Solving a non-convex MIQCP Model for Water Tank Optimization

04.10.2017

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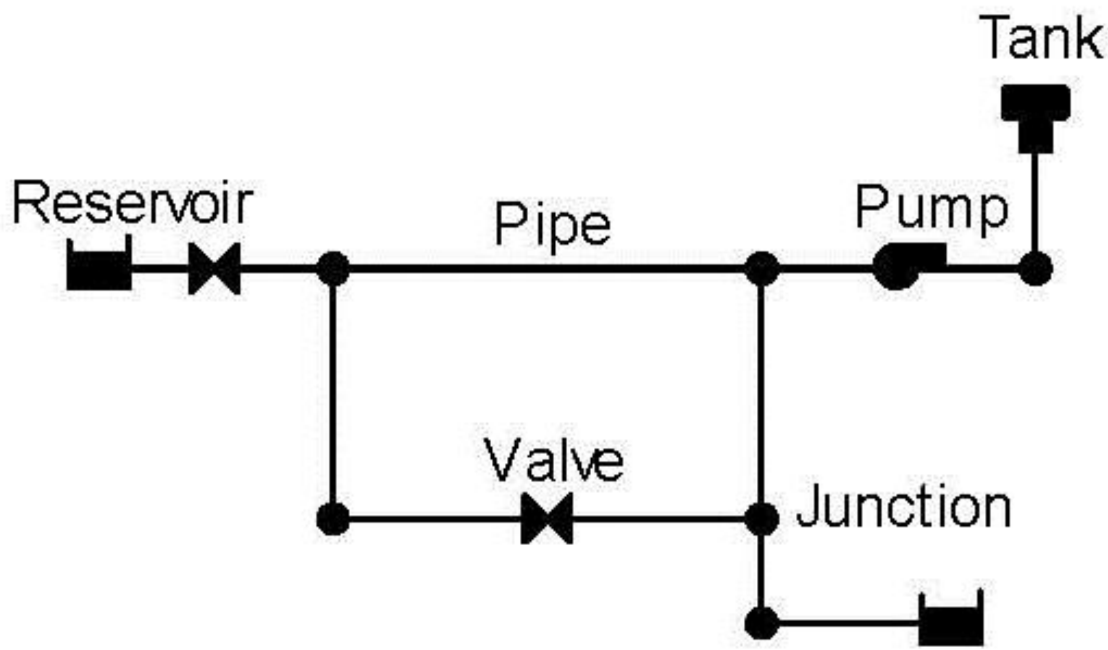
Outline

- Introduction & Motivation
- Solution Process & Results
  - Optimization
  - Network Reduction
  - Hydraulic Simulation & Modification
- Summary & Outlook

Introduction & Motivation

Components of a water distribution system

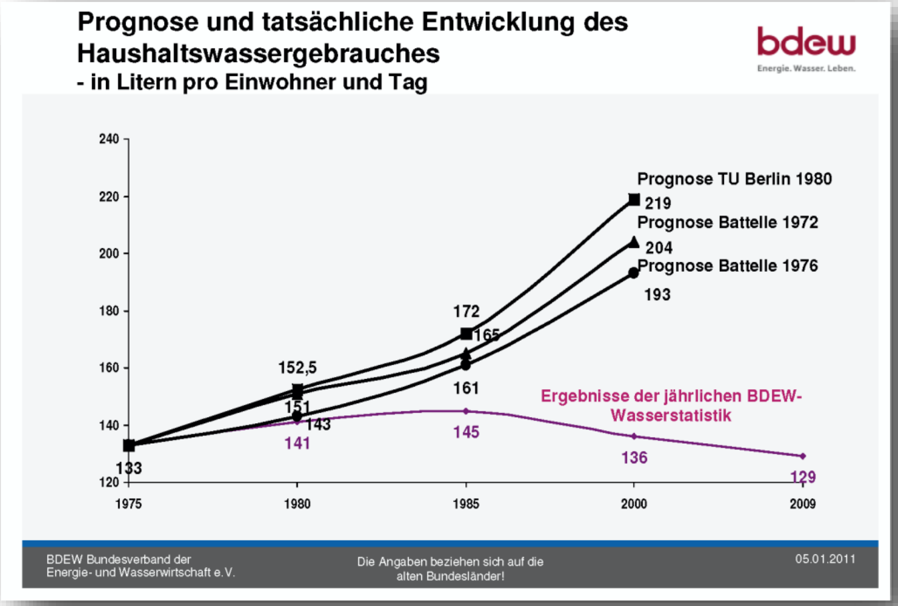
- Nodes:
  - Junction
  - Reservoir
  - Tank
- Links:
  - Pipe
  - Valve
  - Pump
- Network hydraulics:
  - Hydraulic head
  - Flow rate



