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Real Options and Demand Uncertainties in Supply Chain Management

/ ROCE Partners

Final Report

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Executive Summary

This project is about real options and demand uncertainties in the supply chain. The first part of this report is a literature review and the second part is an empirical study of real options. The objective of this work is to study what kind of uncertainties exists in the supply chain and how can they be managed with different real options and contracts. Also appropriate models will be applied to a real case example.

The literature review showed that the main sources of uncertainty in the supply chains were demand, supply and operations. These uncertainties were managed with the following real options or contracts: capacity reservations, flexible order sizes, sanctions, delaying the final order, return option, revenue sharing, channel rebate, backup agreement, futures and forwards.

The case example part studied how two different real options were suitable to a situation, where the production of the final product is stopped and a final order for the components of the product is placed. The components are needed as spare parts for the final product for a known time period, which is often regarded as end-of-life. The final order should be big enough to cover the whole demand during the end-oflife-phase. But the order size should not be really huge because if there is inventory left after the end-oflife-phase has passed, it has no salvage value.

The two studied real options were reorder option and delay option. The reorder option offered a possibility to place a reorder after some time from the final order had passed and the delay option offered a chance to postpone the final order. These two cases were compared to a situation without option and it was found out that the results were always better in the cases with option. However, the premiums of the real options were excluded when calculating the profits/losses of the cases with options. From the difference between the result of the no option case and the real option case was examined how much it would be profitable to pay for the option. The supplier, the seller of the option, has to take account how much costs does the reservation of the capacity cause and price the option according to the cost. The real option cases were mainly examined in the viewpoint of the company, the buyer of the option, and therefore we did not try to estimate the costs on the supplier side.

As a conclusion the use of the real options or contracts offers a possibility to reduce risks in the supply chain. However, there is a price that has to be paid for the reduction of the risks. Therefore the buyer of the option has to think carefully whether it is profitable to pay the premium of the option to the seller.

1

Table of Contents

1 Introduction

1.1 Background

This project work is part of the course Mat-2.4177 Seminar on Case Studies in Operational Research. The work's client is ROCE Partners, a consulting company specialized in supply chain management.

A company's supply chain faces many uncertainties that can cause severe deviations in operations from the planned. The most important is the uncertainty in the forecasts of demand. Other factors of uncertainty in a supply chain are, for example, price and currency risks, delivery disturbances, strikes and weather conditions.

A manufacturing company can, however, hedge from the risks caused by uncertainties, like excess inventories or running out of products, by different means. One way is to use real options. This means that the company can buy itself insurance for the uncertainty by, for example, buying additional capacity from an outside manufacturer, which the company can use if the demand exceeds the forecast. There are plenty of similar options and their control can be a major competitive advantage to a company. This project work's intention is to consider different means of controlling uncertainties. The emphasis, however, is on real options.

1.2 Objectives

The objectives set to this project are as follows:

- 1. Increase ROCE Partners' understanding of what kind of real option and contract models or other methods can be used to reduce the effect of uncertainties associated with supply chain management
- 2. Compare the functionality of different real option and contract models in scenarios regarding different factors of uncertainty
- 3. Apply appropriate models to a real case example of an End-of-life order

1.3 Research questions

The main problem of this project work is to answer to the question of what kinds of uncertainties need to be considered in supply chain management, and how the effects of those can be reduced. To answer this and to reach the objectives, we have divided our research problem into research questions, which we tend to answer during this project. The research questions are:

- 1. What are the uncertainties that need to be considered in supply chain management?
	- 1.1. What parts of the supply chain do these factors of uncertainties affect?
	- 1.2. How can the risk caused by an uncertainty be reduced?
- 2. What kinds of real option and contract models are there, that are designed to reduce uncertainty?
	- 2.1. Which uncertainties can be reduced by which models?
	- 2.2. What are the underlying assumptions in each real option and contract model?
- 3. How can real options be priced?
- 4. What models are appropriate for the case example?
	- 4.1. How do the chosen models work in practice?

It is to be noticed that research questions 1.2 and 2 partly overlap. However, we want to handle them separately, as the former question considers real options as an alternative, whereas question 2 will take a deeper look at them and the different kinds of options.

The literature review will be a base for the analysis and case example, and thus we will limit the review according to the characteristics of the case. This implies that will mainly consider the supply side of the supply chain, i.e. contact to retailer and customer are excluded from the analysis. In addition, the financial option theory will only be briefly mentioned, as the analysis regards real options, which relate to nonfinancial assets.

2 Literature review

2.1 Uncertainties in supply chains

There are several factors that cause uncertainty and risks in supply chains. The most usual and the most evident source of uncertainties is the uncertainty in demand. But that is just one side of the problem, for example the supply of products and parts can be very uncertain as well. In the table below [\(Table 1\)](#page-5-1) are presented the major causes of uncertainty in the supply chains.

Table 1. Sources of uncertainty and their effects

2.1.1 Demand

The varying market demand causes many kinds of risks. The selling price of the products can vary when the demand changes. If the demand decreases usually the price of the product falls too. The change in market demand causes also inventory risks. If the demand grows unexpectedly there may be shortages and part of the customers doesn't get the products they want. On the other hand, if the demand falls significantly, it causes probably overstocking. In both cases the company loses. In the first case it gets less money than it could because of the lost sales. In the second case the overstocking causes additional costs. All these risks concern every part of the supply chain. For example, varying demand affects the retailer, the distributor, the manufacturer and the supplier. However, the effect on the upper stream parties may be delayed. (Martínez-de-Albéniz et al., 2003).

2.1.2 Supply

The uncertainty in the supply causes the same kind of risks that the uncertainties in demand. If the suppliers are not able to supply the amount they have promised the manufacturer can't probably produce as much as its customers need. Again, this causes costs of lost sales. A way to cover these uncertainties is to keep safety stocks but that strategy causes costs too. The price uncertainty in the supply causes risk in profitability. If the suppliers raise the prices unexpectedly it may be impossible to move the rise to the customer prices as fast. The amount and the price of the supplied products and parts are not the only factors causing uncertainty. The delivery time may be too long and affect the production scheduling or the quality of supplied material may be too bad to continue production or distributing the products forwards. Again, these uncertainties concern all the parties of supply chains from suppliers to retailers. (Martínez-de-Albéniz et al., 2003).

2.1.3 Operations and disruptions

Operational risks are an area that requires attention. There are many kinds of technical risks that cause uncertainty and several sources of disruptions in the supply chain. The most usual causes are major accidents, sabotage and economic disruptions, such as strikes. Failure of equipments, such as production machines or transportation vehicles, is a risk in every phase of the supply chain. Failure of products causes also losses. The earlier the failure is noticed the smaller are the losses. That's why faulty products should not be distributed forwards in the supply chain. (Benoussan et al., 2007; Kleindorfer et al., 2003).

2.1.4 Currency rates

If a supply chain is international changes in currency rates can cause uncertainty. A manufacturing firm can have the possibility to select suppliers, plant locations, and customer markets from different regions around the world. Usually, there are different currencies in different countries and continents and fluctuations in the currency rates cause risks. Of course, changes in currency rates can have also positive influence on the profits of a supply chain. (Nembhard et al., 2005).

2.1.5 Double marginalization

Double marginalization is a well known cause of supply chain inefficiency. It means that each party in the supply chain orders less than the optimal order quantity for the supply chain. That is because each party tries to maximize its own profit, not the total profit of the supply chain. That leads to decisions to order less than the optimal quantity in every phase of the supply chain and therefore the chain is not able to distribute as much goods to the markets as customers demand. (Liu et al., 2005).

2.1.6 Characteristics of the high-tech industry

In the high-tech industry the product demand is more difficult to predict than in many other industries because highly volatile and rapidly changing markets. The main factors affecting this are the uncertainty about the market acceptance of new products, changes in technology, changes in competitors' offerings, and fluctuations in spending on high-tech products. Because of the short life cycle of products in the industry the losses from unmet demand or overstocking are extremely high. There are both price and volume uncertainty on the supply side as well in the high-tech industry and the effect of the uncertainty is great. (Billington et al., 2002; Wu et al., 2002).

The capacity costs are major in the high-tech industry. That is why outsourcing is very common in the industry and companies have sharpened their focus on product design, marketing and distribution. Outsourcing has increased the flexibility to exploit uncertainty on both the supply and demand ends of the supply chain. The move to outsourcing has introduced the need for procurement and sales contracts and, with options in them, companies shift particular risks to other companies in the supply chain that are better able to bear the risks. (Billington et al., 2002; Wu et al., 2002).

