AALTO UNIVERSITY SCHOOL OF SCIENCE

NETWORK TOPOLOGY, SYSTEM MECHANICS AND BEHAVIORAL DYNAMICS IN INTERBANK PAYMENT SYSTEMS

Dissertation for the degree of Doctor of Science in Technology

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Abstract:

The financial crisis of 2007-09 showed that financial institutions are highly interconnected and that the dynamic behavior of complex financial systems is hard to foresee. This Dissertation applies and develops new quantitative methods that describe the interbank payment functions of the financial system, both during normal circumstances and during times of crisis. First, it describes the topology of interactions among financial institutions using methods developed in network theory. Using empirical analysis of transaction data from the Fedwire interbank payment system of the Federal Reserve, it finds that the payment networks are complex with scale-free degree distributions and contain core banks that process a very high proportion of the total value. Second, it develops simulation models of the interbank payment systems and provides approximations of their liquidity flow mechanics. The simulation models are used to evaluate the efficiency and safety of alternative system designs. Third, it proposes an agent-based model of the interbank payment system and derives a liquidity demand function for it. The model is also used to forecast the collective behavior of banks when mechanisms that allow them to save liquidity are introduced to the system.

Keywords Network theory, complexity, interbank payment system, liquidity, financial stability, counterfactual simulations, agent-based modeling
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Tekijä
Kimmo Soramäki
Väitöskirjan nimi
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Tiivistelmä:

Vuosien 2007-2009 finanssikriisi osoitti, että rahoituslaitokset ovat tiiviisti kytköksissä toisiinsa ja että kompleksisen rahoitusjärjestelmän dynaamista käyttäytymistä on vaikea ennustaa. Väitöskirjassa sovelletaan ja kehitetään uusia matemaattisen mallintamisen menetelmiä, jotka kuvaavat rahoitusjärjestelmän maksuliiketoimintoja sekä normaaliolosuhteissa että kriisiaikana. Ensiksi väitöskirjassa kuvataan rahoituslaitosten vuorovaikutussuhteita verkostoteorian avulla. Empiirinen analyysi Yhdysvaltain keskuspankin Fedwire -järjestelmän maksuliikeaineistolla osoittaa verkkojen olevan kompleksisia ja mittakaavasta riippumattomia. Verkkojen ytimenä toimivien pankkien havaittiin suorittavan suurimman osan maksuliikkeestä. Toiseksi väitöskirja kehittää maksujärjestelmien simulointimalleja ja approksimaatioita niiden likviditeettivirtojen mekaniikasta. Malleja käytetään erityyppisten maksujärjestelmien tehokkuuden ja turvallisuuden arvioimiseen. Kolmanneksi väitöskirjassa esitetään agenttipohjainen malli maksujärjestelmistä ja johdetaan sen avulla likviditeetin kysyntäfunktio. Mallia käytetään lisäksi pankkien kollektiivisen käyttäytymisen ennustamiseen, kun järjestelmään lisätään likviditeettiä säästäviä toiminta-mekaniikkoja.

Asiasanat Verkostoteoria, kompleksisuus, pankkien väliset maksujärjestelmät, likviditeetti, finanssijärjestelmän vakaus, simulaatio, agenttipohjainen simulointi
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Publications

The Dissertation consists of the present summary and the following articles:

[I] Soramäki K., M. Bech, J. Arnold, R.J. Glass and W.E. Beyeler (2007). *The topology of interbank payment flows*, Physica A: Statistical Mechanics and its Applications 379(1), pp. 317-333.

[II] Johnson K., J. McAndrews and K. Soramäki (2004). *Economizing on liquidity with deferred settlement mechanisms*, Federal Reserve Bank of New York Economic Policy Review 10(3), pp. 51-72.

[III] Beyeler W.E., R.J. Glass, M. Bech, and K. Soramäki (2007). *Congestion and cascades in payment systems*, Physica A: Statistical Mechanics and its Applications 384(2), pp. 693-718.

[IV] Galbiati M. and K. Soramäki (2011). *An agent-based model of payment systems*, Journal of Economic Dynamics and Control 35(6), pp. 859-875.

[V] Galbiati, M. and K. Soramäki (forthcoming). *Liquidity-saving mechanisms and bank behavior in payment systems*, In: Martinez-Jaramillo, S., B. Alexandrova-Kabadjova, A. Garcia-Almanza and E. Tsang (eds), "Simulation in Computational Finance and Economics: Tools and Emerging Applications", IGI Global, Hershey, USA

Contribution of the author

Article [I] was initiated by Soramäki who was the primary author and also responsible for developing and programming the network algorithms and calculations.

Article [II] was equally contributed by all the authors. In addition, Soramäki was responsible for the programming of the simulation model.

Article [III] was equally contributed by all the authors. In addition, Soramäki developed and programmed the simulation model together with W. Beyeler.

Article [IV] was initiated by Soramäki and co-written with M. Galbiati. Soramäki was also responsible for the development and programming of the model.

