Who am I and what am I doing here?

Reena Urban

SAL Monday Seminar
13.05.2024


Hamburg
Bremen
Vechta
Hannover
Berlin
Göttingen
Koblenz
Frankfurt am Main
Kaiserslautern
Landau in der Pfalz
Karlsruhe
Stuttgart

University

Bachelor in Mathematics (BSc), Specialization: Optimization

Master in Mathematics (MSc), Specialization: Optimization
Thesis: Analysis and Computation of Cheapest Paths in Public Transport

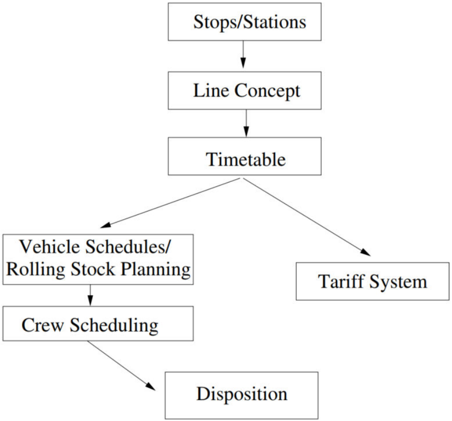
PhD student supervised by Anita Schöbel, optimization research group
Topic: Optimization in public transport planning – fare planning and BRT network design

Optimization research group in Kaiserslautern

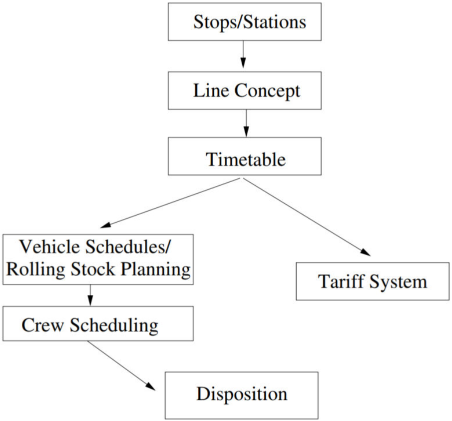
Optimization research group in Kaiserslautern

EURO 2022 at Aalto university

My research


Stops/Stations
Line Concept
Timetable
Vehicle Schedules/
Rolling Stock Planning
Tariff System
Crew Scheduling
Disposition

My research



Fare structure

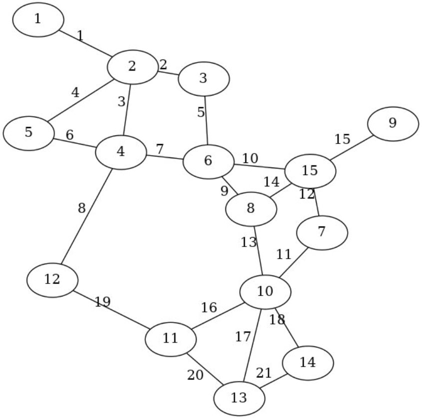
A fare structure is a function $p\colon \mathcal{W} \to \mathbb{R}_{\geq 0}$ that assigns a price to every path in the network.

Different perspectives

- Passenger: comprehensible fare structures, affordable and fair prices
- Operator: financial requirements, no undercutting, increase share of public transport

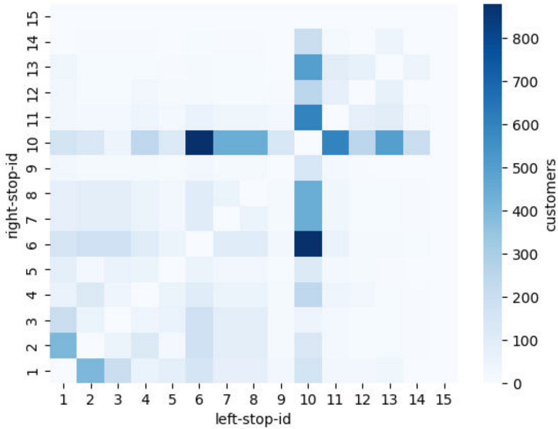
Input data

Public transport network (PTN)
undirected graph (V, E) with stations/stops V and connections E

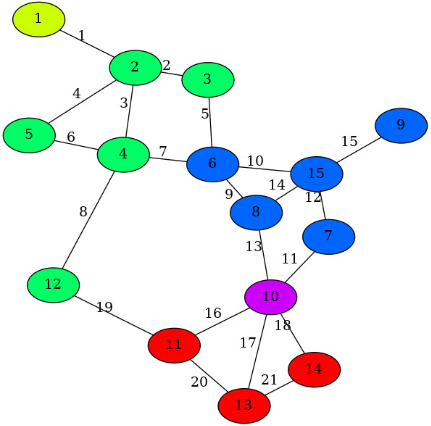


Origin-destination (OD) data

- Set D of station pairs
- Path W_d for every OD pair $d \in D$



Fare structures



Modeling fare structures

Fare structure	Model	Description
Flat	$p_d = f$	Fixed price for all paths
Travel/Beeline distance	$p_d = f + p \cdot l(W_d)$	Base amount plus kilometer price times the length of the path
Zone	$p_d = P(\#zones(W_d))$	Price dependent on the number of traversed zones