2.2 Ways to manage uncertainty in supply chains

2.2.1 Flexibility

Flexibility can be seen as insurance, or the capability to avoid bad outcomes. It means also the readiness to capitalize on opportunities. Flexibility is needed, for example, because of changes in customer demand and shifts in competition. It is also called for disruptions, such as accidents. Flexibility requires prior preparation: backup suppliers, alternative designs, extra capacity and possibility to rearrange the supply chain. It allows managers to recover quickly from bad events, such as disruptions, and to take advantage of opportunities. It can reduce total costs and provide competitive advantage. (Bjornsson et al., 2001; De Neufville, 2003).

Of course, flexibility is costly in many ways. The preparation mentioned earlier costs money. Safety inventories, extra capacity, space for expansions, investments for design and new computer systems are some sources of expenses. Flexibility causes also complexity of management systems and requires time for designing and planning. The essential question is what elements of flexibility are more valuable than their costs and therefore reduce the company's total costs. (Bjornsson et al., 2001; De Neufville, 2003).

2.2.2 Traditional ways

The traditional way to manage uncertainty in demand is to forecast demand and costs for each model and market as well as possible and to act according to the forecasts. The factors considered in the design of the supply chain are normally facility location and sizing, product allocation, inventories and logistics. Some flexibility in the supply chain is ensured by having safety inventories or extra production capacity. Inventory modeling and optimization is an important part of managing supply chains. Another important part is an optimal ordering and transshipment policy. Combining these two helps the management to find the best solutions with the lowest costs. (Birge, 2007; Tsay et al., 1999).

Strategic long-term partnership is a way to lessen uncertainties. The focus of strategic partnerships is on collaboration and it is usual to specify penalties for non-cooperative behavior. The strategic partnership is best applicable in strategic components. For standard, non-strategic, components it is often better to use multiple suppliers and select them and make contracts with them carefully. Penalties for overtime and expediting are general terms in contracts. With multiple suppliers and different contracts it is also possible to get some flexibility in the supply. (Martínez-de-Albéniz et al., 2003; Tsay et al., 1999).

When it comes to disruption risks a way to manage them is to identify the underlying sources of risks and to determine the pathways by which the risks can materialize. Event and fault trees and cause and consequence diagrams are suitable means for this. The potential consequences should be estimated and reduced by designing appropriate emergency systems. By preparing to disruptions beforehand their effects on the supply chain can be softened. Maintenance management and inspections and tests are important in preventing technical risks. (Benoussan et al., 2007; Kleindorfer et al., 2003).

A way to increase flexibility is to select a set of suppliers, plants and markets from different geographical areas. The geographical spread is a way to benefit from the exchange rates and multiple markets, but it is naturally also a more complicated way of forming a supply chain. (Nembhard et al., 2005).

The focus of the traditional way of managing uncertainties is on costs, not on revenues. That's why the policy results in fixed designs, little flexibility and conservative investments. The weakness of the traditional methods is that it makes it hard to succeed in the new competition where customization and responsiveness are key words. (Birge, 2007).

Various ways to manage uncertainties and their advantages and disadvantages are presented in the [Table](#page-9-1) [2.](#page-9-1) Options and contracts are studied below more properly and while the others are studied in this chapter.

Table 2. Ways to manage uncertainties

2.2.3 Incentives

Because every company in a supply chain tries to maximize its own profit, the profit of the whole supply chain is not usually maximized. "The quest for individual benefit leads to collective good" is often a mistaken view. A way to assure maximal profit for the supply chain is to align the incentives in the supply chain so that risks, costs, and revenues are distributed fairly across the network. If the incentives are misaligned it often leads to excess inventories, stock-outs, incorrect forecasts, inadequate sales efforts, and even poor customer service. Instead, if the incentives are well-aligned, the companies work together efficiently to deliver goods and services to consumers. (Narayanan et al., 2004)

2.3 Real options in the Supply Chain

Real options get their name from the financial options, but in the supply chain environment, they differ from their abstract "cousin" in many ways. An option is defined as a right, but not an obligation to take a certain action in the future. Real options are used as extension to financial theory; they relate to nonfinancial assets. When the contract agreement concerns capacity, it is called a capacity option.

Real options can be categorized in many ways, for example concerning timing, growing, production facility extensions, switching of component/products/etc. or shutting down of production. Many of these have similar characteristics, and they can be differentiated by one variable. For example in timing options, it is the time that is the crucial element of the contract as is capacity options; the main issue is often the

optimal quantity. The term of real option actually covers the whole set of contracts used by the parties of the supply chain. These can be further classified in multiple ways and towards all the more specific groups. (Tan, 2001)

2.3.1 Capacity options

We are concerned with options taking place between the parties of the supply chain. More specifically we look at capacity options, which they often follow a similar structure, consisting of decisions taken by the parties of the contract.

Sequence of events

- 1. Supplier announces reservation fee
- 2. Buyer announces the reserved capacity
- 3. Supplier decides the capacity it builds
- 4. Demand is realized
- 5. Buyer is penalized for unused reserved capacity

Also, when analyzing a system of options, assumptions concerning the demand and profit functions have to be put in place. The demand is usually estimated to follow a certain distribution so that its volatility can be expressed with a key figure. (Tan, 2001)

In the base case of a capacity option contract, an initial amount is bought with wholesale price, a number of options are bought with option price and a share of the options is exercised at the exercise price. Here are some extensions to the base case:

2.3.2 Backup Agreement

Backup agreement is a real option or a flexibility contract, where the buyer agrees to buy a fixed amount during one season. If the amount purchased during the season stays below this amount, a penalty is paid for the unbought units*.* (Tan, 2001)

2.3.3 Quantity-Flexibility Contract

A quality-flexibility (QF) contract allows flexibility during the following period by setting minimum or/and maximum levels for the amount of orders bought in the future. (Upward, downward or bidirectional) All orders are purchased with the wholesale price. In the beginning, the buyer decides the initial order size and the number of options. Depending on the characteristic of the option, a payment from or to the buyer takes place at the time of the realizing the option. (Barnes-Schuster et al., 2002)

2.3.4 Pay-to-Delay

Pay-to-Delay reservation means that the minimum amount (called take-or-pay) of the total reservation is fixed. A unit cost is paid for this capacity and an option price is paid for those units exceeding the minimum up to the reservation. And at the event of purchasing these additional units an extra unit cost is paid. (Tan, 2001)

2.4 Real options in high-technology industries

There are many features in high-technology environment that make the use of capacity reservation contracts especially usable. The short product life-cycles together with rapid innovation and volatile demand make flexibility a vital for the supply chain. The production is similarly characterized by capital intensity and high capacity costs. The detailed need for capacity planning has resulted in the creation of capacity contract to optimize the channel coordination. Besides, the phenomenon of double marginalization can be counterattacked by capacity reservation.

As in finance, the diversification of investment reduces risk also in the manufacturing environment. The risks in manufacturing consist mainly of the demand and price. Therefore having a portfolio of real options minimizes the risks. (Martínez-de-Albéniz et al., 2003)

Industry	Option characteristics
High-technology	Capacity reservation
Electronics	QF contracts
Wood products	Option to vary the order size
Computer automobile	Channel rebate
VCD rental	Revenue sharing
Publish	Return option
Fashion, catalogues	Backup agreement

Table 3. Real options in different industries (Kleindorfer et al., 2003; Liu et al., 2005; Wu 2005)

2.5 Real option valuation techniques

Valuation of options in financial markets cannot be fully extended to the case of real options. The market of financial options is a global market place where demand and supply can meet each other and determine the price at each moment. The market dynamics don't work in such a way in the real option environment. In the financial world, option pricing is based on the assumption that the market is complete and frictionless.

The price of a financial option depends on the underlying asset's price, the exercise price, the time period, the risk free interest-rate as well as the volatility of the asset. Correspondingly, the price of the real option can be derived from the asset's price, exercise price, time period as well as volatility of the demand. There are many techniques to value options; from the binomial method to the Black and Scholes –model. But as these have been created for financial option analysis, they require data and assumptions that cannot be made in the real option environment. Thus the valuation of real options should be done in another way. Correlation pricing and lattice techniques are some examples of used methods. Below, there are some examples of real option valuation found in the literature.

2.5.1 Relationship of channel coordination and option valuation

There are three possible channel coordination systems:

- 1. Centralized system (CS), with one decision maker. The prices play no role in the profit function, and the optimal solution is called the first-best solution. This model is used as a benchmark to see if there exist such circumstances where the decentralized system achieves the first-best solution.
- 2. Decentralized system (DS), where supplier is the leader and buyer the follower as in a Stackelberg game. Here the prices are decided by the supplier and the buyer maximizes its profit based on that information.
- 3. Decentralized system with no options (DSNO), which is used to find out if both parties can increase their profits with options.

