$$

where r is the risk-free interest rate, K the strike price, and $E^{S(t),A(t),t}$ the conditional expectation with respect to $S(t)$, $A(t)$ and t. The random variable $A(t)$ includes the sum of log-normally distributed random variables $S(t)$. Given that the sum of log-normally distributed random variables is not log-normal, the probability density function of $A(t)$ is not known. Several approaches to attack this problem exists. Turnbull and Wakeman (1991) approximate the unknown probability density function with an Edgeworth series expansion in which an alternative log-normal distribution is used. The series is developed using an algorithm Turnbull and Wakeman (1991) provide to calculate the first four moments of the true density function. They conclude that their approximation give options values close to Monte Carlo simulations which are used as a benchmark. In contrast to Monte Carlo simulations, the analytical approximation method is cheap in computing time. Other approximative solutions are presented in Levy (1992) and Curran (1994).

Glasserman (2003) develops a simple Monte Carlo algorithm for valuation of path-dependent options by utilizing the fact that a closed-form solution exists for generating spot prices from Equation 1. An overview of stochastic modelling of commodity price processes is provided in Geman (2010). According to Geman (2010), the price trajectories generated by Monte Carlo methods need to look like the observed ones. In addition, the statistical properties of the model, i.e., at least the first four moments, need to match to the empirical properties. Often, a compromise must be made between the complexity and accuracy of the model. A structurally complex model with large number of parameters requires more computing time and can be challenging to calibrate, whereas a parsimonious model may not describe properly the observed data. In operative use, in particular, the robustness of the parameters of the model is important.

Mean-reversion is observed in many commodity spot markets because supply and demand can be adjusted to very low or high prices. Geman (2010) presents an Ornstein-Uhlenbeck process in which the constant mean rate of return μ of the Black-Scholes model is replaced by a linear function.

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t, \tag{4}$$

where $X_t = \log S_t$ and θ the force of mean-reversion. Also, a closed-form solution exists for Ornstein-Uhlenbeck process so that price paths can readily be simulated by Monte Carlo methods. Geman (2010) shows that by replacing the constant mean rate of return μ in Ornstein-Uhlenbeck process with a periodical function, one obtains a mean-reverting price process with seasonality.

$$dX_t = D\mu(t)dt + \theta(\mu(t) - X_t) + dW_t, \tag{5}$$

where D denotes the standard first order derivative. Depending on the function $\mu(t)$, a closed-form solution to this SDE may or may not be available.

In addition to mean-reversion, price peaks and drops are common in stock and commodity markets. Merton (1975) extends the Black-Scholes model by adding price jumps.

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma W_t + (y_t - 1)dP_t, \tag{6}$$

where μ is the mean rate of return, σ the standard deviation of the return and W_t a Brownian motion. The price jump arrivals are modelled with a Poisson process P_t with λ as the mean number of arrivals per unit time. If a Poisson event occurs, then the impact of the event y_t is drawn from a log-normal distribution. Parameter k is the mean relative jump size $E[y_t - 1]$. Price paths can be generated from the closed-form solution of this price process given in Merton (1975).

Geman and Roncoroni (2006) present a model for three major U.S. electricity markets that combines mean-reversion, seasonality and price jumps. Electricity prices in these areas are mean-reverting to a deterministic trend driven by seasonal changes in temperature. Unexpected power plant failures, transmission line outages and very warm temperatures, in particular, can cause very high price peaks which are followed by downward jumps when supply and demand are adapted to very high prices. The price process is represented by the stochastic differential equation.

$$dE_t = D\mu(t)dt + \theta_1 (\mu(t) - E_{t^-})\, dt + \sigma dW_t + h(E(t^-))dJ_t, \tag{7}$$

where E_t is log electricity price, $\mu(t)$ a periodical function, θ_1 force of mean-reversion, and $f(t^-)$ denotes the left limit of f at time t. The number of jumps experienced up to time t is characterised by a Poisson process $N(t)$ with a periodic function $\iota(t)$ as the intensity parameter. The jump sizes are modelled as a compound jump process $J(t) = \sum_{i=1}^{N(t)} J_i$, where J_i are identical and independent draws from an exponential distribution. The jump direction is specified by a Heaviside step function $h(E(t^-))$ with a threshold parameter Γ. Weron (2008) applies a model with similar characteristics to the Nordic power market, and calibrates it to observed futures and Asian options data. Features from both Geman and Roncoroni (2006), Weron (2008) are used to construct the chosen model for electricity.

Besides extending the Black-Scholes model with mean-reversion and jumps, another approach to improve modelling is to introduce additional state variables. Schwartz and Smith (2000) present a two-factor model which decomposes spot prices S_t to short-term deviations χ_t and long-term equilibrium price level ξ_t which are modelled as separate stochastic processes. This configuration allows mean reversion in short-term and stochastic equilibrium level to which prices revert. The model is represented by a system of stochastic differential equations.

$$\begin{cases} d\chi_t = -\kappa\chi_t dt + \sigma_x dz_x \\ d\xi_t = \mu_\xi dt + \sigma_\xi dz_\eta \\ dz_x \cdot dz_\xi = \rho_{x\xi} dt \\ S_t = \exp \chi_t + \xi_t, \end{cases} \tag{8}$$

where κ is the mean-reversion coefficient, μ_ξ the mean rate of return, σ_x and σ_ξ standard deviations, and dz_x, dz_ξ standard Brownian motions which are correlated with the coefficient $\rho_{x\xi}$. Schwartz and Smith (2000) use Kalman filtering with prices of near and long-term futures contracts as observations to estimate the unknown state variables and parameters in the short and long-term processes, respectively. They conclude that their model is better at explaining weekly crude oil data than a geometric Brownian motion or an Ornstein-Uhlenbeck process.

Heston (1993) presents a two-factor stochastic volatility model in which spot prices follow the Black-Scholes model and volatility an Ornstein-Uhlenbeck process. Heston shows graphically that if these two processes are correlated, one can simulate fat tails observed in many markets in the probability density of spot price returns. Bates (1996) adds jumps described by compound Poisson process to Heston's model.

The constant volatility parameter σ in the Black-Scholes model is a function of strike price and maturity in local volatility models presented in Dupire (1994) and Derman and Kani (1994). The functional form of the volatility term is derived from the observed implied volatilities. As a result, the models have a good fit to the observed option prices. However, problems can arise when interpolating or extrapolating to strike prices and maturities that are not quoted at the moment. These models can be computationally heavy, too.

The more complex models include several parameters that need to be estimated from data. In the literature, there are differences to the selected estimation method and used data. Geman and Roncoroni (2006) average historical electricity prices to estimate the periodical function $\mu(t)$ with OLS regression. The mean-reversion parameter θ_1, among others, is estimated using a maximum likelihood function to match the first four moments to the empirical ones. A model calibrated to historical data should be used with care to price options with maturities in the future.

To overcome this, the calibration of the underlying price process can be based on observed option quotations with different maturities and strike prices. According to Gilli et al. (2011), the objective is to find a parameter vector θ so that the option prices from the model C_{model} are consistent with market prices C_{market} with different maturities and strike prices.

$$\min_{\theta} \sum_{i=1}^{N} \frac{|C_{model} - C_{market}|}{C_{market}}, \tag{9}$$

where N is the number of observations. The optimization problem can be solved using commercial solvers, and other objective functions such as squared errors can be employed.

If a closed-form solution does not exist for a price process described by a stochastic differential equation, some error is introduced through the time-discretization when applying Monte Carlo methods. Glasserman (2003) presents the Euler-Maruyama method which can be applied to approximation of stochastic differential equations. The discretization error can be reduced to some extent by using high-order discretisation schemes but these are not common in the literature.

Moreover, Monte Carlo methods involve calculation of n price paths. The standard deviation of the Monte Carlo estimate for the value of an Asian option is proportional to $\sqrt{n}$, and, even with high n, the standard deviation can be too high for operational use. To decrease the computational effort, variance reduction techniques can be employed. Kemna and Vorst (1990) model stock prices with the Black-Scholes model. They arrive at an analytic expression for the value of a geometric average option which serves as a statistic for the unknown arithmetic average price. As Glasserman (2003) shows, the statistic and the original Monte Carlo estimate can be used together to form a new estimator for the price of the Asian option with lower variance. Boyle et al. (1997) discusses several other variance reduction techniques to improve the efficiency of the Monte Carlo methods.

Other methods to price Asian options not discussed in detail include, among others, binomial trees – Hsu and Lyuu (2011) – and Fast Fourier transforms by, for example, Benhamou (2002). Using Monte Carlo methods implies forecasting of spot prices. Thus, also time-series models with exogenous explanatory variables similar to Solin et al. (2011) can be employed. If the process for spot prices cannot be identified, then learning methods presented in de Souza e Silva et al. (2010) and Shin et al. (2013) can be utilised.