Introduction & Motivation

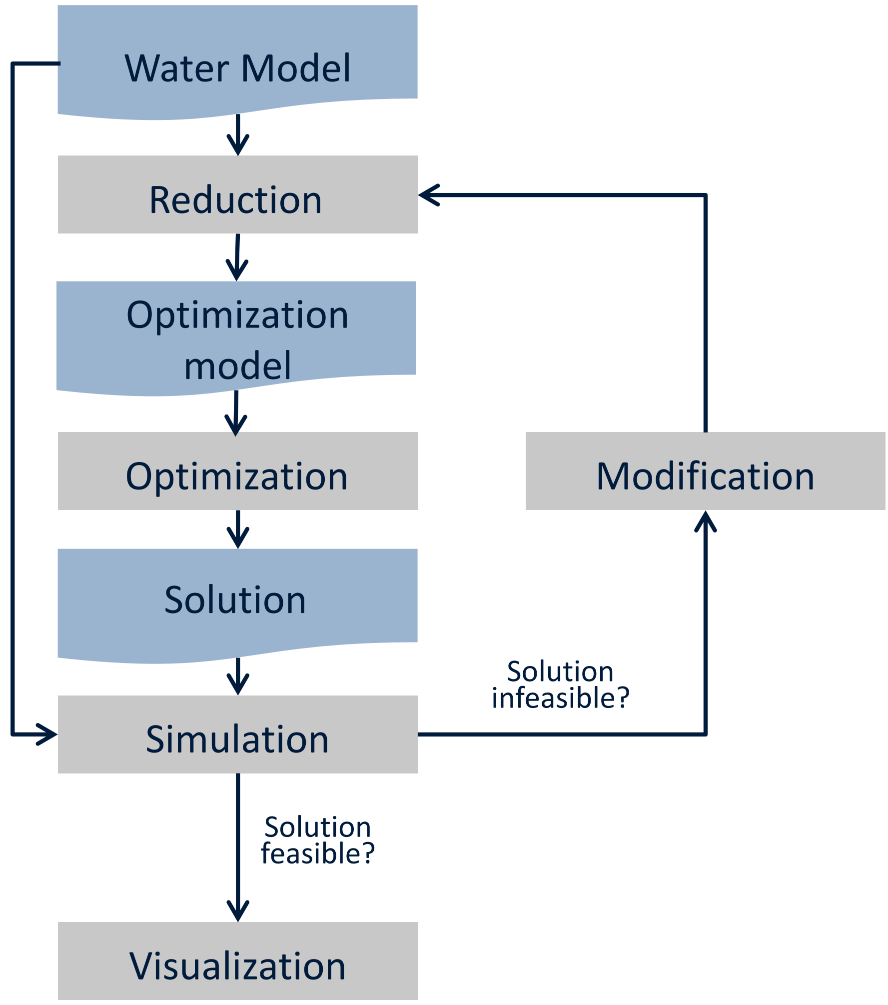
- In recent years, German municipal utilities are facing an increasing cost pressure.

- Decreasing water consumption
  - Many components do not have the right dimensions to work efficiently.
  - Especially tanks and pipes



- Utilities have to
  - Decrease cost
  - Increase efficiency

Goal: Solution Process to support the Planning of Water Tanks



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Optimization

Mathematical Optimization Model[1]

Objective: Minimize investment cost and operational cost of water tanks

Subject to:

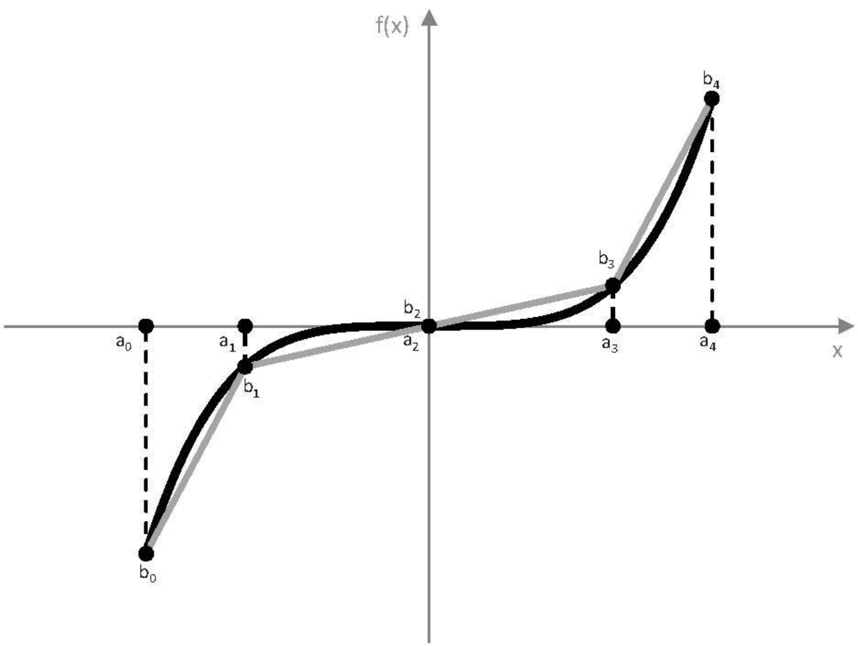
- Tank requirements
- Security of supply
  - Demand
  - Firefighting
- Hydraulic
  - Mass balance equation
  - Nonlinear head loss equation
- Non-convex Mixed Integer Quadratically Constrained Program (MIQCP)