Designing fare structures

Objectives

Literature:

- Minimize deviation from reference prices (Hamacher and Schöbel (1995, 2004))
- Maximize revenue (Otto and Boysen (2017))
- Maximize demand with respect to a budget constraint (Nash (1978), Glaister and Collings (1978), Borndörfer, Karpstein and Pfetsch (2012))

New bi-objective approach:

- Maximize revenue and demand simultaneously $\Rightarrow$ "Attraction model"

Attraction model – Input

Input

As before: PTN and OD data

Additionally: Two groups of passengers for each OD pair $d \in D$

	Captive passengers	Choice passengers
Description	Rely on public transport	Have other options like a car
Number of passengers	C_d^U	C_d^L
Willingness to pay	U_d	L_d

with $0 \leq L_d \leq U_d$.

Attraction model – Objective

Objective function

For $d \in D$, we set

$$f_2(d \mid p_d) := \begin{cases} C_d^U + C_d^L & \text{if } 0 \leq p_d \leq L_d, \\ C_d^U & \text{if } L_d < p_d \leq U_d, \\ 0 & \text{if } U_d < p_d, \end{cases} \qquad \text{(demand of } d\text{)}$$

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$$f_2(p) := \sum_{d \in D} f_2(d \mid p_d), \qquad \text{(demand)}$$

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$$f_1(p) := \sum_{d \in D} f_2(d \mid p_d) \cdot p_d. \qquad \text{(revenue)}$$

Attraction model

Relation-based tariff

Determine individual prices p_d for each OD pair $d \in D$.

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Determine individual prices p_d for each OD pair $d \in D$.

Relation-based tariff attraction model

$$
\text{(RTAM)} \quad \max_{p} \begin{pmatrix} f_1(p) \\ f_2(p) \end{pmatrix} \quad \begin{matrix} \text{(revenue)} \\ \text{(demand)} \end{matrix}
$$
$$
\text{s.t.} \quad p_d \in \mathbb{R}_{\geq 0} \quad \text{for all } d \in D.
$$

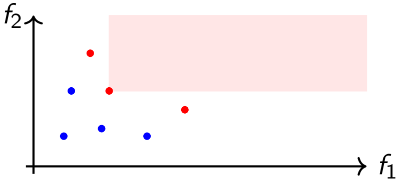
Bi-objective optimization

Efficient solution as concept of optimality

A solution p is efficient (Pareto optimal) and $(f_1(p), f_2(p))$ is a non-dominated point for RTAM if there is no other solution p' such that

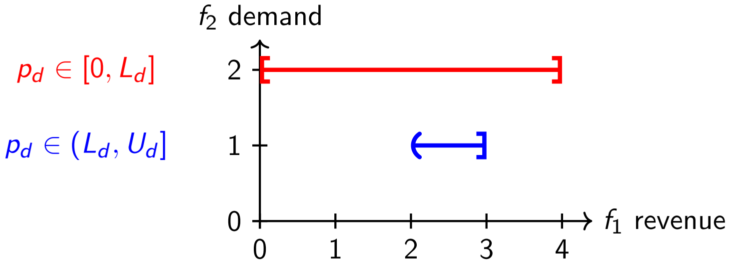
$$f_1(p') > f_1(p) \text{ and } f_2(p') \geq f_2(p)$$
$$\text{or}$$
$$f_1(p') \geq f_1(p) \text{ and } f_2(p') > f_2(p).$$

$\rightsquigarrow$ If one objective improves, another worsens.



Examples

	Captive passengers	Choice passengers
Description	Rely on public transport	Have other options like a car
Number of passengers	$C_d^U = 1$	$C_d^L = 1$
Willingness to pay	$U_d = 3$	$L_d = 2$



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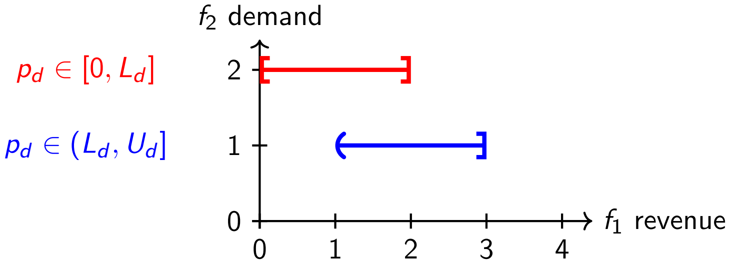

Complexity

RTAM is NP-hard.

Can be proved by a reduction from PARTITION.