To model the situation correctly we have to first make some assumptions concerning the prices, correlations and salvage of the products. We also assume that the demand distribution is common knowledge.

2.5.2 Analysis

The profit functions for the supplier and buyer have to be created. We want to find out the optimal policy for exercising options given a certain initial inventory. The profit function is dependent of the revenues of each period, exercise price of the options, shortage costs and salvage costs. The suppliers profit function is formed with the help of the revenues of the periods, option sales and exercise price, holding cost, salvage value of goods and raw material, raw material costs and labor costs. To get the profit function for the whole system, we have to form a joint profit function. In the end we can derive the optimal number of options under different cost conditions. If, with the help of the joint profit function, one can find the firstbest solution, then channel coordination is said to be in places. This is always the case if the buyer is able to internalize the supplier's costs.

Looking at the pricing of the options we have to make more assumptions, because the marginal costs of exercising options differ in CS and DS. The amount of options and the production and order quantities become identical with a proper setting of the cost and salvage parameters and the correlation coefficient. Thus we are able to achieve linear wholesale, option and exercise prices. Under these conditions we find out that the supplier makes zero profits, which could not be the case in reality. This leads to a second set of assumption, where also a system of return policies is included. With this setting the supplier's profits increase monotonically with the return price that it pays to the buyer. In addition it is the supplier who collects all the profits of the channel.

In addition the results can be analyzed in the light of demand risk, where the coefficient of variation is seen as the measure of this. Other factors to be considered are salvage risk, supplier flexibility and effect of serviceability. As the function as those when option prices and coordination are expected to be linear we derive further results. Naturally the chain's joint profits (and flexibility) increase with options, as is the case with demand correlation as well. (Barnes-Schuster et al., 2002)

2.5.3 Option pricing with multiple suppliers and a single purchaser

We have a look at a situation of one buyer and multiple suppliers. The buyer will construct a real option portfolio of the suppliers' capacities. Alternative of buying in the spot market is discarded here. The

negotiation about the reservation price and the execution price are pictured as a Stackelberg game, where suppliers are the leaders and buyer is the followers.

1. The manufacturer's profit function is concave which means that there is a unique optimum portfolio and the leaders know how the follower will react.

To find out which suppliers will achieve an active position, we have to find out what their optimal bidding strategy would be. The regions, where certain suppliers are active can be illustrated as following: (The active suppliers are expressed with A_{ii})

- 2. The concept of efficiency is defined as when the supplier's cost strategy is a winning point, i.e. the supplier is active in every Nash equilibrium.
- 3. The customer demand function follows a border demand distribution, and thus we conclude that a suppliers cost strategy that is bidding in a region, will have an optimal bid on the border. If a region does not include active suppliers, it is of one's interest to bid there.
- 4. Equilibria with efficient suppliers only: if a supplier has a lower cost of reserved unit, it will also have a lower cost of executed unit. Equilibria with inefficient suppliers: Because the Nash equilibria are only partly related to true costs, an inefficient supplier might be active, but it must be stated

that an active supplier may respond to it. Thus it is possible for inefficient suppliers to set the prices. (Martínez-de-Albéniz et al., 2003).

2.5.4 Real options in material procurement for OSB doors

We are looking at a supply chain consisting of a single buyer and a supplier. The valuation is made from the buyer's point of view, so that we can find out what is the value of a long-term real option embedded supply contract compared to traditional project based spot purchases. The valuation follows these steps. (Bjornsson et al., 2001).

1. Model the demand dynamics of interior commercial doors that use OSB as core:

The demand is expected to follow a geometric Brownian motion, and it is independent of any market price. It is also assumed stationary and normally distributed in one period. The demand dynamics are described with the help of Holtan lattice, which satisfies the behavior as a Brownian motion in a 2 period case. (Bjornsson et al., 2001).

2. Model the price dynamics of OSB.

The price dynamics are found out using Correlation Pricing Theorem (CPT) and Nested projection Theorem. Because the product is not a traded market asset, we have to apply CPT to a traded asset, which is relevant to this product. In the case of OSB doors, we would use the market dynamics of its raw materials, lumber and resin. The beta of those markets can be calculated with CAPM, and with CPT we can imply it also in the case of OSB. And by using correlation and projection pricing in different markets we can come up with characteristics of the OSB market dynamics. (Bjornsson et al., 2001).

3. Find out the optimal policy for a buyer to exercise the option by minimizing expected total cost.

To find out the buyers willingness to use the option we have to come up with the storage cost and shortage cost. Both of these have costs related to the firm's goodwill. By creating a non-option scenario and a scenario for using real options, we can come up with the buyers cost dynamics, i.e. how much it is willing to pay, and what are its savings when using options. (Bjornsson et al., 2001).

2.5.5 Option pricing with downward-sloping demand

Burnetas and Ritchken (2005) have analyzed supply chain options in the case of down-sloping demand. The option is defined as the retailers right to reorder (or return) products at a later period, with a fixed price. The circumstances are such that there is a single supplier and a single retailer, who have a wholesale

contract. The supplier takes part in the risk by offering the retailer the option of purchasing additional units, when the uncertainty has been revealed.

The problem for the retailer is to decide the amount of goods it orders initially and the amount of options it decides to purchase. The manufacturer is concerned with setting the prices in such a way that the retailer's actions become beneficial for the manufacturer as well. The retailer's profit function for each period is formed as a function of the current inventory level, and the number of products released to the market during the period. The demand curve must also be set so that its characteristics, mainly uncertainty, can be observed. Further, the manufacturer's profit is a function of the wholesale price, option price and the option exercise price, as well as the corresponding costs.

At first, we look at a situation with no option contracts. We can picture the behavior of the optimal order quantity, its sensitivity and profits as a function of demand volatility. Moving on to the case with option contracts we can recognize distinct regions, in which the equilibrium situation differs. For example, the situation of period 1 can be separated between the cases where the option exercise price is either higher or lower than the difference of high and low demand. The optimal policies are conditional to the inventory level and the number of exercised options. The first mover is naturally the manufacturer, who sets the prices and is thus able to decide whether the system will use option contracts or not. The use of options will be optimal for the manufacturer only if the variance of demand falls in a certain interval (set by the cost structure of the manufacturer).

The volatility of the demand is beneficial to the use of options; we can derive the value of the options for the chain. Again the volatility of demand is a critical factor, and the sensitivity of the prices, quantities and profits in relation to it are to be analyzed. The behaviors of the profits for each parties are dependent firstly of manufacturer's cost of producing an extra unit. The analysis can be extended to put options, which means that the retailer has the right to return the product for a salvage price. The price of this option can be simple regarded in the light of financial theory and put-call parity.

In conclusion, the manufacturer is greatly dependent on its production and inventory available, when considering offering option contracts. In addition, the price of the options must be set simultaneously with a new wholesale price, and it is the downward sloping demand curve that greatly affects these pricing decisions. (Burnetas et al., 2005).

3 Cases of Real Options

3.1 Framework

The real-life case, to which two real options are applied, is about component orders from a contract manufacturer after the end-products End-of-Life (EOL) point is reached. The EOL point is the moment when manufacturing of an end-product is stopped. Approximately at this point also a final order (i.e. last time buy, LTB) for components is made. The components are ordered for product support and for service firms that use them as spare parts for broken products. The order size is estimated by forecasting the demand between LTB and the time when the commitment to provide spare components ends (i.e. end of service, EOS). The situation is illustrated in [Figure 2.](#page-17-2)

Figure 2. Last time buy without option

The producer's objective is to have zero spare parts inventory level after EOS is reached. However, the inventory level should not reach zero any earlier because that causes stock out costs. On the other hand, the inventory has no salvage value and therefore the amount of the spare parts left in the inventory should be minimized.

In our research we will analyze two different options and compare them with each other and to the situation without an option by their financial implications. The options studied in the EOL context are an option to reorder after LTB and an option to delay the LTB point.

3.2 Contents of the data

We will use real data to test our real options model. We first make the models and then validate them with the data. The data consists of real demand during EOL-phase, forecasts errors, purchasing prices, sales prices, inventory costs, order costs, stock out costs and scrap costs for three different items (item 3, item 4 and item 7). The inventory, stock out and scrap costs are relative to the purchasing prices of the items. The demand forecasts were made with the help of the actual demand and the forecasts errors. We will use 5 different forecasts errors to validate the models: -50 %, -25 %, 0 %, 25 %, 50%. Forecast error is defined as

$$
E = \frac{Q - D}{D},
$$

where D is the total demand during the EOL period. Therefore positive errors result in scrap and negative errors in stock out.

