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Modelling and optimization of polygeneration systems

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Energy systems for communities

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Research interests:

mathematical modelling,simulation, optimization, multicriteria decision support, energy systems

Career

- 81-85 **Helsinki U Tech**, MSc, *Systems & Operations Research*
- 85-90 **Nokia**, *Software methodology development*
- 90-94 **Helsinki U Tech**, PhD in *Applied Mathematics*
- 94-97 **U Jyväskylä**, associate professor in *Maths & SW engineering*
- 97-00 **VTT (Tech Res C Finland)**, professor, *Energy Markets*
- 00-09 **U Turku**, professor in *Computer Science*
- 09- **Aalto**, professor in *Energy for Communities* , 11-15 *head of dept*
- 17- Dual affiliation at ***Systems Analysis Laboratory***

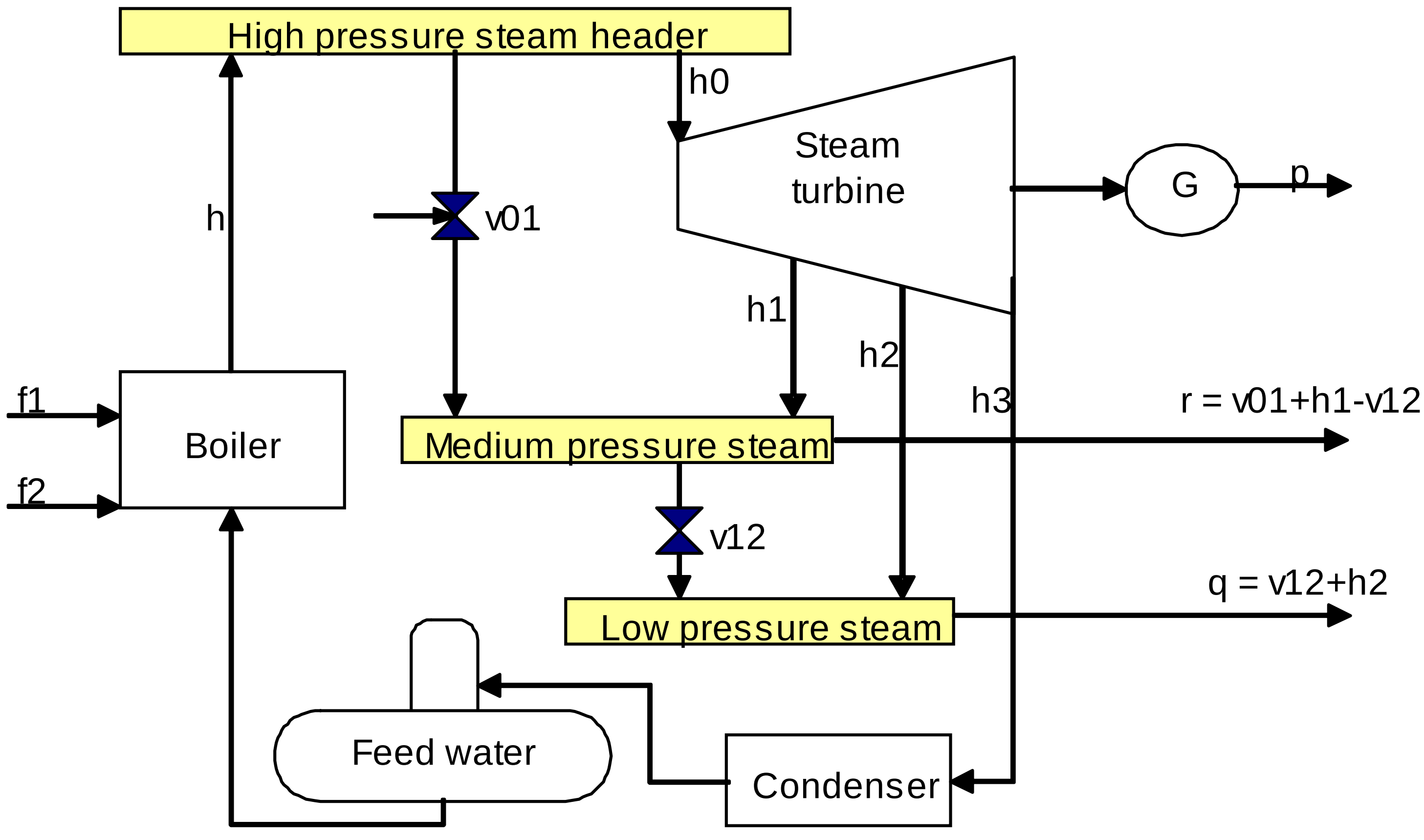
Outline of presentation

- Cogeneration
- Long-term planning model
- Efficient reformulation of model
- Efficient solution algorithms
- Reformulating non-convex models
- Experiences
- Selected publications

Cogeneration

- Cogeneration means production of two or more energy products together in an integrated process
  - CHP = combined heat and power generation
  - Trigeneration:
    - district heating + cooling + power
    - high pressure process heat + low pressure heat + power
  - Technologies: backpressure turbines, combined steam&gas turbines, **heat pumps** …
  - Much more efficient than producing the products separately
    - Energy efficiency increase 40% -> 90%
  - Cost-efficient way to reduce CO_2 emissions
  - Finland is one of the leading countries in cogeneration

Generic backpressure plant for trigeneration



Cogeneration planning

- Objective is to maximize profit subject to production constraints
- Hourly production of the different products must be planned together
