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ELECTRICITY FORWARD CURVE GENERATION

Final report

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Abstract

This study was carried out as a project assignment for the seminar in Operations Research at Helsinki University of Technology and was commissioned by Process Vision, a company specialized in information systems and applications for the energy sector.

We concentrated on the problem of constructing hourly forward curves based on forward market data. Emphasis was especially on researching what kind of accuracy may be obtained and how different complications such as missing or over-lapping data should be handled both from the theoretical and practical point of views. The project consisted of an extensive literature review of methods mainly based on either describing the behavior of the spot price and then constructing the corresponding forward curve or directly modelling the forward curve with a suitable stochastic process. Moreover, statistical analysis of market information has been reviewed and different approaches to fitting the generated forward curve to the current forward/futures market information have been considered. After selecting one relatively straightforward approach suitable for our purposes and implementing it, an ex-post analysis was carried out in an attempt to measure the performance of the model in terms of difference between the forward curve and realizations of the spot price.

The project consisted of a literature review, initial data analysis, forward curve generation algorithm construction and implementation as well as an Excel tool and result analysis and comparison. Finally, we tested how the constructed model was able to forecast actual realizations and compared it to another model from literature.

Table of contents

1 Introduction

1.1 Background

Electricity markets differ from conventional commodity markets in several ways because of the fact that electricity cannot be stored. Electricity is traded in centralized electricity exchanges, such as Nord Pool and European Electricity Exchange (EEX) that offer markets for day-ahead electricity as well as financial products over longer time spans. These products include financially settled electricity forward contracts, options, and contracts for price difference (swaps). The day-ahead market price is usually established hourly and called the spot price. The forward contracts are typically valuated against the spot price. In European electricity markets, a forward curve, calculated over suitable time spans and time step from hourly to yearly using financial forward products and mathematical models, has become an indispensable tool for offers and valuation of electricity contracts and derivatives. There are different algorithms to calculate the forward curve that take into account effects such as overlapping or missing contract periods and seasonality. The algorithms are based on, e.g., least-squares fitting of a profile or a theoretical model. In this project, the construction of such a forward curve using market price data is considered.

1.2 Motivation and goals

The spot price of electricity and electricity load are significantly positively correlated, see [Bunn, 2004] for an in-depth discussion of electricity load and price characteristics. In the deregulated electricity markets, electricity is priced typically for each hour. For an electricity retailer this represents a risk because her contractual obligations are fixed and usually not tied to the hourly spot price. Thus, it is important for the retailer to have an estimate of the hourly spot price to evaluate the value at risk or to consistently price new electricity offers.

The research problem consists of building an algorithm to calculate the forward curve for electricity based on market data, i.e., the spot price of electricity, a number of forward products with given delivery periods and market prices as well as trading volumes (open interest) for these. The emphasis is foremost on electricity markets but some features of other commodity markets may be considered as well.

In concrete terms, the aim of this project was to perform the following activities:

1. Literature survey of

    i. Financial electricity market, forward curve and forward products in Nordic markets,

    ii. forward price curve construction and

    iii. forward and spot price models and their relationship.

2. Analysis of market data and comparison of models and algorithms

3. Construction of an algorithm to calculate a forward curve based on market price time series of forward products

    i. Handling of over-lapping delivery periods and weighting of different forward products

    ii. Handling of periods with no forward price information

    iii. Handling of several correlated reference prices for forwards based on contracts on differences. (Pricing of CfDs left out of scope.)

    iv. Seasonality and profiling the forward curve to hourly-level. Yearly, seasonal, weekly, and daily periodicity.

    v. Combining the effect of external factors on long-term and short-term price development

4. 4.Construction of an algorithm to estimate forward price model parameters based on market price data

Optionally:

5. Calculation of characteristic values for the data and forward curve, e.g., goodness of the fit.

As a result of these activities, an Excel tool that implements the chosen algorithm and models was built. The tool as well as the literature survey and other activities are documented in this final report.

1.3 Research methods and limitations

The research methods consisted of a literature survey to gather an understanding about current approaches to the problem followed by a statistical analysis of market data. The development of the algorithm to generate the forward curve and implementing it was done using MS Excel and available market data. In addition, a model for comparison was built using Java language.

1.4 Structure of the report

This report consists of seven sections. Section 2 presents the literature review of electricity markets and forward curve generation algorithms. Section 3 discusses different algorithms and the chosen algorithm implementation. Section 4 presents the results of test runs. Section 5 concludes and finally, in section 6, a project summary is put together.

1.5 Schedule of the research

The research was conducted between the beginning of February and the end of April in 2005. The project was divided into four separate stages. In the first phase, the project group sharpened the objectives of the project and learned about electricity markets, electricity price forming of both the spot and forward prices as well as models for these and the existing literature and commercial products on electricity forward curve products. The project group then proposed some directions of further research and activity for the rest of the project's duration. This first phase fixed the other three stages of the project. In the second phase, an extensive literature study was conducted on electricity market price models and forward curve generation. In the third phase, the project group concentrated on comparing the models in literature and building a model using Excel. Finally, in the fourth phase, the project group tested the built tool. To conclude, a final report of the project was written.

2 Literature review

The literature on electricity markets, price formation and forward curve generation can be divided in roughly four different categories. First, there are a large number of studies on stochastic spot and forward price models, where often forward curves are finally fitted to current market forward curve, which is referred to as the current term structure hereafter. Another approach is to model the term structure of the entire forward curve as an Itó-process following the framework of Heath-Jarrow-Morton [Heath et al., 1992]. Thirdly, a number of authors consider the fitting of the generated forward curve to the current term structure. This problem is not generally a trivial one, since it involves fitting a parameterized family of forward curve models to an arbitrary term structure; a one interesting approach is however provided in [Landén, 2000]. The last category of the current literature consists of statistical analysis of the market data for both the spot, and forward and /futures prices. Whenever the assumption that underlying interest rates are deterministic holds, the prices of forwards and futures can be assumed to be exactly the same [Cox, Ingersoll, 1981]. In this project it is assumed to hold, i.e., no difference between forward and futures is made.

In addition to the standard approaches there are also some creative combinations of different methods, which include using current market data as a priori information and improving the forecasts using other methods. These are discussed in detail later on in this chapter. This chapter contains some basic discussion about electricity markets, spot price models, modelling the forward curve and the special case of Nordic Electricity Exchange Nord Pool.

2.1 About electricity markets

In 1990, the United Kingdom was the first country in the world to start trading electricity. This began the process of deregulation, which basically means dividing the risk between the supply and demand side of the market by allowing the energy buyers choose their suppliers and usually separating the production and transmission of electricity, which is called unbundling. In many cases, deregulation was first applied to large-scale buyers or the supply side and a few years later to households. The next countries to deregulate were first Norway in 1993 and then Sweden and Finland in 1996, sharing the originally

Norwegian electricity marketplace Nord Pool. Also in 1996, the first two states in the U.S., California and Texas, began deregulation. Most of the Western Europe followed suit within a few years, and another marketplace, European Electricity Exchange (EEX) was established in Germany. Despite these common marketplaces the electricity markets throughout the world are, in general, not centralized, and, e.g., within Nord Pool there are several different area prices due to limited transmission network capacity between countries and even between areas within a country.

The electricity market is inherently different from the markets of other commodities or financial instruments due to the fact that electricity is practically non-storable, which creates a load-matching problem. One needs to decide the optimum amount of reserve margin to provide. An imbalance of demand and supply at one point within the electricity distribution grid can threaten the stability of the whole system. This difficulty to match demand and supply leads to highly volatile prices on the wholesale level. Yet there is usually no real-time pricing in use on the customer level. One significant factor contributing to the high volatility is that electricity production is very capital intensive - it is expensive to maintain the ability to increase production on a short notice. In the Nordic market, roughly half of the electricity is hydro-produced, which provides some flexibility by controlling reservoirs [Audet et al., 2004]. However, there is high variation in the prices between different years, because the total amount of power available depends on the amount of precipitation and the price levels bear a strong seasonal component due to seasonal consumption patterns.