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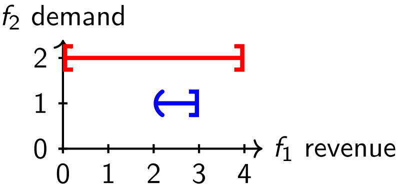
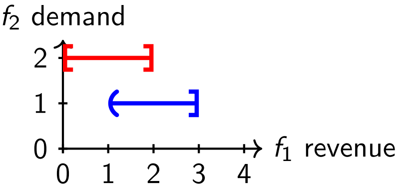
Finite dominating set

If p is an efficient solution for RTAM, then $p_d \in \{L_d, U_d\}$ for all $d \in D$.





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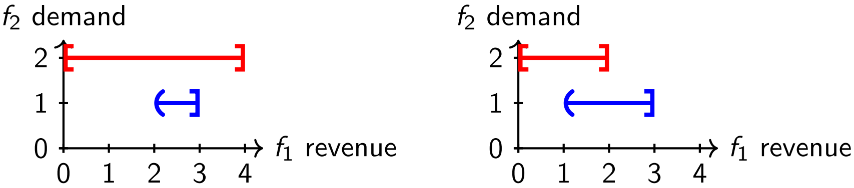

RTAM is intractable.

$\Rightarrow$ There are instances with exponentially many non-dominated points.

Easy cases

Only one passenger group

If $C_d^U > 0$ and $C_d^L = 0$, then $p_d^* = U_d$ in any efficient solution p^* to RTAM.
If $C_d^U = 0$ and $C_d^L > 0$, then $p_d^* = L_d$ in any efficient solution p^* to RTAM.

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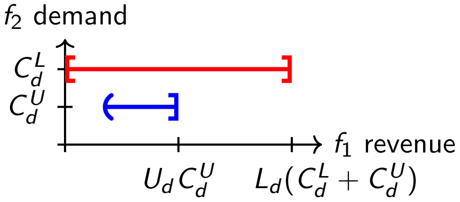
L_d is better in both objectives

If $C_d^U > 0$ and $C_d^L > 0$ and
$f_1(d \mid L_d) \geq f_1(d \mid U_d)$, then $p_d^* = L_d$
in any efficient solution p^* to RTAM.



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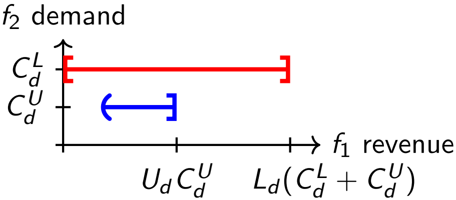

Remaining situation

Need to determine p_d for $d \in D$ with $C_d^U > 0$ and $C_d^L > 0$ and $f_1(d \mid L_d) \leq f_1(d \mid U_d)$.

Computing the Pareto front

Apply the ϵ-constraint method (adapted from Bérubé, Genreau, Potvin (2009)):

- Binary Variable $x_d \in \{0,1\}$ with $x_d = 1$ iff $p_d = U_d$
- Objective: maximize revenue
- New constraint: lower bound on the demand

$$\begin{aligned} \max_{x_d} \quad & f_1(L) + \sum_{d \in D} x_d \cdot (f_1(d \mid U_d) - f_1(d \mid L_d)) \\ \text{s.t.} \quad & f_2(L) - \sum_{d \in D} x_d \cdot C_d^L \geq \epsilon \\ & x_d \in \{0,1\} \quad \text{for all } d \in D. \end{aligned}$$

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$$\begin{aligned} \max_{x_d} \quad & \sum_{d \in D} x_d \cdot r_d \\ \text{s.t.} \quad & \sum_{d \in D} x_d \cdot C_d^L \leq \lambda \\ & x_d \in \{0, 1\} \quad \text{for all } d \in D. \end{aligned}$$

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- Rephrase terms
- Solve IP for $\lambda \in \{0, \ldots, \sum_{d \in D} C_d^L\}$

$$
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& x_d \in \{0,1\} \quad \text{for all } d \in D.
\end{aligned}
$$

Solution method

$$\max_{x_d} \sum_{d \in D} x_d \cdot r_d$$

$$\text{s.t.} \quad \sum_{d \in D} x_d \cdot C_d^L \leq \lambda$$

$$x_d \in \{0, 1\} \quad \text{for all } d \in D$$

- Resulting λ-constraint problem is a knapsack problem.

Solution method

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- Resulting λ-constraint problem is a knapsack problem.
- $\Rightarrow$ Solve it with a dynamic program (DP).
- $\Rightarrow$ DP for $\lambda = \sum_{d \in D} C_d^L$ computes all non-dominated points in $\mathcal{O}(|D| \sum_{d \in D} C_d^L)$.

How to proceed?

Work in progress

- Consider more levels of willingness to pay
- Application to other fare structure types:
  - Integrate willingness to pay into fare structure design problems for flat, distance and zone problems $\Rightarrow$ Pareto front?
  - Use relation-based results as reference prices for fare structure design

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Contact: `rurban@rptu.de`