  - Production of heat & cooling must meet the demand (natural monopoly)
  - Power production is planned to maximize the profit from sales to the spot market (liberalized power market)
- A long-term model consists of many hourly models in sequence
  - E.g. an annual model consists of 8760 hourly models
  - Hourly forecasts for demand and power price
- Various advanced analyses, e.g. risk analysis require solving many long-term models based on different scenarios
  - Solution must be fast!

Long-term planning model: Trigeneration

Min

subject to

$$\sum_{u \in U} C_u(x_u)$$

$$Hx^t = \begin{bmatrix} P^t \\ Q^t \\ R^t \end{bmatrix} \qquad t \in T \text{ (set of hours)}$$

$$x_u \in X_u \qquad u \in U \text{ (set of plants)}$$

where

- U is the set of units (plants, contracts, ...)
- x is the vector of all decision variables
- x_u is the vector of decision variables for plant u
- x^t is the vector of decision variables for hour t
- $C_u(x_u)$ it the production cost function for plant u
- X_u represent plant-specific constraints
- H is a transmission matrix
- P^t, Q^t, R^t are the hourly demands for the three commodities

Decomposition into hourly models

- The long-term model can be decomposed into hourly models

$$\text{Min} \quad \sum_{u \in U} C_u^t(x_u^t)$$

$$Hx^t = \begin{bmatrix} P^t \\ Q^t \\ R^t \end{bmatrix}$$

$$x_u^t \in X_u^t \qquad u \in U$$

  – The necessary decomposition and co-ordination techniques depend on what kind of dynamic constraints are present
    - Without dynamic constraints, decomposition is trivial
  – Traditionally, the hourly model is a general LP/MILP model

Efficient model reformulation

- The idea is to encode the operating area of each plant as a convex combination of extreme characteristic points $(c_j^t, p_j^t, q_j^t, r_j^t)$

$$\min \quad C_u^t = \sum_{j \in J_u} c_j^t x_j^t$$

subject to

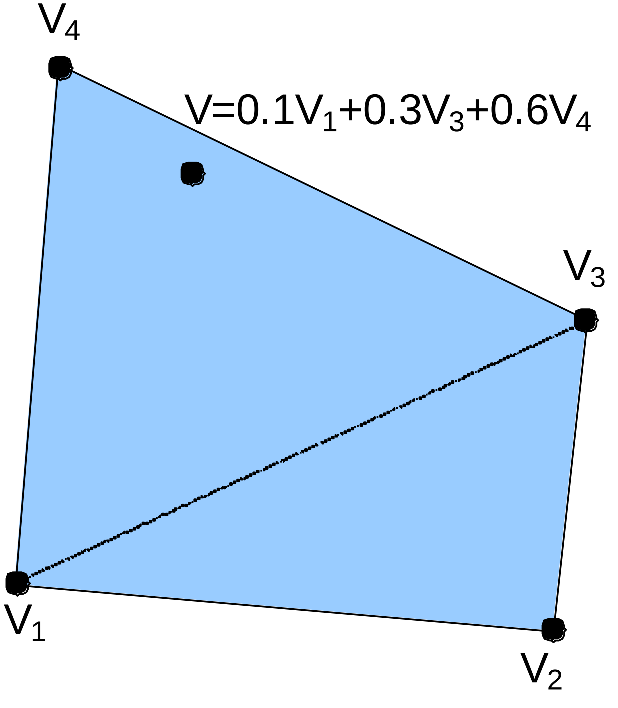
$$P_u^t = \sum_{j \in J_u} p_j^t x_j^t$$

