Aalto University
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Solving the Green Vehicle Routing Problem

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- Andelmin, J., Bartolini, E. (2017). **An Exact Algorithm for the Green Vehicle Routing Problem**. *Transportation Science.* Advance online publication. http://doi.org/10.1287/trsc.2016.0734
- Andelmin, J., Bartolini, E. **A Multi-Start Local Search Heuristic for the Green Vehicle Routing Problem Based on a Multigraph Reformulation.** Submitted to *Computers and Operations Research*

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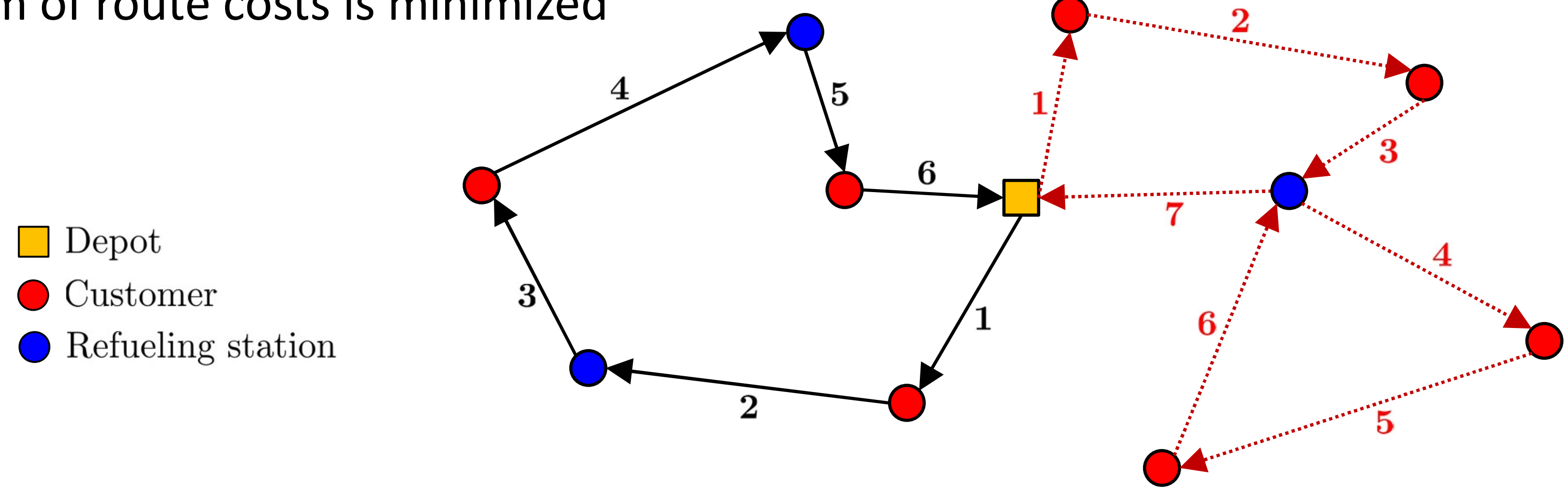
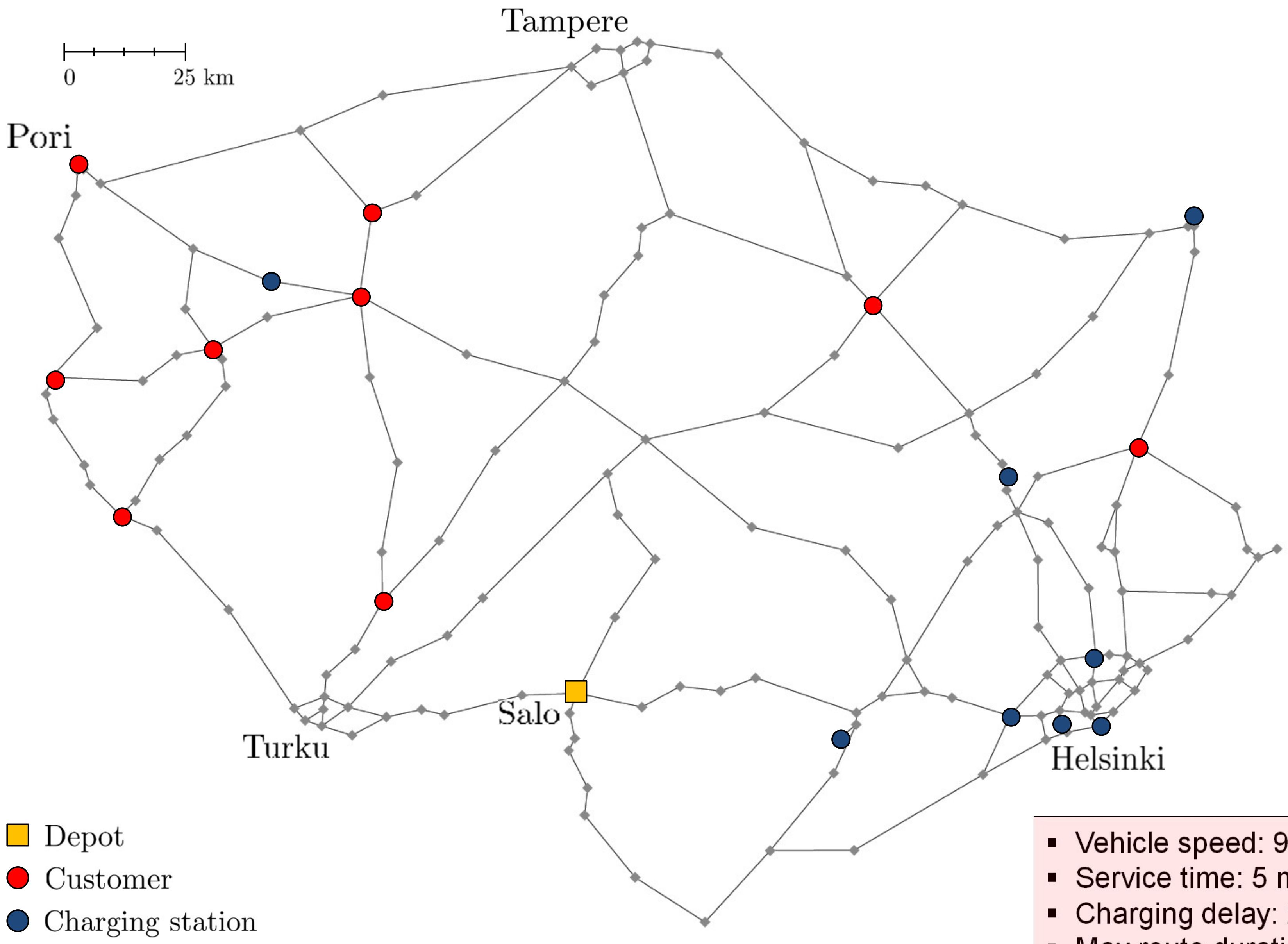
Green Vehicle Routing Problem (G-VRP)

