Background and Research Overview

By
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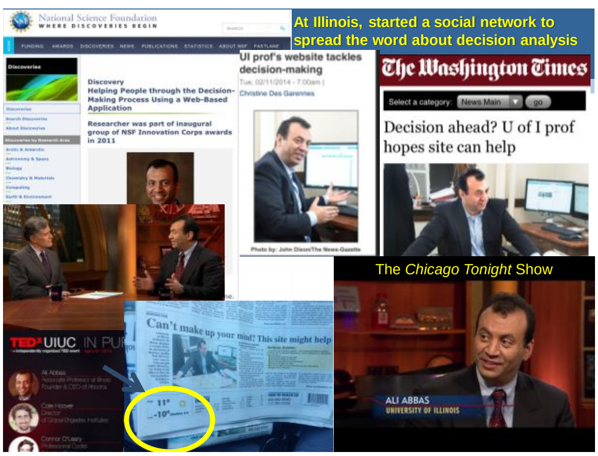
Industrial and Systems Engineering
and Professor of Public Policy

Viterbi School of Engineering
Price School of Public Policy
University of Southern California

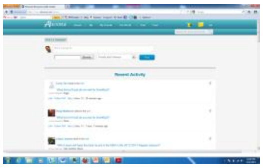
Graduate School at Stanford

At Stanford I received an
MS in Electrical Engineering
MS in Engineering Economic Systems and Operations Research
PhD in Management Science &Engineering
PhD Minor in Electrical Engineering

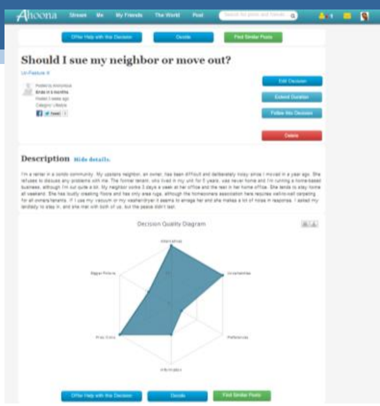
At Illinois, started a social network to spread the word about decision analysis



Building a Decision-Making Social Network



National Science Foundation
Inaugural I-Corps Program
www.ahoona.com



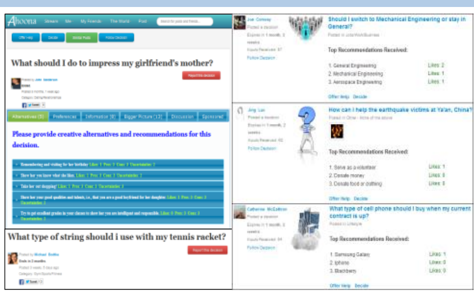
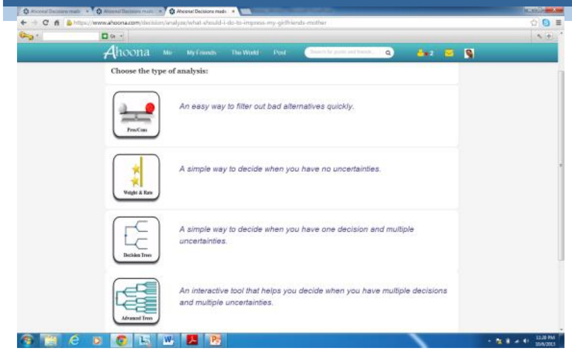
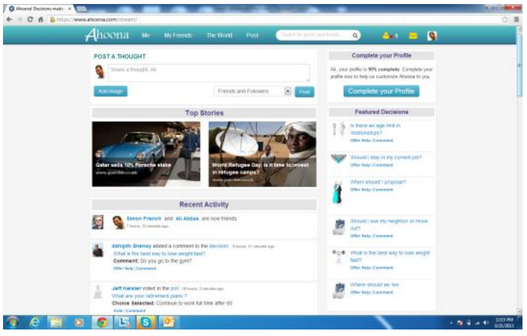


Figure 3. Decisions on Ahoona span a range of topics including social improvement, sporting, education, purchases and philanthropy. The detailed view is on the top left; the quick view is on the right.