$$Q_u^t = \sum_{j \in J_u} q_j^t x_j^t$$

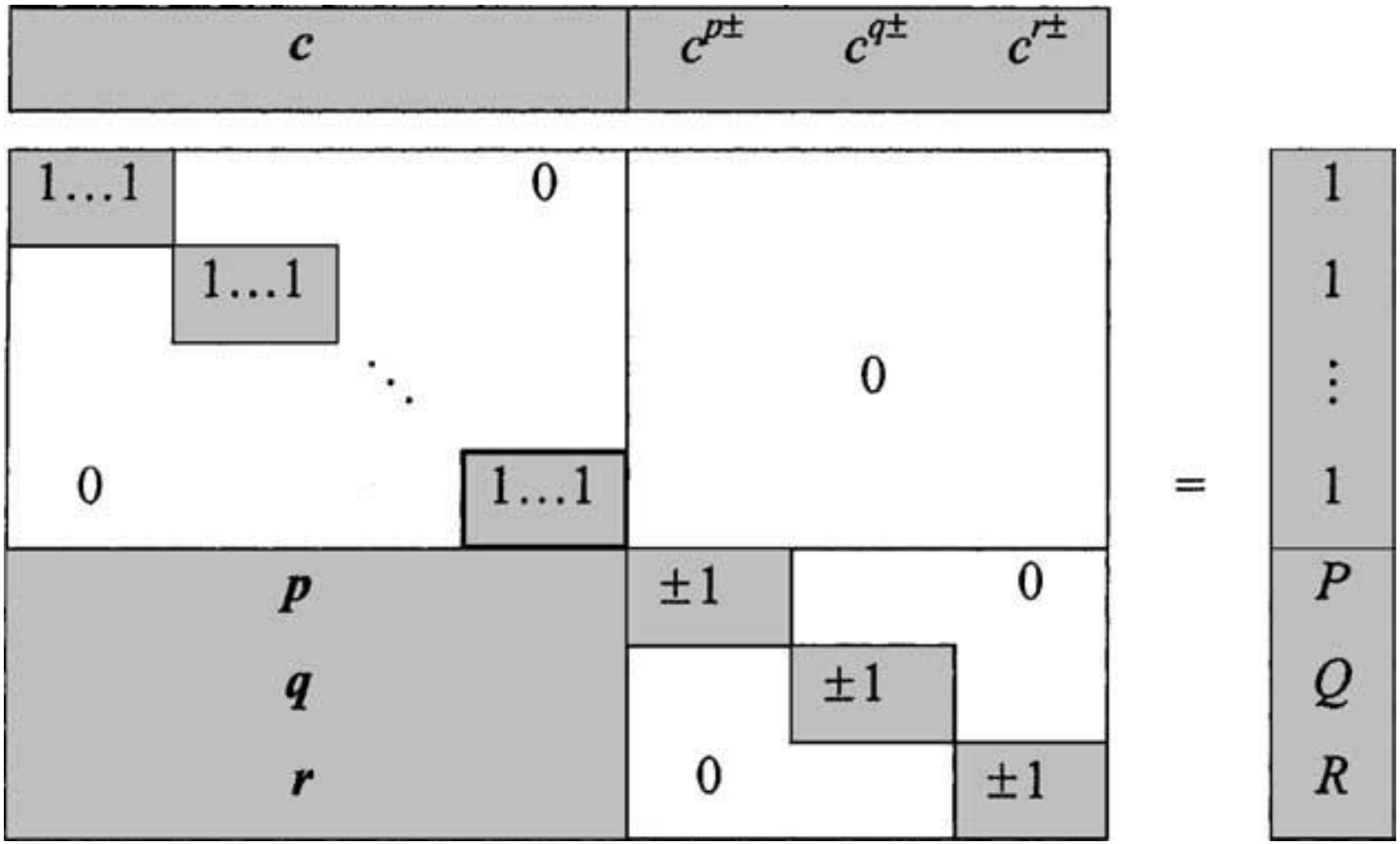
$$R_u^t = \sum_{j \in J_u} r_j^t x_j^t$$