Most players in the financial electricity market are hedgers. Industrial large-volume users and retailers of electricity are seeking to take a long position, i.e., to buy forward contracts in order to set a maximum to their operating expenses and electricity producers are seeking to take a short position, i.e., to sell forward contracts in order to set a minimum to their revenues. Employing forward contracts does not really solve the underlying problem of matching a very inelastic supply with a very inelastic demand, but rather protects market participants from large financial fluctuations when the match happens to be far from perfect. Deregulation may lead to more economical and even environmental decisions on both sides of the electricity market, but participation requires the command of appropriate tools for risk management and derivative pricing.

Even for any kind of energy derivatives, such as forwards on crude oil, the high volatility at the short end of the forward curve and the stability at the long end make it difficult to use the basic Black-Scholes model based on geometric Brownian motion - the actual spot price volatility, e.g., would give unrealistically large option prices [Wilmott, 2000]. For electricity, arbitrage opportunities by exploiting apparent mispricings are negligible, and the correlation between short-term forward (or spot) prices and long-term forward prices is lower than in other commodities or financial markets and no analytical connection exists between them. The traditional way has been to attempt to model the stochastic process of the spot price either with statistical approaches based on explicit formulae for the price process with parameters estimated from the market data or with fundamental approaches based on competitive equilibrium models for the market. The statistical models carry the risk of becoming over-parameterized "black-boxes" and the fundamental models require a complete set of coherent historical data as well as forecasts, which is often difficult to obtain on new and illiquid markets, even in the case of the most liquid Nord Pool market. Moreover, the stochastic process of the spot price is likely to change over time rather than remain constant. Several models can be found in [Pilipovic, 1998], [Deng, 2000], [Lucia & Schwartz, 2002], and [Skantze et al., 2001].

There are also a few asset-pricing models that directly attempt to model the time dynamics of the entire forward price curve, which implicitly also gives the relationship between the spot price and the forward prices, using a multi-dimensional Brownian motion and only a few stochastic factors, taking the initial forward price curve observed in the market. This Heath-Jarrow-Morton framework was originally proposed in [Heath et al., 1992] to be used in interest rate markets. This approach has been developed further, e.g., in [Miltersen et al., 1998], [Lucia & Schwartz 2002], and using principal component analysis (PCA) to analyze the volatility factor structure of the forward curve in [Koekebakker et al., 2001] and [Blanco et al., 2002].

2.2 The case of Nordic power exchange Nord Pool

In this study, we are mainly interested in the financial market dynamics within Nord Pool. However, the work can be generalized to other exchanges as well. The history of Nordic power exchange Nord Pool relates back to early 1990s when new deregulative Electricity act for electricity markets was passed in Norway, where it was the second one in the world.

Nord Pool started as a pure Norwegian exchange in January 1993, but soon it merged with a couple of Swedish electricity grid companies and after extending to Finland as well as Denmark the market area for electricity traded in the twelve-year-old Nord Pool nowadays is the whole Scandinavia. It is a non-mandatory common multinational market, but nonetheless there is still a role for national system operators, namely maintaining the balance between supply and demand.

Power exchange is an organized market place that connects all the market participants together, namely electricity retailers, producers, brokers, traders, and end users. The primary difference between a trader and a broker is the fact that brokers are on a commission, that is, they do not own anything, but act as an intermediate agent bringing sellers and buyers together very much like real estate agents in the property markets. Traders own the assets they are selling or buying and hence are willing to take the risk that nobody else is willing to take, thus having also an important macroeconomic role.

Nord Pool power exchange is operationally divided into four different business areas, which are Elspot, Eltermin, Eloption, and Clearing Services (or Elclearing). However, functional division of Nord Pool is more reasonable. In that context there are two different functions for the Nord Pool exchange. The first function is to provide a trading desk for daily physical delivery of electricity and the other is hedging (since great deal of market participants are hedgers) against certain risks involved in that electricity trading.

Elspot contains Nord Pool's so-called physical market and fulfills the first function. In addition, the very liquid Elspot market gives a reference index for trading with derivative instruments. Eltermin and Eloption provide derivative contracts and these are commonly referred to as Nord Pool's financial market. Nord Pool Clearing Services NEC is a separate business area which guarantees settlement and delivery of all trades made at the market. This is done by entering into the contracts as a legal counterpart for both the buyer and the seller. NEC also provides clearing services of standardized OTC contracts which are registered in the market for that purpose. NEC can thus be used to hedge against counterpart or credit risk possible involved in the delivery contract. Together with financial market (Eltermin and Eloption) NEC (Elclearing) successfully fulfills the second function of the Nord Pool power exchange.

Nord Pool's Elspot exchange contains a pricing mechanism which defines the electricity spot prices for Nordic electricity markets for the following day with hourly resolution by the means of an auction, taking the bids and estimated production quantities of market participants in and calculating a market-clearing equilibrium price, taking not capacity constraints of national transmission lines into consideration. This price is commonly referred as a system price, which is determined separately for each hour for the next day and can be seen as the price for a one-day futures contract, without any extra costs for delivery to a specific area. On the following day, the national system operators organize a so-called regulating (or balance) market to maintain short-term balance. The system price is used only if there are no capacity constraints in national electricity grids, which are owned by several independent transmission system operators, whose activities are controlled by national public authorities. This arrangement guarantees a non-discriminatory access of all market participants to the grid. If there happens to be constraints, meaning that the required electricity flow exceeds the capacity limits of the transmission lines, the respective national operator is responsible for setting up the new zonal price or prices (altogether ten zones in Scandinavia) instead of one system price. The pricing mechanism for this is the very same that the one in Nord Pool's physical market. This arrangement is set up to guarantee the balance between supply and demand in the presence of transmission constraints. This balancing of the usage and the consumption could be done minute-by-minute.

The financial market provides forward and futures contracts for electricity delivery, European options for these contracts and CfDs, that is, contracts for difference between a zone price and the system price. None of futures contracts had entailed to physical delivery since September 1995, but they are settled cash against the system price. During the trading period, i.e., before the delivery period begins, the futures contracts are settled daily by margin accounts, while during the delivery period both the futures and forward contracts are settled daily.

Forward and futures contracts are written on the arithmetic average of the system price[1] and typically specify a constant delivery of electricity over a specified period in the future

[1] Some OTC contracts may have different reference price.

and refer to a base load of 1 MW. The trading period is the period between the first and the last trading day. At Eltermin both forward and futures contracts are traded. The Nord Pool's product category has evolved a great deal during the years and today there are only two delivery periods for Nord Pool's futures products. Those are a day and a week, and forward contracts are written on any other delivery periods. Trading period for daily futures contracts varies between 1 to 8 days, and the last trading day is always the previous weekday before the start of the delivery, since no contract in Nord Pool is traded whose delivery period has already started. For weekly futures, the delivery period is 168 hours and the trading period varies from 2 weeks to 2 months with the last trading day being about a week before the delivery. With regard to Nord Pool's forward products there are currently four different standardized delivery periods. Those are a month, a season, a quarter, means one fourth of the year (each quarter precisely), and a year. The length of the delivery period for monthly is around 670-744 hours. Trading period is 1-6 months before the start of the delivery. In the case of seasonal products, a year is divided into three seasons, Winter 1 (weeks 1- 16), Summer (weeks 17-40), Winter 2 (weeks 41-52), and the delivery period is each season respectively. Trading periods of seasons are usually 7-11 months. Quarter's trading period is lengthy, around two years. Annual contract's trading period is also the same two years. The last trading day for all of these forward contracts is one weekday before the start of the delivery period. Relatively short-term contracts are the most heavily traded, namely weekly futures and two nearest seasonal forward products. On average 20-30 weekly and 30-80 seasonal products are traded daily. The market is the most illiquid for the daily futures contracts. Eloption provides a market place for European options written on annual, seasonal, and quarterly forward contracts. As mentioned earlier there are also contracts for difference between area prices and the system price. These are written on monthly, seasonal, and yearly contracts, respectively. Nord Pool's product category evolves and, interestingly enough, the trading period of the very first contracts on CO2 emissions allowances begun recently. For further information see Nord Pool's product specification[2].

