Aalto University
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Optimization Approaches for Line Planning in Linear Railway Systems

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Master's thesis presentation

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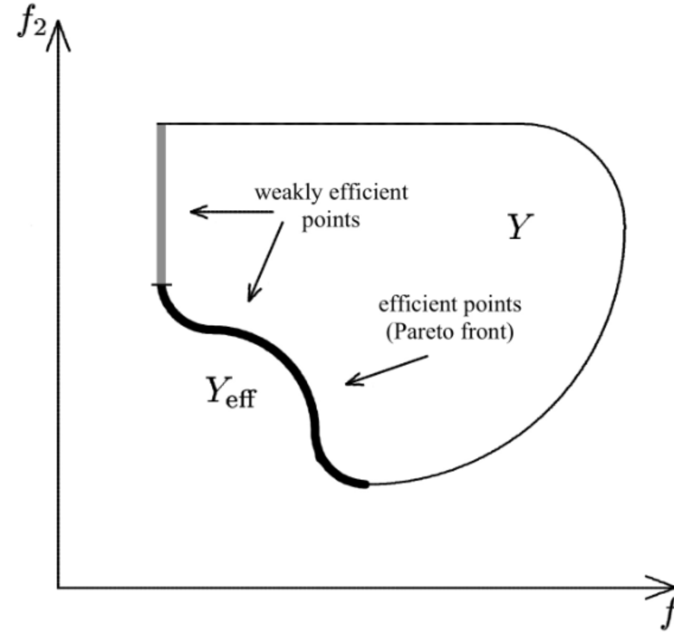
- Models
- Results
- Conclusions

Introduction

- Line planning [1]
  - Part of public transportation planning [2]
  - Selecting lines and setting frequencies
- Two perspectives
  - Passenger: transfers [3], traveling time
  - Operator: costs
- This thesis:
  - Three passenger convenience metrics models
  - Linear PTN: modeling aspects



Pareto front in biobjective setting, adapted from [4]

Base model

- Objective: minimize passengers' inconvenience function *P(y)*
- Common constraints
  - Fleet size (operator's interest) (1)
  - Ensure capacity to all passengers (2)
- $\mathcal{E}$-constraint method [4]
- Optional terminal constraints:

$$\sum_{t \in V^T} z_t \leq z_{\max}$$

$$x_l \leq M z_t \quad \text{for all } l \in L,\ t \in V_l^T$$

$$\min_{x,\, y} \quad P(y)$$

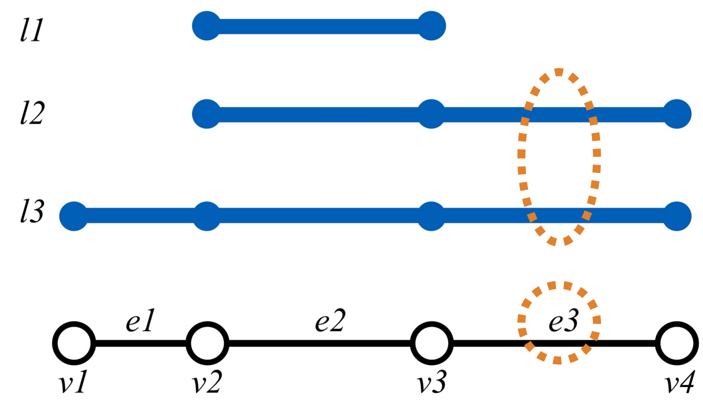
$$\text{s.t.} \qquad \sum_{l \in L} x_l \leq \mathcal{F} \qquad (1)$$

$$\sum_{l \in L : e \in l} C \underbrace{\frac{\tau}{T_l} x_l}_{\textit{Frequency of line } l} \geq w_e \text{ for all } e \in E \qquad (2)$$

$$x_l \in \mathbb{N}_0 \text{ for all } l \in L.$$

Base model in passengers' perspective

Congestion model



- Idea: Minimize congestion on all edges
  - Linearize objective: Maximize minimum edge availability factor λ
  - Require that