Analysis on Ahoona



The AHOONA Stream



Tag Cloud –Objectives of New Graduates (Ages -20-24) Pursuing a New Job

TSAPre Decision Analysis

A Value Measure for Public-Sector Enterprise Risk Management: A TSA Case Study

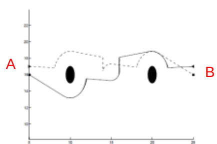
Kenneth C. Fletcher[1,*] and Ali E. Abbas[2]

This article presents a public value measure that can be used to aid executives in the public sector to better assess policy decisions and maximize value to the American people. Using Transportation Security Administration (TSA) programs as an example, we first identify the basic components of public value. We then propose a public value account to quantify the outcomes of various risk scenarios, and we determine the certain equivalent of several important TSA programs. We illustrate how this proposed measure can quantify the effects of two main challenges that government organizations face when conducting enterprise risk management: (1) short-term versus long-term incentives and (2) avoiding potential negative consequences even if they occur with low probability. Finally, we illustrate how this measure enables the use of various tools from decision analysis to be applied in government settings, such as stochastic dominance arguments and certain equivalent calculations. Regarding the TSA case study, our analysis demonstrates the value of continued expansion of the TSA trusted traveler initiative and increasing the background vetting for passengers who are afforded expedited security screening.

National Security Decisions

Tracking and Collision Avoidance

$$\dot{x}_i = g_i(x_i)u_i + h_i(x_i),\ x_i(0) = x_{io},\ \forall t \in [0,\infty),\ \forall i \in \mathbf{N}.$$



Two attributes:

Tracking

Collision Avoidance

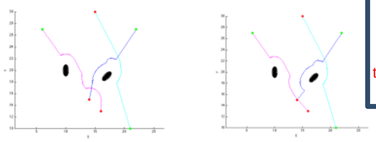
Tracking and Collision Avoidance

$$\dot{x}_i = g_i(x_i)u_i + h_i(x_i), x_i(0) = x_{io}, \forall t \in [0,\infty), \forall i \in \mathbf{N}.$$

Two attributes:

$$v_i^{\mathrm{wt}}(x) := \|x_i - x_i^{des}\|^2, \quad v_i^{\mathrm{wt}} \in [0,\infty)$$ Tracking

$$v_{ij}^{\mathrm{ca}}(x) := \left(\min\left\{0, \frac{\|x_i - x_j\|_{P_{ij}}^2 - R_{ij}^2}{\|x_i - x_j\|_{P_{ij}}^2 - r_{ij}^2}\right\}\right)^2$$ Collision Avoidance

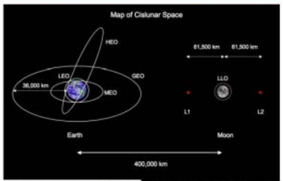


Trade-off between two attributes in a control setting. Changing the trade-offs changes the trajectory.

Published Books

Mission Concept

- Capture and redirect a 7-10 meter diameter, ~500 ton near-Earth asteroid (NEA) to a stable orbit in trans-lunar space
- Enable astronaut missions to the asteroid as early as 2021
- Parallel and forward-leaning development approach



3

NASA's Mission Objectives? Stakeholders?

Who are the stakeholders?

Advance Science
Public Perception
Work toward long term objectives

Safety of crew
Safety of planet

Public Perception
Money

Ability to deflect Asteroid for planetary protection

Possible private investors !

NASA's Mission Objectives?

- *Positive Public Perception/Awareness
- *Advance Science (Return with rich Asteroid)
- *Funding Approval (Money)
- *Safety of Crew
- Public Safety, Deflect Asteroid
- National Security

Meeting Wednesday March 26th with Stakeholders to finalize objectives.

Improving capture mechanism will add significant value to mission.

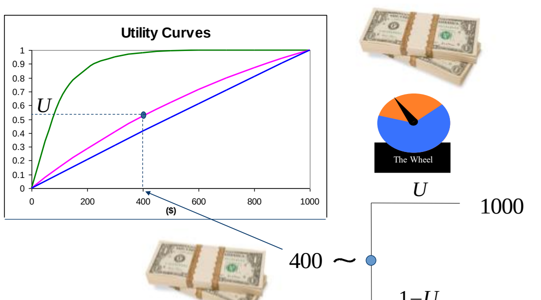
Email from my student last Friday

March 21st 2014

http://www.nbcnews.com/science/space/nasa-wants-6-million-worth-ideas-grabbing-asteroid-n58946