[2] http://www.nordpool.no

2.3 Electricity spot prices behaviour and modelling

Spot price modelling can be divided roughly into two approaches. Fundamental approaches based on competitive supply-demand equilibrium and statistical approaches based on a statistical model with a set of parameters that are calibrated using market data. Statistical approaches model the spot price directly, normally as a regression model, stochastic process or as an auto-regressive time series model. Parameters are conventionally calibrated using market data. Some examples of these can be found in [Räsänen & Ruusunen, 1992], [Lucia & Schwartz, 2002], [Deng, 2000], [Vehviläinen et al., 2003], [Burger et al., 2004] and [Cuaresma et al., 2004].

Hourly spot prices for each day are highly correlated in Nordic electricity market as can be seen from the upper part of the Figure 1. The following hour's price tends to be very close to the previous hour's price (martingale property under convenient probability measure). Figure 1 also shows the daily spot prices from the beginning of 1992 to the mid-April of 2005. As can be seen from Figure 1, during the winter 2002-2003 the spot price was many-fold in comparison with the normal price level. All these figures suggest that there is some kind of a mean-reverting level, that is, the price process is a self-affine anti-persistent random-walk, [Simonsen, 2003].

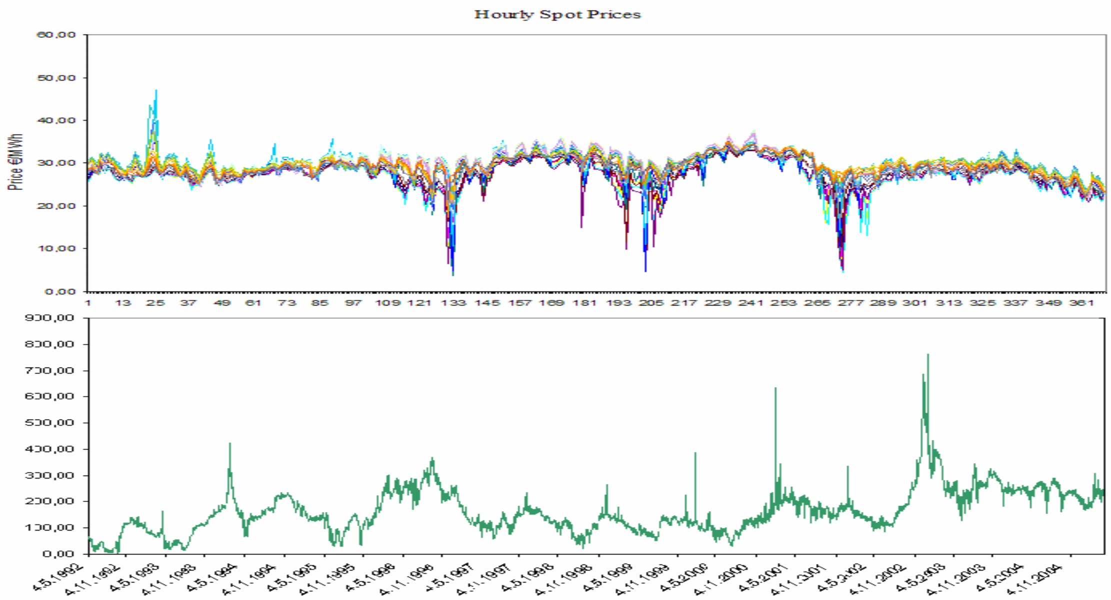

Figure 1, Nord Pool's electricity hourly spot prices in 2004 for each hour of the day (above) and daily spot prices from 1.1.1992 to 12.4.2005 (below).

It has been proposed [Pilipovic, 1998] that a simple sinusoidal function would suffice as a cyclic mean-reverting reference price level for several spot price models. Figure 2, below, shows two realizations of spot prices by Hull-White model for spot prices,

$$dS_t = \lambda(\theta(t) - S_t)dt + \sigma\, dW_t, \qquad (1)$$

with Brownian and Lévy diffusion components, dW_t, respectively.

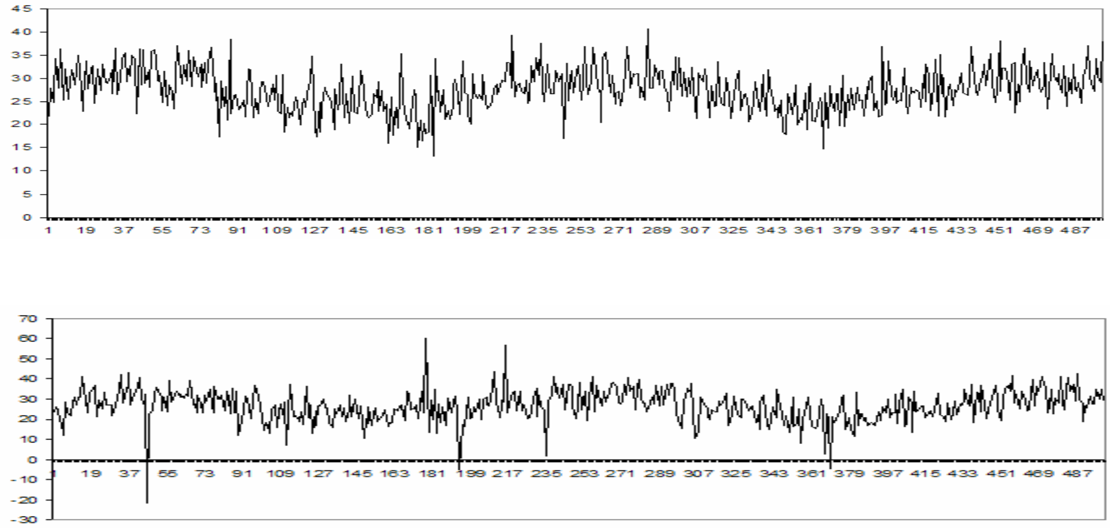

Figure 2, Realization of spot prices implied by Hull-White mean-reverting model with standard Brownian motion and sinusoidal mean-reverting level(above) and with symmetric Lévy motion(below).

Spot price data was retrieved from Nord Pool's FTP server. In the case of spot prices, different hours turned out to be highly correlated pair-wise, as Figure 1 indicates. Linear correlation coefficients between any two time series, i.e., hours, always lie above 0.9521 (data from 1.1.1992 to 12.4.2005). For daily purpose we decided to use averaged hourly price without giving any emphasis to particular daily patterns, very much in the spirit of [Lucia et. al., 2002]. Figure 1 also suggests that the distribution of the diffusion component of the spot price model must be far from normal distribution, or log-normal distribution for that matter. Table 1 shows basic statistical tests for normality and log-normality of daily spot returns, i.e., for $(S_t - S_{t-1})/S_{t-1}$. The normality hypothesis as well as log-normality hypothesis is rejected on the basis of several statistical tests, even the Kolmogorov-Smirnov test, which is not very sensitive to tail densities, rejects the hypotheses.

Normality Test for returns					
	Test	Prob	10% Critical	5% Critical	Decision
Test Name	Value	Level	Value	Value	-5 %
Shapiro-Wilk W	0.2907893	0			Reject Normality
Martinez-Iglewicz	12.69557		0.9948361	0.9949216	Reject Normality
Kolmogorov-Smirnov	0.2535231		0.015	0.016	Reject Normality
D'Agostino Skewness	100.3425	0	1.645	1.96	Reject Normality
D'Agostino Kurtosis	51.2687	0	1.645	1.96	Reject Normality
D'Agostino Omnibus	12697.0895	0	4.605	5.991	Reject Normality

Normality Test for Log-returns					
	Test	Prob	10% Critical	5% Critical	Decision
Test Name	Value	Level	Value	Value	-5 %
Shapiro-Wilk W	9.51E-02	0			Reject Normality
Martinez-Iglewicz	70.41842		0.9948361	0.9949216	Reject Normality
Kolmogorov-Smirnov	0.3350277		0.015	0.016	Reject Normality
D'Agostino Skewness	112.1584	0	1.645	1.96	Reject Normality
D'Agostino Kurtosis	52.4404	0	1.645	1.96	Reject Normality
D'Agostino Omnibus	15329.513	0	4.605	5.991	Reject Normality

Table 1, Normality tests for spot prices and log-spot prices (1.1.1992 to 12.4.2005)

