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End Report

BIDDING OPTIMISATION IN ELECTRICITY EXCHANGES

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ABSTRACT

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The project was done for the course Case Studies in Operations Research and the target company was Process Vision. It offers IT-solutions to companies operating in energy markets. Electricity is sold in exchanges in which participants can buy and sell electricity in a spot market. The goal of this project was to develop a method for bidding optimisation and a pilot application that uses the method.

Background research was conducted to gain an understanding of the problem. Two strands of research were identified from the literature. They were game-theoretic approaches and classical optimisation solutions. A classical approach was chosen.

The developed bidding optimisation method consists of three elements: forecasting and scenarios, deterministic optimisation, and the optimisation method. The market clearing price is forecasted with dynamic regression and price scenarios are created based on it. Deterministic optimisation is used to attain optimal production quantities for each scenario. The optimisation method is used to combine the different scenarios to create the final bid.

The developed pilot application consists of three parts: a MS Excel spreadsheet, GENERIS, and an optimisation module. The system is based on an Excel spreadsheet that is used for data storage and for data input and output. GENERIS is used for handling time series data. The optimisation module is used to produce the quantities for each scenario and finally to create the final bid.

The application was tested with a fairly simple model of an electricity producer. The results indicated that the scenario method can be used if the optimisation module is updated to handle combining the different scenarios better.

The implications of this project were that an optimisation method based on price scenarios can be used for bidding optimisation. However, the pilot application must be further developed.

Key words:	Electricity exchange, bidding optimisation, classical optimisation, scenarios, dynamic regression

TABLE OF CONTENTS

1 INTRODUCTION 1

1.1 BACKGROUND 1
1.2 RESEARCH PROBLEM 1
1.3 OBJECTIVES 2
1.4 STRUCTURE OF THE REPORT 2

2 BACKGROUND RESEARCH 3

2.1 GAME-THEORETIC SOLUTIONS 3
2.2 CLASSICAL SOLUTIONS 4
2.3 STOCHASTIC OPTIMISATION 6
2.4 IMPLICATIONS OF BACKGROUND RESEARCH 8

3 BIDDING OPTIMISATION METHOD 10

3.1 FORECASTING AND SCENARIOS 10
3.1.1 Literature survey on forecasting methods 11
3.1.2 Dynamic regression model 13
3.1.3 Monte Carlo scenarios 15
3.2 DETERMINISTIC OPTIMISATION 17
3.3 OPTIMISATION METHOD 19

4 SYSTEM SPECIFICATION 21

4.1 MS EXCEL 21
4.2 GENERIS 22
4.3 OPTIMISATION MODULE 22

5 APPLICATION TESTING 24

5.1 TEST CASE DESCRIPTION 24
5.2 TEST PLAN 25
5.3 RESULTS 26

6 DISCUSSION 29

6.1 IMPLICATIONS AND APPLICATIONS 29
6.2 DIRECTIONS FOR FURTHER RESEARCH 30

7 REFERENCES 33

1 INTRODUCTION

1.1 Background

This project has been done for the course Mat-2.177 Seminar on Case Studies in Operations Research, arranged by the Systems Analysis Laboratory of the Department of Engineering Physics and Mathematics at Helsinki University of Technology. The objective of the course is to introduce to the students the basics of project work by doing a project related to operations research. Planning and managing a project are the key issues to be learned upon realising the project.

The target company for this project was Process Vision, which offers IT-solutions to companies operating in energy markets. Process Vision participates in this project to eventually develop a new product based on the research done in this project, which focuses on bidding optimisation in electricity spot markets.

Electricity markets were deregulated in the Nordic countries during the mid-1990s. This led to the creation of NordPool, which is an electricity exchange operating in the Nordic region. The European Energy Exchange (EEX) based in Germany is a similar exchange. The exchanges allow the participants to buy and sell electricity in a spot market and in blocks. Supply and demand are calculated for each hour of the next day in the spot market. A market-clearing price is set based on the supply and demand, which are determined from the bids the participants make. The amounts of electricity the participants get or have to deliver are based on the bids. This creates a need for companies to optimize their bids.

1.2 Research problem

The research problem derives directly from the need to optimize bids to electricity exchanges and can be stated as:

How are the optimal bids calculated for a company participating in an electricity exchange?

The research problem can be broken down to a set of research questions:

1. How has bidding optimization been studied before?
2. Based on past studies how could the market be modelled?
3. Which frameworks used in past studies could be implemented in optimisation?

This project aims to answer the research problem by first answering the research questions.

1.3 Objectives

The project objectives are to:

- Build a realistic mathematical model of the underlying problem
- Build a pilot application using the defined model

There are several possible ways of modelling the bidding optimisation problem. A literature survey on the existing methods is needed in order to understand the current solutions and their problems. The literature survey allows the identification of the essential parts of the problem structure.

With a good understanding of the problem and its variations a mathematical model of the bidding optimization procedure can be built. It was acknowledged that several different models might have to be built for different situations.

The model itself has no value without an application. Therefore, a pilot version of a tool that uses the developed model is built and tested. This is required for verifying and validating the model.

1.4 Structure of the report

The report is structured to first give an overview of the research that has already been done on the problem and the implications on this project. The bidding optimisation method is discussed next. The forecasting, optimisation and stochastic properties are covered.

The system specifications are introduced next. After that application testing is described. The report ends with a discussion on implications and applications. Direction for future research is also given.

2 BACKGROUND RESEARCH

2.1 Game-theoretic solutions

Empirical studies have shown that many deregulated electricity markets are characterised by an imperfectly competitive market structure, an oligopoly. Economies of scale in electricity production create significant barriers to entry in oligopolies and provide the market participants the opportunity to exercise their market power (Blake, 2003). Game theory is concerned with examining the interactions of agents within the scope of imperfectly competitive markets and has provided an important evaluation methodology within the area to find optimised electricity bidding strategies.

Game-theoretic models can be classified as static or dynamic with regards to their assumptions about the formation of the market equilibrium. Agents act (bid) simultaneously in the static case so that they maximize their expected gains in "the game". To do this they have to make some assumptions about the other participants' behaviour. In the dynamic case, one player's decision affects the following decisions of other agents and the game is continued until equilibrium is reached. The static equilibrium models can be converted into dynamic equilibrium models by allowing the agents' bidding process to be sequential and allowing cooperation (hence also the distinction between co-operative and non-cooperative models). For example, in a dynamic Cournot equilibrium model, there are two types of market agents: price takers whose bids are not required to meet total demand, and Cournot competitors without whom the total demand cannot be met and that can largely influence the market price. It should be noted that although dynamic and cooperative approaches provide new tools for analysing markets, they entail detailed and complex assumptions on the participants' behaviour and require that the way market mechanisms work is known.

In electricity markets, cooperative game theory is generally used to solve allocation problems in which coalitions or groups are formed to benefit from economics of scale (Singh, 1999). Non-cooperative game-theoretic techniques are more often employed to analyse market positions and especially profit-maximisation strategies.

Game-theoretic approaches used to evaluate electricity market power build on three primary equilibrium models, the Cournot, Bertrand and Supply Function Equilibrium models (Blake, 2003). Quantity- and Price Leadership models have also been examined in the literature. Below are short overviews of the most relevant models used in formulating electricity-bidding strategies. For a comprehensive comparison of different models, see David and Wen (2000).