3 Data and Methodology

We focus on asset classes of electricity and oil. Due to the distinct qualities of these commodities, it is necessary to model them separately. For example, the spot price of electricity has spikes and consistent cycles which cannot be observed in oil prices. Based on the data available, the model for electricity is fitted to historical data of spot prices and oil models are directly fitted to the options prices perceived in the market. The models are developed in Matlab. All the data for the project – including actual market spot prices and implied volatilities of option contracts – was provided by Danske Markets.

3.1 Electricity

Our daily data set for Nord Pool spans from 24 December 2004 to 24 January 2014 and has 3319 observations. Conceptually, we divide Nord Pool spot prices Y_t into three components that build up our model.

$$Y_t = S_t + \exp(J_t) + \exp(X_t), \tag{10}$$

The first component S_t represents seasonal deterministic cycles. Figure 1 shows Nord Pool spot prices (€/MWh) and a first-order Fourier function F_t fitted to them. We find the function F_t with Matlab's Curve Fitting Toolbox. Higher-order functions improve the fit to the historical data at the cost of a possible overfit. Although there is strong variation, spot prices show a cycle that can be traced back to seasonal variations in temperature and hydro reservoirs. Moreover, a weekly cycle is present in the spot prices because demand is higher during work days than weekends. Following Weron (2008), we model the weekly cycle by calculating the median price of each weekday, which are substracted from the average of these median prices. The resulting figures W_t are added to F_t to obtain the seasonal component S_t

The second component J_t represents the strong up and downwards spikes in the time-series which can be attributed to days with extremely high demand and hydro must-run situations, respectively. Spikes are defined as an increase or a decrease in spot price Y_t exceeding $H = 3$ times the average absolute price change $|Y_t - Y_{t-1}|$. Using this threshold level, we find 193 spikes in total, whereas Weron (2008) has only 9 in his dataset spanning from 1996 to 2000. Thus, the recent daily prices are more volatile than the prices in the past.

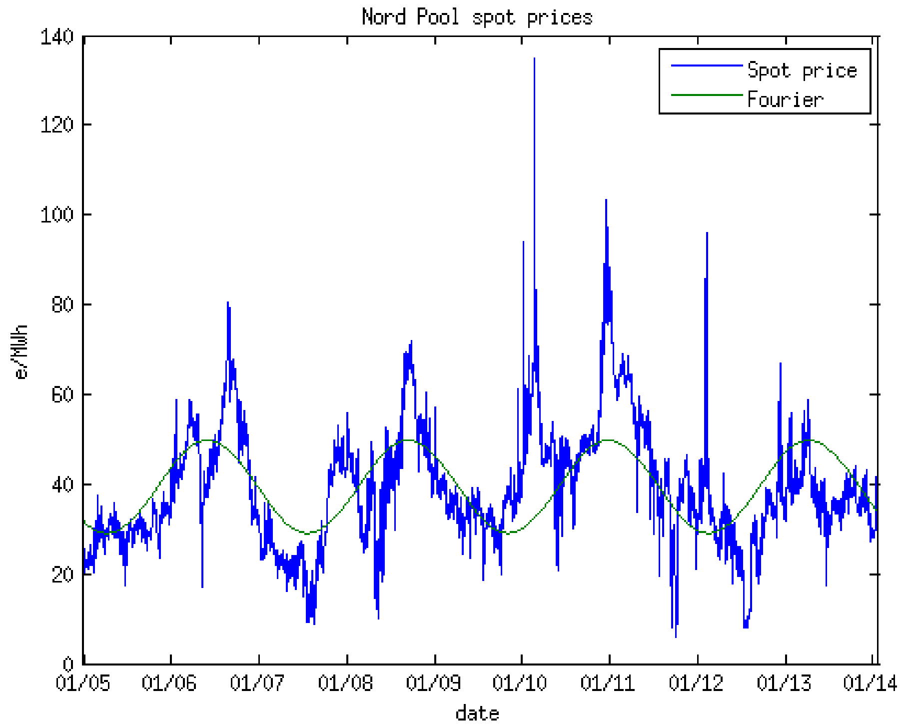


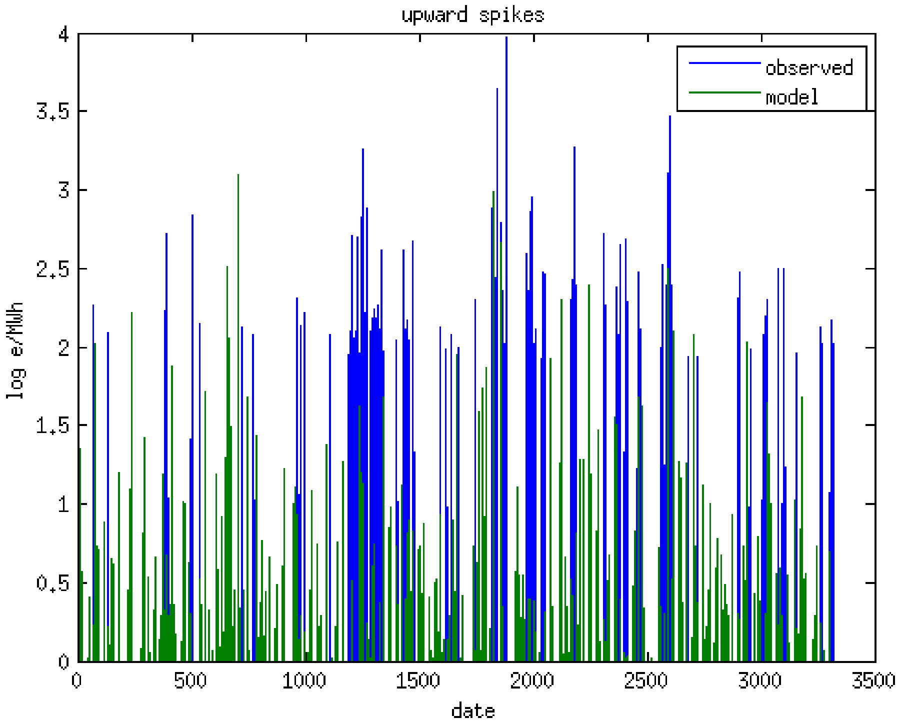
Figure 1: Nord Pool spot prices and a first-order Fourier function

We model both up and downward spike occurrence with Poisson processs P with intensity parameters λ_1 and λ_2, respectively. Spike sizes are drawn from exponential distributions E with rate parameters θ_1 and θ_2. As a result, spikes to both directions are modelled through the following equation

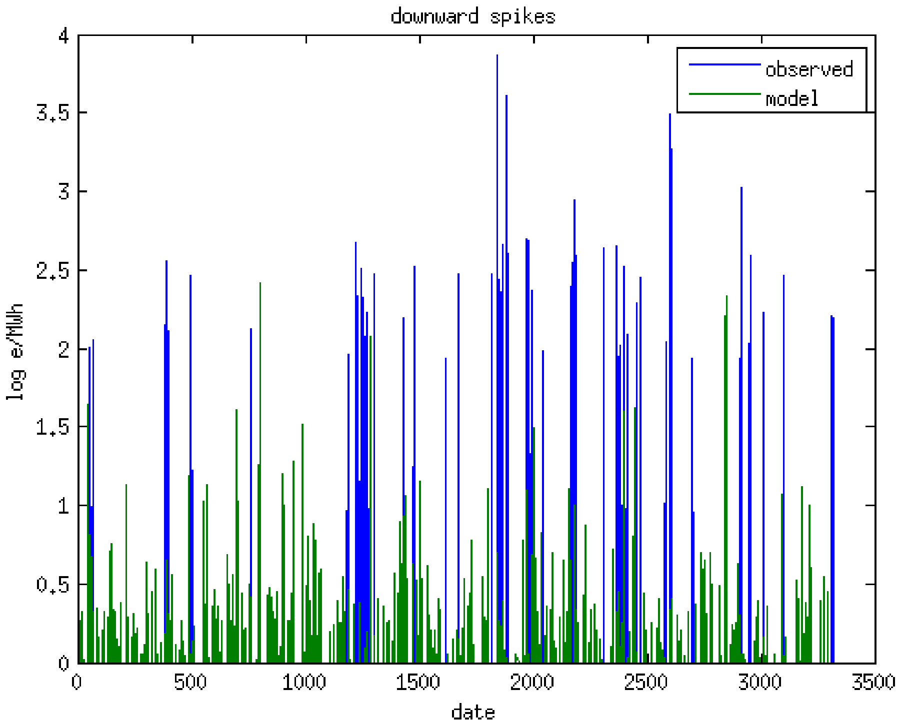
$$J_t = P(\lambda)E(\theta) \tag{11}$$

The models for up and downward spikes are calibrated by minimising the mean square error of the model subject to the parameters λ and θ. We calculate the model results by averaging $n = 100$ independent draws, and use Matlab's patternsearch routine to find optimal values for the parameters. We limit the number of independent draws (n) because higher values of n average the spike occurrence too much. Moreover, we note that manual search is required to obtain good initial values for the parameters. Figures 2(a) and 2(b) show the spikes filtered from the observed data and the model results for up and downward spikes, respectively.