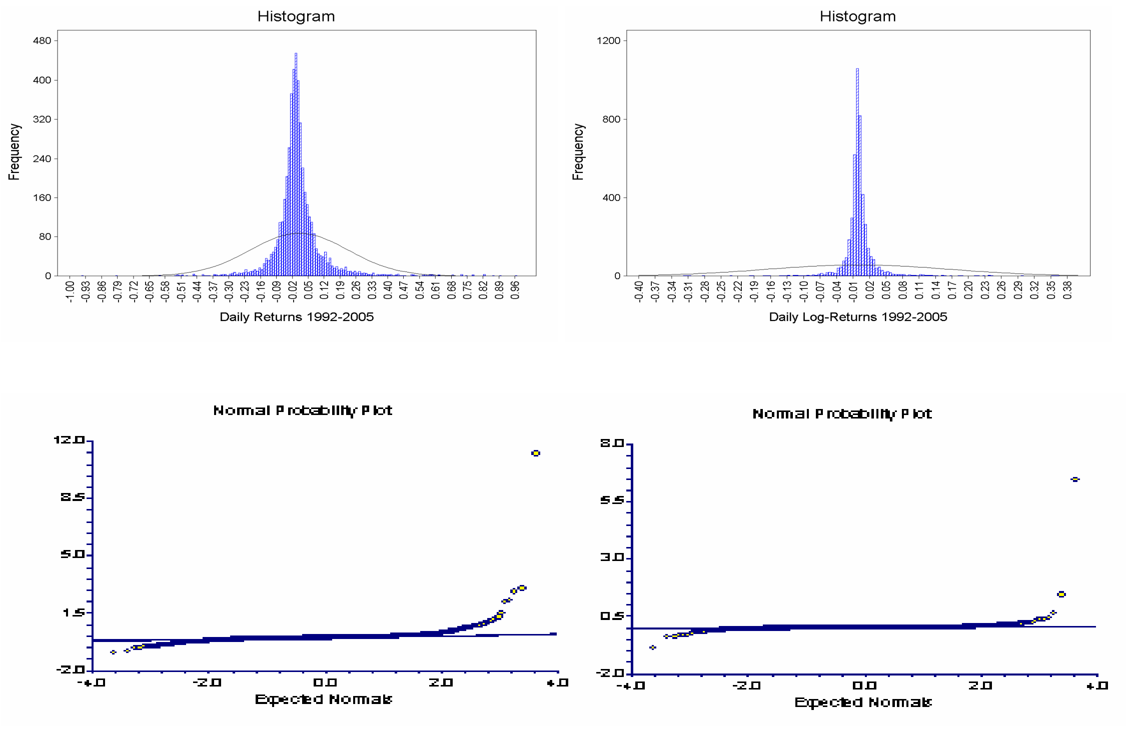

Figure 3, Histograms and QQ-plots of spot returns(left) and spot log-returns(right)(Nord Pool data from 1992 to 2005).

As mentioned in the introduction part, electricity spot prices are hypervolatile in the sense of traditional volatility measures and normality hypothesis as well log-normal hypothesis for spot prices is rejected in the sense of several statistical tests, Table 1 shows this situation. Figure 3 shows QQ-plots and histograms of daily returns and log-returns. The corresponding normal curve is very wide in both cases. The kurtosis is extremely high (1589.3 and 1223.5, respectively), which indicates huge jumps. [Lucia & Schwartz, 2002] found that there is significant variation in distributions between yearly seasons, i.e., cold and warm, wet and dry. Warm seasons turned out to be twice as volatile as cold seasons. In the case of Nordic hydro-driven electricity markets extreme prices (jumps) are relatively frequent. The kurtosis of the whole sample (pure prices) is 7.3233,. Also the skewness estimates (1.6333 for the whole price sample) are statistically significant under the null hypothesis of both normality and log-normality.

The simplest way to model electricity spot prices is to use a one-factor model, traditionally designed to capture the term structure of interest rates. One such model was proposed by [Lucia & Schwartz, 2002]. There the spot price is modelled to be a sum of a deterministic seasonal component and an Orstein-Ohlenbeck process, a mean-reverting stochastic process. The price process is the solution of the stochastic differential equation,

$$dS_t = \lambda(a(t) - S_t)dt + \sigma dZ_t, \quad a(t) \equiv \frac{1}{\lambda}\frac{df}{dt}(t) + f(t), \tag{2}$$

where $f(t)$ is the deterministic function for seasonal patterns. This basic one-factor model manifests two characteristic features of the spot prices, mean-reversion and periodicities, when dZ_t is a standard Brownian motion. Other specific features of electricity spot prices are price spikes, price-dependent volatilities, and long-term non-stationarity. Price spikes can be included by defining dZ_t to be $\alpha - stable$ (Lévy) process (does not necessarily have to be symmetric, and that usually is the case). The volatility parameter can simply be made price-dependent or even stochastic itself, see, e.g. [Deng, 2000], but long-term non-stationarity can be incorporated only by bringing in an additional stochastic factor. Another issue with regard to which the one-factor model is insufficient is the fact that the strong mean-reverting property makes the implied forward prices almost constant. To see this, consider an equation for forward prices (delivery period $[t, t+dt]$, which begins at T),

$$F(t,T) = a(t)(1 - e^{-\lambda(T-t)}) + S_t e^{-\lambda(T-t)} . \tag{3}$$

Now, because of the strong mean-reverting property (reversion back to normal level within a few days) the speed factor λ of the mean-reversion becomes quite large, thus (3) is almost constant in the long-term. Two-factor stochastic model, where the first stochastic factor is responsible for short-term mean-reversion and the other for long-term equilibrium price level can achieve somewhat better results by taking long-term non-stationary effects into account. For that purpose, [Lucia & Schwartz, 2002] proposed the following model:

$$\begin{aligned} S_t &= f(t) + X_t + \varepsilon_t \\ dX_t &= -\kappa X_t dt + \sigma_X dZ_X \\ d\varepsilon_t &= \mu_\varepsilon dt + \sigma_\varepsilon dZ_\varepsilon \\ < dZ_X dZ_\varepsilon > &= \rho dt \end{aligned} \quad . \tag{4}$$

where $f(t)$ models again deterministic seasonal patterns and X_t and ε_t short- and long - term stochastic movements, respectively. ρ tries to capture the correlation structure of these two movements. Another type of a somewhat simpler two factor model was proposed by [Pilipovic, 1998]. In his model there are two stochastic factors and the actual equation reads

$$dS_t = -\lambda (S_t - Y_t)dt + \sigma\, dW_t , \qquad . \tag{5}$$

meaning that the mean-reversion level Y_t itself is a stochastic process. According to [Pilipovic, 1998], an arithmetic Brownian motion would suffice for that purpose. A realization of mean-reversion level another stochastic factor being a somewhat more sophisticated process. A realization of the equation (5) is presented in figure 4 below.

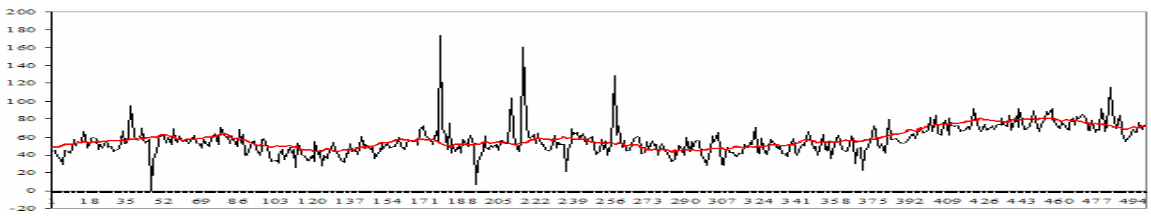

Figure 4, Realization of the above two factor model for electricity spot prices. Center line represents stochastic mean-reverting level modelled by arithmetic Brownian motion, another stochastic factor being asymmetric(skew right) Lévy motion.