Article [V] was initiated by Soramäki who was the primary author. Soramäki was also responsible for the development and programming of the model.

Preface

This Dissertation has been made possible by several people whom I have the privilege to acknowledge here.

First, I would like to thank my supervisor Ahti Salo whose patience and soft pressure have brought the project to this stage. I'd like to thank all my co-authors for collaboration extending in some cases to close to fifteen years. In particular, Morten Bech and Marco Galbiati taught me much of Game Theory and Bob Glass and Walter Beyeler are the sources of my knowledge in Statistical Mechanics and Java programming.

Next, I'd like to thank my past employers for allowing and encouraging research in this area. The first three articles were started or finished when I was seconded to the Federal Reserve Bank of New York's Research Department for the year 2004. I would like to thank Johannes Priesemann and Sirkka Hämäläinen from my employer of the time, the European Central Bank, for making the secondment possible as well as Jamie McAndrews from Federal Reserve Bank of New York for allowing me to focus on these new and risky lines of research while there.

The fourth and fifth papers were written while I was consulting the Bank of England. I'd like to thank Stephen Millard and Anne Wetherilt for making the collaboration possible. I'd also like to gratefully acknowledge the support of OP-Pohjola-Ryhmän Tutkimussäätiö for the last two papers.

I'd like to thank Farooq Akram and Casper Christophersen at Norges Bank for providing a physical and intellectual place to finish this Dissertation and write the summary at hand on my visits to Oslo as a Visiting Scholar to Norges Bank Research Department.

Last but not least, I'd like to thank my wife Inês Soramäki and my parents Anneli and Martti Soramäki for their continuous support during the project.

Barcelona, 20 February 2012

Table of Contents

1 Introduction

> *"When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. [...] As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools."*
>
> in a Speech by Jean-Claude Trichet, President of the European Central Bank, Frankfurt, 18 November 2010

The financial crisis of 2007-2008 showed that financial institutions are highly interconnected and that the dynamic behavior of complex financial systems is hard to foresee. Academia and policy-makers have recently developed a stronger awareness of the need for new analytical methods for monitoring the financial system (see e.g. Colander et al. 2009 and Haldane 2009). Recognition of the fact that stability of the financial system depends on the collective behavior of market participants and on their interconnectedness underpins the recent emphasis on the adoption of a macroprudential view of financial supervision.[1]

The Dissertation covers three aspects of financial systems in an effort to provide a new view for understanding the functioning of the financial system during normal circumstances and crisis. It looks at the topology of interactions among financial institutions, the complex mechanics of these interactions, and economic behavior that both determines and is affected by the first two.

While the ideas presented here can be extended to other areas of the financial system, the application area in this Dissertation is the interbank payment system. Interbank payment systems provide the backbone for all financial transactions. Virtually all economic activity is facilitated by transfers of claims by financial institutions. In turn, these claim transfers generate payments between banks whenever they are not settled across the books of a same bank. These payments are settled in interbank payment systems.

In 2010, the annual value of interbank payments made e.g. in the Pan-European system TARGET2 was \$839 trillion. In the corresponding US system Fedwire, the amount was \$608 trillion - over 40 times its annual GDP (BIS 2010). Due to the sheer size of the transfers, and their pivotal role in the functioning of financial markets and the implementation of monetary policy, payment systems are central for policymakers

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[1] see e.g. preambles 11-13 of the legislations setting up the European Systemic Risk Board (EU 2010).

and regulators. The availability of high-frequency transaction data have made payment systems also widely researched.

This Dissertation consists of this summary and five articles with the following contributions:

The first article models the financial system as a network and studies its characteristics empirically by looking at payment flows between banks in the United States. For this purpose a unique dataset on all individual transactions carried out in Fedwire is used. The article was among the first to model the financial system through the lenses of network theory and the empirical findings of the article were in marked contrast to the (much simpler) interbank network topologies that had usually been considered in theoretical economic and financial models.

The second article develops a simulation model of interbank payment systems and uses it to evaluate the liquidity efficiency of alternative system designs. The same methodology has since been used in a wide body of policy oriented research on payment systems via the means of simulations.[2]

The third article studies the complex dynamics of payment flows in a simulation model that closely resembles the topological and functional characteristics of the system described in the first article. The focus in this article is on how simple rules of settlement can create complex system level behavior. The model shows that the mechanics of the system become unpredictable at low liquidity and emphasizes the need of an interbank lending market for its smooth functioning.

The fourth article adds an economic layer to the analysis. A common critique of simulation models has been that strategic behavior of banks is not adequately taken into account. In the model of this article, banks interact not only via the mechanical dynamics of liquidity flows, but also strategically determine the outcome in an agent-based model where they endogenously decide on the amount of funds to allocate to carry out the payment flows.

The fifth article extends the agent based model of the payment system developed in the fourth article to investigations on the benefits of introducing liquidity saving mechanisms - taking both the complex mechanics of the system and the behavioral responses of its participants into account. The results show that several equilibria with different levels of desirability are possible. Therefore it is important to devise co-

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[2] See compilations edited by Leinonen (2005, 2007 and 2009).

ordination mechanisms when introducing new systems so that the better equilibria are achieved.