Following the procedure in Weron (2008), we remove the seasonal component S_t from the



(a) Upward spikes



(b) Downward spikes

Figure 2: The occurrence and sizes of up and downward spikes

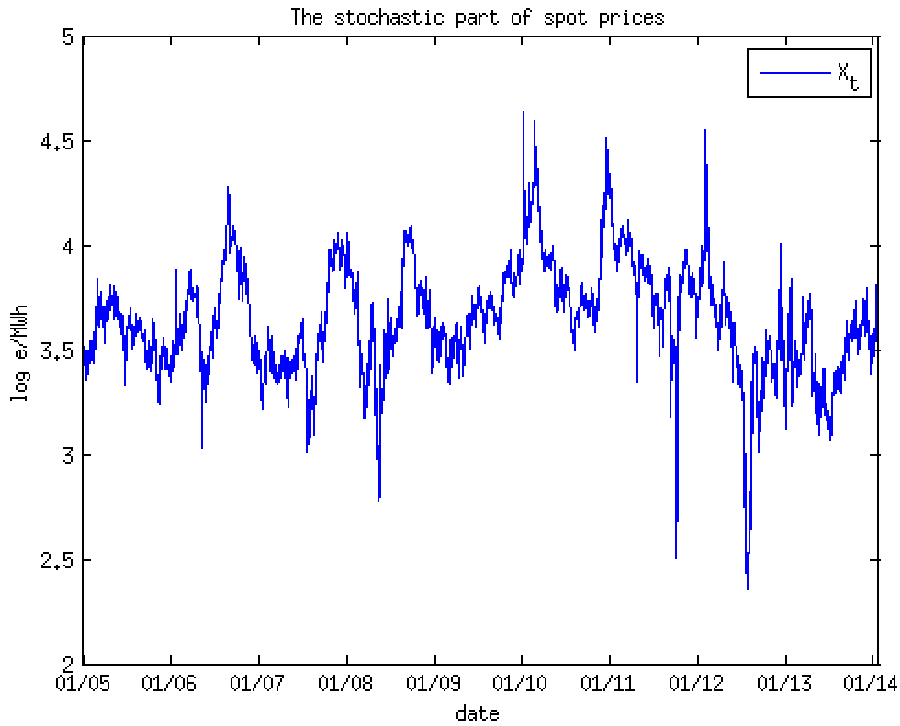


Figure 3: The stochastic part of Nord Pool spot prices

spot prices and take the natural logarithm to obtain log deseasonalised prices d_t. After replacing the values of d_t on dates with a spike with a 6-day moving average, we obtain the third component in our model, namely the stochastic process X_t. Figure 3 shows that there is still spikiness left in the process but we note that these spikes are often associated with longer periods of high or low prices. Furthermore, Figure 3 shows that the process tends to be mean-reverting.

Contrary to Weron (2008), the process X_t we obtain is not normally distributed. To account for the mean-reversion and periods of high and low prices, we combine some of the characteristics of the models presented in Weron (2008) and Geman and Roncoroni (2006). First, we model the mean-reversion through an Ornstein-Uhlenbeck process. Second, we use the methodology presented in Geman and Roncoroni (2006) to model the high and low prices. Our model for X_t is expressed by the equation

$$X_t = (\alpha - \beta X_t)d_t + \sigma dW_t + dK_t - dL_t, \tag{12}$$

where $\frac{\alpha}{\beta}$ is the mean, β the rate of mean-reversion, σ the volatility parameter, and K_t and

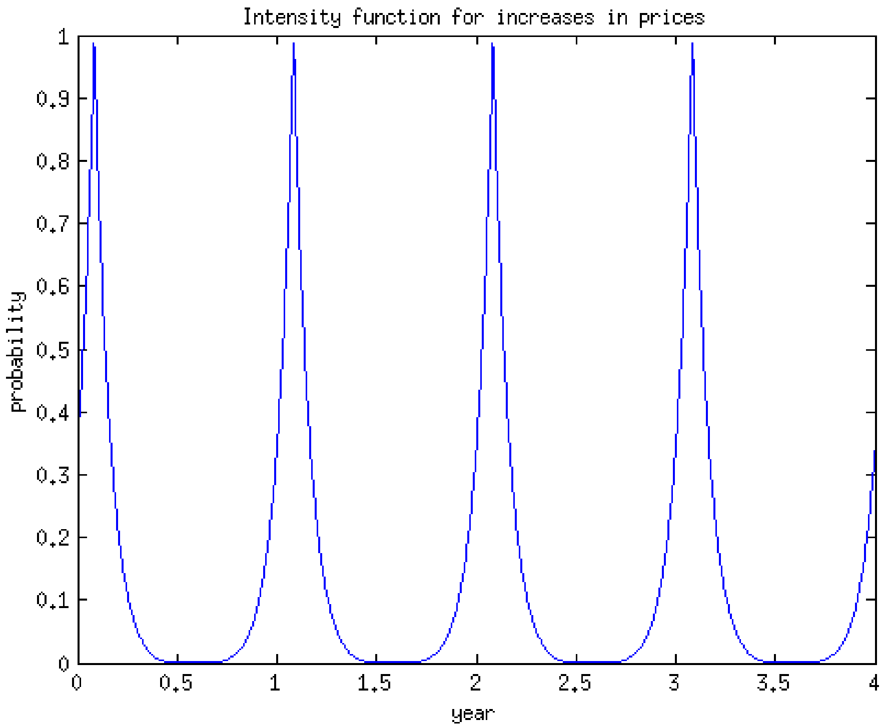
L_t increases and decreases in prices, respectively. The sizes of the increases are modelled as a compound Poisson process $K_t = \sum_{i=1}^{N_t} K_i$, where K_i's are random variables drawn from an exponential distribution with a rate parameter δ_1. The random variable N_t is a Poisson process with a deterministic intensity function depending on time t. The occurrence and the sizes of the decreases L_t are modelled identically with the rate parameter δ_2.

Contrary to Geman and Roncoroni (2006), we let prices remain on a high or low level and dismiss the Heaviside function forcing prices to revert back to the mean. Figures 4(a) and 4(b) show the deterministic intensity functions for periods of high and low prices, respectively. The motivation for the shape of the functions is the fact that, in general, higher prices occur in the winter and low prices in the summer.

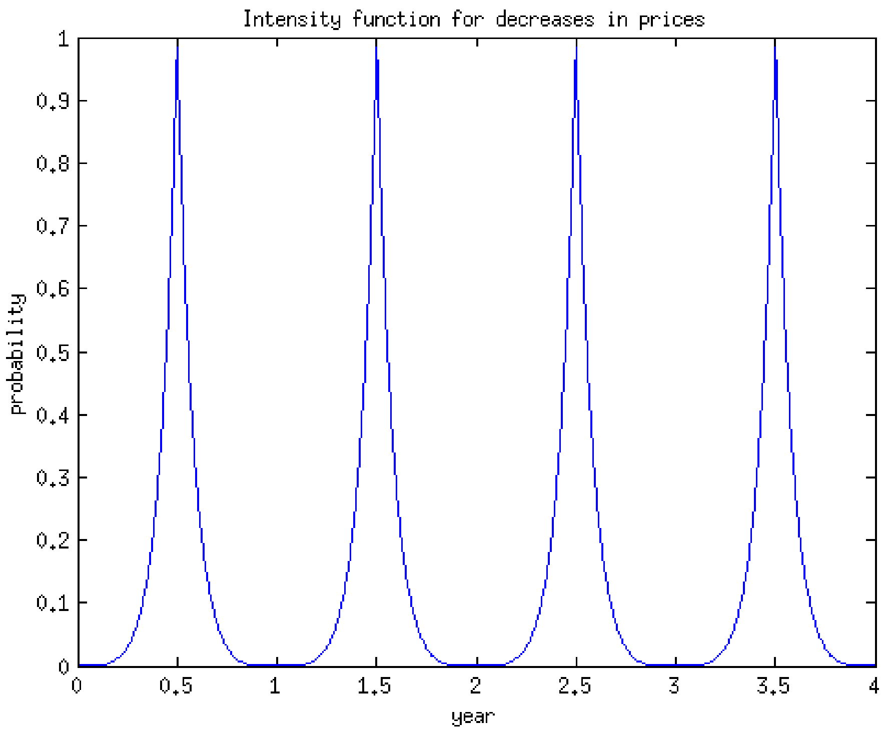
We calibrate our model for the stochastic part by minimizing the mean square error. We use Matlab's patternsearch routine with $n = 1000$ Monte Carlo simulations to determine the optimal values for parameters α, β, σ, δ_1, and δ_2 . When all components are calibrated, we add the seasonal part S_t, spikes J_t and stochastic part X_t to obtain the complete model for the Nord Pool spot prices. Figure 5 shows the fit of our model to the historical prices.