Stochastic volatility models were presented in [Deng, 2000]], where a two-component price process is specified. The first component is for electricity spot price and the second for the price of some commodity needed in electricity production, e.g., fuel, and the third is the stochastic volatility process. It is essential to know the price processes of electricity and some production factor for the purpose of cross commodity risk management. In the case of Nordic electricity markets a cross commodity could be the water in the reservoirs, although it is not a traded commodity, that is, modelling its "price" level would have to rely e.g. on so-called water value models. However, there is clearly a strong dependence between the reservoir levels, weather conditions and electricity , and the electricity spot and futures prices. As a matter of fact, the electricity forward curve mostly reflects the market expectations on the future reservoir levels, since around 50% of all Nordic electricity is produced by hydro plants [Audet et. al., 2004]. There is also evidence [Deng, 2000] that the volatility of the spot price is low when the aggregate demand is low and vice versa, which gives one more plausible argument for using his model. The actual model proposed reads,

$$d\begin{pmatrix} X_t \\ V_t \\ Y_t \end{pmatrix} = \begin{pmatrix} k_1(t)(q_1(t) - X_t) \\ k_V(t)(q_V(t) - V_t) \\ k_2(t)(q_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sqrt{V_t} & 0 & 0 \\ r_1(t)s_2\sqrt{V_t} & \sqrt{1 - r_1^2(t)}s_2(t)\sqrt{V_t} & 0 \\ r_2(t)s_3\sqrt{V_t} & 0 & s_3(t) \end{pmatrix} dW_t + \sum_{i=1}^{2} \Delta Z_t^i \text{ , (6)}$$

, where X_t, V_t and Y_t are the spot price, volatility, and commodity price, respectively. All are mean-reverting to some deterministic level and dW_t's gain matrix models the correlation structure of these three processes. Jumps are modelled by ΔZ_t^i, that is, two kind of jumps, up and down. These can be, e.g., compound Poisson jumps, i.e., $\sum_{j=1}^{N(t)} Y_j$, where $N(t)$ is drawn from Poisson distribution and Y_j are independent random variables. [Deng, 2000] found that this model can capture some real world idiosyncracies (jumps) that are not included in standard models. The model-implied option price volatilities (short-maturity European out-of-the-money calls) are very much the same as observed in the market (which actually shows that the market anticipates jumps). Finally, a different type of approach was put forward in [Burger et. al., 2004]. The model they generated was, of the form

$$S_t = e^{f(t, L_t / v_t) + X_t + Y_t} \text{ ,} \qquad (7)$$

, which they call SmaPS, abbreviated from spot market price simulation. L_t is the load process, which can be divided into two components, i.e.., $L_t = \mathrm{l}_t + L_t^{'}$, where l_t is a deterministic load forecast and $L_t^{'}$ is a SARIMA time series model. v_t models the average relative ability of power plants and f tries to describe the nonlinear (logarithmic in their case) relation between price and load. X_t and Y_t are stochastic processes responsible for short- and long-term variation, respectively. The model was made for option price simulations and was found to perform pretty well, taking more real world phenomena in to account than the standard models.

It should be noticed that these models do not suffice for derivative (e.g., forward) pricing as such. For that purpose, they should be transformed into a risk-neutral framework. Some difficulties may arise since market price per unit risk for short-term electricity is in general challenging to estimate for both one- and multiple-factor models [Burger et. al., 2004] and it is common to assume that the risk factor (non-hedgeable short-term risk) is zero.

Fundamental spot price models approach the problem bottom-up by modeling the asking price or marginal cost of electricity and the demand side dynamics. The demand of electricity is often assumed to be inelastic. Fundamental models can be found, e.g., in [Skantze et al., 2000] and [Vehviläinen et al., 2004]. The model structure is always very market specific. E.g. in [Vehviläinen et al., 2004], the Nordic characteristics are described in detail.

2.4 Forward curve behaviour and modelling

The models are based either on the spot price model or a direct modelling of forward prices. In direct models, the approach is usually based on [Black, 1976]. Ideas from [Heath et al., 1992] are often included to model the dynamics of the forward curve in a complete market setting, see e.g. [Koekebakker, 2001] or [Clewlow et al., 1999]. Spot price –based models fit the model with historical spot market data. Since no arbitrage relation between spot and forward price exists, additional assumptions have to be made to use the model for derivatives pricing [Burger et al., 2004]. Usually this is done by making the rational expectations hypothesis,

$$F_{t,T} = E\left[S_T \mid F_t\right] \tag{8}$$

or by calibrating a market price of risk for each risk factor and changing to an equivalent martingale measure P*, under which the relation $F_{t,T} = \mathrm{E}^*\left[S_T \middle| F_t\right]$ holds. [Audet et. al., 2004] modelled Nordic electricity forward prices using parameterized volatility and correlation structure, that is,

$$df_{t,T} = f_{t,T} e^{-a(T-t)} \sigma(T) dW_T(t), \tag{9}$$

and $dW_{T'}(t) dW_T(t) = e^{-\rho|T-T'|} dt$.

Here ρ tries to capture the well -known effect that the forward prices whose maturity dates are close to each other are significantly correlated and α models the fact that forward price volatilities are lower than the corresponding spot volatilities. There are many restrictive assumptions in this model, some of those are made in order to ease the estimation of the model parameters. According to this model, the forward prices converge to spot prices, that is, rational expectations hold (under objective martingale measure) in continuous framework (delivery at a single point). In a real situation this does not have to be the case since delivery takes place during certain period, and there is also market's risk price added, hence the process is a martingale only under above-mentioned equivalent martingale (pricing) measure P* [Hull, 2000]. Also the spot volatility structure is assumed to be deterministic.

Forward-curve analysis in the literature has typically relied on daily or weekly data. Hourly models are practically non-existent. As in the case of spot prices, a multi-factor model is always more successful in explaining variation in the case of forward prices than a model with just one or two factors. Commodity prices have been successfully described by three - factor models. [Schwartz, 1997] used this type of approach to model oil futures prices. His model was:

$$\begin{aligned} dS_t &= (r_t - c_t) S_t dt + S_t \sigma_S dW_t \\ dc_t &= \kappa_c (\alpha_c - c_t) dt + \sigma_c dW_t \\ dr_t &= \kappa_r (\alpha_r - r_t) dt + \sigma_r dW_t \end{aligned} \tag{10}$$

, where W is a three-dimensional Brownian motion, and relations,

$$\sigma_S \sigma_c = \rho_1 \|\sigma_S\| \cdot \|\sigma_c\|$$
$$\sigma_c \sigma_r = \rho_2 \|\sigma_c\| \cdot \|\sigma_r\|$$
$$\sigma_S \sigma_r = \rho_3 \|\sigma_S\| \cdot \|\sigma_r\| \tag{11}$$

capture the correlation structure. One could try to describe electricity forward prices using this model, but as [Koekebakker et. al., 2001] found, even a ten -factor model is not sufficient to explain 95% or more of electricity forward price differences $f(t,T) - f(t-1,T)$ and in the case of returns $\left(f(t,T) - f(t-1,T)\right)/ f(t-1,T)$ ten factors explain only 93% of the variation. That evidence supported the earlier notion made by [Pilipovic, 1998] about electricity prices' schizophrenic behaviour. [Koekebakker et. al., 2001] used principal component analysis for forward price differences and returns, calculated from smoothed curves, that is, curves fitted to each historical forward term structure using the standard maximum smoothness method. This was done in order to estimate volatility parameters for their K factor models, namely,

$$df_{t,T} = \sum_{i=1}^{K} \sigma_i(t,T) dW_i(t) \tag{12}$$

$$df_{t,T} = f_{t,T} \sum_{j=1}^{K} \sigma_j(t,T) dW_j(t) \, . \tag{13}$$

Since the diffusion components in these models are standard Brownian motions, first of these implies normal distributiveness of forward prices and the other log-normal, respectively.

[Björk et. al., 2000] studied the situation where underlying asset is non-storable. Their approach to fitting was first to assume that forward prices satisfy,

$$F_{t,T} = H_F(t, Z_t, T) \, , \tag{14}$$

,where Z_t is a Markovian process and H_F is C^1 function, which can be obtained as a solution to,

$$\begin{cases} \frac{\partial H_F}{\partial t}(t,z,T) + \mathbf{A}^T H_F(t,z,T) = 0 \\ H_F(T,z,T) = h(T,t) \end{cases} \tag{15}$$

where $h(t, Z_t)$ is the spot price process and, $\mathbf{A}$ is a partial differential operator. The problem can be reduced to solving an ordinary differential equation by defining an affine term structure as,

$$\ln H_F(t,z,T) = A_F(t,T) + B_F(t,T)z\,, \tag{16}$$

where, A_F and B_F are deterministic functions.The main finding was that if the initial term structure $F(0,T)$ (observed) is smooth, that is, C^1, then there exist unique perturbation function which can be added to the standard one-factor model's mean-reverting component thus obtaining a spot-price model whose implied term structure exactly matches to the observed term structure. The assumption behind this is that the original spot price possesses a logarithmically affine term structure, see more [Lánden, 2000]. This is a very powerful result since the problem of fitting is generally very difficult.