The rest of this introductory chapter discusses three themes present in the above articles in more detail: financial networks (article 1), system mechanics (articles 2 and 3) and behavioral dynamics (articles 4 and 5) and provides examples from other realms of financial systems and also from more recent research related to the Dissertation, thereby aiming to set the research in a wider context.

2 Financial networks

The general concept of a network is very intuitive: a network describes a collection of nodes or vertices (e.g. financial institutions) and the links between them, which can be directed (i.e. arcs) or undirected (i.e. edges). The links can denote different relationships between the nodes, depending on the domain of analysis. In finance, these linkages may be related to lending relationships, derivatives trading, insurance agreements, asset correlations or joint ownership of (e.g. toxic) assets. The linkages can exist in different time-scales from intraday liquidity requirements to term loans.

The main premise of network analysis is that the structure of the links between the nodes matters on the performance of each node and the system as a whole. The properties and behavior of a particular node cannot be analyzed on the basis of its own properties and behavior alone, as these may be affected by nodes that have links to it, and also by other nodes that have no direct links, but are linked to its neighbors. Thus, in order to understand the behavior of one node, one must analyze the behavior of many nodes, including those that are, perhaps, several other nodes apart in the network.

In the past the focus of financial risk analysis has been based on the balance sheet of individual institutions and the relationships between institutions have generally not been formally considered. This has changed during the recent financial crisis. Regulators now begin to understand that financial institutions cannot be looked in isolation. In order to understand the riskiness of a particular institution one must consider the riskiness of the institutions that it is exposed to. For example, when extending emergency funding to Bear Stearns the Federal Reserve System justified its decision by

> *"...given the fragile condition of the financial markets at the time, the prominent position of Bear Stearns in those markets, and the expected contagion that would result from the immediate failure of Bear Stearns, the*

> *best alternative available was to provide temporary emergency financing to Bear Stearns...”.*
>
> Minutes of the Board of Governors of the Federal Reserve System, 14 March 2008

In the language of networks, Bear Stearns was a central node in the network (prominent position) and its failure could have led to a cascade of failures (contagion) had it been let to fail. The risk of such chain of events is generally called systemic risk. While the term has many definitions they usually refer to a chain of events that is initiated or magnified in the finance sector and adversely impacts the broader economy. De Bandt and Hartmann (2000) provide a survey on the concept exploring the different forms that contagion can take.

Within financial regulation, systemic risk is currently being addressed by enhancing macroprudential supervision. As of late, many supervisors have been mandated to carry out systemic risk monitoring. Also new ‘systemic risk’ regulators such as the Office of Financial Research (OFR) in the US or the European System of Financial Supervisors (ESFS) have been set up.

2.1 Network analysis

Network models have been are used to explain a wide range of natural and societal systems, ranging from the World Wide Web and the Internet to cellular, ecological and citation networks - to name a few. Jackson (2008) provides a comprehensive synthesis of several strands of network science in sociology, physics, mathematics, computer science and economics

Social network analysis, which developed in the 1950’ within sociology (Freeman 2006), is the older of these fields and has brought forth a number of important findings related, for instance, to the diffusion of ideas, the spread of habits and behaviors, the efficiency of groups based on their social network properties, the origins of power among groups and the concepts of centrality (or importance) of nodes in a network.

The approach that has been taken in physics has focused more on the statistical properties of networks, the resilience of different structures and the processes that take place in networks. Moreover, physicists have tried to explain how networks grow over time and exhibit the complex nonrandom structure that has been uncovered for many empirical networks. Newman (2003) and Albert and Barabási (2003) review advances in the modeling of complex networks, focusing on the statistical mechanics of networks.

The economics literature has only recently looked at the impact of network structure. This has meant that in the past much research assumed complete networks where each node is connected to each other node. Starting with the seminal papers by Allen and Gale (2000) and Freixas et al. (2000), the new literature has looked at the implications that a higher or lower degree of completeness of interbank structures (i.e. of interconnectedness generated by cross-holdings of deposits) could have for financial stability. These papers evaluate the potential for contagion that follows from an aggregate and/or an idiosyncratic liquidity shock or a bank failure, and then analyze the role of the central bank in preventing systemic repercussions. While the results depend strongly on the assumptions of the process that takes place in the network, the common lesson learnt from these models is the importance of understanding the structure of financial flows as a step towards understanding the functioning of the system, and thus to be able to assess systemic stability. The literature in economics has recently started to converge with approaches developed in e.g. biology and physics (see e.g. May et al. 2008 and Schweizer et al. 2009).

More recently, e.g. Allen and Babus (2008) argue that a network approach to financial systems is particularly important for assessing financial stability, and that it can be instrumental in capturing the externalities that the risk associated with a single institution may create for the entire system. It has also been argued that the financial system has become more interlinked. For example, Billio et al. (2011) find that different segments of the finance sector (hedge funds, banks, brokers, and insurance companies) have in the last decade become more connected than before.