Applying a model fitted only to historical data to pricing derivatives on future spot prices is not plausible in electricity spot markets. Future spot price expectations are influenced by the long-run demand growth, planned changes in production capacity and unforeseeable factors such as the climate change. Current market quotations should incorporate this information and, therefore, we wish to adjust our model to it to price derivatives more reliably. Weron (2008) uses directly Asian options data but because these options are not publicly quoted anymore, we utilise only futures data. In general, these contracts are the most traded financial products in Nord Pool.

Similar to Weron (2008), we adjust the stochastic part X_t in our model to account for market expectations. As Weron (2008) shows, the adjustment parameter called the market price of risk is not constant. However, the parameter does not develop consistently for different futures. The forecasting of the development of the parameter is outside the scope of this project,



(a) Upward



(b) Downward

Figure 4: Intensity functions for up and downward jumps.

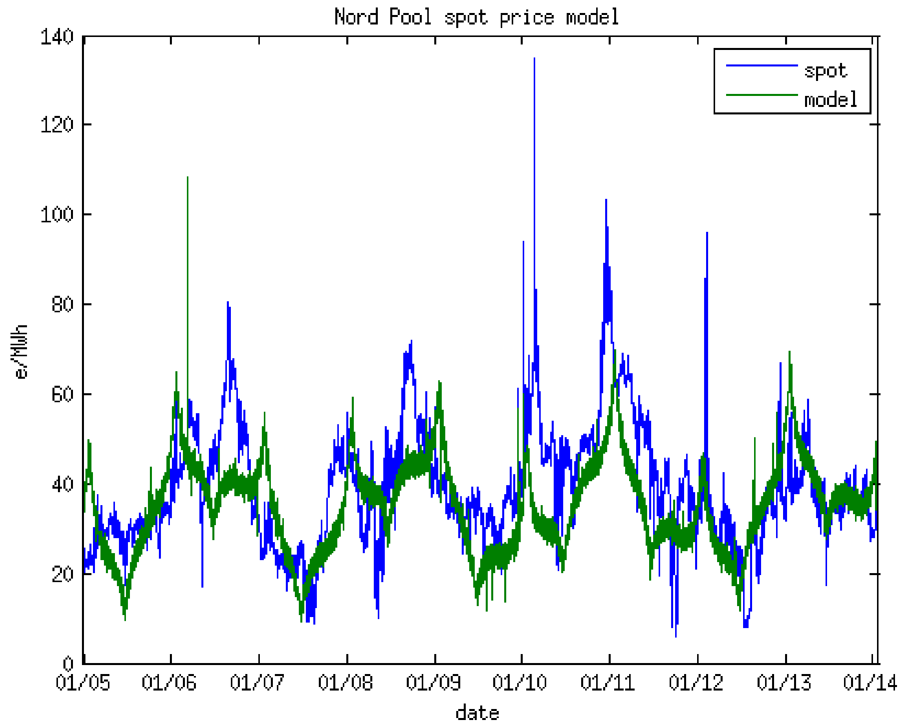
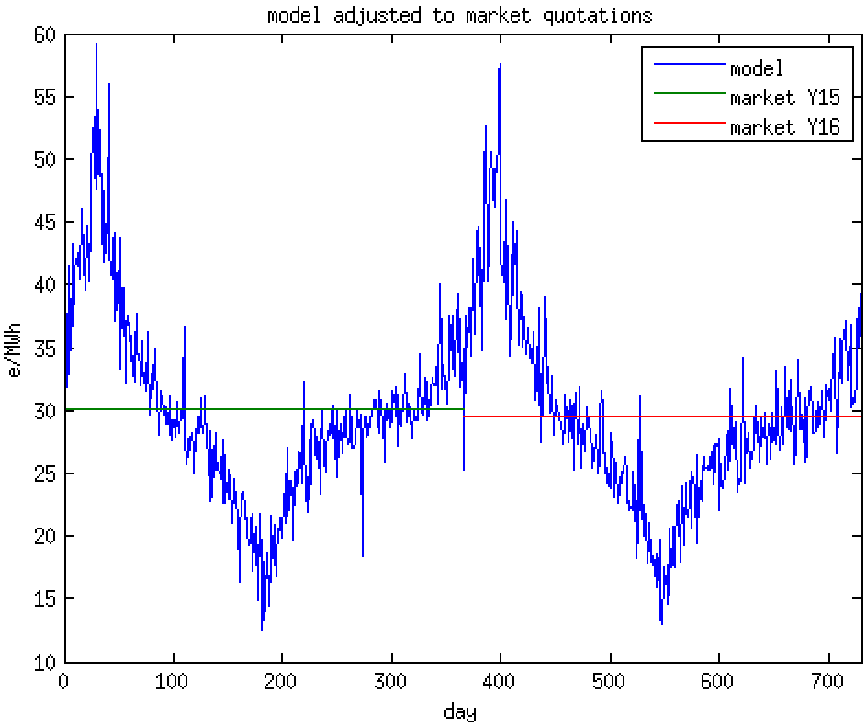


Figure 5: Observed spot prices and the model

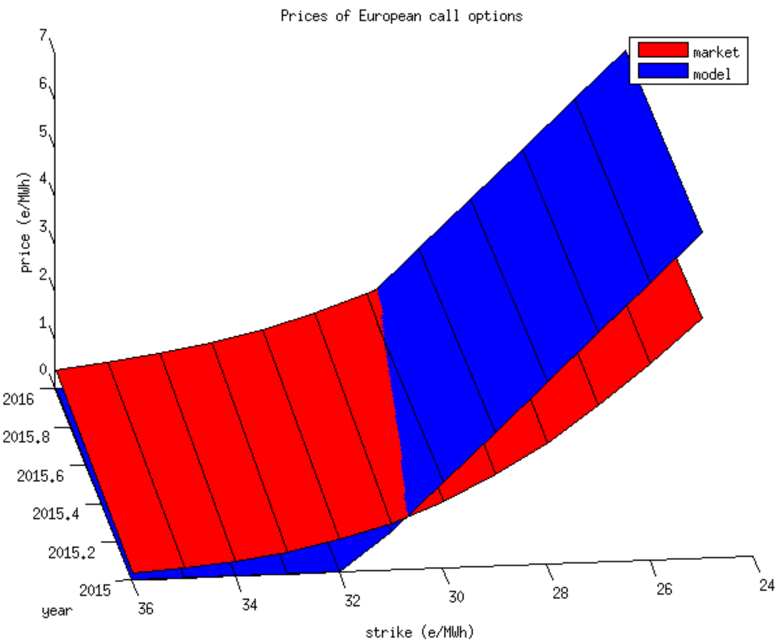
and, therefore, we simplify the adjusment process and introduce a set of constant adjustment parameters a for yearly products. Using these parameters, we adjust the average yearly spot price of our model to match the futures price. In Figure 6(a), we model spot prices in years 2015 and 2016, whereas Figure 6(b) shows the prices for European call options expiring in December 2015 and 2016 and the corresponding results from our model. Our model yields average prices of 34.5 €/MWh and 32.6 €/MWh for December 2015 and 2016, respectively, while the option prices are in the range of $0.15-0.83$ €/MWh for strikes equal to or higher than 33 €/MWh.

3.2 Oil

The data set for Brent oil prices includes 2338 data points from 24 December 2004 to 24 January 2014. Although the period is the same as for the Nord Pool spot prices, now trading takes place only on weekdays. Hence, we use the BUS/252 day count convention. Brent oil spot prices are shown in the figure 7. Historically, oil prices have been subject to many large



(a) Market quotations and the model adjusted to them



(b) European call options prices

Figure 6: Adjustment of the model to futures data and market expectations for December 2015 and 2016. Data are closing prices on 12th May 2014.

shocks. Figure 7 shows a strong increase and drop in spot prices during the financial crisis in 2007-2008. Moreover, Figure 7 shows that Brent oil prices are highly volatile. In general, the price development shows an increasing trend without any seasonality, but during the past three years the mean has been relatively stable.