The most typical game-theoretic models are some form of Cournot models because of their intuitive appeal and simple assumptions (Contreras, Ignacio de la Fuente and Gomez, 2002). In Cournot models, often also referred to as Nash-Cournot models, firms are assumed to believe the competitors will hold their quantities fixed. In addition, products are assumed to be homogeneous and production costs identical. Given this information the agents correctly anticipate others' choices and choose themselves the strategies that maximise their profit. Kian, Cruz and Thomas (2005), for example, have evaluated this approach.

Bertrand models depict a situation in which agents believe the competitors will hold their prices fixed. Otherwise they make the same assumptions as in Cournot models. Therefore Cournot and Bertrand models can be viewed as the two extremes of game-theoretic approaches to behaviour in the electricity markets.

Supply function equilibrium (SFE) models have recently emerged as promising tools for evaluating the behaviour in deregulated markets. SFE models lie somewhere in between Cournot and Bertrand models. In SFE models firms simultaneously maximise their expected profits given their expectations of the competitors' electricity supply curves. The usefulness of SFE models arises from the fact that they offer a more realistic setting to analyse the bidding behaviour of the market participants. For example, Baldick and Hogan (2002) and Li and Keyhani (2004) have taken this approach.

2.2 Classical solutions

The best possible bidding method depends heavily on the market structure, the auction mechanism and the information that is available before bidding. Knowledge of competitors' strategies would have a strong influence on the strategies used but this information is rarely or never available. Madlener and Kaufmann (2002) state that a good strategy can only be developed by modelling and simulating the markets.

Most often the development of a new optimisation method starts with designing a linear programming problem. The technical limitations are usually those that finally make the problem non-linear. According to Thorin, Brand and Weber (2001) the methods are often so complex that compromises between quality and computing time must be made. Thorin, Brand and Weber (2002) found out that in some cases Lagrangian relaxation can be used to speed up the solving process of optimisation problems.

In general, the main purpose of optimising bids in electricity markets is to maximise the bidder's expected profits the following day. The operational costs of production are known but future prices are uncertain. Different bids are made for every hour of the next day. Therefore, the bidder must take into account also the production restrictions of the facilities along with the production costs. The production cannot be halted for one hour and then run at maximum output the next hour. There may also be some start up and shutdown costs to consider in certain occasions.

The problem can be expressed using mathematical notations. This is the model used by Rajaraman and Alvarado (2003) and Rajaraman (2004):

$$\max \sum_{k=1}^{K} E[R_k(x_k, p_k, y_k) - C_k(x_k, p_k, y_k) - c_k(x_k, x_{k+1})] \qquad (2\text{-}2\text{-}1)$$

$$s.t. \left\{ \begin{array}{c} y_k \in Y_k(x_k, p_k) \\ x_{k+1} \in T(x_k, y_k) \\ x_k \in X_k \end{array} \right\} \forall k = 1, \ldots, K$$

in which x_k is the generator state (i.e. UP or DOWN), y_k the generator dispatch quantity and p_k the price vector at moment k. R_k is the revenue function, C_k the cost function and c_k the transition cost function. T is the set of all possible state transitions that can be made and X_k is the set of valid states of production. Y_k contains the allowable dispatch choices of y_k. The model leads to a dynamic programming problem with uncertainty. This optimisation model requires that the market clearing prices for the next day are known.

It should be noted that the variables x_k, p_k and y_k are often vectors. There may be several facilities producing energy and each of them has its own state variable. Along with electricity the power plants often produce, for example, heat which must be included in the vector y_k.

Though this kind of problem is only one approach to optimise electricity bids, it proves to be a common one when leafing through different source literature. It can be solved using nested dynamic programming techniques.

The participants in electricity markets often have fixed contracts which force them to supply some electricity every hour regardless of market prices. This leads to situations in which electricity is sometimes bought from the market instead of only selling it. The buying option and the hourly minimum supply restrictions can be added to the previous model.

According to Vehviläinen (2004) the electricity markets have developed towards typical financial markets due to deregulation. In electricity as well as stock exchanges forecasts must be made and the best possible portfolio is to be approximated. One approach to electricity bidding is to lean on different finance theories such as stochastic modelling, derivative pricing and investment theory.

Hlouskova, Kossmeier, Obersteiner and Schnabl (2002) propose that real option models are also an option when considering how to optimise bids. They are solved numerically using backward stochastic dynamic programming and Monte Carlo simulations. The benefit of this method is the flexibility in adding constraints or sources of uncertainty. The weakness is its enormous need of computing time.

2.3 Stochastic optimisation

Obviously the next day's actual market-clearing price cannot be forecasted precisely. Therefore, the model presented in the previous chapter probably will not give optimal solutions as such. There may be a need to use stochastic optimisation. A stochastic model is an implementation of a non-stochastic model. Weber, Brand and Thorin (2001) give a simple example:

A normal linear model would be

$$\begin{aligned} &\min c^T z \\ &s.t. \\ &Az = b \end{aligned} \qquad (2\text{-}3\text{-}1)$$

When the stochastic part is added the model becomes

$$
\begin{aligned}
&\min c^T x + \sum_s p_s d_s^T y_s \\
&s.t. \\
&Ax = b \\
&x \geq 0 \\
&T_s x + W_s y_s = r_s, \forall s \in S \\
&y_s \geq 0, \forall s \in S
\end{aligned}
\tag{2-3-2}
$$

Here the variable z has been divided into x, which can be decided and y, which depends on some estimated chance. The scalar p_s implicates the probability of scenario s occurring. The advantage of stochastic modelling is the possibility of taking into account all the different scenarios that may occur. The distinct disadvantage is that there is an infinite number of price scenarios that are possible in a power market.

Holmes gives another example of transforming a linear problem to a stochastic one:

Linear problem

$$
\begin{aligned}
\min\ & f(x_1, x_2, ..., x_n) \\
& g_1(x_1, x_2, ..., x_n) \leq 0 \\
& \quad ... \\
& g_m(x_1, x_2, ..., x_n) \leq 0 \\
& \quad x_i \in X
\end{aligned}
\tag{2-3-3}
$$

Stochastic problem

$$
\begin{aligned}
\min\ & \{f_1(x) + E[f_2(y(w), w)]\} \\
& g_1(x) \leq 0, ..., g_m(x) \leq 0 \\
& h_1(x, y(w)) \leq 0 \\
& ... \\
& h_k(x, y(w)) \leq 0 \\
& x \in X, w \in W, y(w) \in Y
\end{aligned}
\tag{2-3-4}
$$

The use of scenario analysis combined with bidding optimisation methods might produce optimal production plans.

2.4 Implications of background research

The true value that game-theoretic considerations give is that the market participants' behaviour may be used extensively and comprehensively in simulations. Typically simulation is a very good tool for analysing the mechanisms and sensitivity of complex systems, but its main purpose is not to provide optimal strategies. The problems of game-theoretic solutions in this project arise from their over-specific assumptions of the market and their concentration on modelling the market behaviour instead of providing optimal strategies at any market situation.

To begin with, NordPool is a highly deregulated market place in which an individual agents' market power is very limited. Although barriers to entry still exist, the market itself consists of several countries each having several electricity producers and other market participants that can be considered as price takers in normal circumstances. In other words, NordPool is a highly competitive environment.