2.2 System topology

Article [I] is an early analysis applying concepts of network theory in empirical research of large-scale financial systems - in particular the payment flows between banks in the United States. The empirical findings of the paper were similar to Boss et al (2004) who analyzed interbank exposures in Austria and De Masi et al (2006) who analyzed the Italian interbank market. They were, however, in marked contrast to the interbank networks that had usually been considered in economic and financial models. The networks were found to be complex with a small number of highly connected large nodes that had connections with a large number of small nodes with few links. The cores of the networks, composed of the most connected banks, processed a very high proportion of the total value (see Figure 1). These networks in fact shared many characteristics with other empirical complex networks, such as a scale-free degree distribution, high clustering coefficient, the small world phenomenon and disassortativeness[3] (Newman 2003).

The article also investigated the impact of the 9/11 terrorist attacks on the network characteristics of the banking system. The attacks caused massive damage to property and communications systems in lower Manhattan and Wall Street area. This made it difficult (and in some cases impossible) for banks to manage their payments and liquidity. It also disrupted the payment coordination by which banks use incoming payments to fund their own transfers to other banks. Once a number of banks began to be short of incoming payments, others became more reluctant to send out payments themselves. Both effects reduced the circulation of funds and collectively banks grew short of liquidity. The analysis also showed that interbank payment systems can provide an almost real-time source of data that could be used in crisis management.

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[3] i.e. where low degree nodes are more likely to connect with nodes of high degree

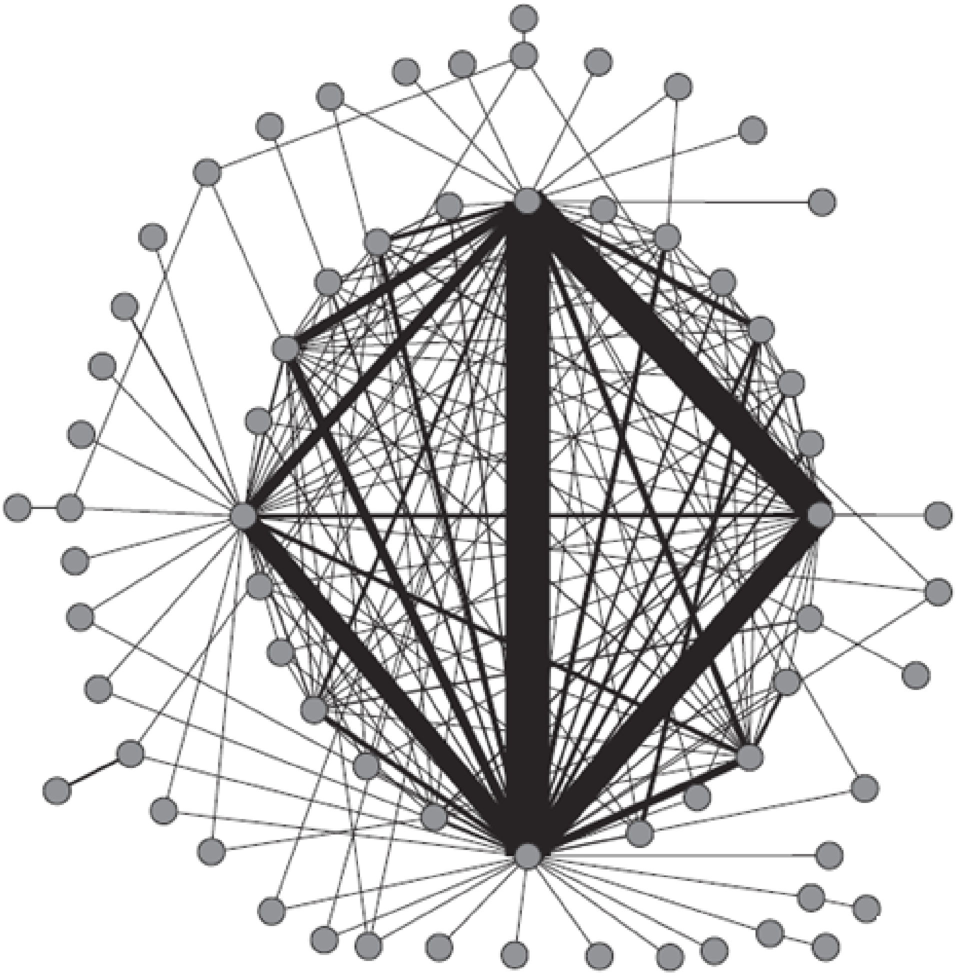

Figure 1: Core of the Fedwire interbank payment network on a representative day. Each node represents a bank and the width of links between the nodes scale with value of payments exchanged by the two banks. The links depicted cover 75% of daily value transferred.