- A fleet of vehicles based at a depot is to serve a set of customers
  - Customers have known service times
  - Vehicles have limited fuel capacity
  - Vehicles can visit refueling stations to refuel
- Objective: Design a set of vehicle routes so that
  - Every customer is served
  - Duration of each route $\leq$ T
  - Sum of route costs is minimized



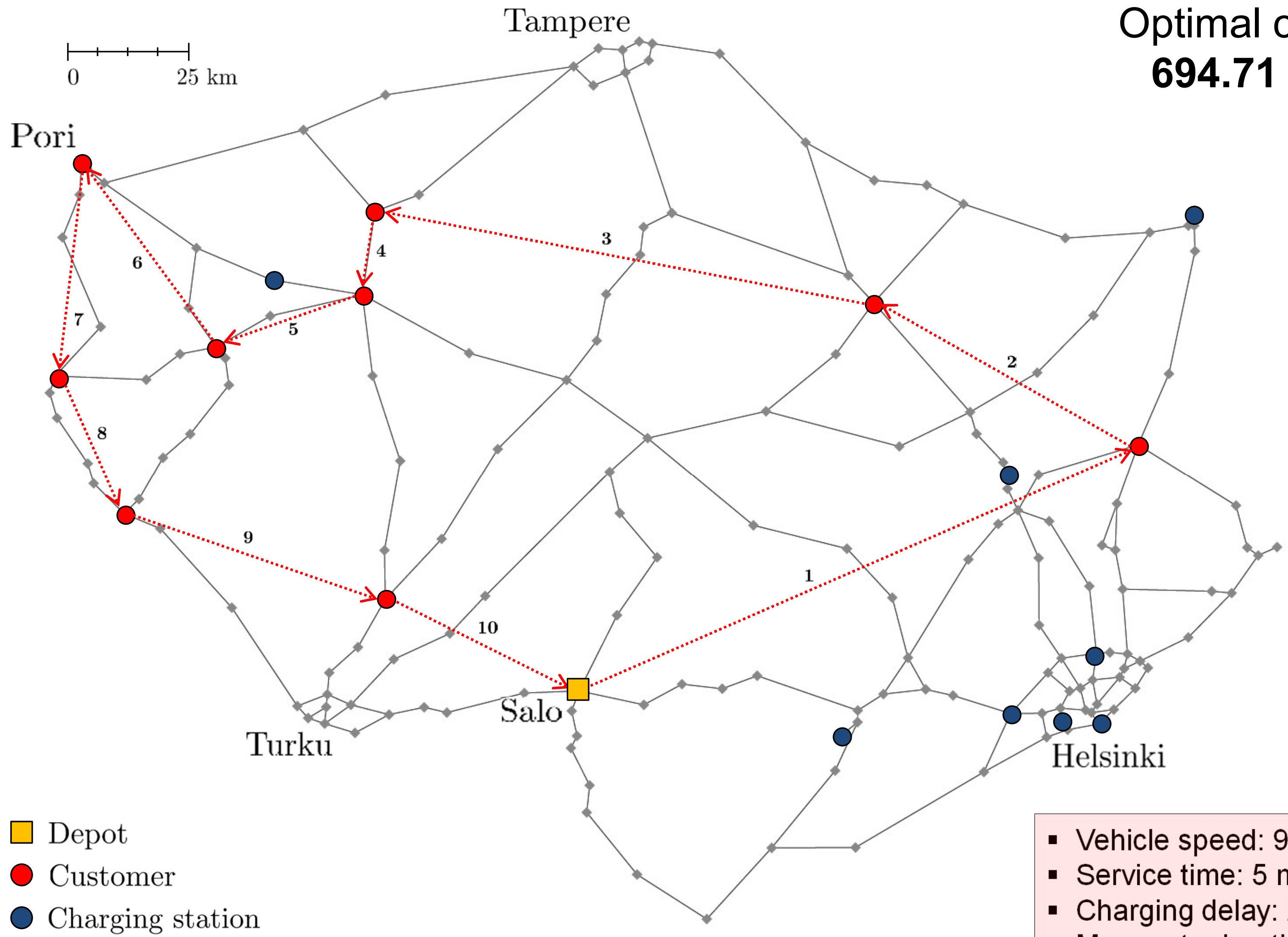
Simple example: 9 customers, electric vehicles



- Vehicle speed: 90 km/h
- Service time: 5 min
- Charging delay: 20 min
- Max route duration: 12 h