Secondly, it cannot be realistically assumed that all participants behave homogeneously in electricity markets with the "same rules of the game", nor is it realistic to assume that all information is symmetric. The complexity of these interactions makes it difficult to determine in advance the strategies that market players will employ in bidding. For example, Wen and David (2001) build a stochastic optimization model based on 'guessing' the competitors' bidding curves. This approach would not serve the aims of this project.

Therefore, to avoid too narrow a consideration, we have decided to take the approach of perfectly competitive markets. It serves better the purpose of this project which is not to analyse the behaviour of the market participants but to develop a usable, understandable tool for decision makers.

Because game-theoretic approaches do not seem to be ideal in developing the bidding tool this project is to be completed using some other viewpoints.

Classical methods such as Rajaraman and Alvarado's (2003) and Rajaraman's (2004) models give us deterministic bidding solutions. They are the same regardless of the competitors' bids to buy or sell. The uncertainty with future market clearing prices adds its own difficulties to the project.

Firstly, the price vector for the next day should be estimated somehow. Secondly, a reasonable estimate could only contain some expectation value of the price and possibly a confidence interval, inside which the price will be with a certain probability.

Still, the methods for estimating the price curve and confidence intervals are not so complex that they could not be attached to the model. Stochastic optimisation methods can be used to strengthen or question the selected strategy.

3 BIDDING OPTIMISATION METHOD

The goal is to find an optimisation method that produces optimal bidding functions for each of the 24 hours of the following day. Mathematically, the problem can be formulated as: find the bidding functions (b_j) for each hour (j) in the optimisation period ($j = 1, \ldots, 24$) maximising the expected profit ($E[P]$) with respect to a given set of scenarios (S)

$$\max_{\beta \in \mathrm{B}} E[P] = \sum_{s \in S} p_s P(\beta, s), \tag{3-1}$$

where β is the set of bidding functions that includes all bidding functions b_j, B is the space of feasible bidding sets, s is a single scenario, p_s is the probability of a scenario and $P(\beta,s)$ is the profit function, which equals the optimal solution of the unit-storage-network model with a given scenario and bidding functions.

This is only the objective function, and now a method for finding the optimal bidding function set is needed. Forecasting is needed for producing the scenarios and calculating the scenario probabilities. Optimisation is needed for calculating the profit function values. In addition, we need a coordinated way of handling the whole optimisation procedure (i.e. how β-instances are created effectively).

3.1 Forecasting and Scenarios

The forecasting of electricity spot prices (i.e. market clearing price) is of utmost importance in optimising bidding, as the bids are very much based on the forecasted values of the market price. Furthermore, any systematic errors or failures in the forecasts are likely to cause the optimal bidding strategy to deviate from the real optimum, which is likely to cause an increase in the procurement or selling costs of electricity. It must be remembered that the forecasts are not the sole factor determining the success of bidding optimisation, but there are other uncertainties present. In order to optimise the profits from selling electricity or minimise the costs due to procurement of electricity, the possible ways of forecasting spot prices are considered in this report. The considerations are made keeping in mind that the electricity spot price must be forecasted for the next 24 hours. The considerations are mainly based on literature survey, but also expert evaluations are used. Based on this study the most convenient and competent method is chosen for forecasting. In choosing the method of forecasting the spot prices, the usability and means of implementation are also taken into account.

3.1.1 Literature survey on forecasting methods

There are several forecasting methods introduced in the literature to be used with this type of problem. However, all the existing methods seem to have some advantages as well as disadvantages, and therefore use of all the methods is quite common. Based on the literature survey, there was no consensus on superiority of a specific method. The most common methods, which are considered in this report, are time-series modelling methods, neural-network method, fuzzy-modelling method, simulation, transfer function and stochastic factor model (Espinola et al., 2005; Vehviläinen and Pyykkönen, 2004; Iyer et al.). Methods that combine at least two of these methods have also been used in practice. It was decided that a dynamic method is needed for forecasting. A dynamic method adapts to and is based on historical information close to the time span to be forecasted. Thus, a static method approach, which would remain the same throughout time, is not considered in this project.

One good dynamic method mentioned in the literature for making forecasts is using Artificial Neural Networks (ANN) (Espinola et al., 2005), which uses both fuzzy-modelling and neural networks. This method produces estimates with relatively low mean errors. Nevertheless, other methods such as an ARIMA-model, a dynamic regression model or a transfer function have been found to provide equally accurate estimates. The project team has, however, no experience with neural networks and therefore the neural network approach was abandoned. This method could be the subject of possible future research as a potential means of forecasting the electricity spot price. Also simulation and using a stochastic factor model were left out of more detailed considerations, as they tend to be very case-specific forecasting methods or need very much a priori information (Vehviläinen and Pyykkönen, 2004). As the object is to find a generic method for forecasting, these methods are not considered more thoroughly in this report. The capabilities of the project team are also somewhat limited within these methods. The transfer function has also been used in actual case studies in literature to generate forecasts and it has proven to provide good estimates. The transfer function approach was found to result in better forecasts than an ARIMA-model, but a dynamic regression model provided equally good estimates (Espinola et al., 2005). The project team has some experience with transfer functions, but knowledge of time-series models is more extensive, and therefore the focus in forecasting was put in them.

Time-series methods are one of the most common methods found in literature, and this may be due to the fact that they have been widely analysed and they are relatively easy to use with some background knowledge of the subject. The project team is also most acquainted with these techniques among all forecasting methods available. The electricity spot prices are also readily available as hourly time-series data, which gives rise to the use of time-series methods. Moreover, these methods are quite efficient and produce accurate short run estimates. The two different time-series methods considered here are the ARIMA-model and the dynamic regression model (DRM). In general these methods are quite close to each other and an AR-model (i.e. only autoregressive part) is a specification of the DRM (Mellin, 2005). As these methods are quite similar, they could be examined together. They are both relatively easy to use and provide quite reliable short-term estimates. The accuracy of the forecasts of the time-series methods decreases exponentially with time, but these methods are still supposed to produce quite accurate estimates for the next 24 hours (Mellin, 2005), which is the case in creating electricity bids. The implementation of these processes differs, however, quite significantly, and therefore they must both be considered, as the motivation is to actually construct a forecasting method, which the end-customer is able to use.

The ARIMA-processes are used very often in forecasting and time-series analysis, as they provide relatively reliable forecasts. The ARIMA-process approach relates the current price to the values of past prices and the current error term of the model to past error terms. ARIMA-processes are usually generated by a three-step method, known as Box-Jenkins, of identification, estimation and examination of results and diagnostics. Although, a well-defined ARIMA-process produces good forecasts, it is relatively hard to come by and requires certain expertise with time-series analysis. As the method for forecasting is required to be dynamic, the parameters of the ARIMA-model would have to be estimated separately every time the bidding optimisation takes place. The construction of an ARIMA-model is not an automated process, but needs a person to make decisions and investigate the structure of the model. The ARIMA-model estimation would also require the use of a statistical program and it is not within the scope of this project to implement the whole statistical method used for forecasting from scratch. For example, it is not possible to estimate the ARIMA-model in MS Excel without any additional programs or scripts. This would require the end-customer to acquire a program for estimating ARIMA-models and having resources to use it. It has also been found in actual case studies that the forecasted mean square error of the ARIMA-model estimates is slightly greater than the one with the dynamic regression model (Espinola et al., 2005).