More recently, a number of other studies have looked at national interbank networks, reconstructed using interbank payment flows and enabled by access to data from interbank payment systems operated by central banks. These include among others Lubloy (2006) on flows in Hungary, Becher et al. (2008) in UK, Boss et al. (2008) in Austria, Pröpper et al. (2009) in Netherlands, Embree and Roberts (2009) in Canada, Akram and Christophersen (2010) in Norway and Jia (2011) in China.

The unsecured overnight money market is another segment of financial markets where network analysis has been applied intensively. Money markets constitute the locus where banks exchange deposits and represent a possible channel of financial contagion. A disturbance in the financial system quickly manifests itself in this market for short term funding.

In order to gain insights into unsecured interbank loan networks, variations of a methodology proposed by Furfine (1999) for matching two temporally separated payments (advance and repayment) as a loan have been applied to payment data. This allows the construction of network time series of the unsecured money market. Loan data of this level of granularity are generally not available from other sources.

A representative paper following this approach is Atalay and Bech (2008), who use data from Fedwire to recover federal funds loans and analyze the network properties of this market. Other applications are e.g. Bech and Bonde (2009) on the Danish interbank market, Wetherilt et al. (2009) in UK and Heijmans et al. (2010) in the Nertherlands. Iazzetta and Manna 2009 identify banks that are important in terms of a liquidity crisis, based on the distribution of liquidity among Italian banks since 1990 and Iori et al. (2008) study the Italian overnight lending markets based on data from eMID.

Empirical research on the other parts of the financial system was initially less common due to the restricted nature of sufficiently detailed data. Bonanno et al. (2004) is an early analysis on networks of financial stocks. Degryse and Nguyen (2007) investigate the extent of systemic risk and network structure in the Belgian banking system over a ten-year period and Castrén and Kavonius (2009) sectorial networks based on flow-of-funds data. Recently, however, the literature has expanded quickly into many areas such as credit networks among banks (Bastos e Santos and Cont 2010), global banking networks (Garratt et al. 2011, Minoiu and Reyes 2011), stock trading networks (Adamic et al. 2009, Jiang and Zhou 2011) or Granger causality networks for contagion in banking (Billio et al 2009) to name a few.

3 System mechanics

From a network perspective, the performance of banks (nodes) is often dynamically dependent on the performance of other banks within the network and upon the structure of linkages between them. A failure by one node in the network, for example, may hinder flows in the network and adversely impact the performance of the other nodes when the disturbance propagates in the network.

3.1 Counterfactual simulations

A natural way to analyze the results of these dynamics is via counterfactual simulations. In this approach the details of the system are replicated in a computational model and real transaction data is used to first replicate the behavior of the real system. In a second step, desired aspects of the system are altered to see how - ceteris paribus - the system performs under the new circumstances.

Interbank payment systems have been an area of financial infrastructures where economy-wide simulations have been carried out for research and policy purposes longer than in other areas of financial research. Article [II] evaluates the effectiveness of alternative methods of settling Fedwire payments in reducing intraday credit extensions. The article carries out counterfactual simulations using various deferred settlement mechanisms that complement RTGS systems - including a novel mechanism entitled Receipt Reactive Gross Settlement (RRGS). The basic idea of RRGS is that banks use only incoming funds to settle their less urgent payments. Each bank has the incentive to submit payments to the RRGS queue as costly liquidity is consumed only when the bank receives funds from other banks. The results suggested that in conjunction with RTGS systems, the RRGS mechanism could significantly reduce daylight credit extensions while modestly delaying the average time of payment settlement.

Subsequent simulation based research work has studied many other dynamics of payment systems, where system rules have varied from simple real-time gross settlement to complex hybrid settlement mechanisms with offsetting and multilateral settlement capabilities. The research can be summarized as trade-off questions between liquidity and speed of settlement (e.g. Leinonen and Soramäki 1999) or risks (e.g. Humphrey 1986, Angelini et at 1996, Galos and Soramäki 2005 and Bech and Soramäki 2005). An overview of this line of research is provided in volumes edited by Leinonen (2005, 2007 and 2009).

Simulations on market infrastructures have also been used in policy. For example, they were an integral part in the regulatory approval of Continuous Linked Settlement (CLS) system which launched in 2003. CLS is currently world’s largest settlement system, settling on peak days almost $9 trillion worth of foreign exchange transactions on the books of 17 central banks (and currencies). Due to the high and time critical liquidity requirements for participants and interlinkages of the system to many currencies and economies, regulators placed special emphasis on assessing its impact on liquidity markets (ECB 2003). For this purpose CLS carried out and regulators evaluated a wide range of simulation scenarios (CLS 2009).

Other examples include Bank of Japan who evaluated alternative liquidity saving mechanisms before their implementation (Imakubo and McAndrews 2006). Also the Eurosystem has recently embraced payment system simulations as an ongoing oversight tool by specifying how the transaction level data may be used (EU 2010) and developing a TARGET2 simulation platform. Another recent example is the project to develop new features for CHAPS interbank payment system in the UK. Denbee and Lafferty (2012) use real payment data to quantify the liquidity efficiency that could be obtained in CHAPS, the UK's large-value payment system, by the implementation of a liquidity saving mechanism.