Optimal solution with driving range = ∞

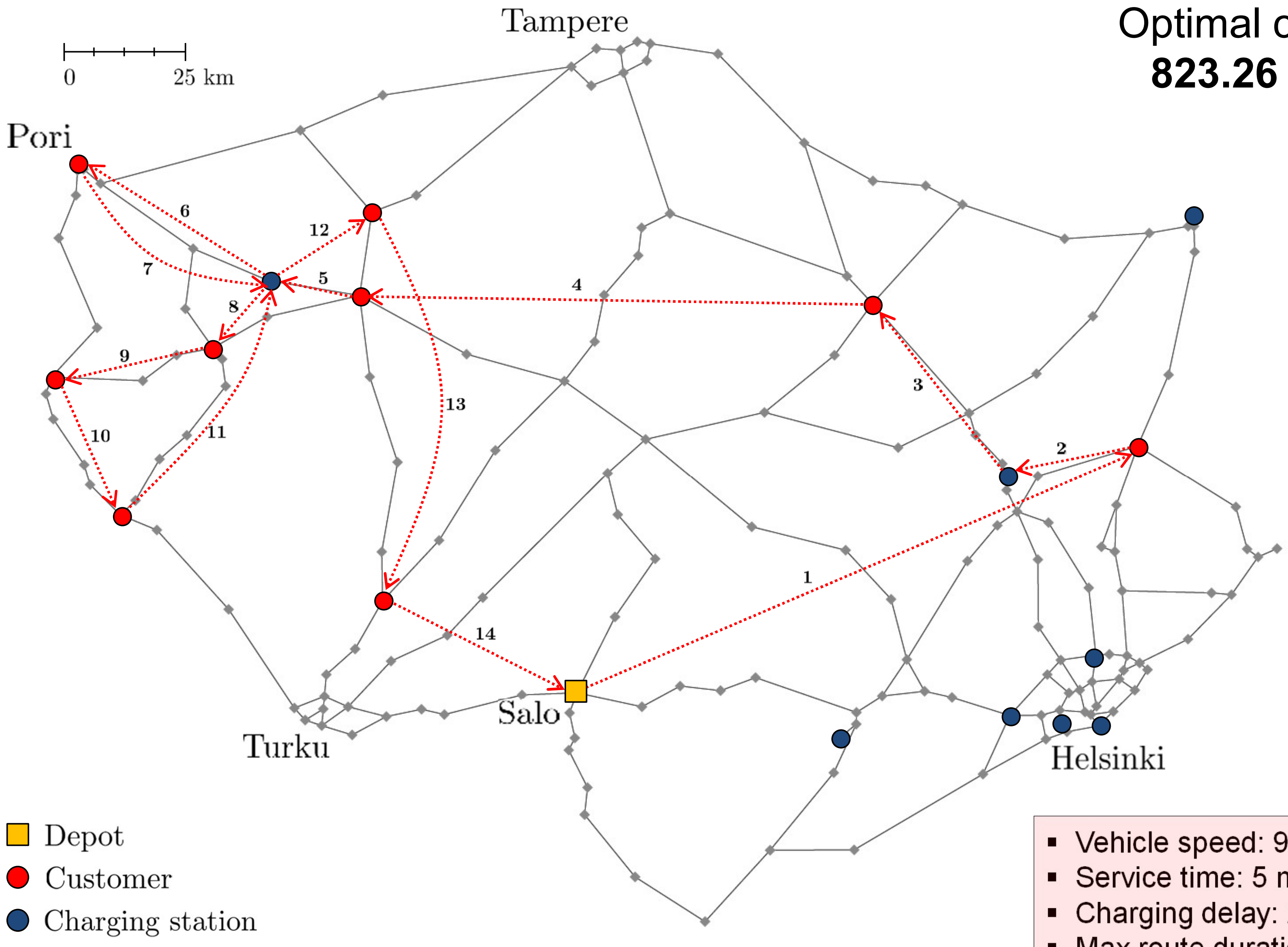
Optimal cost
694.71 km



- Vehicle speed: 90 km/h
- Service time: 5 min
- Charging delay: 20 min
- Max route duration: 12 h