$$\sum_{j \in J_u} x_j^t = 1$$

$$x_j^t \geq 0 \qquad j \in J_u$$



Structure of reformulated trigeneration model



Solving the reformulated model

- The reformulated model is an LP (or MILP) model with a special structure
  - The special structure allows the model to be solved extremely efficiently using tailored algorithms
    - Power Simplex (PS) for CHP (two-generation) models
    - Extended Power Simplex (EPS) for multi-site CHP problems
    - Three Commodity Simplex (TCS) for trigeneration problems
    - On-line and off-line envelope construction algorithms (ECON, ECOFF) for CHP problems under the liberalized market
  - The specialized algorithms can be 20-2000 times faster than generic commercial LP algorithms, such as CPLEX
    - Speed is comparable to specialized network algorithms

Speed of different algorithms

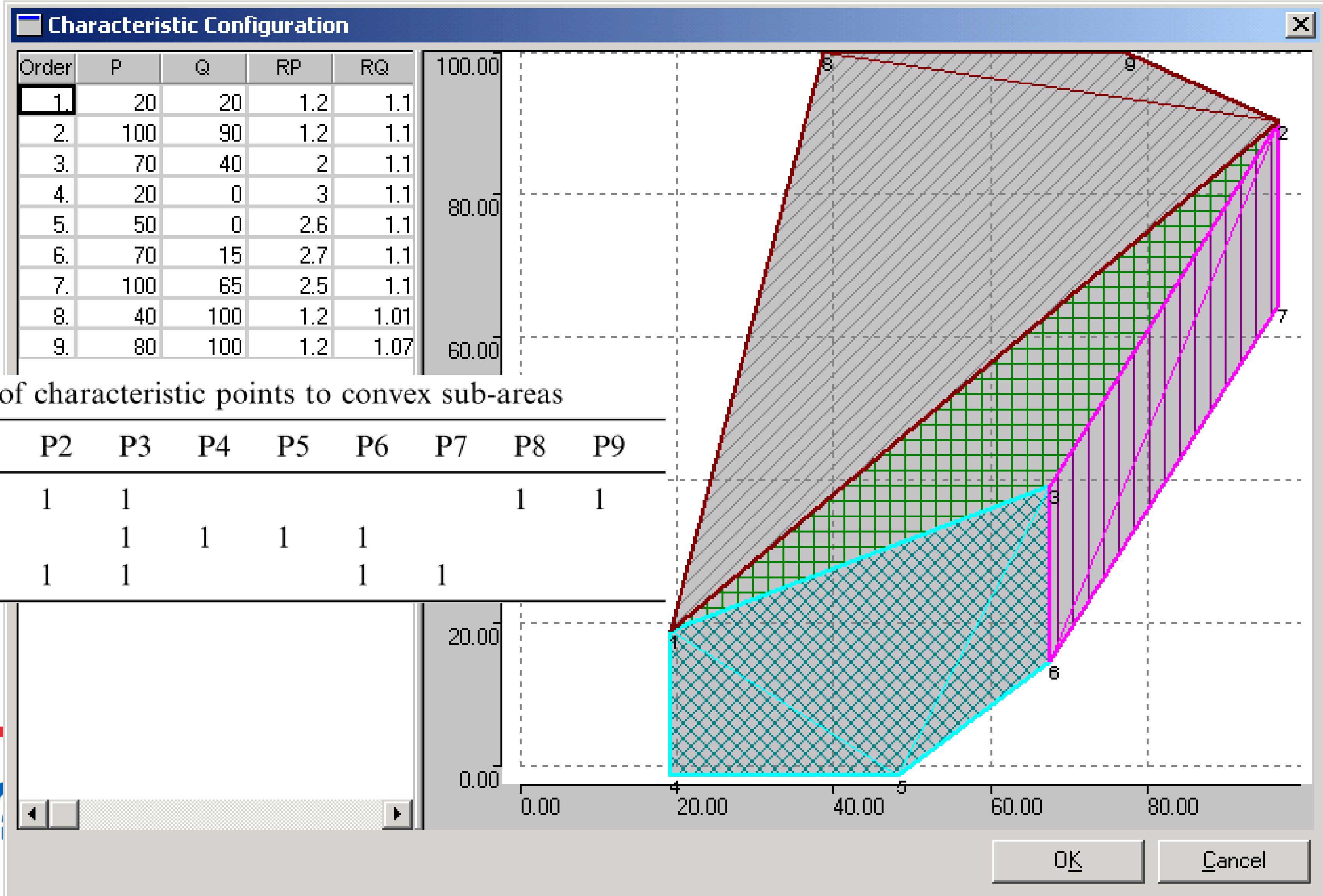
- CHP planning against power market
  - Test models (A1-A6)
    - 3 to 8 convex plants
    - Yearly (8760 h) planning models without dynamic constraints
    - Constraints: 43 800 to 87 600, variables: 420 480 to 1 077 480
  - Solution time on a 2.2 GHz Pentium 4 PC
    - Speedup against CPLEX: 400 to 2300 times