Multi-Attribute Utility Functions

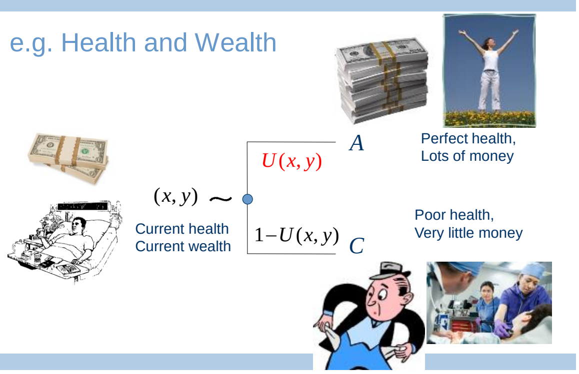
Single-Attribute Utility Function



Trade-off between risk and return.

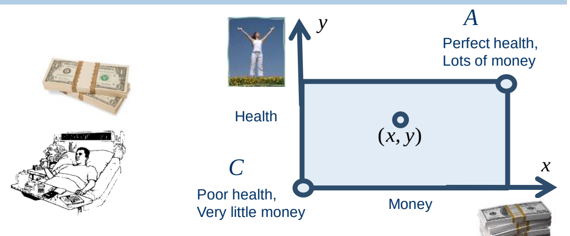
What if the consequences are described by multiple attributes?

e.g. Health and Wealth



Multiple Attributes?

e.g. Health state, y, and money, x.



Assessing even a two-attribute utility function can be a challenge!

Multiattribute Utility Surface

Attributes $X_1, X_2, ..., X_n$, with instantiations $x_1, x_2, ..., x_n$

Consequence $(x_1, ..., x_n) = (x_i, \overline{x}_i) = (x_i, x_j, \overline{x}_{ij})$

Monotonicity Condition

More of any attribute is (weakly) preferred to less.

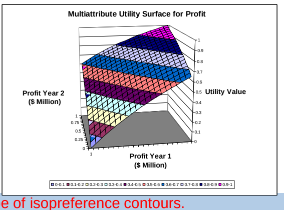
Normalization Condition

$U(x_1^0, x_2^0, ..., x_n^0) = 0$

$U(x_1^*, x_2^*, ..., x_n^*) = 1$

Implies

$0 \le U(x_1, x_2, ..., x_n) \le 1$



Deterministic trade-offs determined by slope of isopreference contours.

Previous Work on Capturing Trade-offs

Utility Independence Decomposition

Given two attributes X, Y

Keeney, R.L., H. Raiffa. 1976. *Decisions with Multiple Objectives:* John Wiley and Sons, Inc.

Utility Independence Decomposition

Given two attributes X, Y

What if I can assert that:

Preferences for any two uncertain lotteries over *X* do not change as we change *Y*.

$\forall X_1, X_2,\ E_{x_1}[U(x,y)] - E_{x_2}[U(x,y)]$ **does not change sign with *y***

X **utility independent** Y

Keeney, R.L., H. Raiffa. 1976. *Decisions with Multiple Objectives:* John Wiley and Sons, Inc.

Utility Independence Conditions

X **utility independent** Y

Preferences for uncertain lotteries over *X* do not depend on *Y*.

$$U(x,y) = k(y)U(x,y_o) + d(y)$$

Decomposition into univariate assessments.

Y **utility independent** X

Preferences for uncertain lotteries over *Y* do not depend on *X*.

$$U(x,y) = k_1(x)U(x_o,y) + d_1(x)$$

Keeney, R.L., H. Raiffa. 1976. *Decisions with Multiple Objectives:* John Wiley and Sons, Inc.

Utility Independence Conditions

Mutual utility independence

Every subset of the attributes is utility independent of its complement.

Multiplicative Form

$$1 + kU(x_1,\ldots,x_n) = \prod_{i=1}^{n}[1 + kk_iU_i(x_i)]$$

Additive Form

$$U(x_1,\ldots,x_n) = \sum_{i=1}^{n} k_iU_i(x_i)$$

If the independence conditions hold, then trade-offs are determined by univariate assessments and normalizing constants.