To with regard to electricity forward curve this problem of finding the perturbation function requires that the initial term structure must be made smooth. This can be done with maximum smoothness criterion, see e.g. [Lim et. al., 2000]. After the term structure is smooth the perturbation function which can be added to the mean-reverting component reads,

$$\mathrm{j}\,(t) = \frac{d}{dt}\ln\left(\frac{F(0,T)}{F^0(0,T)}\right) + \mathrm{k}\,\ln\left(\frac{F(0,T)}{F^0(0,T)}\right) \tag{17}$$

, where k represents original model's mean-reversion speed-factor and $F^0(0,T)$ is original model's implied term structure respectively. This approach has same similarities to the manner in which Hull and White improved Vasicek's short rate model in order to invert the yield curve, see e.g. the classical paper [Hull et. al., 1990].

One creative method that combined old and new was discussed in [Fleten et. al., 2001]. They used current term structure information as a priori and improved this by using some forecast from a bottom-up model (used in regulated markets). Their actual optimization criterion was,

$$\min \quad w_1 \sum (B_i - x_i)^2 + w_2 \sum (x_{i-1} - 2x_{i1} + x_{i+1})^2 \tag{18}$$

, where the first part is a least-square term between the model and a bottom-up forecast, i.e., model fitted to bottom-up forecast whenever $w_1 = 1$ and the second term is a maximum smoothness criterion. Optimization was constrained by bid and ask prices of (discounted) current term structure, that is, some flexibility in fitting is allowed. Bottom-up information could be e.g. data from reservoir levels or in general from hydrological conditions. Bottom-up models themselves cannot capture the market's risk price; and so, it had to be combined with other methods. Finally [Fleten et. al., 2001] found that this model was able to fill the gaps in forward price curve but performed only a slightly better that previously used methods. Modelling the whole forward curve seems reasonable for some reasons, e.g., spot-price models implied forward curve show little or no seasonality, because of high mean-reversion speed factor, HJM framework's popularity in other markets, no problem fitting the model to current forward term structure and the approach gives implicitly the relationship between the spot price and the forward price. However, as [Koekebakker et. al., 2001] concluded, modelling the whole forward curve in electricity markets has much less merit than in other markets. Relation between spot and forward price is indeed more complicated than in most commodity and financial markets. [Fleten et. al., 2001] is, to our knowledge, the only study so far that combines some other methodologies and information to improve the forward or spot price model's implied forward curve, which is then fitted to current term structure using maximum smoothness criterion..

3 Forward-curve generation model

The basis for our model construction was that the model should not depend on a particular time resolution, i.e., that the forward curve could be constructed from at least hourly level to yearly level. Also, the forward products should be defined as simply as possible. The properties for a forward product are commodity, reference price, delivery period, and trading period. These header data are sufficient to calculate the forward curve on an arbitrary time resolution. The only restriction is that the time resolution must be such that the forward delivery periods are multiples of the time resolution base step.

The data for the study was received from Nord Pool ftp server [Nord Pool, 2005]. Data is on a daily-level with values for close, open, highest, and lowest prices. In this project, we decided to use only the close data but it could be interesting to compare the results using the other data as well.

3.1 Information from cascading of products and over-lapping delivery periods

Several Nord Pool products are traded with over-lapping delivery periods. Furthermore, long-term products are cascaded into shorter-term products before their delivery period. For example, yearly forward contracts are cascaded into four quarterly contracts that each in turn cascade into three monthly contracts. When the cascading happens, market information about the seasonality structure can be extracted. However, this information is difficult to compile into the overall model so no models have been proposed to take advantage of this information. For over-lapping contracts, the open interest indicating the liquidity of the product varies significantly. For example, yearly contracts tend to have a bigger open interest than the average open interest of the corresponding quarterly contracts.

Theoretically following the efficient markets hypothesis the cascaded forward prices include all relevant information about future. The information revealed is however only the average expected level of spot price over the delivery periods. There is no information about the shape of the spot price expect for at the very moment when the products are cascaded, assuming that the shorter period contracts are immediately priced efficiently. To avoid arbitrage, the price of the product to be cascaded should equal the average of the shorter-term products. This seems to be generally true – for 2004 the deviation of month product and four week products of same month on the last trading day of the month product were on average 0,3 NOK or in the order of 0,1-0,15 percent of the price of the products. We obtained same kind of results from comparison of monthly and quarter products. Thus, we concluded that the shortest-term products should be used to maximize the information about forward curve profile and longer-term product prices are not used for pricing. This is how we implemented the forward curve calculation algorithm. Although some products are less liquid, there is no significant inefficiency in prices.

Another idea that we considered to implement was to calculate statistically what are the average relative cascading level differences of e.g. yearly product into monthly product, i.e. how on average the price levels of quarter products relate to the price of yearly product at cascading date. This statistical information could be used to *pre-cascade* all long-term products to the shortest short-term maturity, i.e. monthly products in Nord Pool to obtain a forward curve on monthly level that could be further profilized to hourly level. The analysis could be also used regression modeling with some external market situation variable affecting the cascading levels.

3.2 Accounting for seasonality

There are different methods by which the ‘raw’ market forward curve can be converted to the desired time resolution. If the resolution is such that there is no longer-term contract than the resolution, e.g. yearly, the contracts with the same period as the time resolution can be used directly but the shorter-term contracts must be combined into longer ones. We propose doing this by simply taking the average price of the short-term contracts weighted with the delivery period length (equivalent to energy amount of one contract as one Nord Pool forward contract corresponds to 1 MW of base load).

If the chosen resolution is shorter than the longest period forward contracts, seasonality has to be taken into account to break the price of the long-term contracts to different time steps, following the chosen resolution. This could be done up to monthly level (or even weekly or daily level if there is sufficient liquidity, which seems often not to be the case) following the idea presented in Chapter 3.1. For shorter-term, some information about the spot price characteristics must be used. Some appropriate methods were discussed in Chapter 2.2.

3.3 Implemented forward curve calculation model

We have invented a method not used in this context before, although it takes some insipiraiton from existing models. Similar methods have been however applied for load forecasting, see e.g. [Smith, 2000]. We propose here a regression model with dependence on some external explanatory variables, such as reservoir levels or weather. The actual implementation was reduced to a modified time-of-day model due to lacking data of external variables. However, the model has the readiness to easily implement the regression model if such data is available.

The built algorithm works as follows:

1. Choose time of forward curve calculation and get the tradable forward product data of that date.

2. Calculate a standard spot price profile based on historical spot prices, following e.g. [Laukkanen, 2005].

3. Calibrate a regression model with some explanatory variables for the spot profile to account for deviation from the standard spot profile.

4. Using the values of the explanatory variables, calculate the real spot profile of that date

5. Fit the spot profile to actual forward prices simply by fitting the average of the profile over the forward delivery period to the forward price.

We made a simple base profile for spot price by taking a simple average and accounting for weekdays over the period 1992 – 2004. Another approach would have been to use only data from 1996 onwards, when Sweden and Finland joined the exchange. One option for building the base profile could be using (dynamic) regression models. A similar kind of approach has been taken in [Laukkanen, 2005] for forecasting special days' electricity load. Laukkanen reports also a test with an auto-regressive time series model for the modelling of the forecasting error component. This could also be used in here. Furthermore, we omitted the handling of special days. The effect of special days on electricity spot price would be probably relatively similar to that of electricity load as the two have significant positive correlation so in the next implementation they should be accounted for.