3.2.1 Item 3

[Figure 3](#page-18-1) illustrates the demand for the item 3 during the End-of-Life phase. The End-of-Life phase lasts almost three years (150 weeks). It is possible to see that the demand decreases quite much during the first year and during the last year the demand is relatively small, only about 10 % from the demand during the first year. The demand is not smooth; it seems to have some kind of seasonal variation. The total demand for the item 3 during the EOL-phase is 42 216 pieces.

Figure 3. Demand for item 3 during the End-of-Life phase

In the figure below [\(Figure 4\)](#page-19-0) are the sales and purchase prices for the item 3 during the EOL-phase. Both prices decrease exceedingly during the three years. There is not a huge difference between the sales price and the purchase price. The sales price is a little bit higher than the purchase price.

Figure 4. Sales and purchase prices for item 3 during the EOL-phase

3.2.2 Item 4

In figure below [\(Figure 5\)](#page-19-1) is the demand for item 4 during the End-of-Life phase. The EOL-phase lasts one year (53 weeks). The demand decreases quite a lot during the first few months and then remains quite the same during the final time. The demand fluctuates quite much during the whole EOL-phase, but the fluctuations seem to occur regularly, so there is probably some kind of seasonal variation in the demand. The total demand for the item 4 during the EOL-phase is 14 938 pieces.

Figure 5. Demand for item 4 during the End-of-Life phase

In [Figure 6](#page-20-0) are the sales and purchase prices for item 4 during the End-of-Life phase. The decrease in the prices is not so well seen as it is for item 3. This is mainly due to a shorter time period. However, both sales

price and purchase price seem to decrease some. During the EOL-phase the sales price is all the time a little bit higher than the purchase price.

Figure 6. Sales and purchase prices for item 4 during the EOL-phase

3.2.3 Item 7

In [Figure 7](#page-20-1) is the demand for the item 7 during the End-of-life phase. The EOL-phase lasts almost two years (101 weeks). Now the demand decreases at slower pace than the demands for items 3 and 4. The total demand for the item 7 is during the EOL-phase is 110 829 pieces, so it is much higher than the total demand the items 3 and 4.

Figure 7. Demand for item 7 during the End-of-Life phase

In [Figure 8](#page-21-1) are the sales and the purchase prices for item 7 during the EOL-phase. Now the decrease in the prices is clearly seen. At the beginning the decrease is little steeper than at the end. The sales prices are somewhat higher than the purchase prices.

Figure 8. Sales and purchase prices for item 7 during the EOL-phase

3.3 No option

3.3.1 General case

In calculating profit, both costs and revenues must be acknowledged. The costs of the forecast error of LTB in the general situation, where EOL and LTB happen simultaneously and no real option is applied, can be divided into five factors: purchase costs, ordering costs, inventory costs, cost of disposing the scrap and cost of stock out. The revenue in this case consists of the retail price of components. To calculate the profit function, the following variables are needed:

- Unit purchase cost for a component, $C_t \geq 0$.
- Unit retail price for a component, $P_t \geq 0$.
- Order quantity *Q*
- Ordering costs Q_c per order
- Inventory cost per unit per week, $I_c \geq 0$.
- Scrap cost per unit, $SC_c \geq 0$.
- Stock out cost per unit, $ST_c \geq 0$.
- Stock level (Q cumulative sales at time t), $L(t) = \alpha_1 t^3 + \alpha_2 t^2 + \alpha_3 t + \alpha_4 t + \epsilon$, where $\alpha_i \ge 0$, and ϵ is stochastic and its magnitude describes deviation.

Therefore the profit function is defined as

$$
Profit_{LTB \to EOS} = \sum_{t=1}^{EOS} [(L(t-1) - L(t)) \times P_t - 0.5 \times (L(t-1) + L(t)) \times I_c] - QC_0 - Q_c - \left\{ (L(t_{EOS})) \times SC_c, if L(t_{EOS}) \ge 0 - Q_c - \left\{ (-L(t_{EOS})) \times ST_c, if L(t_{EOS}) < 0 \right\} \right\}
$$

The first term within the sigma is the revenue from the weekly demand at time t. The second term is the weekly inventory cost at time t calculated by the average weekly inventory level. The next two terms define the purchase costs of the LTB. Finally in the parenthesis the costs of scrap or costs of stock out are included. If stock level is negative, then no inventory costs occur.

3.3.2 Analysis with the data

Forecast errors in the case where no option is applied affect the value of the profit function in several ways: Negative errors decrease purchase costs and revenue, increase cost of stock out and decrease inventory costs. Positive errors increase purchase costs, decrease revenue, and increase cost of scrap and inventory costs.

In addition to purchase cost, retail price and demand data, the following values were given in the data set for the constants in the profit function:

- Ordering costs per order $Q_c \in [200, 2000]$
- Inventory cost per unit per week $I_c \epsilon \left[\frac{0.01 \times item \ price}{52}, \frac{0.1 \times item \ price}{52}\right]$, a typical value is $\frac{0.1 \times item \ price}{52}$
- Scrap cost per unit, $SC_c \in [0,01 \times item\ price, 1 \times item\ price]$, a typical value is $0,1 \times item\ price$
- Stock out cost per unit, $ST_c \in [1 \times item\ price, 100 \times item\ price],$ a typical value is $20 \times$ item price

The constants in the profit function for items 3, 4 and 7 are presented in [Table 4.](#page-23-0) The values are calculated by setting the ordering cost at 1000, and using the typical values for inventory, scrap and stock out costs. The item price for all items is calculated as an average purchase price in the EOL period.

Item / Constant	$\bm{Q}_{\bm{c}}$	Ιc	\mathcal{SC}_c	ST_c
Item 3	1000	0,101	5,233	1046,593
Item 4	1000	0,034	1,780	356,082
Item ₇	1000	0,042	2,168	433,533

Table 4. Values for the constants in the profit function for different items

The stock level and backlog for item 3, 4 and 7 are illustrated in figures below [\(Figure 9,](#page-23-1) [Figure 10](#page-23-2) and [Figure 11\)](#page-24-0).

Figure 9. Stock level/backlog for the item 3

Figure 11. Stock level/backlog for the item 7

The values of the profit function for all items with different forecast errors are presented in [Table 5](#page-24-1) and [Figure 12.](#page-25-1) As can be seen, even with an accurate forecast, satisfying the EOL demand is not profitable. This is a direct result of the small margins in the beginning of and declining retail prices during the EOL period, which results in items sold with loss. Positive forecast errors are less expensive than negative errors due to high stock out costs. The profit curve becomes more symmetrical if the stock out cost decreases. Positive forecasts become more expensive when stock out costs decrease to the level of about 2 – 4 times the item price. This implies, taken the error is normally distributed, that ordering some amount above the forecast can minimize the overall losses of different items.

Item / Error	$-50%$	$-25%$	0%	25 %	50 %
Item 3	-22 156 029,2	$-11533378,6$	$-1331230,5$	$-2696233,9$	-4 061 286,4
Item 4	-2 659 186,6	-1 341 134,1	-42 107,0	$-141775,2$	$-241955,6$
Item 7	$-24411741,3$	$-12916131,8$	$-1645649,3$	$-2925210,1$	-4 209 844,3

Table 5. Profit function values for items with different forecast errors

Figure 12. Profit/loss for different items with different forecast errors

3.4 Real Option to Reorder

3.4.1 General

The company makes a last time buy for the components of the product when the final product has been produced for the last time. The products have usually one year guarantee, but it has been promised to the customers that the company will have spare parts available for a longer period than the one year guarantee (for example three years). Therefore the objective is to make a right-sized final order for the components from the supplier. If the final order is too small, then stock-outs will occur. Stock outs affect negatively to the company's image and are therefore assumed to be quite costly. If the final order is too large, then the inventory will be left with worthless scrap that incurs expense when disposing it.

In the ideal situation the inventory will reach zero-level after the phase, when the spare parts should be available, has passed. This phase might last quite long time and then forecasting accurately the demand during the whole period is difficult and it includes many uncertainties.

Next we will find out if using a real option would help to manage this situation. We will examine a real option, which offers a possibility to order more from the suppliers after some time has passed from the last time buy-order point. The real option concerns only the buyer, the company, and the seller, the supplier. For simplicity we will analyze only the situation with one supplier. The real option dictates the moment

when to place the reorder, if needed. We have here analyzed the situation where the reorder point is when half of the End-of-Life-time has passed [\(Figure](#page-26-0) 13). The End-of-Life-time is regarded to be the time from the end of the production of the final product to the moment when no spare parts need to be available anymore. So, the flexibility, which the real option offers is for quantity. We have here assumed that either the real option is left unused or the real option has been used in order to order a positive amount of the components. Here it has been assumed that the reorder and the LTB order sizes can be whatever sized as long as they are positive.