The dynamic regression model is more extensive and general than the ARIMA-process, as it is also a time-series method. The dynamic regression model relates the current price to past prices. Thus, it differs from the ARIMA-model, which also relates the current error terms of the price to past errors. In practice, the lags of the spot price (i.e. past hourly spot price values) are the explaining variables and current price is the variable to be explained. Dynamic regression behaves in essence quite similarly to an ARIMA-process, but it is more general. However, there is no specific estimation process (e.g. Box-Jenkins) needed for dynamic regression. The dynamic regression parameters can be estimated like the parameters in a regular multiple regression model (Pindyck & Rubinfeld, 1998). Therefore, the dynamic regression model does not require as much expertise with forecasting as an ARIMA-process and it can be automated to some extent because only the statistically significant coefficients of the lags are chosen in the model. The dynamic regression parameters can be estimated even with MS Excel, which eases the estimation process of the end-customer. The estimation process is supposed to be less time-consuming and to require less expertise of the subject than with an ARIMA-process. As it was found out in the previous chapter, the dynamic regression model also provides results with a lower mean square error than an ARIMA-model.

3.1.2 Dynamic regression model

The dynamic regression model seems to be better than other forecasting methods in the scope of this project, because it has proven to produce very good results and it is quite easy to use and implement (Espinola et al., 2002; Nogales et al., 2002). The extensive time-series data of past spot prices provides a strong basis for estimation, and the members of the project team have theoretical as well as real hands-on experience with dynamic regression models. Thus, the dynamic regression model is chosen to be implemented for forecasting future electricity spot prices. The dynamic regression model can be presented in general form:

$$p_t = c + \omega^p(B)p_t + \varepsilon_t \tag{3-1-1}$$

in which p_t is the current price, c is a constant term and ε_t is an error term. The error term is assumed to obey a normal distribution with zero mean and variance s^2. The function

$$\omega^p(B) = \sum_{i=1}^{K} \omega_i^p B^i \tag{3-1-2}$$

is a polynomial function consisting of the lags of the spot price (i.e. past spot prices), in which

$$B : B^i p_t = p_{t-i} \tag{3-1-3}$$

is a backshift operator, and the sum is calculated to the index K, which corresponds the last lag to be included in the dynamic regression model. The regression coefficients of each lag and 95 % upper and lower bounds and the corresponding P-values may be calculated using MS Excel from the time-series data of the spot price. Based on the P-values, the coefficients that are the most statistically significant are identified and hence the corresponding lags are chosen to be the regression variables in the dynamic regression model used for electricity spot price forecasting. Spectral analysis was also conducted on electricity spot price data from year 2004 to find out any seasonal terms which should be included in the model altogether.

Based on literature (Espinola et al., 2002; Nogales et al., 2002) and careful analysis of 2004 spot price data, ten explaining lags were chosen to be included in the dynamic regression model. The ten regression variables were chosen as a result of analysing different periods of spot price time-series from 1 month to 12 months, and also a dynamic regression model was fitted into the data. The chosen lags are previous hours {1, 2, 24, 25, 72, 73, 120, 121, 168 and 169} of current time and the resulting dynamic regression model for forecasting the electricity spot price is:

$$\begin{aligned} p_t = c_t &+ \omega_1^p p_{t-1} + \omega_2^p p_{t-2} + \omega_{24}^p p_{t-24} + \omega_{25}^p p_{t-25} + \omega_{72}^p p_{t-72} + \omega_{73}^p p_{t-73} \\ &+ \omega_{120}^p p_{t-120} + \omega_{121}^p p_{t-121} + \omega_{168}^p p_{t-168} + \omega_{169}^p p_{t-169} + \varepsilon_t \end{aligned} \tag{3-1-4}$$

The dynamic regression model was built into MS Excel and the estimation of the regression coefficients is left to the end-user along with inserting the appropriate period of spot price time-series. The appropriate period of hourly time-series data for estimating dynamic regression model parameters was assessed to be five weeks (Mellin, 2005). When the time series-data of past spot prices is inserted and the regression parameters are estimated, MS Excel provides the forecast for the electricity spot price for the next 24 hours. Furthermore, four different spot price forecasts are also calculated as corresponding limits for the 35% and 95% confidence intervals of the expected spot price curve. The 95% interval was chosen because it is used frequently and is the most common confidence interval used in statistical calculations. It also corresponds to approximately two times the variance interval in a normal distribution. These calculated limits correspond to the upper and lower limits of the price estimate. The 35 % interval was much less intuitive and it was chosen somewhat arbitrarily, although the motivation with it is that it is approximately half of one variance interval in the normal distribution, 68 %. This interval may therefore be interpreted as a slight deviation interval from the expected value. These four calculated spot price curve estimates are further on used to generate the different scenarios for the analysis.

3.1.3 Monte Carlo scenarios

This chapter introduces the mechanism that is used for creating the electricity price scenarios.