$$
\begin{aligned}
&\min \sum_{b \in B} (InvC_b \cdot y_b + C_b \cdot MaxV_b \cdot \Delta t + C_b \cdot VH_b \cdot y_b) && \\
V_b^t &\leq MaxV_b \cdot \Delta t && \forall b \in B, t \in T \\
y_e &= 1 && \forall e \in E \\
V_b^t &\geq Vmin_b \cdot y_b && \forall b \in B, t \in T \\
V_b^t &\leq Vmax_b \cdot y_b && \forall b \in B, t \in T \\
\sum_{b \in B} V_b^t &\geq Vmin_{to} && \forall t \in T \\
L_b^t - \frac{4 \cdot V_b^{t-1}}{\pi \cdot d_b^2} &\leq (Hmax - E_b) \cdot (1 - y_b) && \forall b \in B, t \in T \setminus \{0\} \\
-L_b^t + \frac{4 \cdot V_b^{t-1}}{\pi \cdot d_b^2} &\leq (Hmin - E_b) \cdot (1 - y_b) && \forall b \in B, t \in T \setminus \{0\} \\
H_b^t &= L_b^t + E_b && \forall b \in B, t \in T \\
\sum_{(i,b) \in P} Q_{i,b}^t - \sum_{(b,j) \in P} Q_{b,j}^t &= (V_b^t - V_b^{t-1}) \cdot \frac{1}{\Delta t} && \forall b \in B, t \in T \setminus \{0\} \\
\sum_{(i,n) \in P} Q_{i,n}^t - \sum_{(n,j) \in P} Q_{n,j}^t &= D_n^t \cdot \frac{1}{\Delta t} && \forall n \in N \setminus (\{B, R\}), \\
& && T \\
& \boxed{H_i - H_j = r_{ij} \cdot Q_{ij} \cdot |Q_{ij}|} && \in R, t \in T \\
& && , j) \in P, t \in T \\
y_b &\in \{0, 1\} && \forall b \in B \\
V_b^t &\geq 0 && \forall b \in B, t \in T \\
MaxV_b &\geq 0 && \forall b \in B \\
Hmax \geq H_n^t &\geq Hmin && \forall n \in N, t \in T \\
Hmax - E_b \geq L_b^t &\geq Hmin - E_b && \forall b \in B, t \in T \\
Qmax_{i,j} \geq Q_{i,j}^t &\geq -Qmax_{i,j} && \forall (i,j) \in P, t \in T
\end{aligned}
$$

[1] Dohle C., Suhl L.: *An Optimization Model for the optimal Usage of Water Tanks in Water Supply Systems*, Proceedings of the International Conference on Applied Mathematical Optimization and Modelling APMOD 2012, Paderborn, pp. 297-302

How to solve this non-convex MIQCP?

- Piecewise linearization of the head loss equation
  - Partitioning the interval with a predefined error bound



$$
\begin{aligned}
x &= a_0 + \sum_{k=1}^{m} z_k \\
\phi(x) &= b_0 + \sum_{k=1}^{m} \frac{b_k - b_{k-1}}{a_k - a_{k-1}} \cdot z_k \\
z_1 &\leq a_1 - a_0 \\
z_k &\geq (a_k - a_{k-1}) \cdot y_k && \forall\, k = 1, \dots, m-1 \\
z_{k+1} &\leq (a_{k+1} - a_k) \cdot y_k && \forall\, k = 1, \dots, m-1 \\
z_k &\geq 0 && \forall\, k = 1, \dots, m \\
y_k &\in \{0, 1\} && \forall\, k = 1, \dots, m-1.
\end{aligned}
$$

Test Set

Small Networks

Name	# Nodes	# Links	# Tanks	# Reservoirs
S01	8	10	3	1
S02	10	12	1	0
S03	11	12	4	1
S04	12	11	8	1
S05	12	11	5	1
S06	14	13	8	0
S07	14	15	7	1
S08	17	16	10	0
S09	18	17	6	1
S10	20	21	15	0
S11	21	20	15	0

