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Mat-2.177 Project Seminar in Operations Research

Grid Computing in Home Environment

Interim report

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1 Introduction

This paper discusses the state of the project for course Mat-2.177 Project Seminar in Operations Research, ordered and supported by Nokia Research Center. The intention is to give a realistic view of the work done so far, shifts in project focus, a brief roll-out plan and updated review on project's risks.

2 Present state of the project

Starting point for the project was more challenging than the team first conjectured. Project team did not receive any concrete and strictly delimited research problem from its client NRC. Instead, NRC provided us a more general topic of interest. The practical approach of how to go about the topic and to formulate a project problem out of it was left to the project group. Possible solutions to the problem and relevant questions shaped during the meetings with Nokia's contact person and professor Salo as well as during literature study. This process took more time than the project team initially assessed and also caused some changes to the project plan.

Initially it was intended to decide upon appropriate cases, from which a mathematical model could be derived to find out if it would be lucrative to decompose the selected computational problems. Restructuring the problem led to the analysis of defining the limits to specify when it would be reasonable to consider adapting grid computation. Appropriate case problems will be selected after this task is completed. Thus, project's focus has shifted form the deep analyses of particular cases to the formulation of parametrized mathematical model.

Another change, although a minor one, was project team's decision to allocate more resources to writing the final report.

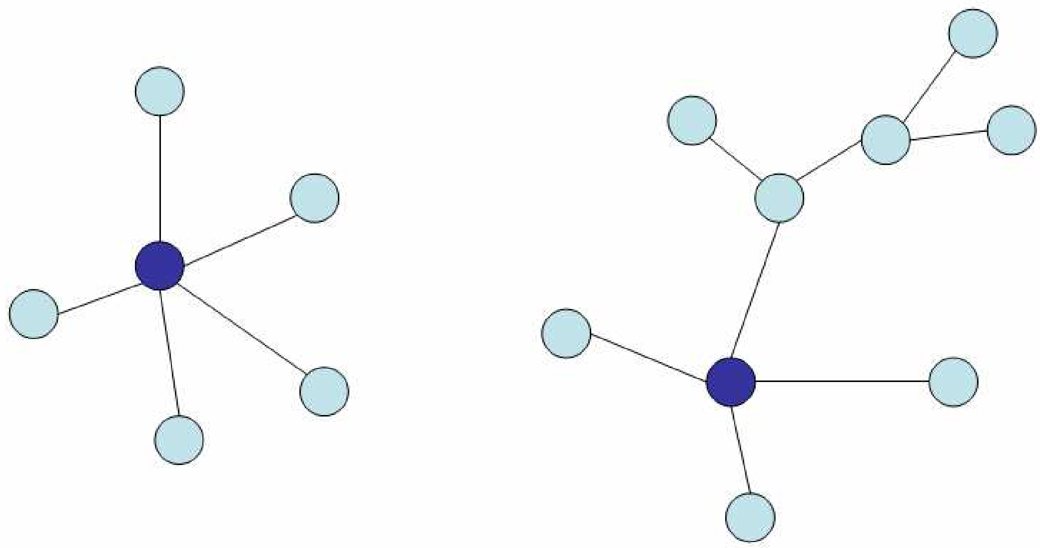
The project group can, after defining the actual project focus and scope, state that the project is on schedule. Literature study over the subject matter is completed and the relevant issues on the Nokia's topic of interest have been understood. Data concerning required model parameters is gathered. This includes the technical information on the different computing devices and possible communication links between these devices.

Project team has rescheduled the case selection task to start at week 13 when the mathematical model should be ready. Correspondingly, the structuring of mathematical model has been partly completed. Results related to the model are described in more detail in appendix A. Project group wishes that our opponents could have a constructive and critical look over our model.

3 How to continue

The project group must thoroughly examine if the formulated mathematical model has an analytical solution. If the analytical solution exists the connection architecture of the computing devices can be expanded to include star topology with branching nodes i.e. a tree topology.

If the analytical solution does not exist, we have to abide by the current version of the model and settle with the simple star topology.



Stage 1. Simple star topology (current situation)

Stage 2. Generalized tree topology (used if the analytical solution can be formed)

It is possible to evaluate the model with appropriate case problems either analytically or numerically and this is done to the extent of a reasonable workload. One case that has already partly been tested is the conversion of video signal. Writing the final report is also to be started soon.

4 Review on the project's risks

The major problem has been finding the balance between the functionality and outright usefulness of the mathematical model constructed. The model should be adequate description of the domain of grid computing as well as a handy tool for analyzing decompositions of selected grid computing problems with the different arrays of computing units and in arbitrary star architectures.

Appendix A – Mathematical model

This section deals with the question "what is an optimal solution strategy in a grid network for a perfectly decomposable problem". More formally, solution strategy means finding an allocation of a problem among devices that minimizes the time *T* in which a problem *P* can be solved in a grid network. The topology assumed is a star topology (*Figure 1)*. Later on we hope to expand this to a tree topology.