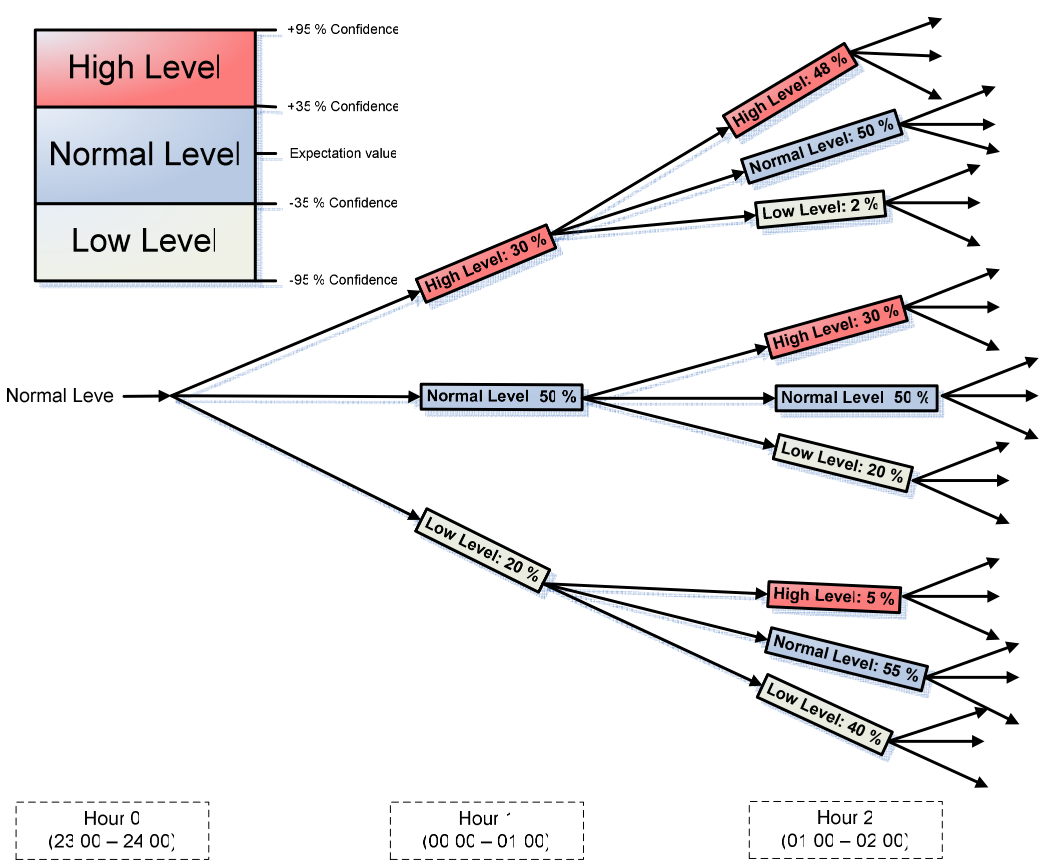

Figure 3-1-1. Defining the price levels in the scenario randomisation process

The price level of a specific hour of a Monte Carlo scenario depends only on the price level randomised for the previous hour. The level at hour 0 defines the probabilities for the levels for the first hour. The transfer probability *p(m, n)*, $m = 1, 2, 3$; $n = 1, 2, 3$ tells the probability for the price level moving from level m to level n in an hour and it is estimated to be a time-invariant value. The probabilities are shown on the figure above. Even though the symmetrical confidence intervals are used, the price is estimated to be more likely at the high level than at the low due to typical market behaviour.

The actual hourly prices are taken from uniform distributions of the given intervals after the price levels are randomized. The actual 35 % and 95 % hourly price confidence limits as well as expected values are calculated during the forecasting phase.

The optimisation module needs a confidence interval for every scenario analysed. Here the standard mean error of the hourly prices is used to determine the upper and lower confidence limits.

For instance, if the price level of hour 1 is low in a certain scenario, the actual price *X(1)* will be randomized by

$$X(1) \sim Uniform(Forecast_{-95\,\%}(i), Forecast_{-35\,\%}(i)). \tag{3-1-5}$$

After the prices *X(i)* for the day are known the scenario is built by

$$X(i) = [X(i) - e, X(i) + e],\ i = 1, \ldots, 24, \tag{3-1-6}$$

In which *e* is the standard mean error of *X(i)*:s.

With the given transfer probabilities the chance of a scenario *i* to actually occur is given by

$$P(i) = \prod_{j=1}^{24} p(l(j-1), l(j)), \tag{3-1-7}$$

in which *l(j)* is the price level of the hour *j* and *p(m, n)* the transfer probability defined earlier. As resources do not enable the testing of all the possible scenarios, the probabilities *P(i)* must be normalised before the optimisation process to get

$$\sum_i P_n(i) = 1. \tag{3-1-8}$$

The normalised probabilities are

$$P_n(i) = \frac{P(i)}{\sum_j P(j)}, \qquad (3\text{-}1\text{-}9)$$

The probability of the expected price curve is $P(1) = p(2,2)^{24}$ and $P(2), P(3), \ldots$ are the probabilities of the Monte Carlo scenarios.

3.2 Deterministic Optimisation

A variety of methods have been applied to the deterministic short-term procurement optimisation problem. The chosen solution method depends typically on the problem instance at hand. Typically, a division between generation utility types is made in power generation. Purely thermal power or purely hydropower problems use different techniques in optimisation because the underlying problems are so different in nature. In addition, procurement contracts offer yet another extension to the problem.

In this project, the optimisation method that will be used is the one introduced by Niemelä in his Master's Thesis (2005). The method has already been implemented at Process Vision, which means that only a new optimisation model must be configured in this project. This method has also the advantage that it can be used with different kinds of procurement portfolios, so the resulting solution is not limited only to specific types of market players. The goal of this project is not to introduce a sophisticated algorithm for solving the deterministic optimisation problem. The idea is that the developed solution utilises the existing solutions that are available to Process Vision.

The bidding optimisation procedure is independent of the internal details of the optimisation module, which means that any available optimisation method could be used as long it offers a well-defined interface for parameter setting and reading results. It is reasonable to use an existing method instead of a totally new one to avoid work overhead.

Another possibility would have been to implement the same (or some other) optimisation method in the MS Excel environment. A totally independent solution has no additional value for Process Vision because in the product development phase, that method must be integrated to the overall solution anyway.

Niemelä (2005) introduces the unit-storage-network model and describes the mathematical relationships in that model. This chapter gives only a brief summary about the main principles. The interested reader is referred to Niemelä (2005) for more information.

The basic idea in the unit-storage-network model is that the whole procurement optimisation problem is modelled as a network flow problem between two types of components: units and storages. Units are components, which use some commodities as fuels and produce others; a typical combined heat and power (CHP) plant could use oil and gas as fuels and produce heat and electricity. Storage components are used to model the dynamic properties of the system. That is, all balance equations and different flow mechanisms are modelled through storages. A storage is always associated with a single commodity, i.e. all inflows and outflows carry the same commodity and the storage properties can be used to restrict these flows.

The model cannot be solved with classical network flow algorithms, though, because there are complex restrictions on the flows, which have a unit as an endpoint. The restrictions are needed to model the link between production and fuel consumption. Makkonen and Lahdelma (2003) address the problem of non-convex optimisation and a similar technique was applied by Niemelä (2005) as well.

It should be emphasized that there is a need for two types of optimisation:

1. optimise the procurement portfolio with a fixed market price and
2. optimise the procurement portfolio with fixed spot deals (i.e. electricity load).

The first optimisation case can be used to find an upper limit for the function $P(\beta,s)$ in (3-1). The electricity market component is present in this model, thus allowing the optimisation to use the electricity market as a part of the procurement portfolio. Therefore, this optimisation case utilises the electricity market in an optimal way for the given scenario, i.e. it fixes the optimal spot deals for a given price scenario. If some other deals are fixed for the same price scenario, optimisation against this load will, by definition, produce lower profits.

The second optimisation case is used to calculate maximum profits when the spot deals are fixed. This corresponds to calculating the value for function $P(\beta,s)$. This model is used when a bidding function set β is evaluated. For a given scenario and a bid function set, it can be easily determined which kinds of deals would take place in each hour. The bid function is thought to be defined by a set of price-quantity points, which are ordered in ascending order with respect to their price value. The function is defined to be piecewise linear between the points. Below the minimum (above the maximum) price value the function is defined to obtain the minimum (maximum) quantity. This corresponds to a low-risk policy, because no extrapolation is made outside the range of analysis. After the spot deals have been calculated, the electricity market is omitted from the model and the balance node is associated with the deal quantities as electricity loads.

3.3 Optimisation Method

There are various methods for handling stochasticity in bidding optimisation problems. Analytical probabilistic considerations are almost never used because the source data does not necessarily yield distributions that could be directly used and even if it did, analytical results are still very hard to find. Some methods try to tackle the stochastic problems through direct modelling (Shrestha et. al., 2004). That is, they try to optimise the whole problem over all stochastic parameters in a single step. The advantage is that optimality can be guaranteed but on the other hand the size of the problem could well increase along with the memory and computing time requirements.

In this project, scenario analysis will be used for handling stochasticity. The optimisation module that is being used has no way to account for stochasticity, but it can find the optimal solution for any given scenario.

The following method is suggested for finding the optimal bidding functions.