Optimal solution with driving range = 200 km

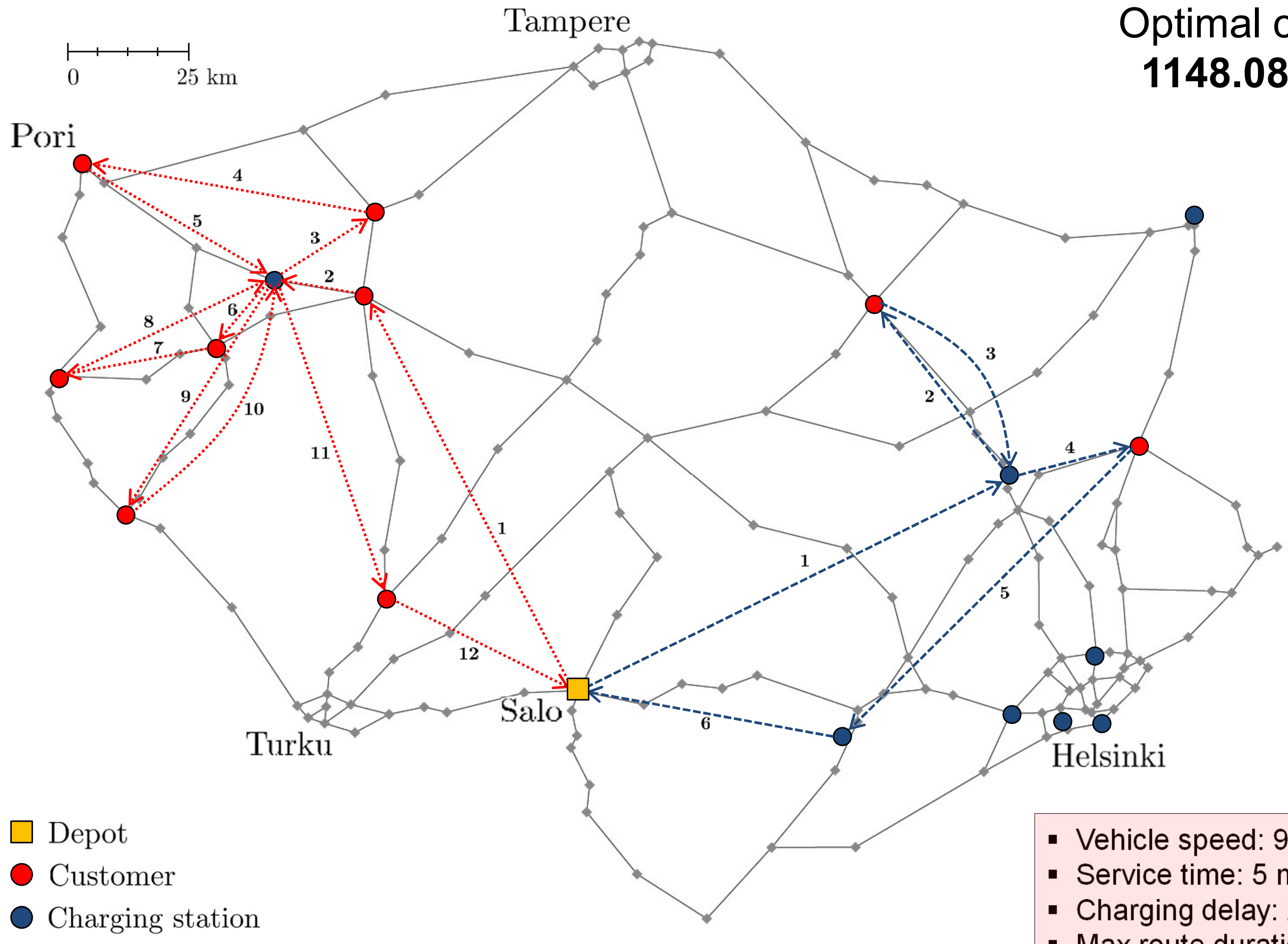
Optimal cost
823.26 km



- Vehicle speed: 90 km/h
- Service time: 5 min
- Charging delay: 20 min
- Max route duration: 12 h

Optimal solution with driving range = 160 km

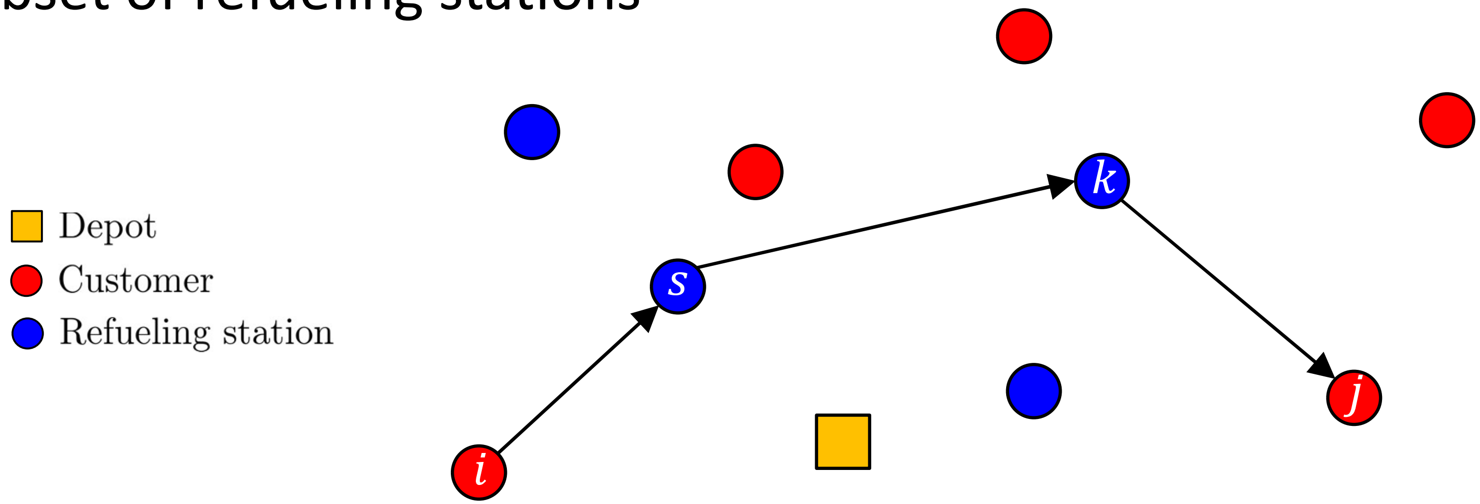
Optimal cost
1148.08km



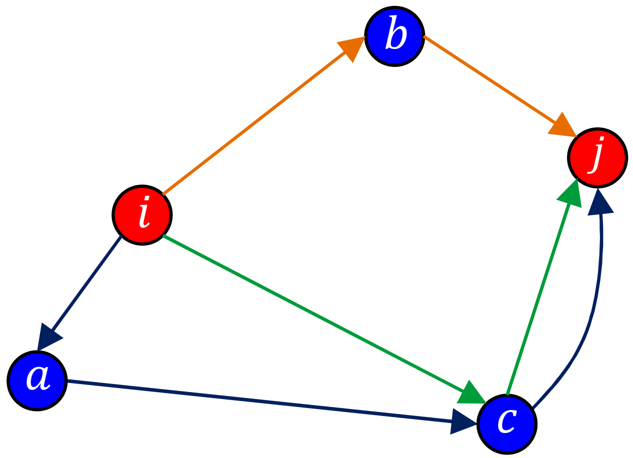
- Vehicle speed: 90 km/h
- Service time: 5 min
- Charging delay: 20 min
- Max route duration: 12 h

Refuel paths

- ***Refuel path***: a simple path between two customers that visits a subset of refueling stations