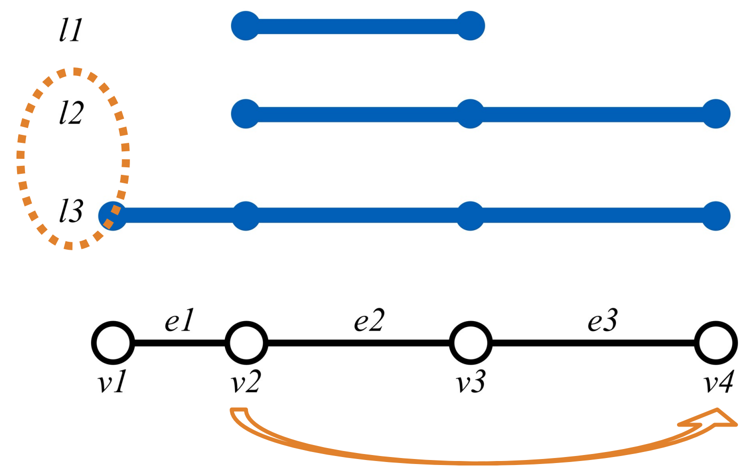
    $$\lambda \leq \lambda_e \text{ for all } e \in E$$
- Linear PTN:
  - Simple path between all stations
    - *No routing*

$$\lambda_e = \frac{\text{available seats on edge } e}{\text{passenger load on edge } e}$$

$$\lambda_e = \frac{1}{w_e} \sum_{l \in L : e \in l} C \underbrace{\frac{\tau}{T_l}}_{\text{Frequency of line } l} x_l$$

Direct passengers model

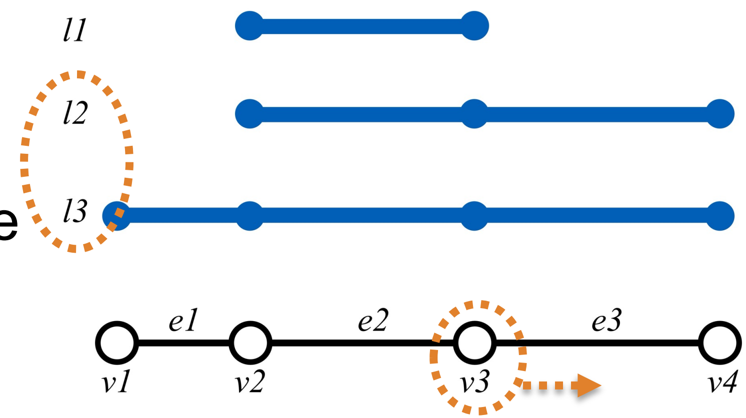
- Idea: Maximize the number of direct passengers
- Constraints:
  - No more direct passengers than demand between u and v
  - No more direct passengers than capacity on edge e
- Again: Linear PTN
  - No routing



$$\max_{x,\,d} \sum_{u,v\in V} \sum_{l\in L^{uv}} d_{uvl}$$

Direct passengers traveling from u to v using line l

Initial waiting time model



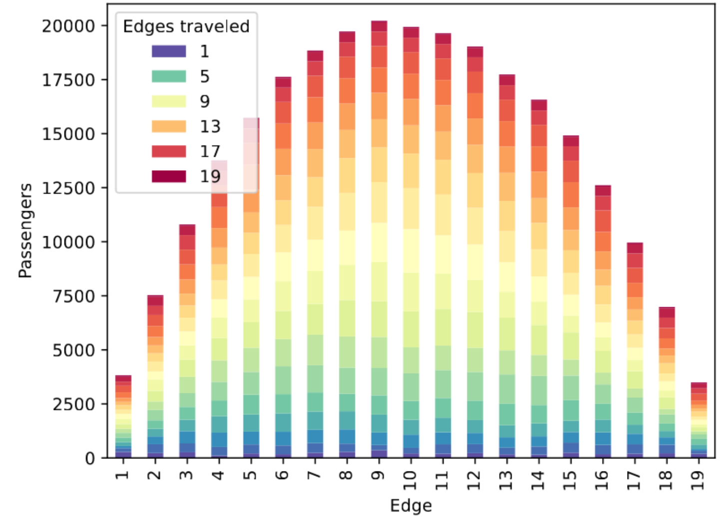
- Idea: Minimize the average waiting time for the first train (weighted by demand)
- Assume:
  - Passengers board the first train
  - Passengers arrive uniformly distributed
  - Trains arrive uniformly distributed
- Objective function linearized with binary variable F_k:

$$\left(\sum_{l} f_l\right)^{-1} = \sum_{k} \frac{1}{k} F_k$$

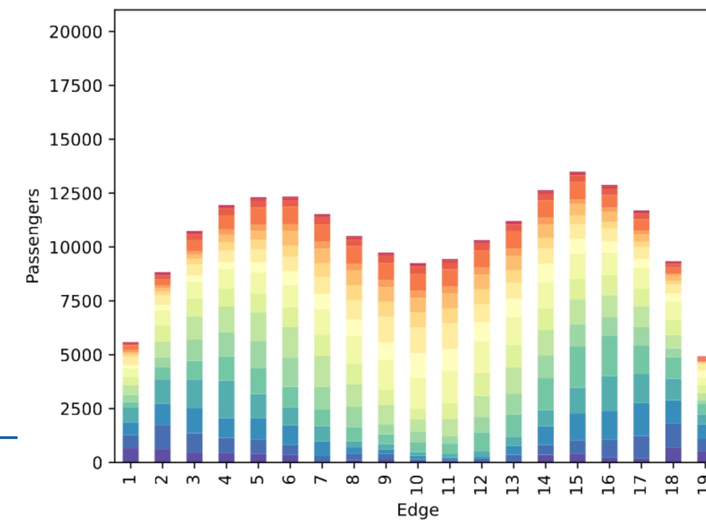
$$\sum_{u \in V}\left(\sum_{v \in V} OD_{uv} \frac{1}{2} \tau \underbrace{\left(\sum_{l \in L: e_{uv} \in l} f_l\right)^{-1}}_{\text{Average headway between trains at station } u}\right)$$

Data

- Generated data (PTN and demand)
- 20-station PTN
  - Two 8-terminal stations configurations
- Two demand profiles
  - Unicentric
  - Bicentric
- Demand data represented in two ways
  - Origin-destination-matrix (OD)
  - Edge loads (w_e)



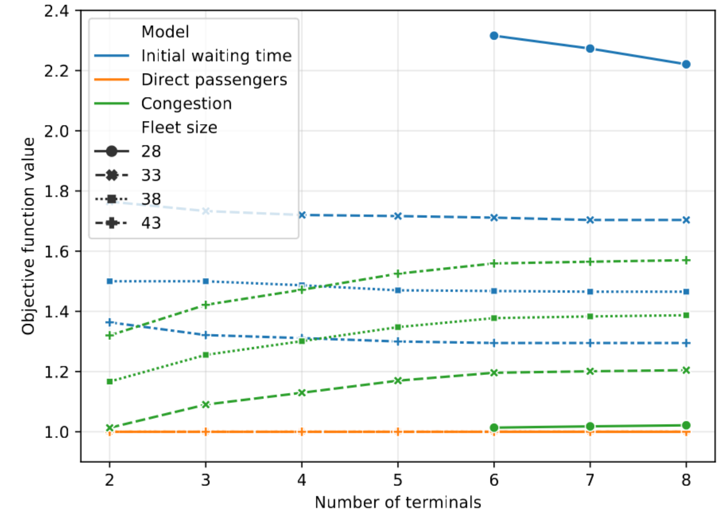
Unicentric demand profile



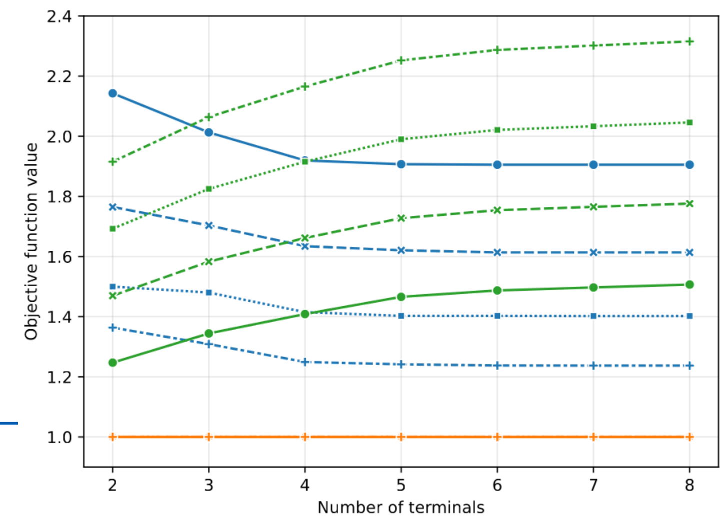
Bicentric demand profile

Results: Terminal stations

- Passengers' convenience improves when
  - More terminals are available
  - More trains are available
- ~100% of passengers can travel directly
  - If dictated by the objective function
  - If feasible
  - Regardless of fleet size & terminals