1. Order the scenarios in descending order according to their probabilities and choose the limit coefficient k for a satisfying solution (e.g. k = 0,9 if it is sufficient that the result is at least 90% from the theoretical upper limit).
2. Calculate the upper limits for each scenario. The optimal bidding functions are saved for each scenario.
3. Take the optimal strategy of the next unused scenario and calculate the values $P(\beta,s)$ for each scenario s. If there are no unused scenarios, go to step 6.
4. Use the results from step 3 to calculate the expected profit with current strategy (i.e. calculate the expected profit in equation (3-1)).

5. If the ratio between results from step 4 and step 2 exceeds *k*, terminate the procedure. Otherwise go back to step 3.
6. Form a set of new strategies from the best-found solutions.
7. Evaluate the new strategies (analogy to step 3).
8. If maximum iteration count (or time limit) is exceeded or a good-enough solution is found, terminate the procedure. Otherwise go back to step 6.

This method consists of two parts. The steps 1-5 form the basic part of the method and steps 6-8 form the genetic part. In this project, the focus of the implementation will be entirely in the basic part. The genetic part is discussed, but no genetic algorithm will be implemented in this project.

The basic idea is that an optimal solution for a scenario with high probability is a good candidate for an optimal solution. It is, by definition, optimal for the scenario for which it was calculated, but it has to be ensured that it does not cause infeasible controls if the price scenario is altered. Each scenario produces a candidate for the solution and all the candidates can be evaluated, so we can directly search for the best candidate among the controls, which are optimal for a single scenario.

The second part was engineered to be able to handle the situations, where none of the original candidates cause satisfactory results. No specific method for performing the second part of the algorithm was implemented within this project.

4 SYSTEM SPECIFICATION

The implemented system uses three different applications: MS Excel, GENERIS (Process Vision, 2005) and a separate optimisation module. This chapter clarifies the needs of each individual component as far as the implemented application is concerned.

4.1 MS Excel

The whole system solution is based on a MS Excel spreadsheet model. The model consists of sheets and macros. The sheets hold the data and macros are used for data I/O. There are six sheets, which are labelled as

1. SPOT History,
2. Forecasts,
3. Scenarios,
4. Controls,
5. Deals and
6. Bid Curve.

SPOT History is separated into its own sheet because it is merely raw data and it should not be mixed with produced or modified data. Forecasts are produced with a separate mechanism and therefore this entity is also separated into its own sheet. Scenarios are created from the forecasts, and again, because this mechanism is a well-defined, independent procedure, the whole concept is stored in its own sheet. The next sheet, Controls, is reserved for reading data from the optimisation module. That module produces the optimal control values, which are then imported to MS Excel. Another sheet is reserved for the calculation of spot deals. When some scenario is selected as the reference scenario, the deals that would occur with other scenarios can be calculated. These results must be saved for further calculations and that is done in this sheet. Bid Curve sheet is the actual result sheet, in which the optimal bidding curves are represented.

In addition to the actual data storing, the use cases in Figure 4-1 must be implemented within the spreadsheet model.

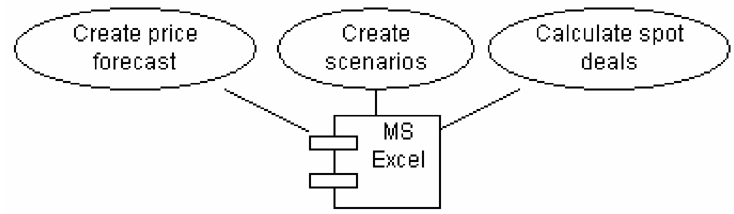

Figure 4-1 MS Excel use cases

4.2 GENERIS

The implemented system relies on GENERIS solely because it handles time series. GENERIS offers a Component Object Model (COM) –library (Microsoft, 2005) for time series handling, which allows one to build Visual Basic macros handling GENERIS time series. This COM-library is a part of the GENERIS product family and there is no need to discuss all the properties of the library in this context. The library is only used in this project, i.e. there is no development that must be made to this module in this project. The use cases of the library in this project are shown in Figure 4-2.

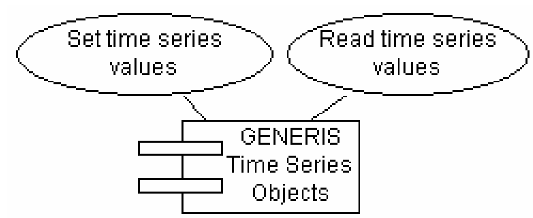

Figure 4-2 GENERIS use cases

If the suggested procedure is integrated into the GENERIS system as a whole, the dependency of the additional components to GENERIS architecture will be much stronger, of course. GENERIS has a well-tested platform, which offers readymade frameworks for different aspects of the application development, including database operations, user interface configuration, calculation, automated processing to name a few.

4.3 Optimisation Module

The optimisation module must also be callable from the Visual Basic macros. The first idea was to build a COM-object that embeds the optimisation model of this project and offers two different functions: optimisation against electricity market price and optimisation against electricity load. The advantage of this solution is that the COM-objects are fully synchronized with the program that runs the script.

There were some problems with object instantiation in Visual Basic macros, which led to an alternative solution. The optimisation model was embedded within a command line program, which takes the information as a parameter whether the electricity market is being used or not. This executable is then started from the Visual Basic macros with the standard Shell-command that allows macros to perform command line commands. Figure 4-3 shows the use cases of the optimisation module.

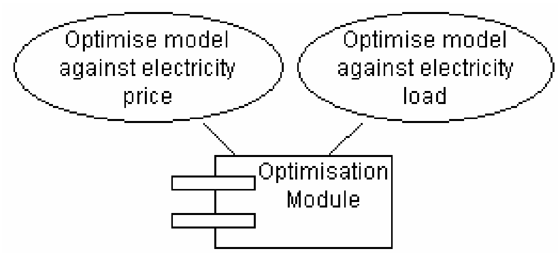

Figure 4-3 Optimisation module use cases

The problem that arises from this solution is that the executable is run fully asynchronously from the program that is running the Visual Basic script. This means that the program continues from the next program statement immediately after instantiating the executable (i.e. starting the optimisation procedure). Result reading is the next procedure that is run after the optimisation and if this procedure were included in the same script as the optimiser call, the results would be from the old optimisation, because they would be read before the optimisation has stored the new results. To avoid this problem, result reading must be manually started after the optimisation has finished, which makes the application much more difficult to use.

This problem is purely a problem in the pilot application and has no effect on further development of the real application. The failure to produce a better solution in this project was due to the fact that only one project group member had the possibility to develop this part of the pilot application. This risk was identified earlier in this project but no measures could have been taken to tackle the problem. Also, a workaround was found, which minimised the impact of the problem.

5 APPLICATION TESTING

The pilot application that was built in this project was also tested. This chapter describes what was tested and how. In addition, the obtained results are described and discussed.

5.1 Test case description

The test case includes a fairly simple optimisation model consisting of one fuel source, two production units, a heat balance, an electricity balance and an electricity market. Figure 5-1 shows the test case configuration. The test could be realistic for an energy company in a small town which has to take care of the heat load in its local district-heating network. It may or may not function as an electricity provider as well, but that is not relevant in the test case. Simplicity is an asset because it helps in result verification; for a simple model, the optimal procurement strategies can be easily derived and the solution of the bidding optimisation procedure can be verified more easily.