Figure 13. The real option to reorder

The existence of the real option offers the buyer a chance to perceive the demand during the first half of the EOL-phase and make the reorder according to the realized demand and the new forecasts for rest of the time. If the demand for the components has been very different from the forecasts during the half, then the real option offers a good possibility to react to the change.

The existence of this real option obliges the supplier to keep up the needed component production resources for the possible reorder after half of the EOL-phase has passed from the Last-Time-Buy, even though the supplier may not have any other use for this capacity. Therefore it is understandable that the company has to pay a certain payment to the supplier for this capacity reservation. This payment is paid regardless of the company's decision to exercise or not to exercise the real option.

With the opportunity to do a reorder after the Last-Time-Buy, it is possible to place a smaller order in the last time buy point. This means that the average inventory level in the first half will be lower and the holding costs of the inventory will also be lower.

For simplicity the delivery time of the components from the supplier is assumed to be negligible. In reality it takes some time to produce the all ordered components and then to deliver them to the buyer. The accurate time to produce the components is not known now and therefore we use this simplification.

3.4.2 The Model

The model has the following parameters in addition to the parameters previously presented:

- Last time buy order quantity, $Q_0 \ge 0$
- Reorder quantity, $Q \ge 0$
- Length of EOL-phase in weeks, $T \ge 0$
- Components in the inventory in period t, $L_t \geq 0$
- Demand in period t, $D_t \geq 0$

Here it has been assumed that there is no beginning inventory. At the beginning of the EOL-phase a last time buy is made and the order size is 90 % of the normal LTB-order size because now there is a possibility to order more components later. The realized demand might be smaller than forecasted and then it has been beneficial to order less than the forecasts suggest. The cost of the last time buy is:

(1) Cost of the last time buy = $0.9 * Q_0 * C_0 + Q_c$

The cumulative profit from the beginning of the EOL-phase to the midpoint is:

(2)
$$
Profit_{0\rightarrow0,5T} = \sum_{t=1}^{0,5T} (D_t * P_t - \frac{L_{t-1} + L_t}{2} * I_c)
$$

If the stock runs out before reaching the midpoint, then some stock out costs occurs (T_E is the time when the stock level reaches zero):

(3) Stock out $costs_{0\to 0,5T} = \sum_{t=T_F}^{0,5T} D_t * ST_c$ $t = T_E$

At the midpoint it is possible to exercise the real option and order more components. Before placing the order, new forecast is made. The new forecast is more accurate than the forecast that was made in the beginning of the EOL-phase for the whole demand. Here it has been assumed that the forecast error will be cut down to half because the forecast is now made for shorted time period and the demand has also decreased from the start. If the real option is exercised and a new order placed, then the following cost will occur:

(4) *Cost of the reorder* =
$$
Q * C_{0,5T} + Q_c
$$

The cumulative profit from the midpoint to the end is:

$$
(5) \ \text{Profit}_{0,5T \to T} = \sum_{t=0,5T}^{T} (D_t * P_t - \frac{L_{t-1} + L_t}{2} * I_c)
$$

If the stock runs out before reaching the endpoint, then stock out costs will occur (T_F is the time when the stock level reaches zero):

(6) Stock out costs_{0,5T}
$$
\rightarrow
$$
 $T = \sum_{t=T_E} T_t * ST_c$

If there are components left in the inventory after the EOL-phase has passed, then scrap costs will occur:

(7) *Scrap cost* =
$$
L_T * SC_c
$$

The profit/loss is the sum of the equations (1)-(7). However, this sum excludes the premium of the option. We have not estimated any value for the real option because we will later calculate the value that should be maximally paid for having the real option. If there is not enough stock to satisfy the demand during some week, then the customers do not receive the components and stock out costs occurs. After receiving the replenishment to the stock, the backlog orders are not delivered to the customers. The stock is only for satisfying new demand.

The possible savings resulting from the use of the reorder option may be calculated using the following formula:

(8)
$$
Savings = \frac{E[Profit_{NO}]-E[Profit_{OD}]}{E[Profit_{NO}]} * 100\%.
$$

3.4.3 Analysis with the data

We will analyze the model with five different forecast errors and three different items. A forecast for the demand of the components during the EOL-phase is made first and according to it a last time buy is made. The forecast is made with the help of the forecast errors and the real demand. With the real option to order it is possible to make a smaller last time buy at the beginning of EOL-phase because a reorder can be made later. The demand forecasts can be some times higher than the actual realized demand and therefore it might be better to order a little bit less than the forecasts suggests. Therefore the last time buy is now 90 % of the whole demand forecast during the EOL-phase.

3.4.3.1 Item 3

In figure below [\(Figure 14\)](#page-29-0) are the stock levels/backlogs with different forecast errors. If the forecast errors were 25 % or 50 %, then the real option was not exercised. If the forecast errors were zero or smaller, then the real option was exercised and a reorder placed. With -50 % and -25 % forecast errors the stock was too small to satisfy the demand and that caused some stock out cost. With 25 % and 50 % forecast errors there were some components left in the inventory after reaching end of service point and that inventory caused come scrap cost. When the forecast error was 0 %, then no scrap or stock out costs occurred.

Figure 14. Stock level/backlog for item 3

In [Table 6](#page-30-0) are the profits/losses for item 3 without option and with a real option to reorder. The difference between the no option and the reorder option cases is the amount of money that is saved when there is a possibility to do a reorder and also a smaller LTB. The holding of the real option has at least the value of 281 000 EUR, so this is the price that can be paid for the option and still end up saving some money regardless of the forecast error. In the situation with -50 % or -25 % forecast error, the real option would provide over 4.5 million EUR savings. The forecasts are very rarely 100 % accurate, and therefore the reorder option is more valuable than 281 000 EUR, which is the difference between the results when forecast error is 0 %.

Item 3	Forecast error				
	$-50%$	$-25%$	0%	25 %	50 %
No option		-22 156 029 -11 533 379	-1 331 231	-2 696 234	-4 061 286
Reorder option	-17 640 059	-6 111 755	-1050226	-2 013 754	-3 242 224
Difference	4 515 970	5 421 624	281 005	682 479	819 062
Difference %	20,38 %	47,01 %	21,11 %	25,31 %	20,17 %

Table 6. Profit/loss for item 3

In [Figure 15](#page-30-1) are the profit percentages for the item 3 with different forecasts errors when the premium of the option is not included. The profits with the real option were always at least 20 % (20.17-47.01 %) better then the profits of the no option case. When the forecast error was -25 %, the use of the option enabled over 47 % better result.

Figure 15. Profit/loss during the EOL-phase

3.4.3.2 Item 4

In [Figure 16](#page-31-0) are the stock levels/backlogs with different forecast error for item 4. Now the real option was exercised when the forecast errors were zero or smaller. With a positive forecast error the real option was not exercised.

Figure 16. Stock level/backlog for item 4

In [Table 7](#page-31-1) are the profits/losses for item 4 without option and with real option to reorder. The difference between the no option and the reorder option cases is the amount of money that should be maximally spent when buying the real option from the supplier. However, the forecasts error is not known beforehand and therefore it is hard to estimate whether it is profitable to pay for example 70 000 EUR for it. If the forecast error is 0 %, 25 % or 50 %, then it would not be profitable to buy the option with that price. But if the forecast error is -25 % or -50 %, then the use of the real option would decrease significantly the losses.

Item 4	Forecast error					
	$-50%$	50 %				
No option		-2 659 187 -1 341 134 -42 107 -141 775 -241 956				
Reorder option	-1977018			-516 405 -39 567 -91 907 -182 095		
Difference	682 168	824 730	2 540	49 868	59861	
Difference %	25,65 %	61.49%		$6,03\%$ 35,17 %	24,74 %	

Table 7. Profit/loss for item 4

In the figure below [\(Figure 17\)](#page-32-0) are profits/losses relative to the situation with no real option. When the forecast error is 0 %, the profit with the real option is only 6 % better than the profit without option. With other forecast errors the difference between the situation with the option and the situation without the option is much greater (25.65-61.49 %). If the forecast error is -25 %, then the use of the real option offers 61.5 % better profit.

Figure 17. Profit/loss for item 4

3.4.3.3 Item 7

In [Figure 18](#page-32-1) are the stock levels/backlogs with different forecast errors for item 7. As it was with the cases of the items 3 and 4, the real option was exercised if the forecast error is zero or smaller. If the forecast error was greater than zero, then the LTB-order size was big enough to cover the demand during the whole End-of-Life-phase and it was not needed to exercise the real option.