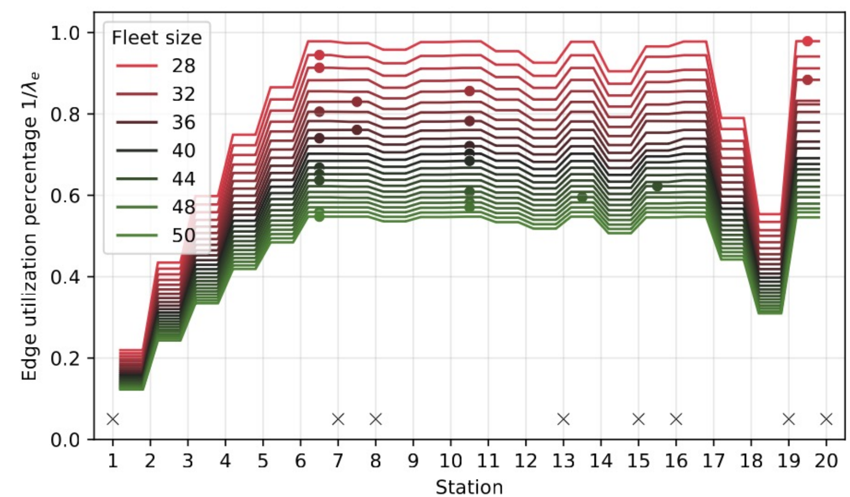
Unicentric demand profile



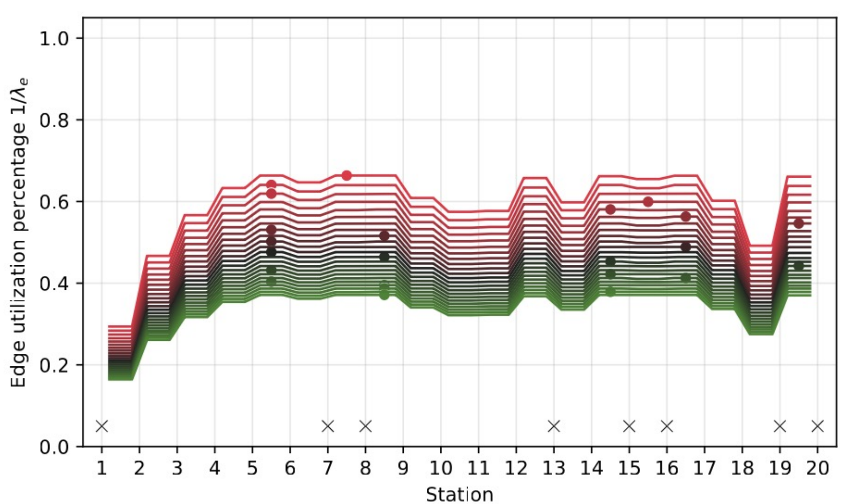
Bicentric demand profile

Results: Fleet size in congestion model

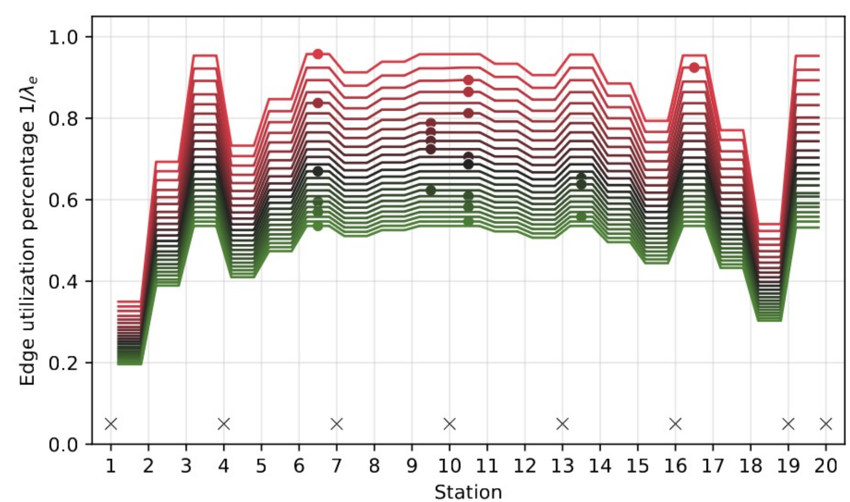
- Optimal shape in edge utilization, independent of fleet size
- Congestion peaks adjacent to important terminals
- Active edge changes