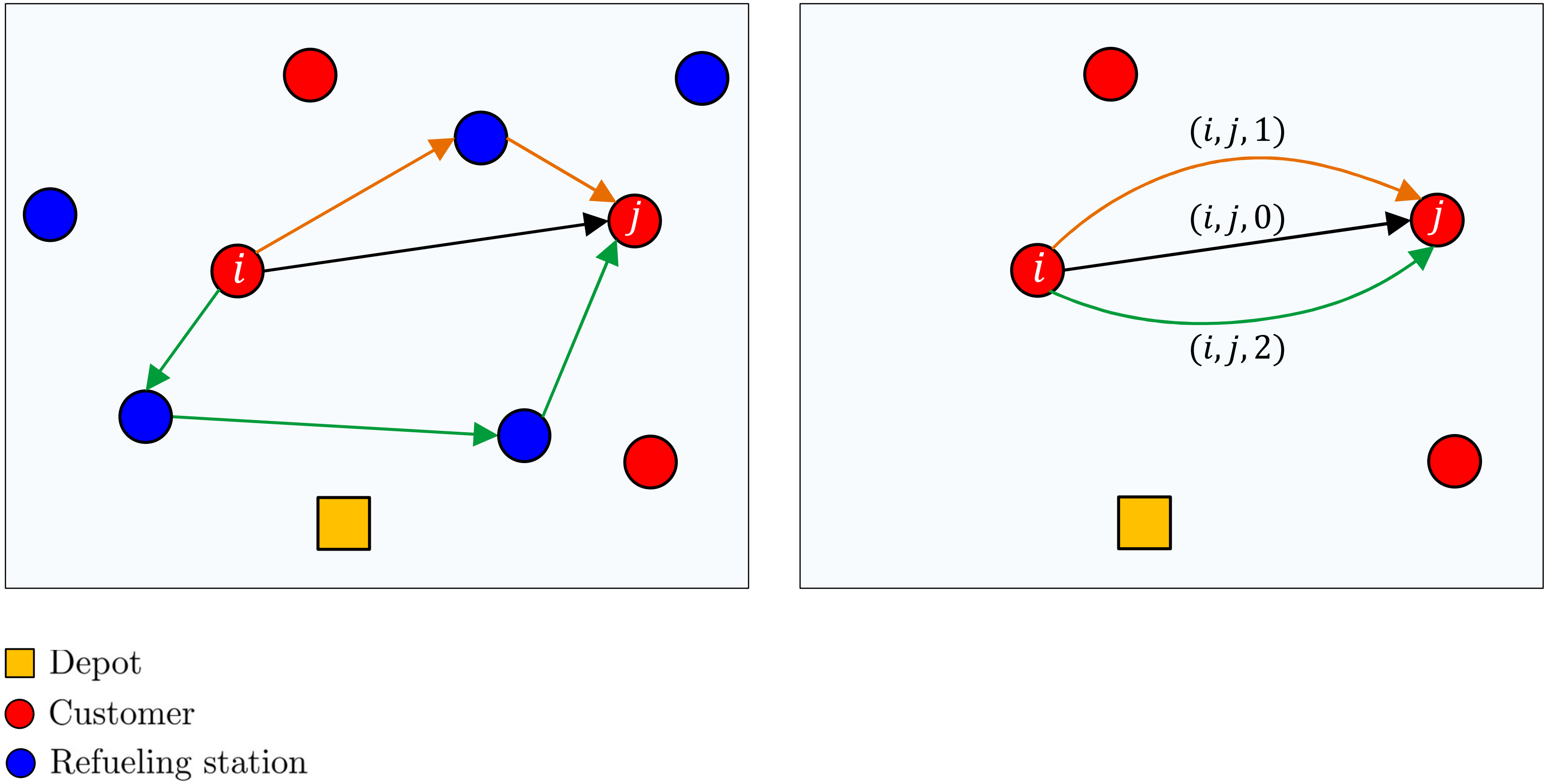
- Many refuel paths are dominated
- Example:
  - **Green path** $i \rightarrow c \rightarrow j$ is dominated by **orange one** $i \rightarrow b \rightarrow j$



Multigraph

- We model the G-VRP on a multigraph $\mathcal{G}$ with one arc for each non-dominated refuel path

Two refuel paths + direct arc from i to j — Three corresponding arcs in $\mathcal{G}$

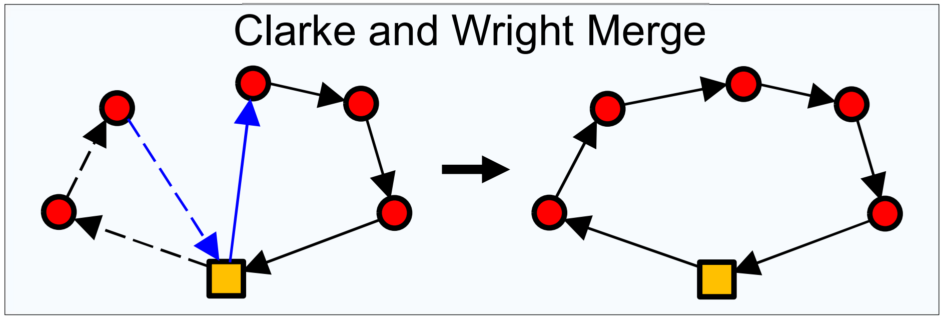


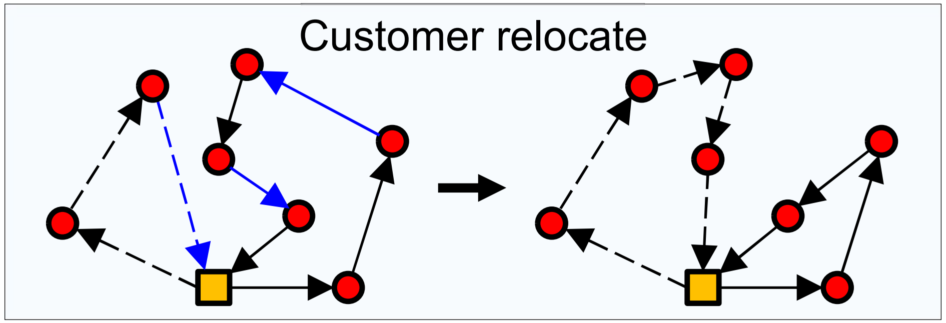
Multi-Start Local Search Heuristic (MSLS)

Three phases

1) Iteratively construct new solutions
2) Store vehicle routes forming these solutions in a pool $\mathcal{R}$
3) Find a set of routes in $\mathcal{R}$ that gives least cost solution