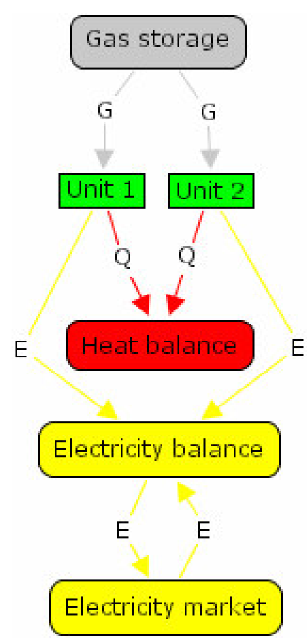

Figure 5-1 Test case topology

As stated before, optimisation must be performed both against electricity price and electricity load. In Figure 5-1, the electricity market is shown, but it can be disabled with a single parameter. The resulting model is used when optimisation against electricity load has to be performed.

5.2 Test Plan

The testing procedure is depicted in the sequence diagram in Figure 5-2. The picture clarifies the relationship between the components and their use cases, which were introduced in the previous chapter.

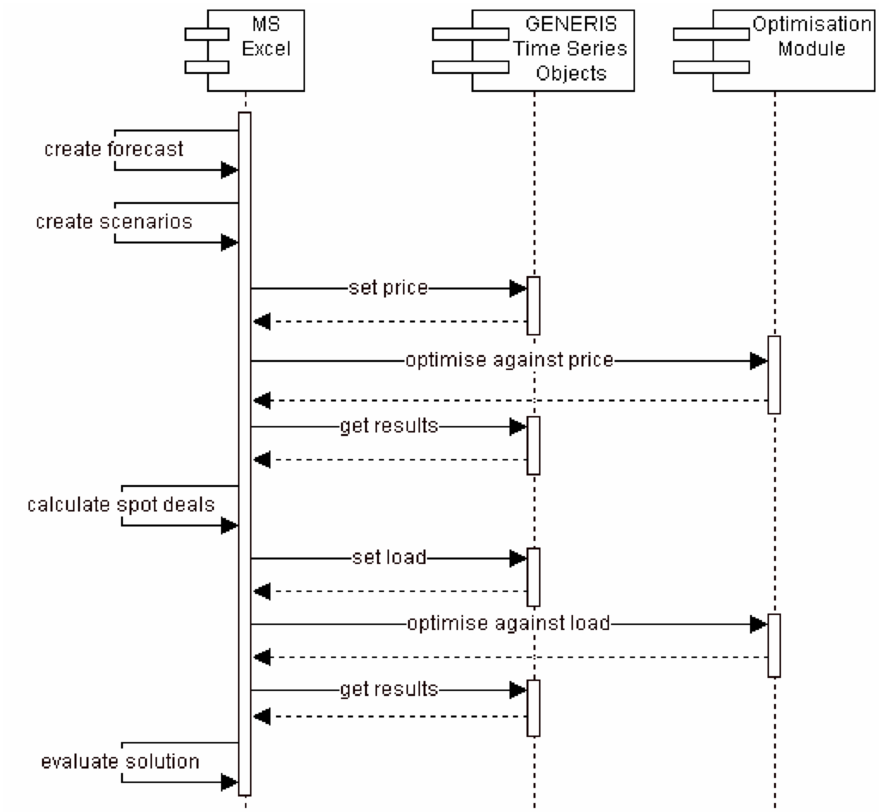

Figure 5-2 Sequence diagram for test procedure

Testing the whole bidding optimisation concept extensively is a very challenging task because there are so many variables. The first restriction that is made in this project is that the model configuration is considered to be constant (i.e. no analysis is made considering the configuration of the units' characteristic areas). Another restriction is that the only forecasted parameter is electricity price. In real life, fuel prices and load functions must be forecasted as well, and according to experienced professionals within Process Vision organisation, forecasting the heat load is the cause for most of the problems with optimisation. This project was initiated to consider the bidding optimisation in electricity exchanges and therefore only electricity market behaviour was investigated. Therefore, the two restrictions mentioned above are reasonable within the scope of this project. Once the optimisation method is validated, the tests can be extended so that these restrictions can be omitted.

The plan was to test the method for a period of one week, which means that the basic test routine that is shown in Figure 5-2 is repeated seven (7) times. For each day five (5) scenarios are created in addition to the original forecast. Moreover, for each scenario, the bid curve is formed through three (3) variants of the scenario. This means that the model is optimised 126 (7*(5+1)*3) times against the market price. In addition, when evaluating how good a single bid curve is, all the other scenarios must be optimised once against electricity load. There are always five (5) scenarios to be evaluated and this is done maximally for all scenarios (6). This means that the model is optimised 210 (7*6*5) times against electricity load. It is easy to calculate the maximum number of iterations I for a single day (i.e. the optimisation period in bidding optimisation)

$$I = (S+1)B + (S+1)S = (S+1)(S+B), \qquad (5\text{-}2\text{-}1)$$

where S is the number of created scenarios and B is the number of points in the bidding curve. It is important to notice that this is the maximum number of iterations needed. The maximum number is needed only if the algorithm ending criteria is not fulfilled during the algorithm.

The numbers S and B were kept rather small in this case because the full automation of the test procedure failed as was stated in chapter 0. Considerably more extensive testing is possible when the whole concept is implemented within the GENERIS product family and testing can be fully automated.

5.3 Results

The results were almost identical for every day. The first thing that can be directly seen in the results is that for a given day and hour, the bid curves of all scenarios contain exactly two values for sold/purchased quantities. One value is "low" and the other one is "high". This is due to the fact that the optimisation module does not make dynamic considerations in its current state. Because the hourly problems are not linked to each other, the optimal control are so called bang-bang controls (Kirk, 1970), which produce either as little or as much as possible depending on the quote between fuel and market price. This is not a problem of the bidding optimisation procedure but makes the results somewhat meaningless, because the bid curves vary only marginally between scenarios.

The second result is that the first bid function set candidate that is obtained from the original forecast produces nearly optimal results in all other scenarios. Table 5-1 contains statistical information about the relative deviation that occurred when the created scenarios were optimised against the electricity load and the original forecast is the reference scenario. The numbers represent to which extent the theoretical optimum was achieved. It is evident that in the examined case, further optimisation is always unnecessary as the scenarios produce practically optimal solutions.

Table 5-1 Deviations from theoretical optimum

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Day 1	0.99968	0.99962	0.99931	0.99888	0.99958
Day 2	0.99937	0.99952	0.99921	0.99911	0.99937
Day 3	0.99917	0.99910	0.99942	0.99987	0.99976
Day 4	0.99855	0.99966	0.99924	0.99948	0.99948
Day 5	0.99970	0.99927	0.99889	0.99945	0.99927
Day 6	0.99919	0.99976	0.99960	0.99993	0.99948
Day 7	0.99934	0.99925	0.99961	0.99963	0.99955
AVG	0.99941				
STDEV	0.00029				

This is also partly due to the fact that the optimisation module does not make dynamic considerations. Also, if the electricity price were more volatile in the study period, the scenarios would vary much more and the probabilistic effects would become more visible in the solutions. In the used study period, there were no abnormalities and the scenarios are only slightly different from the original forecast.

The third result is that with the used scenario transfer probabilities, the original forecast obtains a very large probability.

Table 5-2 contains the probabilities of the different scenarios for different days as well as some statistical figures calculated from the data.