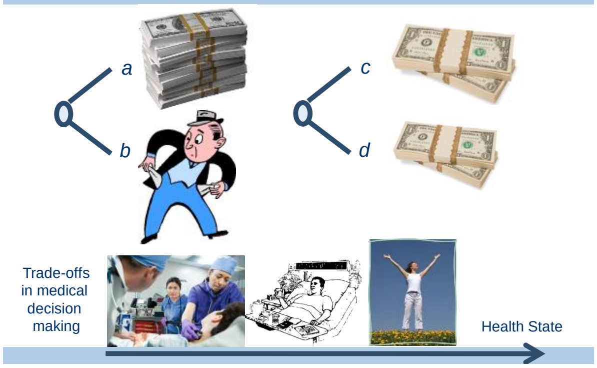
Keeney, R.L., H. Raiffa. 1976. *Decisions with Multiple Objectives:* John Wiley and Sons, Inc.

Beyond the Independence Condition

Utility independence is a powerful property that serves well for simplifying the assessments of the multiattribute utility function.

But if it is not appropriate in a given situation, what options does the analyst have?

Healthcare: Preferences for Investments may Change with Health State



Climate Change: Utility Independence Conditions May Not Exist

Trade-offs in energy and climate change require a specification over a wide range of the attributes.

If utility independence is not appropriate in a given situation, what options does the analyst have?

- **Utility Copula Functions***
  Match the boundary Assessments with a generating function. Degree of freedom to vary the trade-offs.
- **Briefly: One-Switch Independence**
  Determine the trade-offs by asserting the maximum number of switches.
- **Briefly: Utility Trees and Diagrams**
  Derive the functional form when the full independence conditions are not present.

Attribute Dominance Utility Functions

E.g.: Utility function for health and consumption

(1) $0 \le U_{xy}^{d}(x,y) \le 1$ (Normalized)

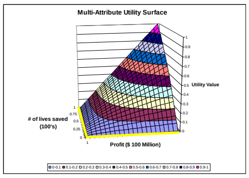
(2) $U^{d}(x_{\min}, y_{\min}) = U^{d}(x_{\min}, y) = U^{d}(x, y_{\min}) = 0$

(attribute dominance condition)

(3) Non-decreasing with arguments

$x_1 > x_0 \Rightarrow (x_1, y) \succ (x_0, y) \forall y \in (y_{\min}, y_{\max}]$

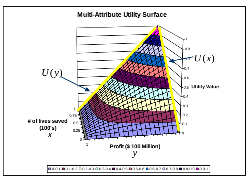
$y_1 > y_0 \Rightarrow (x, y_1) \succ (x, y_0) \; \forall x \in (x_{\min}, x_{\max}]$



Abbas, A.E. and R.A. Howard, Attribute Dominance Utility, Decision Analysis, 2 (4) 185-206, 2005

Marginal and Conditional Attribute Dominance Utility Functions

$U_{x}^{d}(x) \triangleq U_{xy}^{d}(x, y_{\max})$ $\quad$ $U_{y}^{d}(y) \triangleq U_{xy}^{d}(x_{\max}, y)$

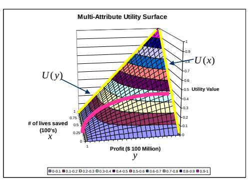


Abbas, A.E. and R.A. Howard, Attribute Dominance Utility, Decision Analysis, 2 (4) 185-206, 2005

Marginal and Conditional Attribute Dominance Utility Functions

$U_{x}^{d}(x) \triangleq U_{xy}^{d}(x, y_{\max})$

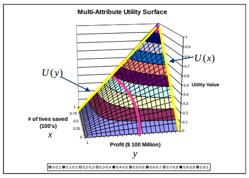
$U_{y|x}^{d}(y \mid x) \triangleq \; ?$



Marginal and Conditional Attribute Dominance Utility Functions

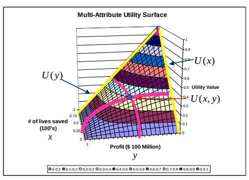
Similarly,

$$U^d_{x|y}(x \mid y) \triangleq \frac{U^d_{xy}(x,y)}{U^d_y(y)}$$



"Bayes' Rule" for Attribute Dominance Utility

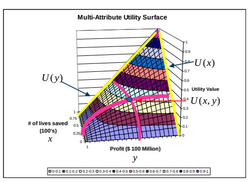
$$U^d_{xy}(x,y) = U^d_y(y)U^d_{x|y}(x \mid y) = U^d_x(x)U^d_{y|x}(y \mid x)$$