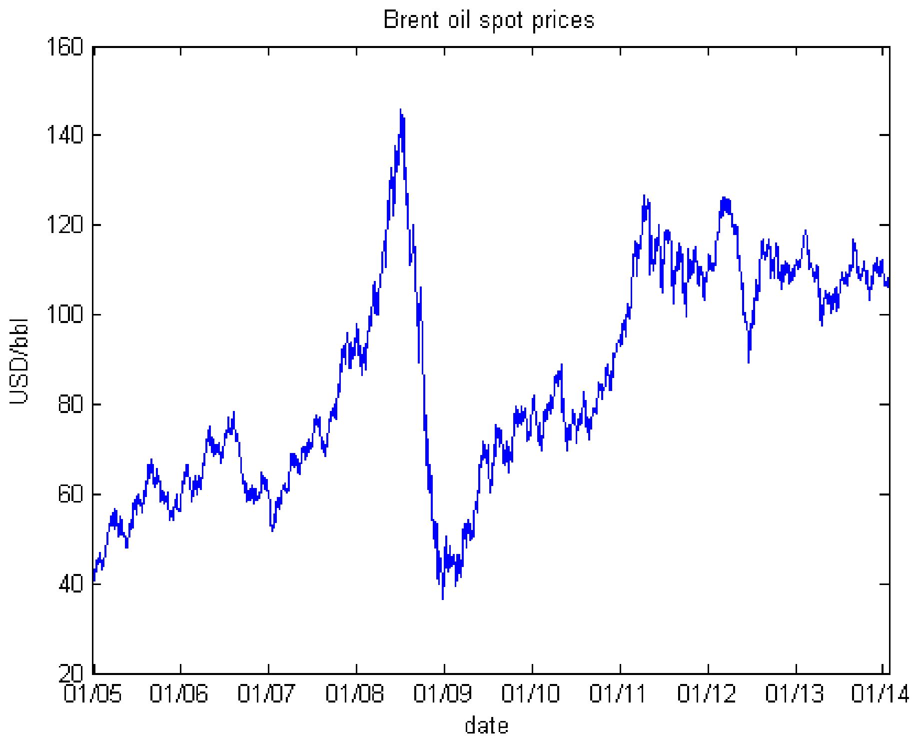


Figure 7: Brent oil spot prices

3.2.1 Models

Next, we summarize the models we use for pricing Asian oil options and their use in generating price paths. The stochastic processes described by the models cannot be used straightforwardly for pricing derivates. Only risk neutral versions of price processes can be used for pricing. In addition, we need to discretize processes before Monte-Carlo simulation.

Geometric Brownian motion is a widely used model for financial processes such as stock prices. We use it as a benchmark for more complex models. The process is presented in Equation 1. There are only two terms: a constant drift μ and volatility term σ. The simulation is done by Equation 13, where process is made risk neutral by replacing the drift by a risk-free rate r. Δt is the length of time step $(t_i - t_{i-1})$ and ΔW is random number from normal distribution.

$$X_{t+1} = X_t e^{r\Delta t - \frac{\sigma^2}{2}\sqrt{\Delta t}\Delta W} \tag{13}$$

We employ the arithmetic Ornstein-Ohlenbeck model in Equation 4 because it is a commonly used model for commodities and interest rates. The main feature of the process is mean-reversion, which holds for many commodities. The exact discretization of the Equation 4 is given by Equation 14, where parameter η is the force of the mean-reversion. The risk neutral version has also a new term $(\mu - r)/\eta$, which models market price of the risk.

$$X_{t+1} = X_t e^{-\eta\Delta t} + (\overline{X} - \frac{\mu - r}{\eta})(1 - e^{-\eta\Delta t}) + \sigma\sqrt{1 - \frac{e^{-2\eta\Delta t}}{2\eta}}\Delta W \tag{14}$$

The model presented in Heston (1993) replaces the constant volatility term σ in geometric Brownian motion with a separate stochastic process v_t following an Ornstein-Ohlenbeck-process. The volatility v_t has mean θ with reversion rate κ. The volatility of volatility is constant ξ and the changes in volatility and price are correlated with the correlation coefficient ρ.

$$\begin{cases} dX_t = \mu X_t dt + \sqrt{v_t} X_t dW_t^X \\ dv_t = \kappa(\theta - v_t) + \xi\sqrt{v_t} dW_t^v \\ dW_t^X dW_t^v = \rho dt \end{cases} \tag{15}$$

The discretization procedure follows that of geometrian Brownian and Ornstein-Uhlenbeck processes presented above. A major difference is that, in the Heston model, the volatility process v is real, not risk neutral version. Consequently, there is no market-price of risk term in Equation 16. However, the price process X_t is risk neutral, i.e., $\mu = r$.

$$
\begin{cases}
dX_t = X_{t-1}e^{(r-\frac{v_{t-1}}{2})\Delta t+\sqrt{v_{t-1}\Delta t}\Delta W_t^X} \\
dv_t = X_t e^{-\kappa\Delta t} + \theta(1-e^{-\kappa\Delta t}) + \xi\sqrt{1-\frac{e^{-2\kappa\Delta t}}{2\kappa}}\Delta W_t^v \\
dW_t^X dW_t^v = \rho dt
\end{cases}
\tag{16}
$$

Finally, Schwartz and Smith (2000) model commodity prices with separate processes for short-term deviations χ_t and long-term equilibrium price level ξ_t. Because the short-term process is an Ornstein-Uhlenbeck process and the long-term process a geometric Brownian motion, discretization can be done as before. The discretization is presented in Equation 17, where the parameters are the same as in Equation 8

$$
\begin{cases}
d\chi_t = \chi_{t-1}e^{-\kappa\Delta t} - \frac{\lambda}{\kappa}(1-e^{-\kappa\Delta t}) + \sigma_\chi\sqrt{1-\frac{e^{-2\kappa\Delta t}}{2\kappa}}dz_t^\chi \\
d\xi_t = \xi_{t-1}(1+\mu_\xi\Delta t) + \sigma_\xi\sqrt{\Delta t}dz_t^\xi \\
dz_t^\chi dz_t^\xi = \rho_{\chi\xi}dt.
\end{cases}
\tag{17}
$$

3.2.2 Implementation and results

The Asian option price for maturity K can be computed using Equation 3, when the average spot price A for maturity T is known after generating spot price paths from a model. However, the models presented in the previous section include several free parameters that need to be calibrated. We use observed oil options prices for several strikes and maturities, and choose the parameters of each model by minimising the squared relative error to these observed prices. The parameter search is done using Matlab's *patternsearch* routine. The option prices given by the models are computed by averaging $n = 100000$ price paths to reduce the standard deviation of the parameter estimates.

The number of free parameters and independent draws n affect the required computing time of the optimization procedure. Moreover, the parameter search was sensitive to changes in initial values of parameters and options of the *patternsearch* algorithm, which increases the total calibration time. The computing time can be reduced to some extent by variance reduction techniques presented in Glasserman (2003) but these need to be specified for each process separately and were not employed.

Using the Black-Scholes formula for pricing European options, we transform the option prices to implied volatilities. The residual sum of squares (RSS) for each model for two distinct observation sets are presented in Table 1. The best fit is provided by the Heston model in both data sets. The simple geometric Brownian motion was used as a benchmark and we note that all the models with special characteristic fit to the implied volatility surface better than the benchmark process. Some Asian option prices in the market and model results are presented in Table 2. We note that the prices of the models are closer to market values for options with short maturities. With longer maturities, the error increases. The observed implied volatility surfaces and the ones yielded by the calibrated models are presented in Figures 8 to 11. Figure 8 shows that the implied volatility surface of the geometric Brownian motion is flat with long maturities, while the other models follow the tilted market surface better. None of the models is able to reproduce the high implied volatilities with small maturities or the skewness with different strikes. The high short-term implied volatilities can be affected by trader behaviour and high uncertainty due to fundamental factors. These effects may be reduced in the long-term, where implied volatilities are lower and more stable. The spot price paths generated by the Heston model are presented in Figure 12.

Model	RSS (data 1)	RSS (data 2)
Geom. Brownian	0.1763	0.4002
Heston	0.0636	0.0506
Schwartz-Smith	0.1356	0.1656
Ornstein-Uhlenbeck	0.1437	0.1378

Table 1: Comparison of the residual sum of squares of the models.