Unicentric demand profile



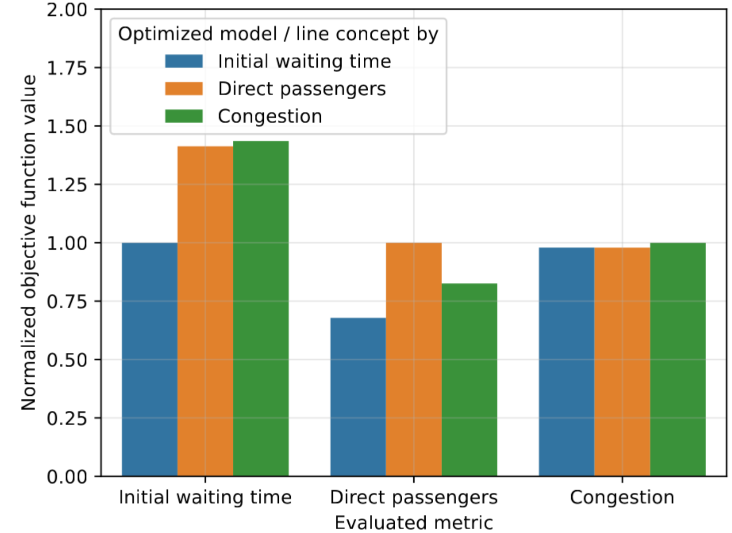
Bicentric demand profile



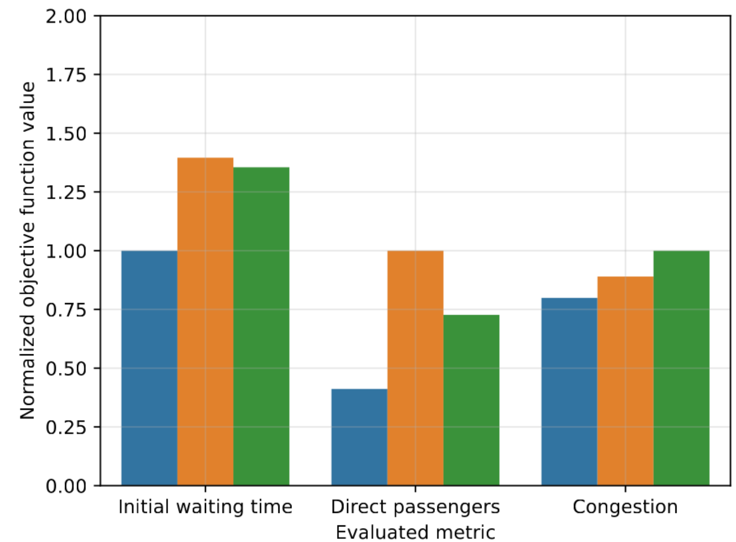
Unicentric demand profile, even terminals

Results: Intercorrelations

- One model optimized, other models evaluated with the line concept
  - Optimized metric scaled to 1
- Evaluated metrics much worse when not considered in the objective function
  - Initial waiting time more so
- Direct passengers far from 100% when other models are optimized
  - Why: next slide