"Bayes' Rule" for Attribute Dominance Utility

$$U^d_{xy}(x,y) = U^d_y(y)U^d_{x|y}(x \mid y) = U^d_x(x)U^d_{y|x}(y \mid x)$$

$$U^d_{x|y}(x \mid y) = \frac{U^d_{y|x}(y \mid x)U^d_x(x)}{U^d_y(y)}$$



Utility Inference

Utility Independence is symmetric for attribute dominance utility

Multiattribute Utility Surface

Attributes $X_1, X_2, ..., X_n$, with instantiations $x_1, x_2, ..., x_n$

Consequence $(x_1, ..., x_n) = (x_i, \overline{x}_i) = (x_i, x_j, \overline{x}_{ij})$

Monotonicity Condition

More of any attribute is (weakly) preferred to less.

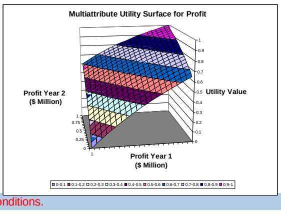
Normalization Condition

$U(x_1^0, x_2^0, ..., x_n^0) = 0$

$U(x_1^*, x_2^*, ..., x_n^*) = 1$

Implies

$0 \leq U(x_1, x_2, ..., x_n) \leq 1$



Relax the grounding condition and the n-increasing conditions.

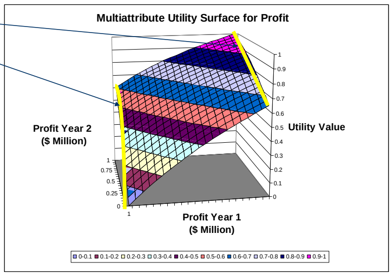
Question:

Can we construct continuous and strictly increasing multiattribute utility functions that incorporate utility dependence using single-attribute utility assessments?

Spreadsheet

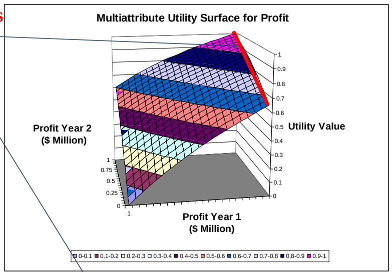
Definition: Marginal Utility Function

Two Marginal Utility Functions



Definition: Marginal Utility Function

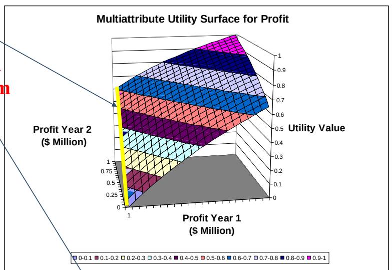
Choose this one then normalize it.



$$U(x_i \mid \bar{x}_i^*) = \frac{U(x_i,\bar{x}_i^*) - U(x_i^0,\bar{x}_i^*)}{1-U(x_i^0,\bar{x}_i^*)} \text{ (at max complement)}$$

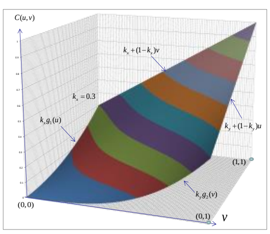
Definition: Marginal Utility Function

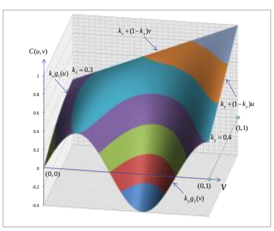
This one is a conditional at minimum value. Normalize it.



$$U(x_i \mid \bar{x}_i^0) = \frac{U(x_i,\bar{x}_i^0)}{U(x_i^*,\bar{x}_i^0)} \text{ (at min complement)}$$

Utility Copulas (*u*-*v* space)



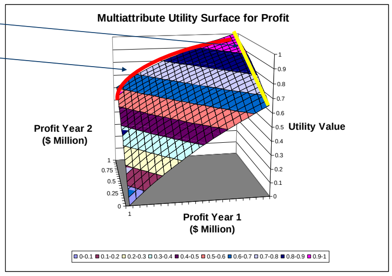


In *u*-*v* space, these copula functions are linear at the upper bound.

Abbas, A. E. 2012. Utility Copula Functions Matching all Boundary Assessments. Forthcoming in *Operations Research*.

Single-Sided Utility Copula

Match
Only one
Marginal
Utility
Function
For each
Attribute



Abbas, A. E. 2009. Multiattribute Utility Copulas. *Operations Research*, 57 (6), 1367-1383.