In Figure 13 the implied volatility surface which is calculated by using market yield curve for risk-free rate is shown. From the figure it can be observed that having risk-free rate as a

Option	Strike ($)	Market ($)	Geom. BR ($)	O-U ($)	Heston ($)	Schwartz-Smith ($)
May '14	109.8	1.5	1.15	1.28	1.95	1.65
June '14	109.2	2.5	1.81	2.02	2.22	1.90
Q3 '14	107.9	3.9	2.25	2.50	2.34	1.73
Q4 '14	106.3	5.0	2.89	3.12	2.65	1.83

Table 2: Comparison of the market prices to prices given by the models

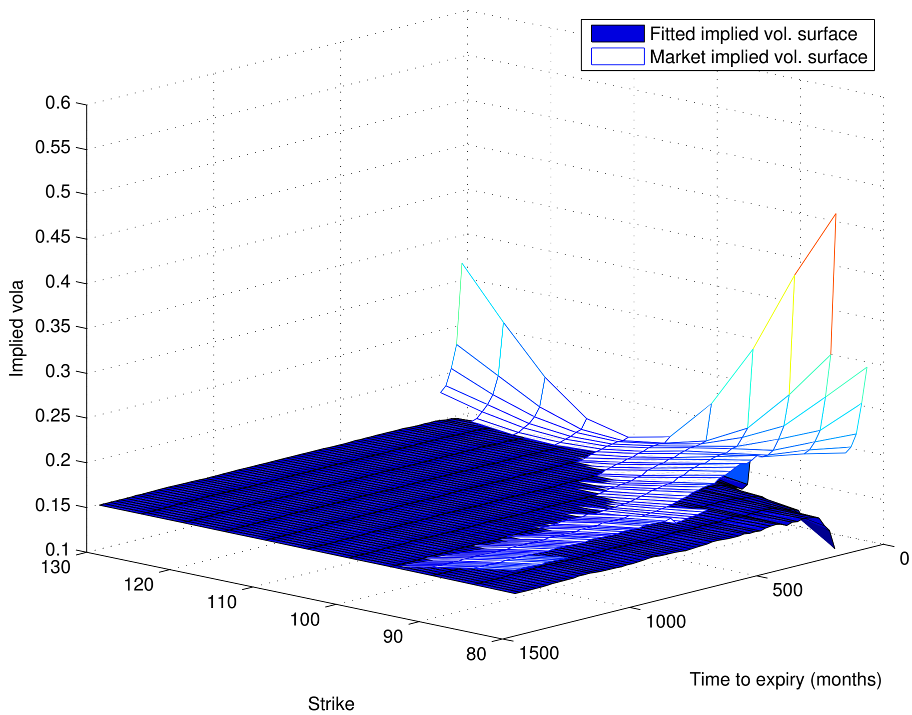


Figure 8: Implied volatility surface in the market and by the geometric Brownian motion

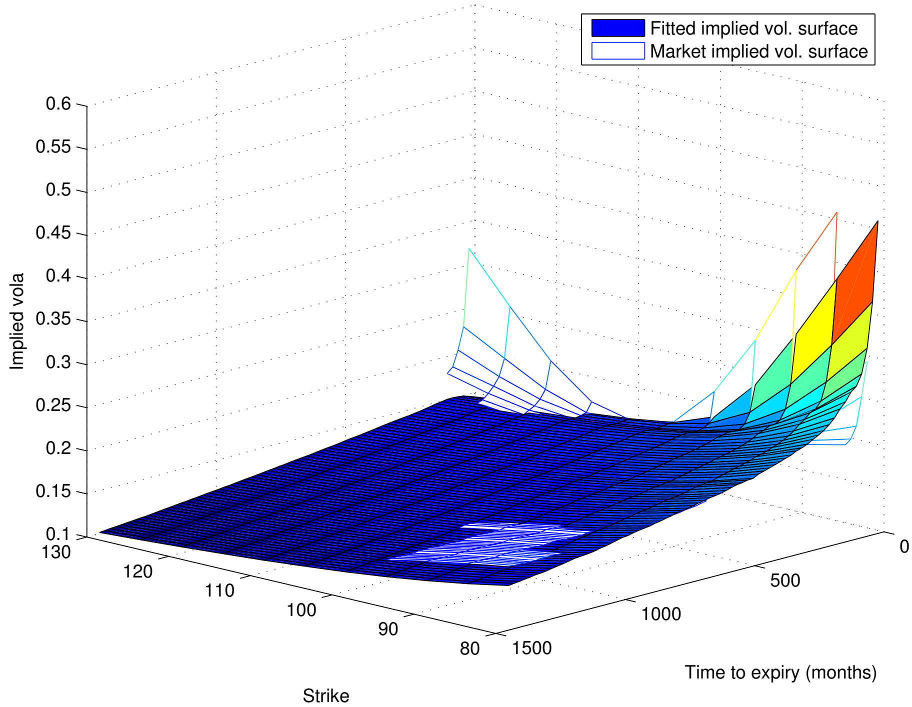


Figure 9: Implied volatility surface in the market and by the Ornstein-Uhlenbeck process

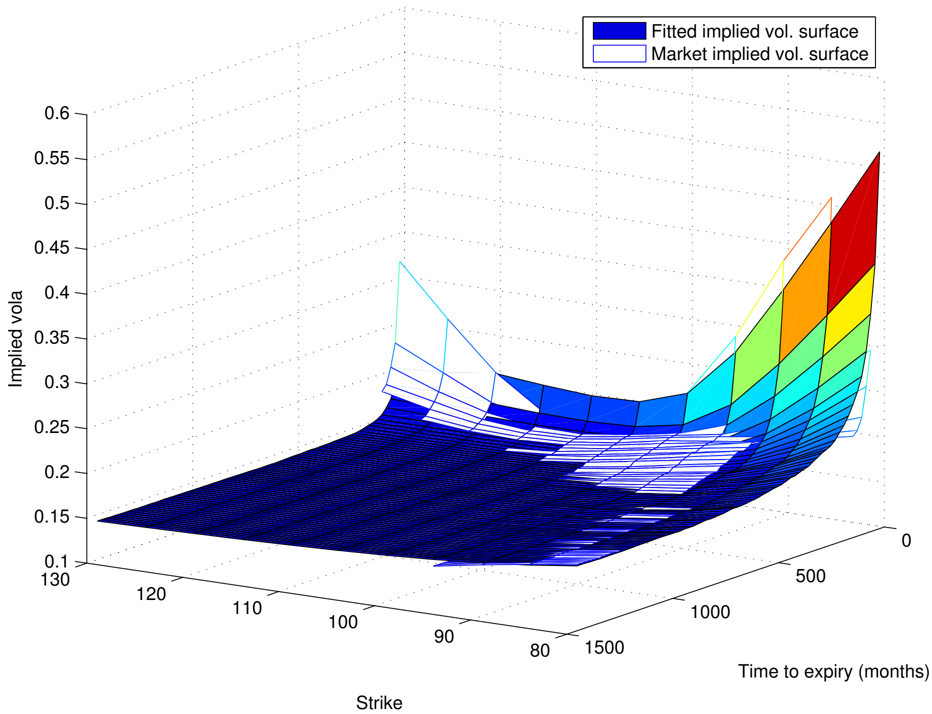


Figure 10: Implied volatility surface in the market and by the Heston model

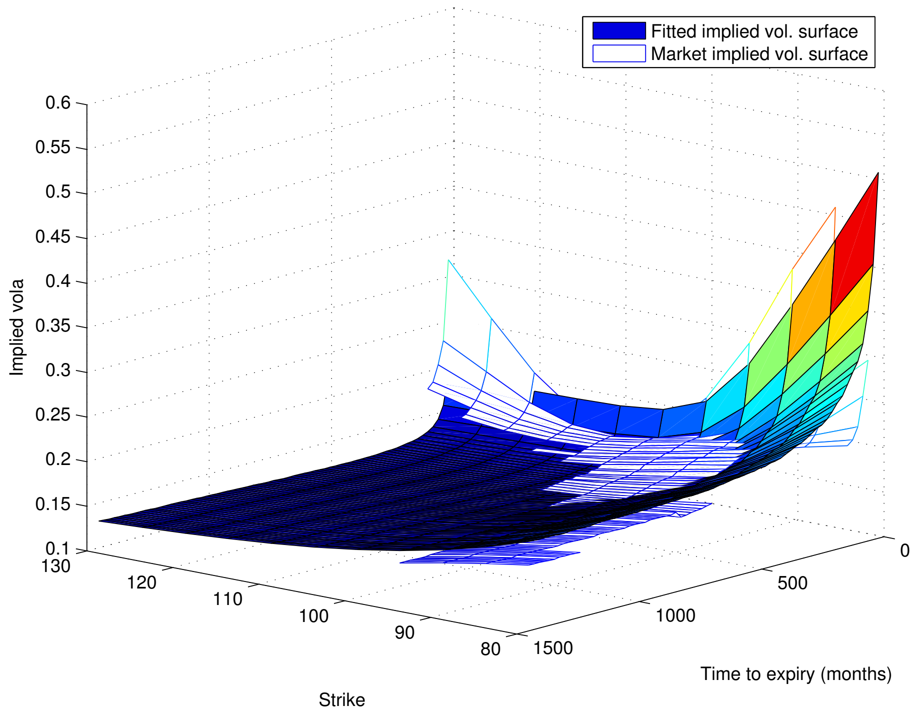


Figure 11: Implied volatility surface in the market and by the Schwartz-Smith model