3.2 Congestion and cascades

Counterfactual simulations presented in the previous section are close to various percolation and cascade models developed within statistical mechanics and network theory. One branch of this literature has investigated the resilience of processes in different network topologies in terms of a connectivity (or percolation) threshold (see e.g. Bollobas 1985, Moore and Newman 2000 and Callaway et al. 2000) at which a network dissolves into several disconnected components. A well-known finding is that scale-free networks are more robust to random failures than other types of networks. However, they are very susceptible to the removal of the very few highly connected nodes. However, such static failure models are more applicable to networks where the interest is the availability of paths between nodes in the network (e.g. in transportation) - but are less applicable to networks of monetary flows which contain both flows via the shortest paths as well as any walks within the network.

Another branch of the literature has studied the impact of perturbations that cascade through the network on the basis of established theoretical or domain-specific rules (see Sachtjen et al. 2000 and Kinney et al. 2005 for power networks). In these dynamical models nodes generally have a capacity to operate at a certain load and, once the threshold is exceeded, some or all of the node's load is distributed to neighboring nodes in the network. While the detailed dynamics depend on the rules applied for the cascades, generally the most connected nodes (or nodes with highest load in relation to overall capacity) are more likely than an average node to trigger cascades.

Cascade models have been applied to various systems in fields such as geology, biology and sociology (e.g. Jensen, 1998). This research has demonstrated that models made of very simple agents, interacting with neighboring agents, can yield surprising insights about system-level behavior.

In the spirit of these cascade models, article [III] formulates a simple agent-based model for liquidity flows within a simulated payment system. The paper applies methods from statistical mechanics to describe dynamics of the interbank payment system. Other cascade models on financial systems have been developed e.g. in Nier et al. (2007) who also used simulated exposure data, and Bastos e Santos and Cont

(2010) who use data from the Brazilian credit registry. Recently also others (Lorenz at al. 2009, Georg 2011) have applied cascade models to systemic risk analysis.

The model in article [III] combines counterfactual simulation research with cascade models to create an agent-based simulation with static rules of behavior for the model's agents. The model consists of depositors who hold accounts at banks and randomly instruct their banks to make unit payments to other depositors in the economy. Banks hold accounts at the central bank, and these balances can be used to transfer funds between them (i.e. forming the interbank payment system). Banks are reflexively cooperative: they submit a payment for settlement to the central bank if their central bank account balance has enough cover; otherwise they place the instruction in a queue for later settlement. If a bank that receives a payment has instructions in its queue, the payment it just received enables it to remove a queued instruction and submit it as a payment. If the bank that receives that payment is also queuing instructions, then it can make a payment, and so on. In this way, a single initial payment made by a bank can cause many payments to be released from the queues of the downstream receiving banks. This is an example of the cascade processes typically studied in other models of self-organized criticality (Bak et al. 1987).

The model has two parameters that control bank interdependence: overall liquidity and conductance of a liquidity market. Abundant liquidity allows banks to operate independently; reducing liquidity increases the likelihood that a given bank will exhaust its balance and begin queuing payments. A bank that has exhausted its balance must wait for an incoming payment from one of its neighbors. When liquidity is low a bank's ability to process payments becomes coupled to its neighbors' ability to process. The output of the payment system as a whole is no longer determined by overall input, but instead becomes dominated by the internal dynamics of the system.

The model was elaborated to explore how liquidity markets reduce coupling among network neighbors and thereby reduce congestion. Liquidity market transactions were represented as a diffusive process, where a bank's balance plays the role of a potential energy or pressure. Banks with high balances tend to contribute liquidity to the market, while banks with low balances tend to draw liquidity from the market. There is no decision-making or price setting in this simple market model, but it reflects two essential features of a real market: liquidity flows from banks with surplus funds to banks that need funds, and liquidity can flow from any bank to any bank - flows are not confined to the links of the payment network. It creates a separate global pathway for liquidity flow.

With a liquidity market included, the number of payments closely tracks the number of instructions as the coupling between banks is weakened and the size of the settlement cascades is reduced. The rate of liquidity flow through the market relative to the rate of flow through the payment system is very small. The performance of the system can be greatly improved even when less than 2 per cent of the system through-put flows through the market. However, this also shows the vulnerability of the system. If the flow of liquidity is severed, the performance of the system is greatly deteriorated.

Renault et al. (2009) has subsequently extended the model to two interdependent payment systems connected via a foreign-exchange market. In this case, further interdependence is created by a Payment versus Payment (PvP[4]) constraint that links the two legs of the foreign exchange transactions and can import instabilities from one currency area to the other.