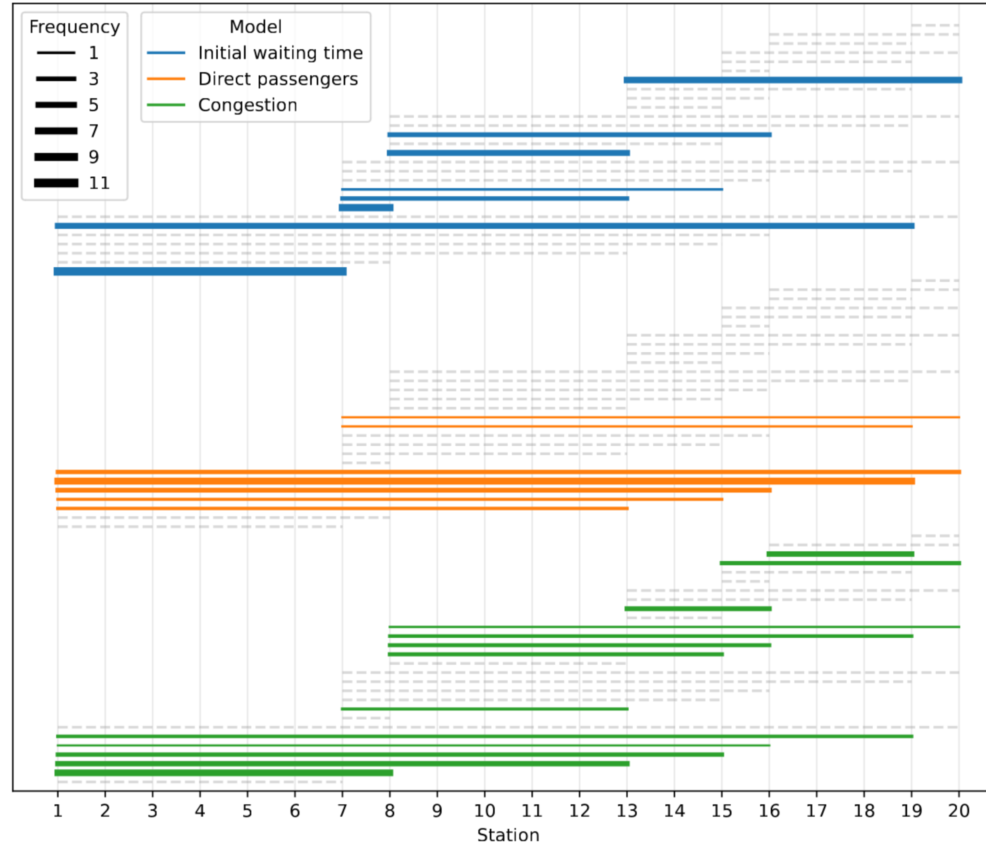
Fleet size 28



Fleet size 38

Results: Line concepts

- Line concepts very different between models
- Initial waiting time model
  - Only departing trains considered
- Direct passengers model
  - Long lines
- Congestion model
  - Most unique lines
    - *Optimal edge utilization shape*



Unicentric demand
Fleet size 28

Conclusions

- The three models yield quite different line concepts
- Linear PTN simplifies modeling
- Limitations
  - Generated data
  - Elementary models for applications
    - *For instance, high number of unique lines*
  - Conflicting passenger objectives

References

[1] Schöbel, A. (2012). Line planning in public transportation: models and methods. *OR spectrum*, 34(3):491–510.

[2] Desaulniers, G. and Hickman, M. D. (2007). Public transit. *Handbooks in operations research and management science*, 14:69–127.

[3] Bussieck, M. R., Kreuzer, P., and Zimmermann, U. T. (1997). Optimal lines for railway systems. *European Journal of Operational Research*, 96(1):54–63.

[4] Ehrgott, M. (1999). Multicriteria optimization. *Lecture Notes*.

Appendix: Congestion model

$$
\begin{aligned}
\max_{x,\lambda} \quad & \lambda \\
\text{s.t.} \quad & \sum_{l \in L} x_l \leq \mathcal{F} \\
& \sum_{l \in L : e \in l} C \frac{\tau}{T_l} x_l \geq \lambda w_e && \text{for all } e \in E \\
& x_l \in \mathbb{N}_0 && \text{for all } l \in L \\
& \lambda \in [1, \infty).
\end{aligned}
$$