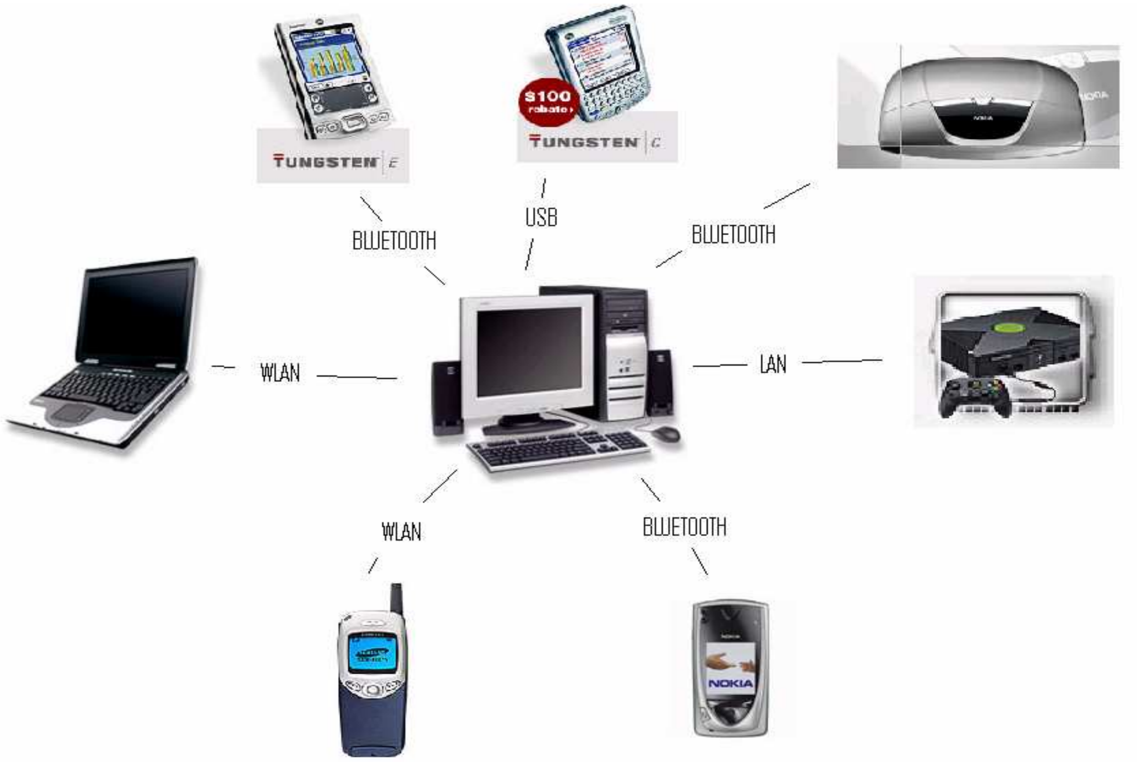


Figure 1. Devices found in homes connected in a star topology.

The Grid Environment

In this part we assume only a star topology for the grid network, like the one seen in *Figure 1*. The network consists of *n+1* computing units, where the central device corresponds to index zero and the surrounding devices correspond to indices 1,..,*n*. Each unit in the grid has a computing power c_i. Between the central device and each surrounding device there exists a symmetric communication link with bandwidth s_i and latency l_i.

About the Problem

In this environment we have a problem *P*, which we can solve on the host computer of the problem (assumed to be the central unit) or in a distributed fashion in the grid network of devices. The problem has a complexity *Y*, a description, which has a size *X* (later refered to as size or description merely), and an answer, which has a size *R* (later refered to as answer merely). We assume that the complexity can be divided into wanton independent parts, i.e.

$$Y = \sum_{i=0}^{n} y_i \tag{1}$$

and we call this kind of problem perfectly decomposable. Each sub problem p_i with the complexity y_i has a description x_i and an answer r_i.

Objective and Costs

The objective is to find the minimum time taken to solve the described problem by distributing the problem to different devices. The costs incurred when solving a problem in a distributed fashion consists of decomposition costs, solving costs and composition costs. The following will discuss the costs incurring when doing this.

Decomposition and Composition Costs

Before a problem can be solved in a distributed fashion the problem must be decomposed into independent solvable parts of positive sizes $(x_0,\ldots,x_n)$. $T_{Decompose}$ is the cost incurred or time taken to perform the decomposition. After decomposing the problem, it can be solved in the grid, which takes a time T_{Solve}. This cost will be discussed in more detail in the next chapter. As all independent parts of the problem have been solved, all independent answers must be composed to the complete answer, which takes time $T_{Compose}$. Thus, the time taken to solve a problem is taken to be the sum of the decomposition cost, the solving cost and the composition cost.

Solving Costs

The time taken to complete the i^{th} sub problem takes time t_i. The time t_i has a different composition or structure for the host device and for the surrounding devices. Before looking at the individual durations of completing individual sub problems we define the time taken to send a description over a communication link, the time taken to compute an individual sub problem, and the time taken to send an individual answer problem over a communication link.