Figure 18. Stock level/backlog for item 7

In the table below [\(Table 8\)](#page-33-0) are presented profits/losses without option and with option to reorder. The difference between the profit/loss of the no option case and the profit/loss of the reorder option is the amount of money that the use of the option saves. For example, if the forecast error is -25 %, then the reorder option would enable over 9 million EUR savings. If the premium of the option is 224 000 EUR, then it would be profitable to buy it regardless of the forecast error.

Item 7	Forecast error				
	$-50%$	$-25%$	0%	25 %	50 %
No option	-24 411 741	-12 916 132	-1 645 649	-2 925 210	-4 209 844
Reorder option	-15 727 760	-3 267 165	-1 421 534	-2 286 321	-3443126
Difference	8 683 982	9648966	224 115	638 889	766 718
Difference %	35,57 %	74,70 %	13,62 %	21,84 %	18,21%

Table 8. Profit/loss for item 7

In [Figure 19](#page-33-1) are the profits with different forecast error relative to the no option-case. Also now the profits were always better with the real option than without option (13.62-74.70 %). The greatest savings was when the forecast error was -25 %. Then the profit with the real option is 74.7 % better than the profit without the real option. However, the real profit with the real option is slightly smaller because now the premium of the option is not included.

Figure 19. Profit/loss for the item 7

3.4.4 Results

If the forecast error were -50-0 %, then the real option was exercised and components were bought at a lower price than at the beginning of the EOL-phase. If forecast error was negative, then some stock out cost occurred and if the forecast error was positive, then scrap cost was caused. The stock out was more expensive than scrap, so therefore the losses were much greater when the forecast error was negative. As illustrated previously [\(Figure 4,](#page-19-0) [Figure 6,](#page-20-0) [Figure 8\)](#page-21-1) the purchase prices decreases during the EOL-phase. The stock out costs were also smaller when using the real option because it was possible to order more, if the stock ran out or it seemed that the stock would run out before the EOL-phase will have passed. If the forecast error were 25-50 %, then the real option was not exercised, but the last time buy was only 90 % of

the normal LTB order size and therefore the inventory and scrap costs were smaller than in the situation without the real option.

The real option to reorder helps to manage uncertainties relating to demand. If the demand is higher than expected, then it is possible to order more at the middle of the End-of-Life-phase. It is also possible to make a smaller initial last time buy and later order more, if needed. Due to the smaller LTB, there will not be left so many components in the inventory if the demand is lower than expected.

Any clear pricing strategy for the real options was not identified with the real option to reorder. The pricing of the options depends a lot on the costs that are caused to the supplier. The main focus in these real option cases was on the buyer of the option. However, we were able to calculate how much it would have been profitable to pay for the real option. Of course the rate of the profitability depends quite much of the forecast error and as a rule of thumb it can be said that the greater the forecast error, the better it is to have a real option or contract in order to reduce the risks. The use of the reorder option was also very beneficial when the forecast error was 0 %. This is because of the smaller LTB and buying more components when the prices of them have decreased. Also average inventory level was lower and therefore also the inventory costs were lower.

3.5 Real Option to Delay

3.5.1 General

The option to delay is a kind of interesting variation of real options in the capacity contract category. It has ability to provide flexibility to the buyer side, but it doesn´t necessarily require the supplier to source more capacity compared to the case where option to delay has not been agreed. However, the risk of overstocking at the final time is partly transferred to the supplier side. It is now supposed that the supplier has better access to reuse or recycle excess components and thus may take more cost-efficient actions with them. The buyer is asked to pay some premium for the possibility to reduce its own risk and to balance the costs with supplier. The real option to delay framework resembles the situation where no option has been agreed. However, the essential difference is that now the final decision point is not fixed. As shown in [Figure 20,](#page-35-0) it may be delayed until the ultimate replenishment order point, which supplier has accepted in the supply contract.

Figure 20. The real option to delay framework in component´s end of life phase

There are now three distinct scenarios that may occur. First, the stock is already running out. In this case, the option to delay is not utilized, but replenishment order should be done immediately as stock out is very costly. This is done according to known future demand estimates. Thus this case is reduced to one where no option is available. However, the premium paid for the flexibility is lost. Second, the stock out has not yet been reached at the ultimate replenishment order point. In this case, the remaining future demand estimate is updated and the current stock level will be reduced from the order quantity. In the third case the stock out is encountered somewhere between these two extremes and the final replenishment decision is done accordingly. Thus the real option to delay may have a great potentiality in reducing the risk of having excess stock of components at final time. In case the option to delay can be utilized, it also reduces the buyer´s warehousing costs, as lower level of stock is allowed compared to the case without option.

3.5.2 The Model

In this part of the work, the aim is to develop rough-cut heuristic model for finding easily understood practical capacity contract solutions. Only the value of the option for the buyer side is considered. Basically, there exists a long term contract over many months, but here only the end of contract is analyzed. Initially, the buyer had made a total capacity reservation, of which it should buy at least some amount, less than the

35

whole capacity reserved. The contract may involve fixed prices for the component, but the buyer has the right to vary order size during the contract lifetime. However, if the buyer orders less than certain amount in a month, it will cause a loss of goodwill. For simplicity it is supposed that the delivery time lag is one week.

In this framework, the 'End of Life' stock level function reflects the component´s demand. The stock level´s initial level is overall EOL demand forecasted initially. This is gradually reduced by diminishing and stochastic demand. Thus the stock level function is stochastic and downward sloping as well. In this general part, stock level function is modeled with a piecewise continuous third order polynomial function with a step size of one week. This can capture essentially the effect of diminishing EOL period´s demand on stock level.

As was mentioned above, if the initial stock is zero, the replenishment postponement is not acceptable and order should be done immediately. The case reduces to one where no option has been agreed. Now one extra parameter for the initial stock level S_0 has to be included into no-option model, as the delay option results are compared to no option cases. Otherwise the parameters for the no-option case are similar as before. Thus modified no option model has the following parameters:

- Unit purchase cost for a component, C_t .
- Unit retail price for a component, P_t .
- \bullet Unit inventory cost per week, I_c .
- Ordering cost, Q_c .
- Scrap cost per unit, SC_c .
- Stock out cost per unit, ST_c .
- EOL overall demand estimate, D_{e1} .
- Initial 'End of Life' stock quantity level S_0 , which is some percentage of overall demand estimate, D_{e1} .
- No option order quantity $Q_{NO} = D_{e1} S_0$.
- Forecast error, δ . It describes percentage difference between estimated and realized demand, and for simplicity, it is supposed to accumulate constantly over the EOL period.
- Essential time points are reflecting 'End of Life'-phase initial moment t_0 and the end of EOLphase $t_2 \in [t_0, \max(156)]$ in weekly accuracy.
- EOL stock level is supposed to follow roughly some third order polynomial functions
- (1) $G(t) = \alpha_1 t^3 + \alpha_2 t^2 + \alpha_3 t + \alpha_4 t + \epsilon(t)$

- Now $\epsilon(t)$ is stochastic and its magnitude describes deviation. It follows some unknown distribution. Its expected value is supposed to be zero in calculations, and instead all the deviations are described by forecast error δ , which gives some interval where the stock level could be expected to fall.
- Thus for the period $[t_0, t_1]$ stock level is expected to follow

$$
(2) \ \ F(t) = \begin{cases} (1-\delta)G(t) + \delta. & \delta \ge 0 \\ (1+\delta)G(t) - \delta, & \delta < 0 \end{cases}.
$$

Thus the expected 'No Option' profit function has the form

(3)

$$
P = \begin{cases} \sum_{t=t_0}^{t_1} \{ P_t[F(t-1) - F(t)] - I_t[F(t-1) + F(t)]/2 \} - Q_{NO}C_{t_0} - S_0C_{avrg} - Q_c - \delta D_{e1}SC_c, & \delta \ge 0\\ \sum_{t=t_0}^{t_1} \{ P_t[F(t-1) - F(t)] - I_t[F(t-1) + F(t)]/2 \} - Q_{NO}C_{t_0} - S_0C_{avrg} - Q_c - \delta D_{e1}ST_c, & \delta < 0 \end{cases}
$$

Next the profit function for the delay option is created. It has the following parameters in addition to readjusted no option model:

- Premium paid for the option to delay, $F_c \geq 0$.
- Essential time points are reflecting 'End of Life'-phase initial moment t_0 , delayed order point t_1 \in [t_0 , max delay], and the end of EOL-phase $t_2 \in [t_1, max(156)]$ in weekly accuracy.
- Initial 'End of Life' stock quantity level S_0 , which is some percentage of overall demand estimate, D_{e1} .
- EOL overall demand estimate at t_1 , D_{e2} .
- Delayed order quantity $Q_{OD} = D_{e2} S_0 + \delta S_0$, $\delta \ge 0$, and $Q_{OD} = D_{e2} S_0 \delta S_0$, $\delta < 0$.
- EOL stock levels are supposed to follow roughly some third order polynomial functions

(4)
$$
G_1(t) = \alpha_1 t^3 + \alpha_2 t^2 + \alpha_3 t + \alpha_4 - (1 - S_0) + \epsilon(t)
$$

for the period $[t_0, t_1]$, and

(5) $G_2(t) = \alpha_1 t^3 + \alpha_2 t^2 + \alpha_3 t + \alpha_4 t \epsilon(t)$

for the period $[t_1, t_2]$.