Appendix: Direct passengers model

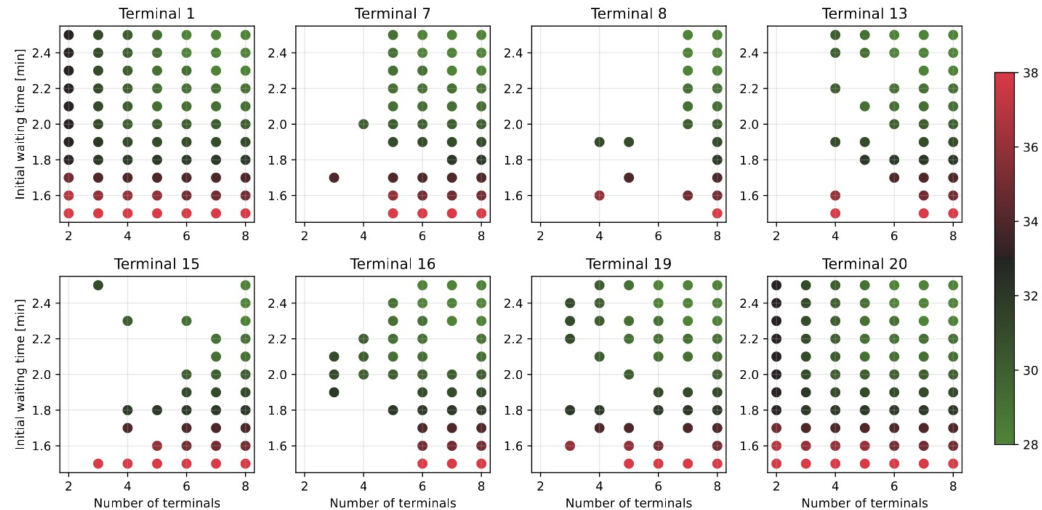
$$
\begin{aligned}
\max_{x,\,d} \quad & \sum_{u,v \in V} \sum_{l \in L^{uv}} d_{uvl} \\
\text{s.t.} \quad & \sum_{l} x_l \leq \mathcal{F} \\
& \sum_{l \in L^{uv}} d_{uvl} \leq OD_{uv} && \text{for all } u, v \in V \\
& \sum_{l \in L : e \in l} C \frac{\tau}{T_l} x_l \geq w_e && \text{for all } e \in E \\
& \sum_{\substack{u,v \in V : l \in L^{uv}, \\ e \in l^{uv}, u > v}} d_{uvl} \leq C \frac{\tau}{T_l} x_l && \text{for all } e \in E, l \in L \text{ with } e \in l \\
& \sum_{\substack{u,v \in V : l \in L^{uv}, \\ e \in l^{uv}, u < v}} d_{uvl} \leq C \frac{\tau}{T_l} x_l && \text{for all } e \in E, l \in L \text{ with } e \in l \\
& x_l \in \mathbb{N}_0 && \text{for all } l \in L \\
& d_{uvl} \in \mathbb{N}_0 && \text{for all } u, v \in V, l \in L.
\end{aligned}
$$

Appendix: Initial waiting time model

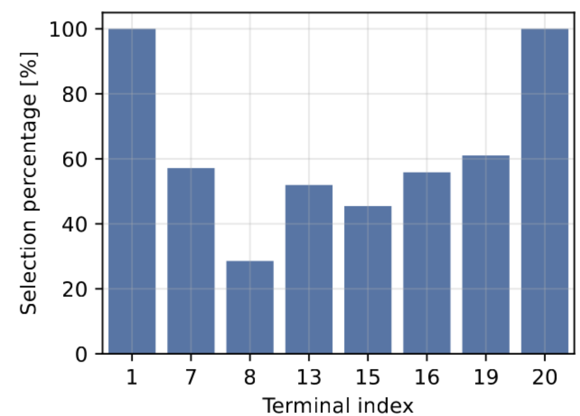
$$
\begin{aligned}
\min_{x, F, f} \quad & \sum_{u \in V} \left(\sum_{v \in V} OD_{uv} \frac{1}{2} \tau \left(\sum_{k}^{F_{\max}^{euv}} \frac{1}{k} F_{e_{uv} k} \right) \right) && \\
\text{s.t.} \quad & \sum_{l} x_l \leq \mathcal{F} && \\
& \sum_{l \in L : e \in l} C \frac{\tau}{T_l} x_l \geq w_e && \text{for all } e \in E \\
& \sum_{k}^{F_{\max}^{e}} k F_{ek} = \sum_{l \in L : e \in l} f_l && \text{for all } e \in E \\
& \sum_{k}^{F_{\max}^{e}} F_{ek} = 1 && \text{for all } e \in E \\
& f_l \leq \frac{\tau}{T_l} x_l && \text{for all } l \in L \\
& x_l \in \mathbb{N}_0 && \text{for all } l \in L \\
& f_l \in \{0, \ldots, f_{\max}\} && \text{for all } l \in L \\
& F_{ek} \in \{0, 1\} && \text{for all } e \in E, k \in \{1, \ldots, F_{\max}^{e}\}.
\end{aligned}
$$

Appendix:
Terminal importance

- Some terminals more important than others
- No regularity in selecting individual terminals
- Terminals closer to PTN ends are more important
  - Unicentric
  - Lines halting in the middle not as useful



Unicentric demand profile



Unicentric demand profile