Class 1: Utility Copula Functions

$$C[u_1, 1, 1, ..., 1] = au_1 + b$$

Assess conditional utility functions at the upper bound

$$U(x_1, ..., x_n) = C[U_1(x_1 \mid \bar{x}_1^*), ..., U_n(x_n \mid \bar{x}_n^*)]$$

Single-Sided Utility Copula Functions

Archimedean Utility Copulas

Abbas, A. E. 2009. Multiattribute Utility Copulas. *Operations Research*, 57 (6), 1367-1383.

Extended Archimedean Functional Form

$$C_\lambda(v_1,\ldots,v_n) = a\eta^{-1}[\prod_{i=1}^{n}\eta(l_i + (1-l_i)v_i)] + b$$

η **is a continuous and strictly monotone function**

If $\eta(1) = 1 \Rightarrow$

$$C_\lambda(1,\ldots,1,v_i,1,\ldots,1) = a(l_i + (1-l_i))v_i + b, \forall v_i$$
$$= a_i v_i + b_i$$

Linear Transformation at maximum value of complement arguments.

Presentation Contents

1. Introduction
   a. von Neumann-Morgenstern Utility Functions
   b. Implications of Utility Independence
2. Utility Copula Functions
3. Other Methods
   a) One-Switch Independence (with David Bell)
   b) Utility Diagrams
4. Conclusions

Recall: Utility Independence Decomposition

Given two attributes X, Y

What if I can assert that:

Preferences for any two uncertain lotteries over *X* do not change as we change *Y*.

$\forall X_1, X_2,\ E_{X_1}[U(x,y)] - E_{X_2}[U(x,y)]$ **does not change sign with *y***

X **utility independent** Y

Keeney, R.L., H. Raiffa. 1976. *Decisions with Multiple Objectives:* John Wiley and Sons, Inc.

What if Preferences Can Change, but..

can change only once?

Jointly with David Bell
dbell@hbs.edu

Abbas, A. E and D. E. Bell. 2011. One-Switch Independence for Multiattribute Utility Functions, *Operations Research*, 59(3) 764-771.

One-Switch Utility Independence

Definition: Two attributes *X,Y*, and a utility function *U(x,y)*.

X is one-switch independent of *Y* if the ordering of any two lotteries over X switches at most once as *Y* increases.

I.e.

$$\forall X_1, X_2, E_{x_1}[U(x,y)] - E_{x_2}[U(x,y)]$$

can cross zero only once as we increase y.

What must be the functional form of *U*?

Relation to Single-Crossing Property in Economics !!

Abbas, A. E and D. E. Bell. 2011. One-Switch Independence for Multiattribute Utility Functions, *Operations Research*, 59(3) 764-771.

One-Switch Utility Independence

Definition: Two attributes *X,Y*, utility function *U(x,y)*. *X* is one-switch independent of *Y* if and only if the ordering of any two lotteries over X switches at most once as *Y* increases.

Theorem 1 *X* 1S *Y* *if and only if*

$$U(x,y) = g_0(y) + g_1(y)[f_1(x) + f_2(x)\phi(y)]$$

$g_1(y)$ does not change sign

$\phi(y)$ monotone

Five univariate assessments.

Abbas, A. E and D. E. Bell. 2011. One-Switch Independence for Multiattribute Utility Functions, *Operations Research*, 59(3) 764-771.

Presentation Contents

Probability Independence

$$(Z \text{ Probability Independent of } Y \mid X)$$

Use Bayes' Expansion Theorem and substitute

$$F(x,y,z) = F_x(x)F_{y|x}(y \mid x)F_{z|xy}(z \mid x,y)$$

$$F_{z|xy}(z \mid x,y) = F_{z|xy}(z \mid y^*,x) \triangleq F_{z|x}(z \mid x)$$

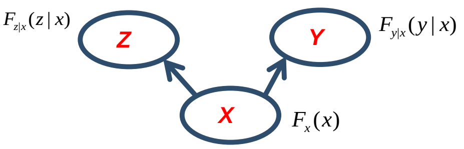
$$\boxed{F(x,y,z) = F_x(x)F_{y|x}(y \mid x)F_{z|x}(z \mid x)}$$

Probability Independence is a symmetric property!!

$$(Y \text{ Probability Independent of } Z \mid X)$$

Diagrams

$$(Z \text{ Probability Independent of } Y \mid X)$$



$$\boxed{F(x,y,z) = F_x(x)F_{y|x}(y \mid x)F_{z|x}(z \mid x)}$$

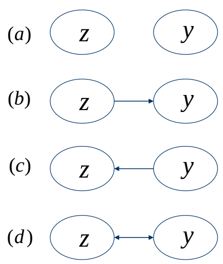
Probability Independence is a symmetric property!!

$$(Y \text{ Probability Independent of } Z \mid X)$$

How does this concept extend to Multiattribute Utility Functions?