Table 5-2 Scenario probabilities

	Forecast	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Day 1	0.96075	0.00364	0.00089	0.01422	0.02020	0.00030
Day 2	0.91168	0.00825	0.00570	0.05780	0.00435	0.01221
Day 3	0.95107	0.01727	0.00480	0.02027	0.00051	0.00608
Day 4	0.94992	0.00277	0.00031	0.02875	0.01485	0.00340
Day 5	0.97993	0.00638	0.00952	0.00002	0.00032	0.00383
Day 6	0.95487	0.00051	0.00373	0.03942	0.00000	0.00148
Day 7	0.96406	0.00006	0.00265	0.00019	0.01427	0.01876
AVG	0.95318	0.00936				
STDEV	0.01938	0.01246				

It is easy to conclude that the control that is derived from the original forecast is very likely to be the best candidate, because the results are weighted with the scenario probability in (3-1).

Result verification has been done for the optimisation and the suggested procedure only utilizes those results, so the optimisation results can be said to be computationally correct. The problem is that all the three results that were mentioned above make it impossible to validate the suggested procedure, because no realistic case could be built within the scope of this project.

6 DISCUSSION

6.1 Implications and applications

The scope and object of this project was to find a justified mathematical model for the bidding optimisation problem frequently encountered in competitive power markets in order to offer a solution for key decision makers participating in procurement and selling of electricity. The foundation of this project was laid on a very extensive literature survey, which covered several vital aspects in terms of this project. The available means of optimisation were researched and based on the findings the classical linear programming methodology was adopted. After careful consideration, a method of scenario analysis was chosen for modelling the possible price scenarios, on which the optimal bid curve is based. The literature survey also covered the different ways of forecasting the spot prices, which is essential for creating trustworthy bids. Several methods were considered, and a dynamic regression model was chosen as the forecasting method used in this project. The dynamic regression model estimated produces upper and lower confidence limits of 35 % and 95 % for the expected 24-hour spot price curve. The confidence limits are used in the scenario analysis further on.

The scenario analysis method was implemented in MS Excel, and it uses the forecasted prices as an input. This part of the pilot application produces the likely market clearing price scenarios, which are then used for optimisation. These price scenarios are used via GENERIS by the separated optimisation module to optimise the bidding function against the electricity price and to take into account the specific production constraints. Accordingly, the algorithm for finding the optimal bidding functions was introduced. The constructed pilot application was finally tested on a realistic bidding optimisation problem. The test case resulted in a very discrete bidding function, which is equal to a bang-bang control. This somewhat unsatisfactory result is, however, due to the optimisation module's inability to make dynamic considerations. Thus, it may be concluded that the constructed pilot application for devising optimal bidding curves is expected to function properly. The results, however, give rise for an optimisation module, which can make dynamic considerations, to be implemented.

6.2 Directions for further research

One interesting aspect in the optimisation procedure is to find ways to improve the already found solution candidates. The original (i.e. most probable) strategy candidate is likely to be nearly optimal since it is optimal in the most probable scenario. Problems occur when that solution yields non-feasible solutions for some scenarios. This occurs, when some electricity price scenario has a very large peak. The suggested strategy tries to maximise the profits during each hour, which could cause the bids to become non-feasible when such peaks occur. One could try to solve this by smoothing the bidding curve. This way, the production would not increase or decrease so drastically when the price peaks occur. If the smoothing makes the non-feasible scenarios feasible, the expected profit should increase because non-feasibilities are avoided with large penalty functions. One way to do the smoothing is to assign a smoothing variable for the bidding curve and perform a one-dimensional search for this variable. This search increases the solving time significantly so it is important to identify the hours, where this smoothing is needed. These hours can be identified with the help of the solution candidates. If the bidding functions vary significantly from scenario to scenario for some hour, there is probably a need for smoothing. If, on the other hand, all the bidding functions are quite similar, the smoothing will probably not add value.

An important commercial aspect regarding the generated pilot application is not only its improvement of the current state, but also integration to the existing solution by Process Vision. Further research should be done on implementing the product of this project into GENERIS. It should also be confirmed that the existing solution produces congruent results with the pilot application of this project, and comparison between results of the pilot application and the existing method is advisable. Further considerations should also include whether the pilot application and the existing method could and should be integrated or a new method based on both earlier approaches should be introduced.

The dynamic optimisation aspect of the optimisation module should also be investigated. As the optimisation module may not consider the generated realistic price scenarios dynamically at its current state, the method for producing optimal bidding functions introduced in this project may not be fully taken advantage of. Therefore improvement of the current optimisation module should be considered, and especially the augmentation of the dynamic features into the module. Other implications and enhancements of the dynamic optimisation module in bidding optimisation should also be reflected.

A promising viewpoint to be analysed is the scenario correlation between prices and price levels of electricity and other forms of energy. It should be investigated whether there are any such interdependencies adjacent to the market. This is to say that finding positive correlation between some other commodity (e.g. heat, gas etc.) and electricity would mean scenario correlation between the prices of these commodities. This correlation and the magnitude of it could then be used in the scenario analysis of this project in order to generate better estimates for the optimal bidding functions. Thus, investigating prices of other commodities and determining possible cross-correlation factors between these commodities and electricity could improve the price forecasts used in optimisation. Therefore, the impact of scenario correlation should be more closely investigated in the future.

In this project the expected price curve is calculated using processes with which the variance of the price can also be estimated. The problem is, however, that the variance is equal for each hour in an individual forecast. Even though any research was not done concerning the differences in variance between separate hours it is expected that the variance is higher in peak hours and lowest during the night time. If the variance estimation was more specific, the scenario analysis would probably give more accurate results. Another problem is that when using the transfer probability $p(2,2) = 0.5$ the probability of expected price curve to realize is $P(1) = 0.5^{24}$, that is much higher than the probabilities $P(i)$, $i = 2, 3, \ldots$ When the scenario probabilities are normalized the expected curve gets a weight much higher (often over 95 %) than the other $P_n(i)$:s. The problem should be corrected by estimating the weighted probabilities some other way.

During the testing session only five scenarios were created and analysed in the optimisation phase. When having only three different price levels the number of different possible scenarios is $3^{24} \approx 10^{11}$ when transfers between all levels are possible. Of course not all the scenarios could or should be gone through, but at least there should be clearly more than five of them to be analysed in the optimisation process.

As the number of scenarios grows there should be also a control, which eliminates similar or same kinds of scenarios to decrease the computing time needed in the optimisation process. Brand, Thorin and Weber (2002) introduce an algorithm for scenario reduction.

The scenario creation process could also be more sophisticated. The transfer probabilities between price levels should be estimated from the historical data by using some kind of mathematical approaches rather than only approximating them by common sense. In addition the transfer probabilities are probably somehow time-dependent unlike what was assumed in this project.

When the number of scenarios is grown it enables the possibility of testing how the profits of the optimised production and bidding plan are changed for instance if the price is unexpectedly high or low. In this project all the five price scenarios are created using really modest changes in price. In reality there often exist some unexpected high peaks in the electricity price curve.

Another area for further development would be to extend the formula (3-1) to contain some form of risk analysis term. That would mean that not only the solution's expected value but also its stability would increase the solution's value. This would require that the risk attitudes of the energy companies were examined and that proper risk metrics were calculated during the optimisation procedure.

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