4 Results of test runs

Two different types of tests for validation of our model were conducted, namely ex-post forecast, i.e., the calculation of the sum of square errors between the forecast and the realized data, and few simple European style call option pricings with Monte Carlo simulations. Ex-post tests for the validation of our forward curve generation algorithm were conducted using two different models. First, a sinusoidal function was fitted to the historical spot data from 14.4.2003 to 14.4.2004 with least-squares method.

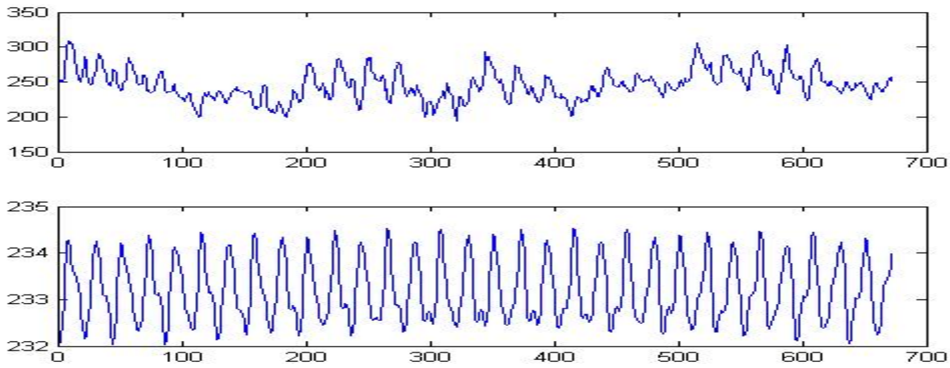

Figure 6, realised spot data and sinusoidal fit from 14.4.2004 to 14.5.2004.

Figure 6 shows realized hourly spot prices and sinusoidal function's predicted prices from 14.4.2004 to 14.5.2004. The actual fitted model was,

$$q(t) = \min_{a_j, b_j, c} \sum_{i=1}^{N} \left[x_i - c - \sum_{j=1}^{K} (a_j \cos(f_j t) + b_j \sin(f_j t)) \right]^2 \qquad (19)$$

, where x_i is realized hourly spot-data from 14.4.2003 to 14.4.2004, c is an additive constant, a_j, b_j are coefficients for sinus and cosinus terms and f_j are the K most characteristics frequencies to electricity spot-prices. In this case $K = 3$ and the characteristic frequencies describe the seasonal, weekly and daily patterns of electricity spot-prices. Figure 7 below shows the periodogram of hourly spot-prices for estimation period.

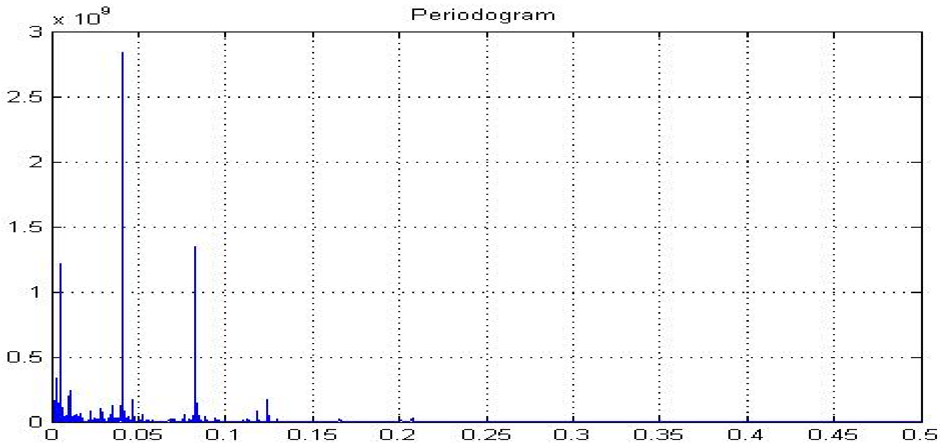

Figure 7, periogram of spot-prices from 14.4.2003 to 14.4.2004.

The actual spot-prices are supposed to realize randomly around this fitted sinusoidal curve. Secondly, our hourly forward curve profile was fitted to the term structure at 14.4.2004, using the weighted average method, meaning that when the profile price at time i is y_i and the term structure price is x_i, then the fitted profile price is

$$z_i = \frac{x_i}{(1/n)\sum y_i} y_i \quad . \tag{20}$$

This guarantees that the average prices of fitted profile and forward curve are the same in any given constant level of FC. Figure 8 below illustrates this straightforward fitting method.

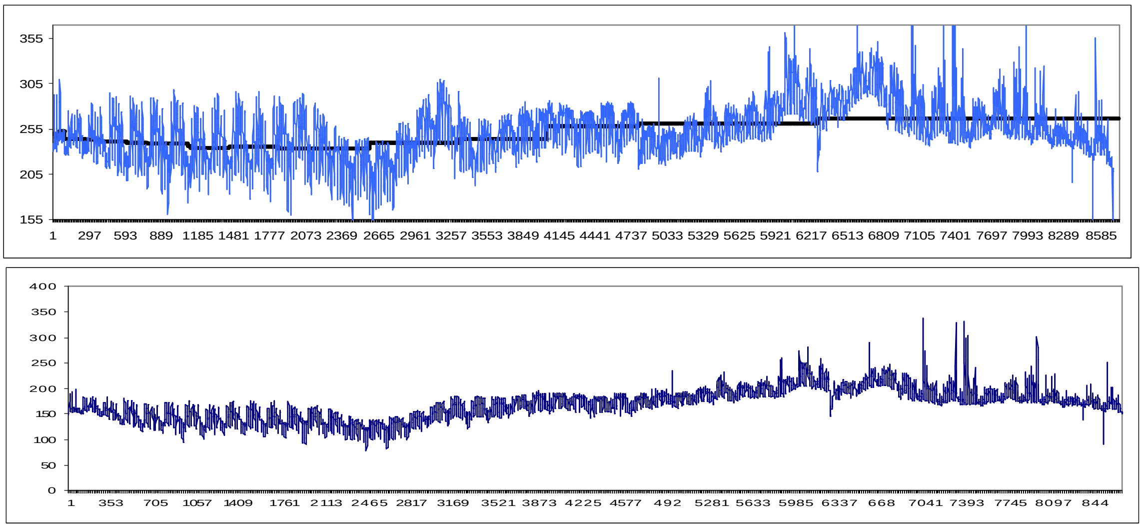

Figure 8, original forward profile and fitted forward profile.

Unfitted profile, which is obtained from averaging yearly spot data, is the lowest curve. Above it is the term structure and the fitted profile. The sum of the square errors between realized spot data from 14.4.2004 to 14.4.2005 was calculated for both methods, sinusoidal fit and profile fit. Table 2 below shows the results as a percentage shares of sum of the square errors between term structure at 14.4.2004 and realized spot prices between 14.4.2004 and 14.4.2005.

Model	Profile fit	Sinusoidal fit
Share	144.4%	135.7%

Table 2, shares of sum of the square errors between realized data and the models' forecasts'.

An option pricing test was conducted to few European style call options. The derivatives were priced at 18.4.2005. Their expiry dates were in mid-September 2005 and the underlying contract was the last seasonal product for the year 2005, that is, the next autumn's electricity. Our fitted profile was used as a mean-reverting component in a one-factor Hull-White model,

$$dS_t = \kappa(\alpha_t - S_t)dt + \sigma\, dW_t \tag{21}$$

,where the diffusion component was modelled as an $\alpha - stable$ motion, whose parameters were estimated from spot data for the same time period one year earlier. The moment matching technique was used whenever possible (since Lévy distribution's kurtosis and variance are formally infinite this is not always possible) and when not some heuristic was used to match the descriptive statistics of the simulated diffusion component and that obtained from the historical spot data. Figure 9 below shows the histogram of the diffusion component obtained from historical spot data.