The time taken to send an individual description over a communication link is defined as

$$t_{send,i} = \frac{x_i}{s_i} + l_i \tag{2}$$

and similarly the taken to send an individual answer over a communication link is defined as

$$t_{receive,i} = \frac{r_i}{s_i} + l_i \tag{3}$$

As the latencies are negligible in comparison to the transmission times, in the problems we wish to solve, we will drop latencies l_i from the analysis from here forth. Finally, the time taken to compute an individual sub problem is defined as

$$t_{compute,i} = \frac{y_i}{c_i} \tag{4}$$

Time taken to solve sub problem p_0 on the host computer

The host device may start to solve the sub problem p_0 immediately after the decomposition of the problem is done. The time t_0 consists of the computing time on the host device. Still, the sending and receiving of data by the host computer results in the CPU time being divided between computing the sub problem p_0 and sending and receiving other sub problem descriptions respectively answers. We assume each communication link will require a percentage of the total computing power denoted by α_i. Thus, the time taken to solve p_0 on the host is

$$t_0 = \frac{y_0}{c_0} + \frac{1}{c_0}\sum_{i=1}^{n} \alpha_i \frac{(x_i + r_i)}{s_i} \tag{5}$$

Time taken to solve sub problems p_i

We assume the completion of sub problem p_i can be divided into three discrete phases, namely, sending of the sub problem over the communication link, computing of the problem p_i, and sending of the corresponding answer over the communication link. Thus, the time taken to solve the sub problem p_i is

$$t_i = \frac{y_i}{c_i} + \frac{x_i + r_i}{s_i} \qquad i = 1,\ldots,n \qquad (6)$$

Total solving cost

As we assume the composition of the final answer can not begin before all answers have been received by the host, the total solving cost of the problem is

$$T_{Solve} = \max_{i \in \{0,\ldots,n\}} (t_i) \qquad (7)$$

Simplifications

In order to derive an analytical solution we need to make a few simplifications. First we assume each description to be such that

$$X = \sum_{i=0}^{n} x_i \qquad (8)$$

Additionally we assume there exists a bijection f that maps each x_i onto y_i. Based on the above the function f must be linear. More specifically we assume the dependency to be

$$y_i = kx_i \qquad (9)$$

where k is the complexity coefficient or factor, which is problem or algorithm dependent. Each sub problem has an answer, which has a size r_i, such that

$$R = \sum_{i=0}^{n} r_i \qquad (10)$$

and

$$r_i = \beta x_i \qquad (11)$$

Substituting (9) and (11) into (5) and (6) yields

$$t_0 = \frac{kx_0}{c_0} + \frac{1}{c_0}\sum_{i=1}^{n} \alpha_i \frac{(1+\beta)}{s_i} x_i \tag{12}$$

$$t_i = \frac{kx_i}{c_i} + \frac{(1+\beta)}{s_i} x_i \qquad i = 1,\ldots,n \tag{13}$$

Furthermore we assume the decomposition and composition costs to be dependent only on the total size X of the problem. Hence, if we choose to solve the problem in a distributed fashion the decomposition and composition costs may be omitted when deriving the optimal allocation, as they do not impact it. On the other hand the solution derived in this manner must be compared to the time taken to solve the problem only on the host device, before optimality can be justified.

The optimization

As the total time computing and data transmit time for device i is t_i, the total amount of time needed to compute the whole problem is the maximum of the individual device times, as mentioned. The task is to minimize the total time needed for the problem. Thus, the problem can be written in the form

$$\min \max\{t_0, t_1, ..., t_n\} \tag{14}$$

If t_0 is of the form[1]

$$t_0(x_0, x_1, x_2, \ldots, x_n) = a_{00}x_0 + a_{01}x_1 + \ldots + a_{0n}x_n + b_0 \tag{15}$$

and the computing times of other devices $t_1, \ldots, t_n$ [2]

$$t_i(x_i) = a_i x_i + b_i \tag{16}$$

The problem is therefore

$$\min \max\{t_0(x_0, x_1, x_2, \ldots, x_n), t_1(x_1), t_2(x_2), \ldots, t_n(x_n)\} \tag{17}$$

$$\text{s.t.} \qquad \sum_{i=0}^{n} x_i = X$$

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[1] Equation (12) is of this form when $b_0 = 0$
[2] Equation (12) is of this form when $b_i = 0$

$$x_i \geq 0 \qquad i = 0,\ldots,n$$

Writing

$$z = \max_i \{t_i\} \tag{18}$$

the problem can be formulated as

$$\min z \tag{19}$$

$$\text{s.t.} \quad a_{00}x_0 + a_{01}x_1 + \ldots + a_{0n}x_n + b_0 \leq z$$

$$a_i x_i + b_i \leq z \qquad i = 1,\ldots,n$$

$$\sum_{i=0}^{n} x_i = X$$

$$x_i \geq 0 \qquad i = 0,\ldots,n$$

The above problem is to be solved with the Karush-Kuhn-Tucker optimality conditions and an analytical solution may be found.