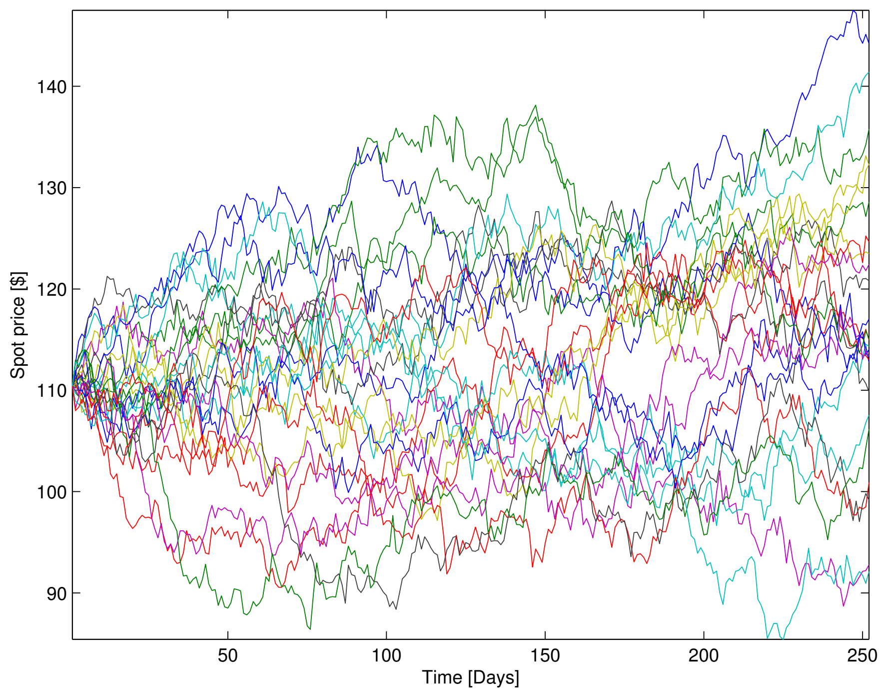


Figure 12: Realisations of the simulated risk neutral paths using the Heston model

variable for maturity does not necessarily produce satisfactory results. In this case, it is quite the opposite because clear discrete deviations exists between market prices and model's prices.

Figure 14 shows different pricing processes fitted to the termine-curve when constant risk-free rate of (0.2%) and convenience yield of (4%) are assumed. Maturities are the same as in the above implied volatility surfaces. Again we can observe good results from the Heston model, but now also Geometric Brownian motion produces surprisingly well fitting curve.

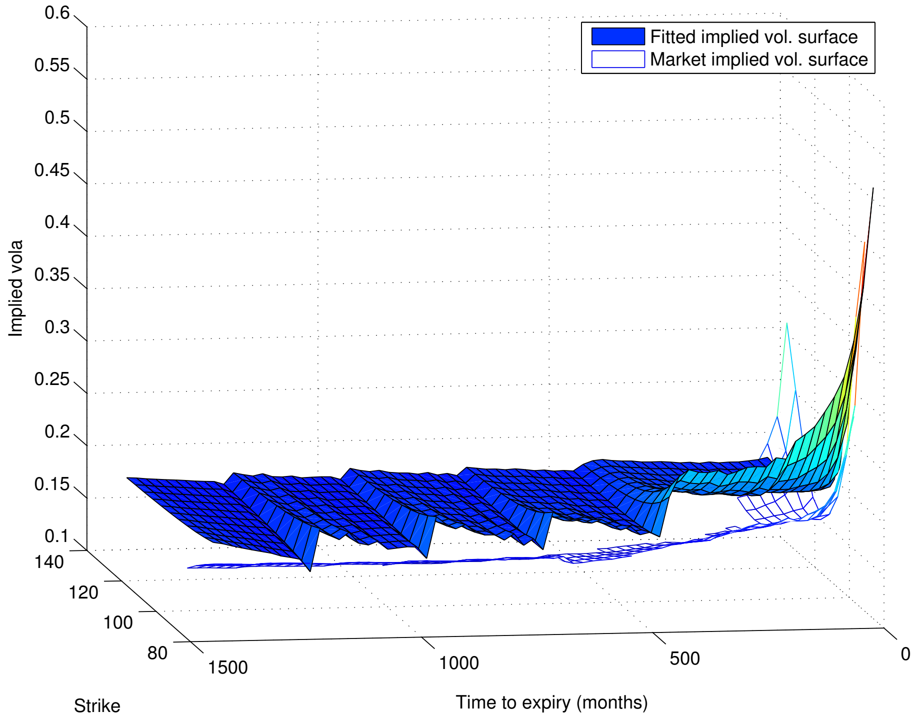


Figure 13: Implied volatility surface using market yield curve for risk-free rate

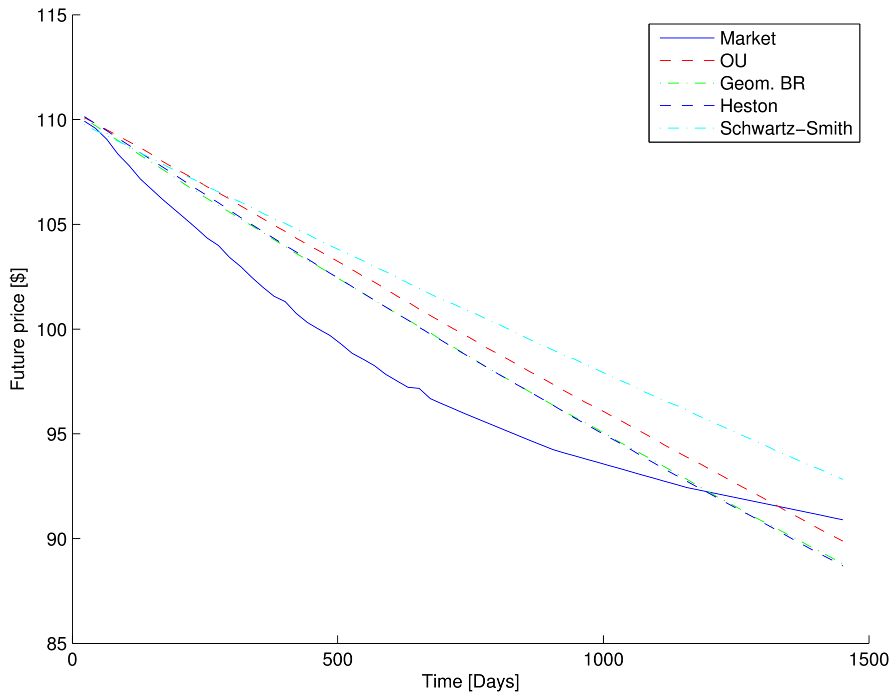


Figure 14: Termine-curves by markets and models. The constant risk-free rate (0.2%) and convenience yield (4%) are assumed.

4 Validation

4.1 General about validation

The results are validated using two distinct criteria whenever possible:

1. Simulated option prices compared to real market prices
2. Robustness of the model

Comparison to market prices is important since it is likely that prices from a good model are similar to those found in market. This is due to the fact that the markets are widely considered to be able to price in all the relevant information related to the prices. This idea is a building block of the *Efficient market hypothesis* (EMH) that was conceptualized in Fama (1965).

On top of the theoretical aspects there are also practical viewpoints to be considered. In practise, the prices may differ from theoretical ones especially if the market is very illiquid. If the supply is limited this will have an effect on the prices. Due to the nature of the market, we argue that the direction of the possible differences is such that simulations will yield lower prices. This stems from the fact that the sell-side has understandable incentives to avoid selling at an expected loss at all costs. On top of that the sell-side market participants are able to add a premium to the price. The size of the premium is likely to be relative to the supply-demand imbalance.

The second validation check that is done to estimate the robustness of the models. An optimal model would yield accurate predictions in various situations. The robustness can be evaluated by visually comparing the resulting surface to the one of markets. An overfitted model may converge extremely well at one place, and have unrealistic predictions elsewhere. A robust model should have approximately similar accuracy in all the data points. However, a model providing extremely accurate predictions in one place and nonsensical at others can still be valuable. The main thing is to know the limitations of the models and to understand when to use a certain model type.

In the project the main focus was to study and find suitable models. Due to finite resources (data and manpower) a few things had to be left for future work. Interesting avenues for further validation would include more comparisons to market data and further robustness checks for different time periods to see how models fared under different circumstances.