4 Behavioral dynamics

The previous section discussed the complex mechanics of financial systems. It did not address how the banks interact strategically with each other on those systems - possible in a repeated manner. The difference is that most mechanical rules do not take into account how the reactions of rest of the system influence one's own actions - i.e. are not strategic. The actions of participants have the potential to either mitigate or augment the system's stability. Much of the literature on counterfactual simulations has been subject to a form of 'Lucas critique', i.e. that the behavioral response of banks in the system has not been engodenized in the research - leading to possibly wrong conclusions. Hence, understanding how the participants interact and react when faced with operational adversity will assist operators and regulators in designing countermeasures, devising policy, and providing emergency assistance, if necessary.

The first approach to study bank behavior in interbank payment systems was to use standard game theory. Angelini (1998) uses a setup derived from earlier literature on precautionary demand for reserves and shows that in a RTGS system, where banks are charged for intraday liquidity, payments will tend to be delayed and that the equilibrium outcome is not socially optimal.

Bech and Garratt (2003) analyze strategic incentives under different intraday credit policy regimes employed by central banks and characterize how the Nash equilibria depend on the underlying cost parameters for liquidity and delays. They use a stylized

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[4] PvP is mechanism which ensures that the final transfer of a payment in one currency occurs if – and only if – the final transfer of a payment in another currency or currencies takes place.

two period - two player "liquidity management game" to analyze intraday liquidity management behavior of banks in a RTGS environment. It turns out that two classical paradigms in game theory - the Prisoner’s Dilemma and the Stag Hunt emerge from the analysis - thus permitting policy questions to be understood in terms of well-known conflicts and dilemmas in economics.

More recently, Martin and McAndrews (2008) develop a two-period model where each bank in a continuum has to make and receive exactly two payments of unit size. Banks have to choose when and how to make payments. They can choose to pay either via the a “Real-time gross settlement” (RTGS) stream, or via a “liquidity saving mechanism” (LSM). Both delayed payments and use of liquidity generate costs. Banks may be hit by liquidity shocks - i.e. the urgency of certain payments is unknown ex-ante. The model is solved analytically under assumptions about the pattern of payments that may emerge.

Analytically solvable models need by design be simple in their mechanics and they cannot capture the rich interaction between the decisions of the system’s participants, its internal mechanics and the feedback of this back to the participants. Other approaches that have been applied to similar problems of repeated interaction among a large number of players stem from evolutionary game theory, reinforcement learning or agent-based modeling (ABM) in general.

In an ABM, the system under study is modeled as a collection of autonomous decision-making entities called agents. Each agent assesses its situation individually and makes decisions on the basis of a set of rules. The behavior of the system is an emergent property of the behavior of its constituents. The rules by which the agents operate can be static, or the agents may learn over time to improve their actions.

Agents who learn about each others' actions through repeated strategic interaction is a topical theme in evolutionary game theory (see a recent discussion in Kirman 2010). Often existing literature looks at the players' asymptotic behavior in situations where the payoffs are some known function of players' strategies. In theory of repeated games, this knowledge is a prerogative of the players, who can therefore use adaptive rules of the type "choose a best reply to the current strategy profile". In a reinforcement learning tradition (see e.g. Sutton and Barto 1998 and Fudenberg and Levine 1998), the rules do not require knowledge of the payoff function on the part of the learners. Such rules are instead of the kind "adopt more frequently a strategy that has given a high payoff".

4.1 Agent-based model of payment systems

Article [IV] uses fictitious play (Brown, 1951) to numerically solve a model with interactions among a number of banks that settle payments on a continuous basis under imperfect information, stochastic payoffs and a finite but long sequence of settlement days. It is a dynamic multi-agent model of an interbank payment system where payments are settled on the basis of pre-committed funds. In the model, banks choose their level of committed funds on the basis of private payoff maximization. It builds on the previous articles in the Dissertation by adding a layer of economic behavior to interactions in a given network topology and complex system mechanics.

The literature developing agent-based models for understanding financial systems stems back to the Santa Fe Artificial Stock Market in the early 90's (Arthur et al. 1997). Modeling financial markets and prices has remained an active field. As for models on financial stability, Martinez-Jeremillo (2007) and Martinez-Jaramillo and Tsang (2009) study extensively evolution in agent-based financial markets. Markose et al. (2010) develop agent based model for contagion from Credit Default Swaps. Adams et al. (2010) and Galbiati and Giansante (2010) look at the formation of interbank payment systems.

The model in article [IV] consists of a sequence of settlement days. Each of these days is a simultaneous-move game in which each bank chooses the amount of liquidity to commit for payment processing, and receives a stochastic payoff. Payoffs are determined by means of simulating the settlement day with the amounts of liquidity chosen by the banks. Instructions to be settled by the banks arrive on the basis of a Poisson process and they are ex-ante unknown to the banks. As discussed in the previous sections, the relationship between instruction arrival and payment settlement is very complex. Adaptation takes place through reinforcement learning with Bayesian updating, with banks maximizing immediate payoffs.

Through the process of individual pay-off maximization, banks adjust their demand for liquidity up (reducing delays) when delay costs increase, and down (increasing delays), when they rise. It is well known that the demand for intraday credit is generated by a tradeoff between the costs associated with delaying payments, and liquidity costs. Simulating the model for different parameter values, we find that the demand for intraday credit is an S-shaped function of the cost ratio between intraday credit costs and the costs associated with delaying payments .