- Example operators used in phase 1





Exact algorithm

- Set partitioning formulation (SP)
  - Each possible vehicle route serves a subset of customers
  - Find least cost set of routes serving each customer exactly once

$$(SP) \quad \min \sum_{l \in \mathcal{R}} c_l x_l$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{R}} a_{il} x_l = 1 \quad \forall i \in N$$

$$x_l \in \{0,1\} \quad \forall l \in \mathcal{R}$$

c_l: cost of route l

x_l: 0-1 variable equal to 1 if route l is in solution

a_{il}: 0-1 coefficient equal to 1 if route l serves customer i

$\mathcal{R}$: index set of all possible vehicle routes

N: set of customers

Phase 1:

- Compute lower bound LB by solving Linear Programming relaxation of SP with Subset Row [4], Weak Subset Row [1], and *k*-path cuts [6]
- Compute upper bound UB with the MSLS heuristic

Phase 2:

- Enumerate all routes $\mathcal{R}^*$ having reduced cost $\leq$ UB – LB
- Solve SP using only the routes in $\mathcal{R}^*$ → **optimal solution**
- If all routes $\mathcal{R}^*$ cannot be enumerated optimality not guaranteed

Computational results

- Benchmark problems:
  - 56 instances with 20-500 customers and 3-28 stations
- Heuristic: best new solutions to instances with 111-500 customers
  - Compared to 7 state-of-the-art heuristics [2][3][5][7][8][9]
- Exact algorithm:
  - Instances up to 111 customers 28 stations solved to optimality
  - Best exact from literature [5] solves up to 20 customer instances

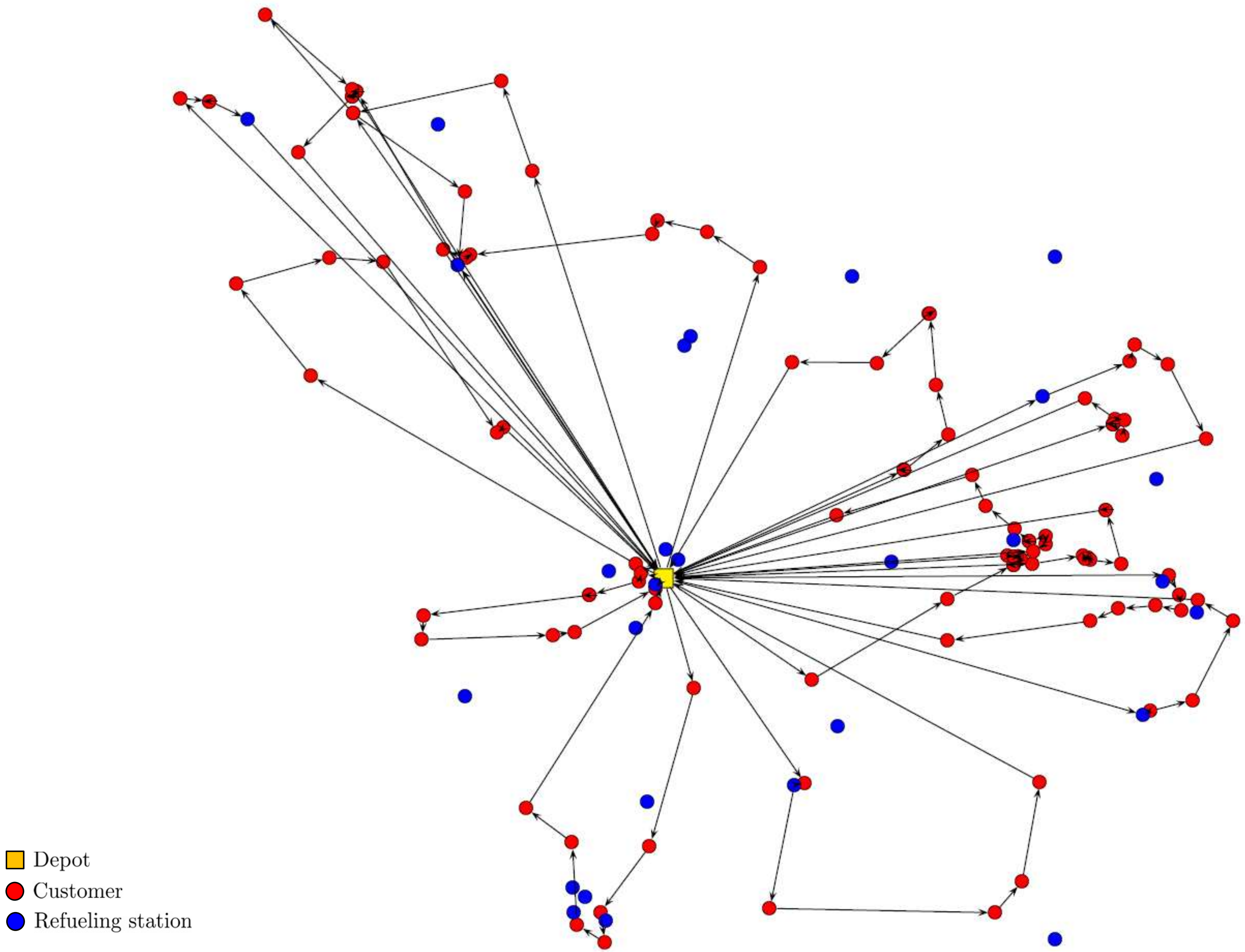
Instance name example:

75c_21s: 75 customers 21 stations

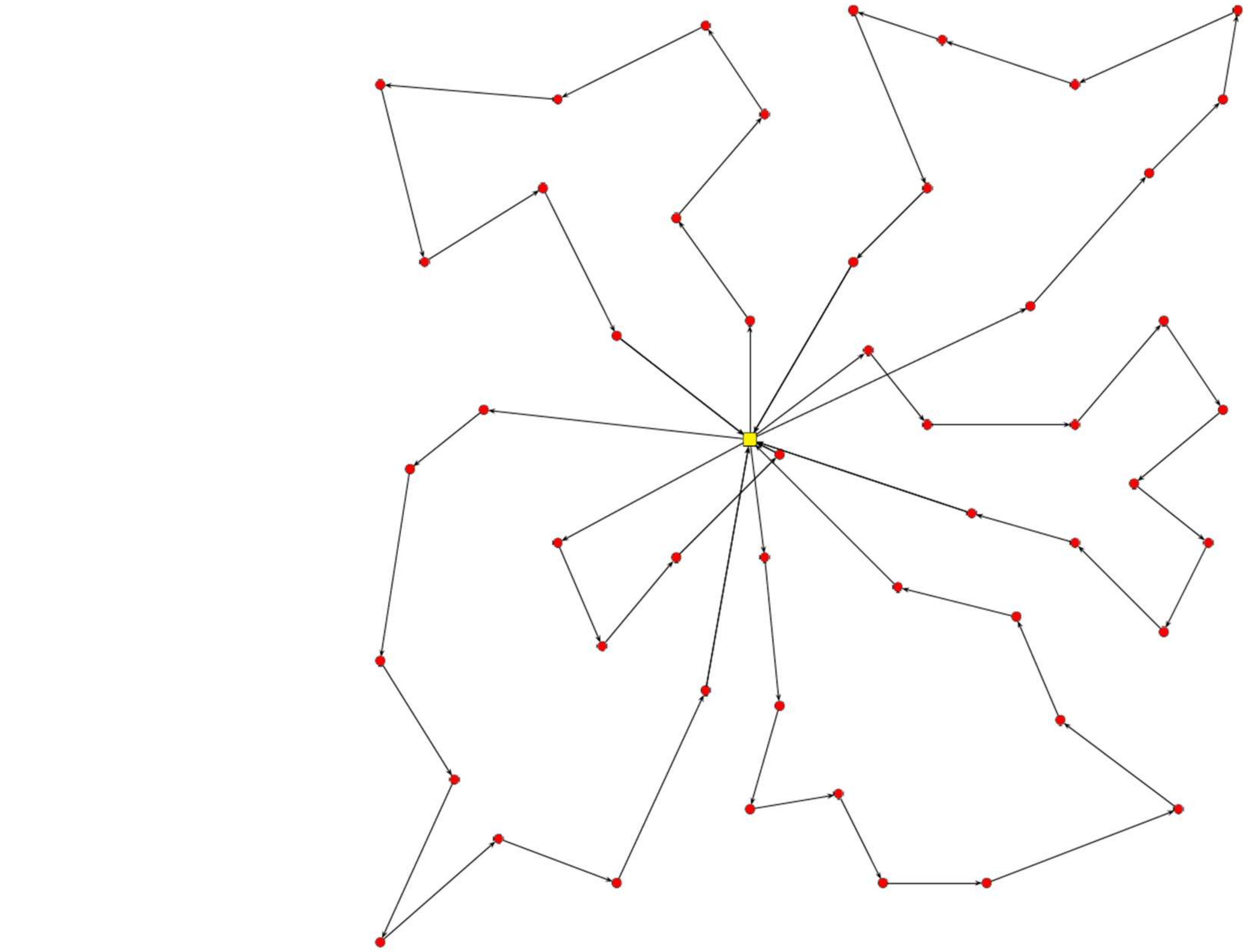
$$\%LB = \left(\frac{\text{UB} - \text{LB}}{\text{UB}}\right) * 100\%$$

Inst.	Opt	$\%LB$	Time(s)
75c_21s	*	0.00	5217
75c_28s	*	0.00	5582
100c_21s	*	0.00	5206
100c_28s	*	0.00	6531
111c_28s	*	0.13	11265
200c_21s		0.60	28135
250c_21s		0.57	27939
300c_21s		0.84	25903