Thus the stock level is expected to follow

(6)
$$
F_1(t) = \begin{cases} (1 - \delta)G_1(t) + \delta S_0, & \delta \ge 0 \\ (1 + \delta)G_1(t) - \delta S_0, & \delta < 0 \end{cases}
$$

for the period $[t_0, t_1]$, and

(7)
$$
F_2(t) = \begin{cases} (1 - \delta)G_2(t) + \delta(1 - S_0). & \delta \ge 0 \\ (1 + \delta)G_2(t) - \delta(1 - S_0). & \delta < 0 \end{cases}
$$

for the period $[t_1, t_2]$.

The expected profit function will be considered in two components; one for the delay period, and the other for the remaining of EOL period. Thus

$$
(8) \;\; Profit_{EOL} = Profit_{Delay} \;\; + Profit_{Final} \; , \ \, \underset{area}{\underset{area}{\text{Phase}}}
$$

where

$$
(9) \; Profit_{Delay} = \sum_{t=t_0}^{t_1} \{ P_t[F_1(t-1) - F_1(t)] - I_t[F_1(t-1) + F_1(t)]/2 \} - S_0 C_{avg} - F_c,
$$

and for final phase

(10)

$$
P = \begin{cases} \sum_{t=t_1}^{t_2} \{P_t[F_2(t-1) - F_2(t)] - I_t[F_2(t-1) + F_2(t)]/2\} - Q_{0D}C_{t_d} - Q_c - \delta D_{e2}SC_c, & \delta \ge 0\\ \sum_{t=t_1}^{t_2} \{P_t[F_2(t-1) - F_2(t)] - I_t[F_2(t-1) + F_2(t)]/2\} - Q_{0D}C_{t_d} - Q_c - \delta D_{e2}ST_c, & \delta < 0 \end{cases}.
$$

The possible savings resulting from the use of delay option may be calculated using the following formula

(11)
$$
Savings = \frac{E[Profit_{NO}]-E[Profit_{OD}]}{E[Profit_{NO}]}100\%.
$$

Next, this general model is simulated by third order stock level function, which have the following coefficients a=-0,00131, b=0,03305, c=-0,286, and d=1,000.

Savings are calculated with some parameter values compared to no-option case. If

- Unit purchase cost for a component, $C_t = 100(1 0.0050t)$. $C_0 = C_{a v r a} = 100$.
- Unit retail price for a component, $P_t = 115(1 0.0051t)$.
- Unit inventory cost per week, $I_t = 0.1C_0/52$.
- Ordering cost, $Q_c = 1000$.
- Scrap cost per unit, $SC_c = 0.1C_0$.
- Stock out cost per unit, $ST_c = 20C_0$.
- EOL overall demand estimate, $D_{e1} = 50000$.
- Forecast error, $\delta = 0$, $\delta = \pm 0.25$ and $\delta = \pm 0.5$.
- Initial 'End of Life' stock quantity level, $S_0 = 0.1D_{e1}$, $S_0 = 0.2D_{e1}$, and $S_0 = 0.3D_{e1}$
- No-option order quantity $Q_{NO} = D_{e1} S_0$.
- 'End of Life'-phase initial moment $t_0 = 0$ and the end of EOL-phase $t_2 = 155$, where step size 1 means one week.
- Order quantities Q_{OD} for delay period are shown in [Table 9](#page-39-0)

Savings from delaying replenishment order are gathered in results [Table 10.](#page-39-1)

Table 10. Total savings from delay option

The effects of the option to delay to stock level are demonstrated in figure below [\(Figure 21\)](#page-40-0) with initial stock level $S_0 = 0.2$ and forecast error $\delta = 0.5$. Stock levels are percentages of overall EOL demand D_{e1} . The uppermost dashed line illustrates no option case with +50% forecast error. The second dashed line below is the stock level with option to delay. Solid line illustrates no option case without forecast error.

Figure 21. Effect of option to stock level at the end of EOL-phase

3.5.3 Analysis with the data

Next, the profit model is evaluated with previously presented real life data and parameter values. Now the stock level function is replaced with discrete weekly data, which consists of overall EOL demand gradually reduced by weekly demand. Initial stock level S_0 is given value 20 % of overall EOL demand and held constant during simulations. Other constants, as presented previously, are ordering cost Q_c , scrap cost per unit SC_c , stock out cost per unit ST_c , and inventory cost I_t . Purchase cost of a component C_t and retail price of a component P_t have been previously presented and they have highly fluctuating values for different items. This affects to profits, as replenishment order unit prices are calculated with the prices of the order date. Initial stock´s value is calculated with the price of EOL initial moment plus added 5% to reflect early acquisition´s higher unit prices. Each of the three item data are simulated with forecast error values, $\delta = -50 \%, \delta = -25 \%, \delta = 0 \%, \delta = 25 \%,$ and, $\delta = 50 \%$. Finally, profit values are compared to the case where no option has been available. The results are given in percentage form with meaning of percentage savings from the no option case.

3.5.3.1 Item 3

In [Figure 22](#page-41-0) the stock levels/backlogs are shown for item 4 with forecast errors for ±25 % on left and for ±50 % on right side of figure. Percentage numbers are presented in same order as plots appear in figure. Here it is assumed that forecast errors are not known in advance. Thus stock levels/backlog have been plotted with different realized demand, which finally may signify scrap or stock out in presence of forecast error.

Figure 22. Stock level/backlog for item 3

In [Figure 22](#page-41-0) can be seen that stock levels or backlogs are reduced consistently with use of delay option. In [Table 11](#page-41-1) are the profits/losses for the item 3 without option and with real option to delay. The difference between the no option and the delay option cases is the amount of money that is saved when there is a possibility to do a order postponement. The use real option has at least value of 12 617 EUR, so this is the price that can be paid for the option.

In [Figure 23](#page-42-0) are the profit percentages for the item 3 with different demand forecast errors. The profits with the real option were always positive, but varied between 0.92 – 46.08 %.

Figure 23. Profit/loss during the EOL-phase

3.5.3.2 Item 4

In [Figure 24](#page-42-1) the stock levels/backlogs are shown for item 4 with forecast errors for ±25 % on left and for ±50 % on right side of figure. Percentage numbers are presented in same order as plots appear in figure. Here it is assumed that forecast errors are not known in advance. Thus stock levels/backlog have been plotted with different realized demand, which finally may signify scrap or stock out in presence of forecast error.

Figure 24. Stock level/backlog for item 4

In [Table 12](#page-43-0) are the profits/losses for item 4 without option and with real option to delay. The difference between no option and the delay option cases is the amount of money that is saved when there is a

possibility to do a order postponement. The use real option has at least value of 47 392 EUR, so this is the price that can be paid for the option.

In [Figure 25](#page-43-1) are the profit percentages for the item 4 with different demand forecast errors. The profits with the real option were always positive, but varied between 9.58-114.11 %.

Figure 25. Profit/loss for item 4

3.5.3.3 Item 7

In [Figure 26](#page-44-0) the stock levels/backlogs are shown for item 7 with forecast errors for ±25 % on left and for ±50 % on right side of figure. Percentage numbers are presented in same order as plots appear in figure. Here it is assumed that forecast errors are not known in advance. Thus stock levels/backlog have been plotted with different realized demand, which finally may signify scrap or stock out in presence of forecast error.

Figure 26. Stock level/backlog for item 7

In [Table 13](#page-44-1) are the profits/losses for item 7 without option and with real option to delay. The difference between the no option and the delay option cases is the amount of money that is saved when there is a possibility to do an order postponement. The use real option has at least value of 1 117 397 EUR, so this is the price that can be paid for the option.