Bidirectional Utility Diagrams

Two Attributes



(a) Mutual utility independence

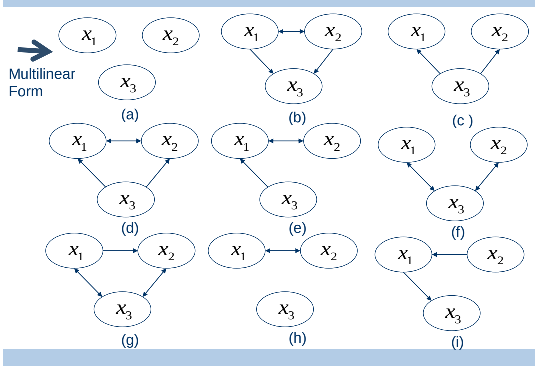
(b) Directional utility independence
z utility independent of *y*

(c) Directional utility independence
y utility independent of *z*

(d) No independence assertions

Abbas, A. E. 2011. General Decompositions of Multiattribute Utility Functions. *J. Multicriteria Decision Analysis*, 17 (1, 2), 37–59.

But there are many more diagrams even for three attributes



Can we tell the functional form when only partial utility independence conditions exist?

One-Step Expansion

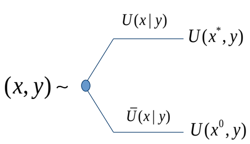
$$U(x\,|\,\bar{x}) \triangleq \frac{U(x,\bar{x}) - U(x^0,\bar{x})}{U(x^*,\bar{x}) - U(x^0,\bar{x})}$$

$\bar{U}(x\,|\,\bar{x})$ = Normalized conditional disutility for x at $\bar{x}$
$\triangleq$ 1-$U(x\,|\,\bar{x})$.

$$U(x,\bar{x}) = U(x^*,\bar{x})U(x\,|\,\bar{x}) + U(x^0,\bar{x})\bar{U}(x\,|\,\bar{x})$$

Abbas, A. E. 2011. General Decompositions of Multiattribute Utility Functions. *J. Multicriteria Decision Analysis*, 17 (1, 2), 37–59.

One-step utility tree for two attributes



$$U(x,\bar{x}) = U(x^*,\bar{x})U(x\,|\,\bar{x}) + U(x^0,\bar{x})\bar{U}(x\,|\,\bar{x})$$

Expansion around two attributes

$$U(x,y,\overline{xy}) = U(x^*,y^*,\overline{xy})U(x\,|\,\overline{x})U(y\,|\,x^*,\overline{xy}) +$$
$$U(x^*,y^0,\overline{xy})U(x\,|\,\overline{x})\bar{U}(y\,|\,x^*,\overline{xy}) +$$
$$U(x^0,y^*,\overline{xy})\bar{U}(x\,|\,\overline{x})U(y\,|\,x^0,\overline{xy}) +$$
$$U(x^0,y^0,\overline{xy})\bar{U}(x\,|\,\overline{x})\bar{U}(y\,|\,x^0,\overline{xy}).$$

Compare to Bayes' Expansion Theorem!

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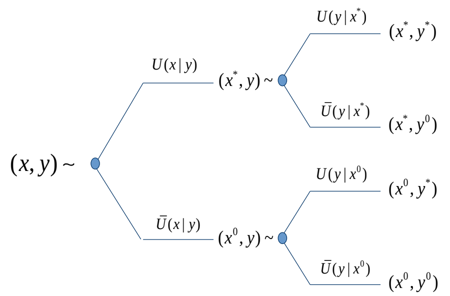
Two-attribute Utility Tree



$$U(x,y) = U(x^*,y^*)U(x|y)U(y|x^*) + U(x^*,y^0)U(x|y)\bar{U}(y|x^*) + U(x^0,y^*)\bar{U}(x|y)U(y|x^0) + U(x^0,y^0)\bar{U}(x|y)\bar{U}(y|x^0).$$

Abbas, A.E. 2011. The Multiattribute Utility Tree, *Decision Analysis*, 8 (3), 180-205.