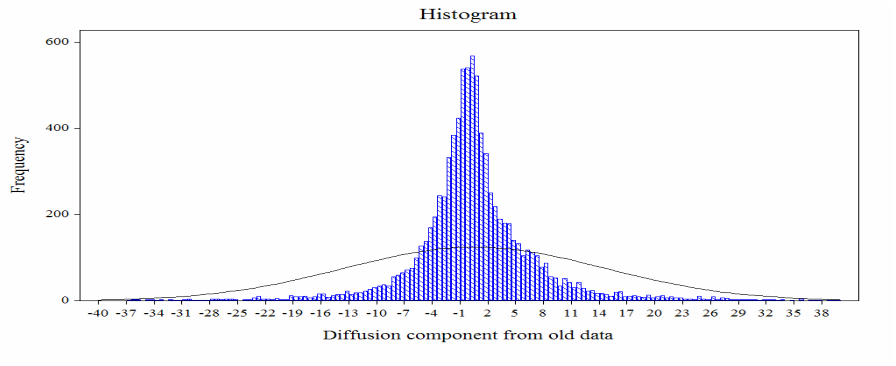

Figure 9, diffusion component from historical spot data.

This model was then used to price described option. Used pricing formula was,

$$C(S_T, K, t, T) = E^*\left[e^{-r(T-t)} \max(S_T - K, 0) \mid F_t\right] \tag{22}$$

where S_T is the spot price at T, t is the current time, K is the strike price, r is the risk-free rate (here assumed to be 4,5% annually), and F_t is the filtration representing the information available at the market. Call options were chosen since their prices can be evaluated analytically. Figure 10 shows the results.

Figure 10, European call-option prices, observed and evaluated.

Black-Scholes prices the options with low strike below the market price and options with higher strike price market price. Black-Scholes formula is designed to price the market risk caused by normal fluctuation, not the risk of randomly occurring price spikes. However because of the relatively high volatility Black-Scoles gives very satisfying evaluations.

For the pricing purposes the amount of simulations used were 10^4 rounds for each strike with variance reduction technique. Simulation model prices options constantly below market price albeit Lévy variates were used and mean-reverting level rises due to expiry. To see this consider figure 11, our fitted profile anticipates prices to rise right after the summer season.

There is a significant model risk involved in simulation model. All the prices implied by the model are very sensitive to parameter values, which are all estimated from historical data. This problem is a general one when estimating parameters from historical data, as a matter of fact simulation models are usually calibrated to price all the plain vanilla options correctly daily by using so called implied volatility function, and after the calibration simulation models are used to price exotics, see[Hull, 2000]. Nonetheless some quantitative tests for validation have been carried out. Figure 11 below shows the term structure on 18.4.2005(to mid-September) and the fitted profile for pricing purpose(to which jumps etc. can be added).

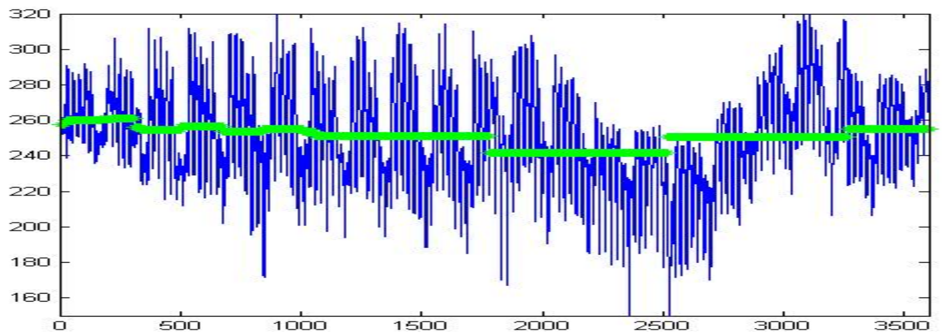

Figure 11, term structure at 18.4.2005 and fitted profile to be used as a mean-reverting component in stochastic pricing model.

Despite our model's simple homegrown nature it performs quite well in comparison to standard models in the sense of square error. To with regard to derivative pricing it shows the models sensitivity to parameters (especially changes in Lévy variates' tail parameters may cause hazardous scenarios).

This work gives a good starting point for developing this idea further as well as improving some other models using ideas presented in this document.

5 Conclusions

The electricity markets were introduced from a Nordic perspective and its peculiarities were discussed in the context of Nord Pool. We presented the modelling of spot and forward prices of electricity that has been performed in the literature using a variety of classical and more modern approaches. We presented the approaches found in literature for price process modelling as well as forward term structure modelling. Then, an algorithm for building an hourly forward curve was proposed. This algorithm was implemented in Excel. The implementation was tested against a number of other models using data from Nord Pool. The model seemed to perform ok despite its evident simplicity. However, the advantages of the model are its transparency as well as easy combination with other models, such as those based on time series or regression models. The lack of time resulted in somewhat over-simplified model. However, the development ideas are numerous so the work will be continued separately after the project to research the ideas invented during the project.

5.1 Future activities

In addition to testing the model, more sophisticated ways to combine the overlapping forward prices and to accommodate external factors should be explored. Overlapping of prices is an issue for week, month, quarterly, monthly, quarterly, seasonal, and yearly products. Trading and delivery are scheduled so that daily products (whose delivery always starts from 1 to 6 days from now) provide the only forward price for the corresponding day. The overlapping prices can be handled by averaging, as in the basic model, or by using the product with the shortest delivery period priced (e.g., use only the week product's price for next week and neglect the possibly available longer products that overlap next week) or a weighted average of the prices, where more weight is given to shorter products. Theoretically, the prices should be consistent. However, prices corresponding to shorter periods may include factors that affect only that period in particular, but whose effect fades applied to average over a longer period. Thus, the use of shorter periods seems justified. The accuracy of the model should be tested with different weighting schemes to find the best one.

External conditions could also be utilized in more detail to improve the accuracy of the model. In addition to the reservoir levels, we could use, for example, fuel prices, supply and demand curves, and production capacity available. The effects of these external state variables on the spot price profile could be estimated from historical data. Then, by first observing the current state of the external conditions, we could use a spot profile which corresponds more accurately to the current state of nature, and use this profile in making the forward curve. A simple or dynamical regression model could be used here, potentially combined with time series models. The method of long-term reversion from the initial situation to “normal” situation should be researched. This should be based on each external variables’ characteristics.

Analyzing in detail the cascading of products could be an enlightening exercise. Using that information for building a synthetic forward curve by *pre-cascading* longer products based on statistical data should be evaluated.

The most significant “out of scope” idea that came up in the project was to use the forward curve, which is generated based solely on forward prices (i.e.., future expectations), as a basis for the deterministic parameters in stochastic mean-reverting models. In this approach, the expectation of the process is taken as the best guess available in the market, instead of a historical mean. This model, too, should be tested ex post to analyze its accuracy and applicability.

The short comings of Excel in time series manipulation became evident. In future development, the programming language should be Matlab or some lower –level language, such as C, C++ or Java.

6 Project summary

The project resulted in an innovative, simple, and feasible model for forward curve construction, as well as ideas for using this model to calibrate existing spot-price models. The forward curve model was implemented for testing in an Excel-VBA macro. This macro (algorithm) calculates the yearly spot-profile from historical data and creates the hourly forward curve by normalizing that section of the profile which corresponds to the delivery period of a forward product with the respective forward price. Furthermore, the project delivered this final report which includes a description of the Nordic power market, a thorough literature survey on both spot price and forward price models, as well as a documentation of the built model.

However, the initial project objectives – which were set rather ambitiously – were not all met completely. Specifically, the model has not yet been tested and validated thoroughly, and the Excel-implementation is rather defective, both in terms of computational performance and design in general. Furthermore, the handling of over-lapping delivery periods and periods with no forward price information, handling of different reference prices, and accommodation of external variables (e.g.., the weather) were considered only on theoretical level rather superficially and were not implemented into the algorithm due to missing data.

These slight shortcomings in project results followed from the team members insufficient knowledge and experience in Excel-VBA programming. Furthermore, it seems that Excel is not the most applicable platform for running the model in the first place. These factors were initially identified as the main risks in the project, wherefore they were anticipated to some extent. Despite of some preventive actions, these risks did come in effect partially.

The project work was clean so that all important dead lines were met, with a cut down scope in some respects, however. In addition to the physical deliverables, the project team members gained knowledge about electricity markets and financial products. The team worked well together, all members were knowledgeable and enthusiastic about the subject, and the group gatherings resulted in fresh and visionary ideas. The team is interested in doing further research with the development, validation, testing, and use of the model after the end of the actual project. In these extensions, however, the programming platform should be changed to Matlab, for example.

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