Optimal solution to 111c_28s

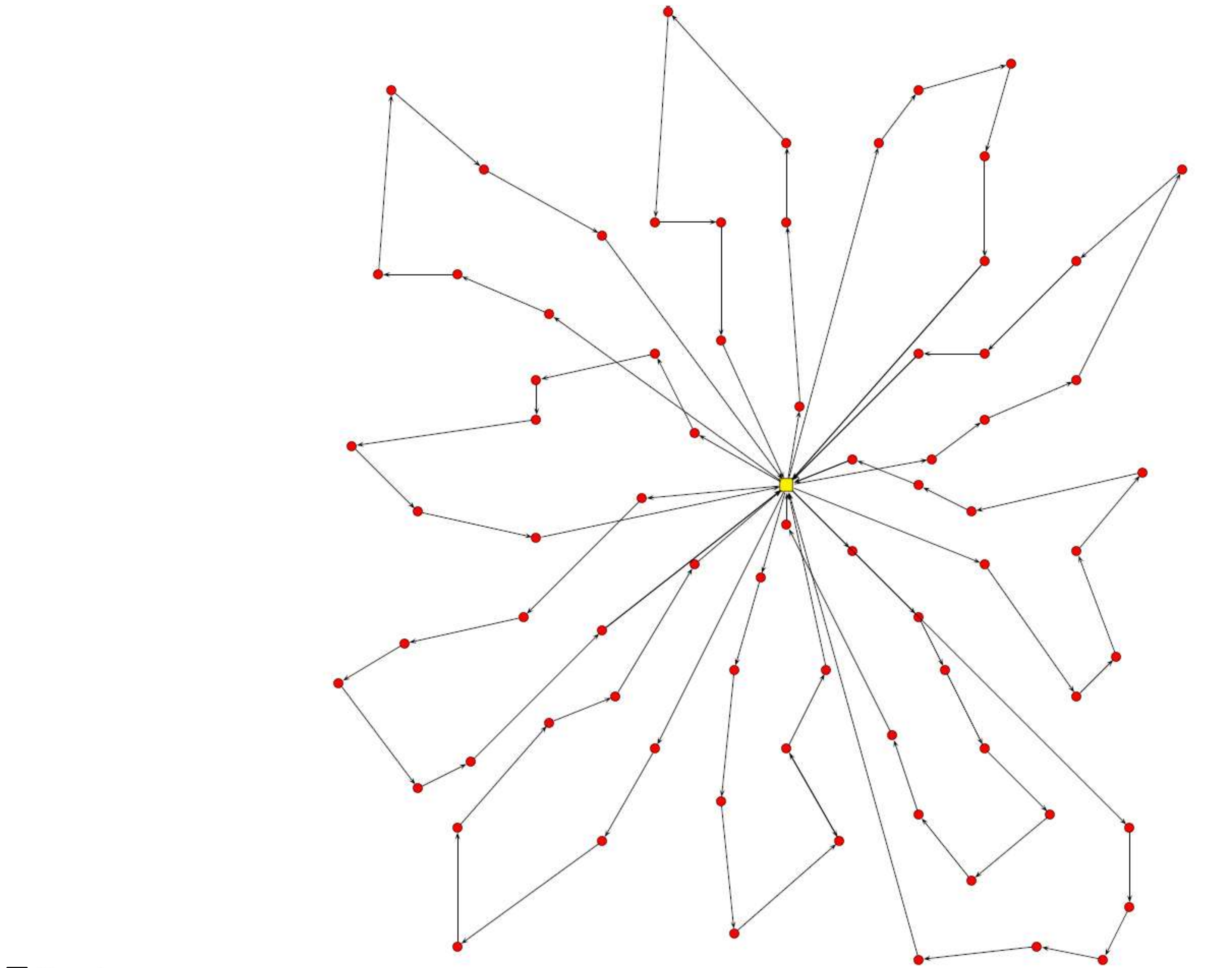


Optimal solution to Distance-constrained CVRP instance CMT6



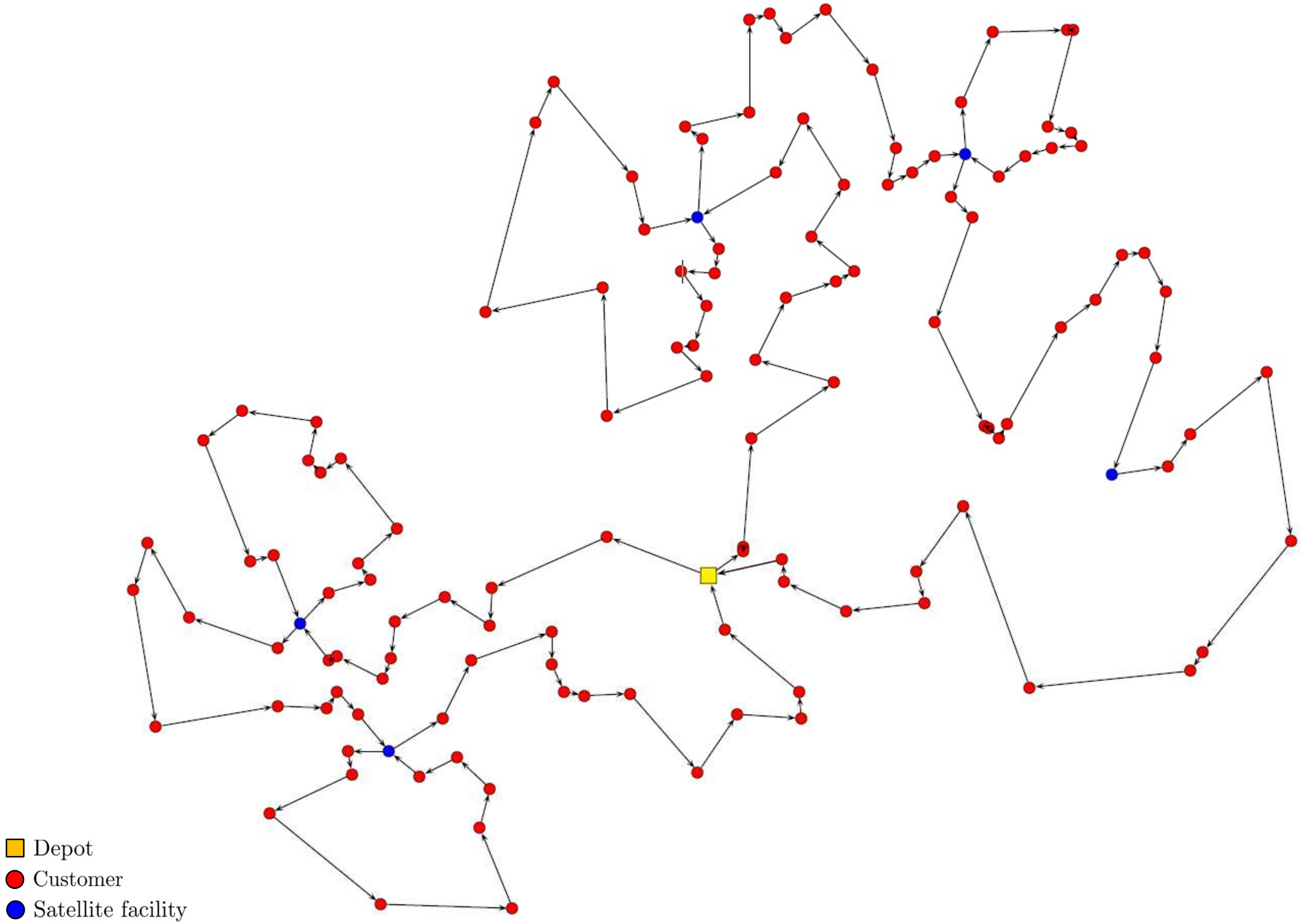
Instance	n	s	UB	Opt	$\%LB$	$\%LB_+$	Time(s)
CMT6	50	0	**555.43**	*	3.17	0.00	573

Optimal solution to Distance-constrained CVRP instance CMT7



Instance	n	s	UB	Opt	$\%LB$	$\%LB_+$	Time(s)
CMT7	75	0	**909.68**	*	1.65	0.15	1290

Heuristic solution to VRP with satellite facilities instance



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