4.2 Liquidity-saving mechanisms and bank behavior

The model of article [IV] was further extended in article [V] for the study of liquidity saving mechanisms. In this model, banks make a choice on the amount of funds to commit for settlement, and an additional choice on which method to use for settling the payments - normal real-time gross settlement or an alternative stream where payments are offset against each other (based on an algorithm developed in Bech and Soramäki 2001). The second stream is called a Liquidity Saving Mechanism (LSM).

The liquidity dynamics of such a system are not immediately obvious as both streams exhibit economies of scale. The more payments are submitted to each stream, the faster it can settle payments. Due to the fact that payments have different levels of criticality (i.e. penalties for settlement delays) all payments cannot be settled efficiently in the LSM - which suits better payments of lower criticality. This means that some payments are better settled in the first stream, but how many?

The article finds that liquidity saving mechanism can improve the system performance by moving the system to a better equilibrium compared to a system with only real-time gross settlement. This equilibrium is, however, not socially optimal. Even better outcomes would be achieved if system participants would commit more liquidity - a resource all of them are economizing on but everyone collectively would benefit from.

The article looks at the equilibrium liquidity and routing choices. A typical equilibrium has banks routing part of their payments to RTGS, and part into the LSM, with the reliance on the LSM increasing with the price of liquidity. Despite the fact that such an outcome is inefficient, it can still be better than the one emerging without the LSM. Depending on prices, the planner would choose to settle payment in the first stream and allocate a lot of liquidity or in the second stream and allocate no liquidity - but never settle payments in both. Thus, an LSM may lead to a “second best” outcome, improving on the plain RTGS system.

The system with an LSM, however, also possesses some “bad” equilibria. These equilibria feature the somehow paradoxical mix of high liquidity usage, intense use of the LSM, and costs which exceed those of the basic RTGS system. The reason behind the existence of such equilibria is probably the following: if many payments are sent in the LSM, this can be self-sustaining, in the sense that each bank finds it convenient to do so. However, the RTGS stream may become less expedite (as fewer payments are processed there), which may in turn imply that the equilibrium level of liquidity is also large. This suggests that LSMs can be useful, but they may need some co-ordination device, to ensure that banks arrive at a “good” equilibrium.

5 Concluding remarks

This final section provides some views how this Dissertation may be useful for future research on financial systems, how it can be operationalized into policy, and what challenges lie ahead.

In a way, payment system research has been an area of financial research where simulation and network models have been applied first - probably because data of the necessary granularity has been available from interbank payment systems. Results gained in this field have in also some cases been used to argue for regulatory change in others (see e.g. Lo 2009). Better data on other infrastructures and markets will allow regulators to develop more complete maps of the financial system. These maps will be invaluable for decision support of crisis management, when time is too short to develop more robust models and when intuition can be assisted by data mining. Financial network analysis and network visualizations are likely to be good tools for gaining this intuition and for developing more objective network based metrics - as presented in the first article. The challenge for regulators is to develop these systems so that they can be monitored over time. Financial relationships are in a continuous flux and can rapidly change form in the event of a financial crisis.

In addition to maps of the financial system at a given point of time, research and policy makers need to take into account the mechanics of the financial system in order to be able to make informed forecasts of likely courses of events. These mechanics have become more complex and automated with continuous advances in technology. Human decision making and therefore economic behavior is in many cases pre-programmed as algorithms and as a consequence bounded in addition to rationality also by the operational IT framework. Modern financial systems involve a high degree of automation both in how decisions are made and how they are executed - including algorithmic trading, straight-through-processing of settlement and automated rules for liquidity management of the settlement process. These automated infrastructures have become ubiquitous but are not well understood. For example, under stressed market conditions, the automated execution of trades can trigger extreme price movements and the interactions between automated orders and algorithmic trading strategies can be unpredictable[5] - and once the rules have been set up they are part of the system mechanics. The system is highly complex and even in the absence of any human decision-making, the system exhibits many feedback loops that affect the system's performance - as also shown by the second and third article in the

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[5] CFTC-SEC joint report (2010) on the 6 May 2010 "Flash crash"

Dissertation. Simulations can bring an improved understanding of the interaction of the mechanics of a system.

Finally, no description of an economic phenomenon is complete without strategic and economizing agents. Agent-based models such as those presented in the fourth and fifth article of the Dissertation are necessary to more completely describe the behavior of a system. In both papers the equilibrium behavior deviated from any corner solution or the planner’s choice absent of strategic behavior. The distinction to mechanics is sometimes, however, arbitrary. The vast majority of trades in most advanced financial systems are executed by algorithms of varying sophistication that may or may not be considered to act strategically. However, in crisis situations human decision making may produce very different outcomes than algorithms that have been trained to operate on historical or steady-state data - and intuition both by market participants and regulators will continue to play a key role.

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