Table 13. Profit/loss for item 7

Item 7	Forecast error				
	$-50%$	$-25%$	0%	25 %	50 %
No option	-3435528	-2561365		-1 687 202 -13 310 229 -25 168 463	
Delay option	-1761456	-1443968		-558 686 -11 224 638 -21 448 810	
Difference	1674072	1 117 397	1 128 516	2 085 592	3 719 653
Difference %	48,73 %	43,63 %	66,89 %	15,67 %	14,78 %

In [Figure 27](#page-45-0) are the profit percentages for the item 7 with different forecasts errors. The profits with the real option were always positive, but varied between 14.78-66.89 %.

Figure 27. Profit/loss for item 7

3.5.4 Results

Clearly, the option value depends strongly on volatility of the demand. Also, the initial 'End of Life' stock level is important, as was demonstrated with general case model. Basically, in the model constructed here, the option to delay can increase profits through three different mechanisms. First and most importantly, it reduces the final deviation from the forecasted demand, as later demand forecasts have been updated and because deviation is calculated as a percentage from the initial level, when replenishment has been ordered. If at the moment t_0 stock level is considerably over zero, it should be utilized before doing the replenishment order thus allowing the order postponement. Now if the stock level is supposed to reflect the true demand, the delayed replenishment order size Q should be calculated in a way that takes into account forecast deviation, as was defined above.

Secondly, if component´s purchasing and retail prices follow some downward sloping stochastic functions, where retail prices descend on average faster than purchasing prices, may option to delay reduce disadvantage of diminishing marginal returns caused by early purchases with later time retailing. Thirdly, the option to delay reduces the general inventory cost and the stock´s capital costs by reducing the stock level compared to no-option situation.

The option to delay may not be beneficial, if the premium paid for it exceeds the gains. Also, in the case of low deviation between forecasted and realized demand with fixed selling and purchasing prices, as well as the case of initially low stock level triggering immediate replenishment order, may both considerably reduce the benefits of this option category.

When analyzing the model with real data, the fluctuations of the selling and purchasing prices had some distortion effects on the delay options profitability. However, in practice the purchasing and selling prices

are often fixed in advance. Also, in simulations, the stock out cost was 200 times larger than the scrap cost. Its effect on savings can be seen in figures 23, 25, and 27. The positive forecast errors, meaning higher than forecasted demand, resulted stock out situations and thus lower savings compared to negative demand forecast errors, resulting excess stock at the end of EOL period. This should be considered at ordering moment and it should depend on decision makers risk attitude or company policy. Stock outs should be tried to avoid with this high gap between stock out and scrap costs.

4 Reliability

The literature review was made mostly based on articles that had been published in specialist magazines. They all seemed to be quite reliable because the authors were mostly the same persons who had done the research and almost everything was well justified. The oldest source had been published in 1999 and the rest were published after 2000. All in all the used literature was up to date and provided well the latest information about the uncertainties in the supply chains and about the different real options.

The real option models were first made without using the real data so that the models would not be influenced by the data. The aim was also to make models that would be usable with different datasets, not just with the data that was used in this research. After building the models, the models were tested with the data. The sensitivity analysis of the models was made by testing the models with five different forecast errors: -50 %, -25 %, 0 %, 25 %, 50 %. In the no option case the forecast errors influenced highly the profits during the EOL-phase. In the real option to reorder case, the forecasts errors influenced quite much whether to exercise the real option or not. With positive forecast errors the real option was not exercised, but with negative forecast error the real option was exercised. The option to delay model behaved well in ideal situations. It managed consistently to reduce forecasted demand deviation from the realized one. However, when simulated with real data, the results were not that consistent, even though they generated always positive savings. The profits were mainly affected fluctuating purchasing and selling prices.

The two different option models constructed and simulated here fit better to slightly different situations. Option to reorder model presupposed zero stock level at the beginning of the EOL. On the contrary, the delay option needs some initial stock level allowing order postponement. Thus savings resulting from different option types´ cannot be compared directly due to different presumptions.

The used real data has been given us from the client's database and it has been also used in some other research previously. The data can be assumed to be reliable. However, the data influences only the numerical results of the models. So, if the used data would have been faulty, it would have not influenced significantly this work.

47

5 Summary and conclusions

In this chapter the results of this research are summarized by answering each of the research questions. After that some final conclusions derived from the case results are made.

The first research question's aim was to gain knowledge of the most important uncertainties that affect supply chain management. From the literary review we can list the following uncertainties as the most important: Demand volatility; volatility, price increase, delivery time and quality of the supply; equipment failures in production and distribution and product failures; currency rate fluctuations. The sub questions asked what parts of the supply chain do the previous factors of uncertainty affect, and how the risk of those factors could be reduced. In the literary review all the affected parts were considered. Also the mechanisms by which losses occurred were discussed.

The second research question examined what kind of real option and contract models exist in the literature that are designed to reduce uncertainty. The field of real options being wide, the focus here was turned on options and contracts that were somehow related with capacity (i.e. capacity options). The focus was chosen partly because of the nature of the real-life case but also because demand and supply volatilities are the most common uncertainties supply chain management faces. Besides recognizing the general supply chain methods of reducing the risk of uncertainty, the literature review found several kinds of capacity options and contracts: the base case capacity option, backup agreement, quantity flexibility contract and pay-to-delay.

The third research question focused on what kind of pricing models are found in the literature. The validity and applicability of models found in the financial theory was discussed. Then several examples of ways to price real options were presented in greater detail. No single way or even guidelines of pricing real options were found in the literature. This and the different examples suggest that the valuation techniques of real options should be determined case by case.

The fourth research question examined what real options can be used with the case example. The focus in the case was on the end-of-life demand. Two options were chosen: the option of delaying the last time buy order and the option of one reorder after the last time buy. The options were analyzed in a general form and applied to the data. The delay option was also analyzed using a $3rd$ degree curve as a demand approximate, since it had the best resemblance found for the demand in the case data.

The options were analyzed using five different forecast errors. The options were found valuable for the buyer with all the forecast errors. The option to reorder decreases the base case loss independent of whether the option is exercised due to smaller ordering amount at the time of LTB, because it results in

lower inventory costs. We found both the options deliver up to 114 % increases in profit (or decreases in loss) the usual amount being somewhere between $10 - 40$ % compared to the normal case where no option was agreed on. The profits delivered by the options naturally change as a function of the constants in the profit function. Especially the small scrap / stock out cost ratio affects the results greatly.

Valuating the options was not discussed in the case analysis. It can however be concluded that it greatly depends on the volatility of the demand and the balance of stock out costs and the costs of paid inventory as scrap. In the reorder option case, already the knowledge of the possibility to exercise the option creates value for the buyer, since it will be possible to order a little bit under the forecasted demand. In this research 90 % was used, but the optimal percentage could be lower or higher depending on the stock out costs and demand volatility. The supplier may also set a minimum for the percentage. The reorder option is exercised with non-positive forecast errors. If we think of the demand volatility as normally distributed, the option will create value approximately with the probability of \mathcal{Y} . In the case of the delay option the stock level can be thought as evenly distributed between 0 and the economic order quantity (EOQ) at the time of EOL. Therefore the option is practically always exercised and creates value as a function of the demand volatility and the stock level, which has the expected value of EOQ/2.

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7 Appendix

Appendix 1: The evaluation of the project

The project was roughly divided into two parts:

- 1. Literature review
- 2. Case examples and analysis

Two persons, Hietaniemi and Roine, were mainly responsible of the first part and the three other persons, Niinistö, Tiainen and Venesmaa, were mainly responsible of the second part. The first and the second part of the project were divided into smaller tasks and the responsibilities of them were divided among the project team during the planning phase. It was a good thing to have a named person in charge of some area of the project because it was on his/hers shoulders to take care of doing that part. However, the project members did many parts together and while doing the project the communication between the team was active.

In figure below [\(Figure 28\)](#page-51-1) is presented the structure of the project work. The project started with planning phase, which was followed by literature review. The literature review was split into different parts: uncertainties in supply chains, ways to manage them, real options and valuation of the real options. After the planning phase also the making of the models was started. The models were finished after the literature review had been done because that provided useful information of different real options. The option models were first made and after that they were tested. Finally we simulated the models with real data. The analysis was made on the base of the literature review and the results of the option models' simulation. From analysis we made conclusions.

Figure 28. The structure of the project work

In table below [\(Table 14\)](#page-52-0) is presented the estimated work hours involved to this project. The work hours were distributed among the project members quite equally.

Table 14. The estimate of the work involved

The objectives of this project were met and the research questions were answered. The project was managed to finish on time. Some parts of the project were made a little bit later than planned in the project plan, but they were still managed to finish on time.

If something should be done differently than the project team did, it would probably be stricter time control and more detailed planning. All in all the project work gave the project members a better understanding of the topics that this project covered. The real options were not beforehand familiar to the project members, so this project work clarified well the importance of them, especially in supply chain management.