Theorem 1:

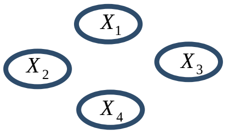
Basic Expansion Theorem for Multiattribute Utility Functions

$$U(X) = \sum_{x_K^{*0} \in X_K^{*0}} U(x_K^{*0}, \overline{x}_K) \prod_{X_i \in X_K} g(x_i \,|\, x_{iP}^{*0}, x_{iF}).$$

$$g(x_i \,|\, x_{iP}^{*0}, x_{iF}) = \begin{cases} U(x_i \,|\, x_{iP}^{*0}, x_{iF}), \text{ if } x_i = x_i^* \text{ in the unexpanded term } U(x_K^{*0}, \overline{x}_K) \\ \bar{U}(x_i \,|\, x_{iP}^{*0}, x_{iF}), \text{ if } x_i = x_i^0 \text{ in the unexpanded term } U(x_K^{*0}, \overline{x}_K). \end{cases}$$

Does not make any assumptions of utility independence.

Example 1 : The Multilinear Form



$$U(x_i \mid \bar{x}_i) = U(x_i \mid \bar{x}_i^0),\ i = 1,2,3,4.$$

$$U(X) = \sum U(x_N^{*0}) \prod_{i \in N} g(x_i \mid \bar{x}_i^0).$$

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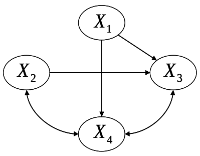
Example 2 : Canonical Form



$$U(x_1 \mid x_2, x_3, x_4) = U(x_1 \mid x_2^0, x_3^0, x_4^0),\quad U(x_2 \mid x_1, x_3, x_4) = U(x_2 \mid x_1^0, x_3^0, x_4),$$

$$\Rightarrow U(x_1, x_2, x_3, x_4) = U(x_1^*, x_2^*, x_3, x_4) U(x_1 \mid x_2^0, x_3^0, x_4^0) U(x_2 \mid x_1^0, x_3^0, x_4) +$$
$$U(x_1^*, x_2^0, x_3, x_4) U(x_1 \mid x_2^0, x_3^0, x_4^0) \bar{U}(x_2 \mid x_1^0, x_3^0, x_4) +$$
$$U(x_1^0, x_2^*, x_3, x_4) \bar{U}(x_1 \mid x_2^0, x_3^0, x_4^0) U(x_2 \mid x_1^0, x_3^0, x_4) +$$
$$U(x_1^0, x_2^0, x_3, x_4) \bar{U}(x_1 \mid x_2^0, x_3^0, x_4^0) \bar{U}(x_2 \mid x_1^0, x_3^0, x_4).$$

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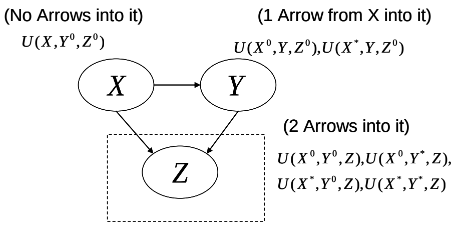
Can we tell the assessments needed?

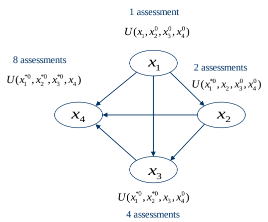
Example

By Inspection: Determine the size and number of utility assessments needed for the following diagram



Example:

Determine the size and number of utility assessments needed for the following diagrams



	x_1	x_2	x_3	x_4
x_1	○			
x_2	○	○		
x_3	○	○	○	
x_4	○	○	○	○

This idea generalizes many theorems in K&R!

Conclusions

1. Great opportunities lie ahead of us in the field of decision making.
2. Good decision making requires all elements of decision quality.
3. Determining the right trade-offs between two or more attributes is extremely important in many fields.
4. If utility independence conditions do not apply, we offer

   one-switch independence, utility copulas, and utility trees and diagrams as additional tools that the analyst might use.
5. Analogies often create new research ideas.

Thank You !