4.2 Electricity

Figure 5 shows that the model for Nord Pool spot prices provides a relatively good fit to the historical prices. The model is able to capture the general price development but not the very high or low prices and sudden spikes. When the model is fitted to historical data, it is possible that the model is overfitted and lacks predictive power. However, Figure 6(a) shows that the model is able to reproduce a reasonable yearly profile.

For electricity no comparable Asian option prices were available. Therefore, futures data was used to adjust the model to market prices. These adjustments, called the market by of risk by Weron (2008), are different for different times of year due to uncertainties related to weather, for example. Moreover, the market price of risk depends on the financial product. Therefore, calibration to futures prices introduces some error when pricing Asian options. However, we note that this error is likely to be relatively small compared to other error sources because both futures and Asian option prices are settled against the average price during the delivery period.

One of the major drawbacks of the model is that it has several free parameters that are used to calibrate the jump diffusion and mean-reversion characteristics of the model. However, as the model is fitted to a large number of observations, the parameters does not change very fast when new data comes in. Consequently, recalibration is needed, when the market price of risk changes due to updated expectations.

4.3 Oil

For oil comparable Asian prices were available and presented in Table 2. The results seem to be generally in the right ballpark. One big trend is that the difference between market prices and simulated prices seems to increase as the period length increases. The direction of the difference is identical to the one hypothesized above, which is encouraging. The simulation results tend to price options lower than markets, which can be partly explained especially in the case of longer and more illiquid options. Nevertheless, this will not provide a completely satisfactory explanation as to why the differences are tens of per cents at times.

Table 1 provides insight on how the models perform with different datasets. This gives a sense on the robustness of the models. It should be noted that results are not expected to be identical with different datasets. Due to the limitations of the models different models may work better during different days, but a robust model should work relatively well during these similar two days.

It is evident that geometric Brownian does not appear to be very robust as the RSS changes quite a bit between datasets. For other models the difference is of much smaller magnitude. Ornstein-Uhlenbeck fares the best as the relative change of its RSS is less than five per cent. For Heston and Schwartz-Smith the relative difference is around twenty per cent. Curiously the accuracy of Heston and OU improves in dataset 2, whereas the the opposite happens for geometric Brownian and Schwartz-Smith. The difference is likely to stem from the different nature of the data. Certain models are able to explain certain features better.

When visually examining the results it is clear that certain models are able to explain the more illiquid out of money and in the money options better. The benchmark geometric Brownian is flat and unable to accommodate the skewness of the real data. Heston and Schwartz-Smith are able to explain the most of the skewness out of the models tested.

In conclusion some models are better at modelling features of the data in certain circumstances. For example, we found that the benchmark model (geometric Brown) underperformed other models presented here, which provides at least some validation for the choice of our models. No model was able to completely explain the market surface, but as the liquidity of the extreme points is questionable, the economic significance of such underperformance can be questioned. For the most liquid and important in the money options the results seemed to generally agree with the markets.

5 Limitations of the models

5.1 Electricity

The model for Nord Pool has very limited predictive power due to the extremely dynamic nature of electricity spot market. In the short-term, factors such as changes in weather, power plant or transmission line failures and production pricing decisions can have a considerable impact on spot prices. In the long-term, unpredictable factors such as demand growth, new investments and climate change drive the price level. None of these short or long-term factors are considered explictly in our modelling but they are contained implicitly in the statistical properties of the historical data and current market expectations for the future. Therefore, changes in the current fundamentals or market expectations should trigger a recalibration of the model.

We note that the model for Nord Pool is better suited for options with long maturities (at least a quarter ahead). As the model is based on the historical data, it assumes an average yearly profile for the spot prices. However, the actual spot prices often fluctuate around this profile, which reduces the applicability of the model in the short-term. We use only yearly parameters to adjust our model to market prices, and thus, the information on short-term risk premiums is lost. Moreover, when the maturity of the option is longer, the impact of extreme events reduces and the statistical properties start to dominate.

5.2 Oil

The models for Brent oil have different features. Certain models, like Heston and Schwartz-Smith, are able to predict the skewness rather well, whereas others are unable to replicate the results of the market data. This is the most evident in the case of geometric Brownian, which provides completely flat surface. This means that certain models have rather strict limitations on their predictive power on options that are strongly either out of money[1] or in the money[2].

The oil models here are based on a finite time cycle. The period includes several crashes, as one can see from Figure 7. This is important as the models' initial parameters are obtained using this period, and thus it includes an implicit assumption that the future is relatively similar to past. For example, models trained only with pre-crash data are not able to take into account the risk of financial crash as they have no knowledge of such an event. Similarly, if the future is

[1] If the option was to expire today it would not make money

[2] If the option was to expire today it would make money

to bring completely new dynamics to the oil market, like during the 1970s oil crisis, the models trained with historical data may be of little use.

6 Conclusions

The main objective of this project was to produce pricing models for Asian commodity options. From the modelling perspective, this meant that we needed to examine simulation models because of the Asian options. Particularly, Monte Carlo methods were pointed out as possible solution models for us by the client. Modelling process was supposed to be carried out in the following way. Firstly, a pricing process was to be implemented so that it would simulate paths for a commodity spot price. To accomplish this, historical data of the spot prices could be used. Secondly, a model should be calibrated according to implied volatilities of the European options specific for every commodity. A model would be complete with its estimated parameters and could be used to price options, surely Asian options also. Lastly, model could be assessed against Asian option prices observed in the market. Data would be supplied by the Danske Markets to perform each step in the modelling process. We note here that this was only a suggestion for the modelling part and that other kinds of modelling processes could be used as well. Apart from actual implemented pricing models, documentation of a literature review and of modelling process with an assessment of the models were requested as end results.

At the end, we managed to build models for two of the asset classes – specifically electricity and oil. We chose to implement the models with Matlab which is known to be extremely suitable programming language for simulation purposes. Minor checks on the code were made for the verification of our models. Also, we qualitatively observed our results to carefully verify that they made sense (e.g. that Geometric Brownian motion model produced flat implied volatility surface).

The modelling process was highly different for each of them. The model for electricity is based on historical spot price data of the commodity and it was built of fundamental components such as seasonality and spikes. This was also one of the wishes of our clients that the models would include fundamental factors. Lack of data about the option prices, however, forced us to validate the model against futures prices.

The models regarding oil are a bit simpler. They do not have connection to the historical data in the similar way as in the electricity model, but they are future-oriented so that parameters of the models are fitted according to European option prices perceived in the market. Thus estimating parameters from historical spot prices for various components are not needed. Fortunately, we got real Asian option prices for the Brent oil.

The electricity model showed rather good fit to the observed electricity prices. The model had similar cycles and spikes as perceived in the markets. Yet, the model was not overly fitted for the market prices which would suggest robustness in the model. Concerning the oil models, the best fit to European options prices was achieved with the Heston model. The advantage of the model is its flexibility which rules in the steep curvature at the short maturities. Similar results were reached with the OU-model and Schwartz-Smith-model. Only the Geometric Brownian motion model could not capture any of the curvature as its implied volatility surface is flat. Validating our models with the prices of Asian options proved that the models are quite off compared to reality. The closest model to a certain market price is Schwartz-Smith-model for May'14 option which is produces 10% higher price than the market price. It was observed that the models do not fit the market prices so well as the maturity increases. However, more data would be needed for more thorough validation.

7 Appendix: Self-assessment

Our team made stable progress throughout the project. We set meetings for almost every week. In those meetings, we discussed status of the project, solved problems and thought about next steps. Nevertheless, the major part of the work was done on each team member's own time. We went to see our client approximately once in a month to discuss our progress, possible issues and general direction of the project. We also reported to and received data from them via email. All the documents for the course itself were delivered on time and we always had at least a few members of the team to deliver the presentations in seminars.

The real amount of work was expected to be large in this course from the start. Especially, we thought that the whole modelling process would take long time. The workload proved to be heavy and the project really required everyone's contribution. The modelling part of the work was quite heavy, but maybe a bit less than we initially guessed. However, we were satisfied with the overall amount of work in the project as everyone was quite excited to work on the subject.

Modelling and implementation part of the work was successful in our minds. We were able to produce models for the two different asset classes. Furthermore, the electricity model and oil models are totally different types. By taking into account fundamental factors in the model of electricity and then having future-oriented oil models, we think objectives regarding the models were achieved. Even though we built many functioning option pricing models, we could not validate them properly. This was mainly due to the fact that it is difficult to find the kind of data that we needed. Thus our results lack some credibility. Also, we probably did not focus on the argumentation concerning the models as much as would have been beneficial. Instead, it is likely that we wasted some time searching more models than what was useful.

There is always room for improvement. For our project that improvement would have been focusing more on details of only a few models. This could have allowed us to search for more scientific papers and data in them. And hence, a more comprehensive validation of the models could have been possible.

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