CPU (ms)	From scratch					Reusing previous results			
Model	CPLEX	PS	ECON	ECOFF		CPLEX	PS	ECON	ECOFF
A1	22548	127	30	13		24136	44	25	10
A2	24638	191	47	21		29937	52	36	10
A3	30373	191	60	27		34967	60	49	16
A4	33410	260	75	37		38090	77	60	20
A5	37134	315	88	40		46619	80	66	21
A6	40933	403	105	50		51797	96	78	24
Average	31506	247	68	31		37591	68	52	17

Non-convex CHP model

- Can be formulated as a Mixed Integer Linear Programming (MILP) model
- Necessary when either (or both)
  - The cost function is non-convex
  - P-Q the characteristic is non-convex
- Idea
  - Partition the objective function into convex parts
  - Partition the characteristic into convex parts
  - Use **binary variables** to choose in which area to operate
  - A binary variable can only have value 0 or 1

Sample non-convex cogeneration model



Order	P	Q	RP	RQ
1.	20	20	1.2	1.1
2.	100	90	1.2	1.1
3.	70	40	2	1.1
4.	20	0	3	1.1
5.	50	0	2.6	1.1
6.	70	15	2.7	1.1
7.	100	65	2.5	1.1
8.	40	100	1.2	1.01
9.	80	100	1.2	1.07

Allocation of characteristic points to convex sub-areas

Area	P1	P2	P3	P4	P5	P6	P7	P8	P9
A1	1	1	1					1	1
A2	1		1	1	1	1			
A3		1	1			1	1		

Non-convex cogeneration model

- Characteristic is partitioned into 3 convex parts

Allocation of characteristic points to convex sub-areas

Area	P1	P2	P3	P4	P5	P6	P7	P8	P9
A1	1	1	1					1	1
A2	1		1	1	1	1			
A3		1	1			1	1		

- A_j is **set of areas** to which x_j belongs
- Define zero-one variables y1, y2, y3, and allow exactly one of them to have value 1.
- y-variables select which corner points are allowed in convex combination
- U* is set of non-convex plants

$$x_j \leqslant \sum_{a \in A_j} y_a, \quad j \in J_u, \quad u \in U^*,$$

$$\sum_{a \in A_u} y_a = 1, \quad u \in U^*,$$

$$y_a \in \{0, 1\}, \quad a \in A_u, \quad u \in U^*.$$

Experiences

- Experiments with other plant models have showed that the number of extreme points typically varies
  - from 10 to 70 for trigeneration plants
  - from 5 to 20 for CHP plants
- This number of extreme points is very reasonable
- The method works well in the case of 1, 2 or 3 commodities
- In more complex models the combinatorial explosion may become a problem
- Similary, the method can handle efficiently only a fairly small number of 0/1 variables.

Selected publications

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