Medium Networks

Name	# Nodes	# Links	# Tanks	# Reservoirs
M01	27	26	15	1
M02	30	31	10	1
M03	34	38	20	0
M04	42	44	10	1
M05	46	50	15	1
M06	52	73	15	1
M07	56	77	20	0
M08	86	114	15	3

Large Networks

Name	# Nodes	# Links	# Tanks	# Reservoirs
L01	219	250	20	0
L02	300	345	30	2
L03	916	973	25	1
L04	932	1014	30	2
L05	1913	2487	20	2

Goal: Solution Process to support the Planning of Water Tanks

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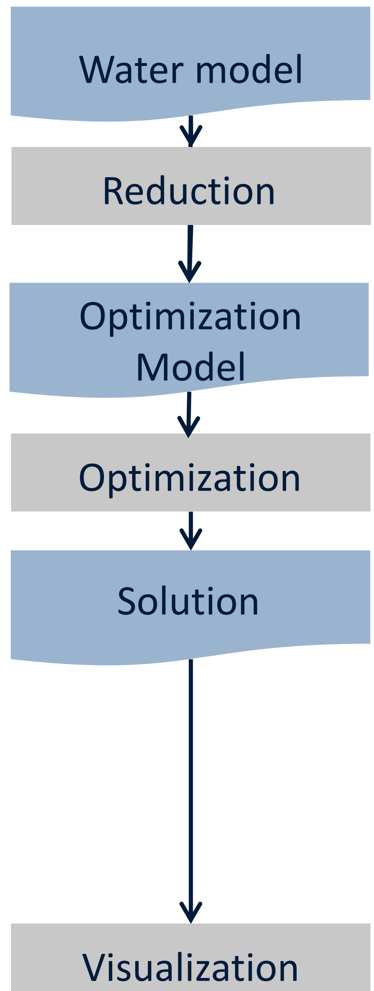

Network Reduction

Techniques

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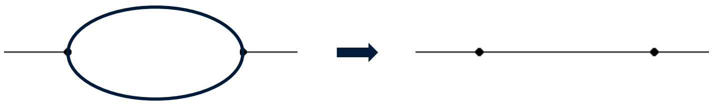
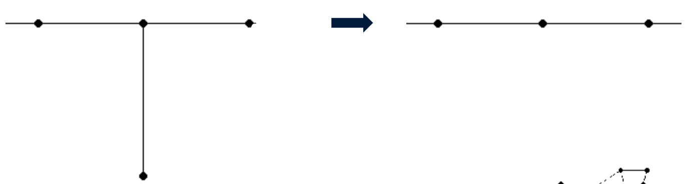
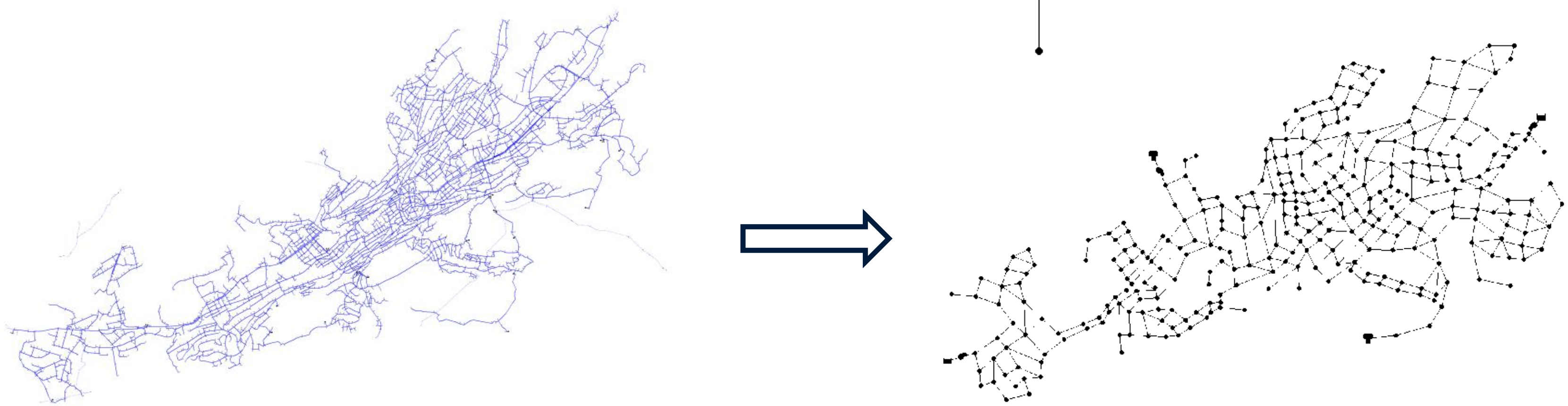
- Apply Network Reduction techniques to reduce the number of nodes and links in the network model:
  - Elimination of pipe sequences

  - Elimination of parallel pipes



  - Elimination of end nodes



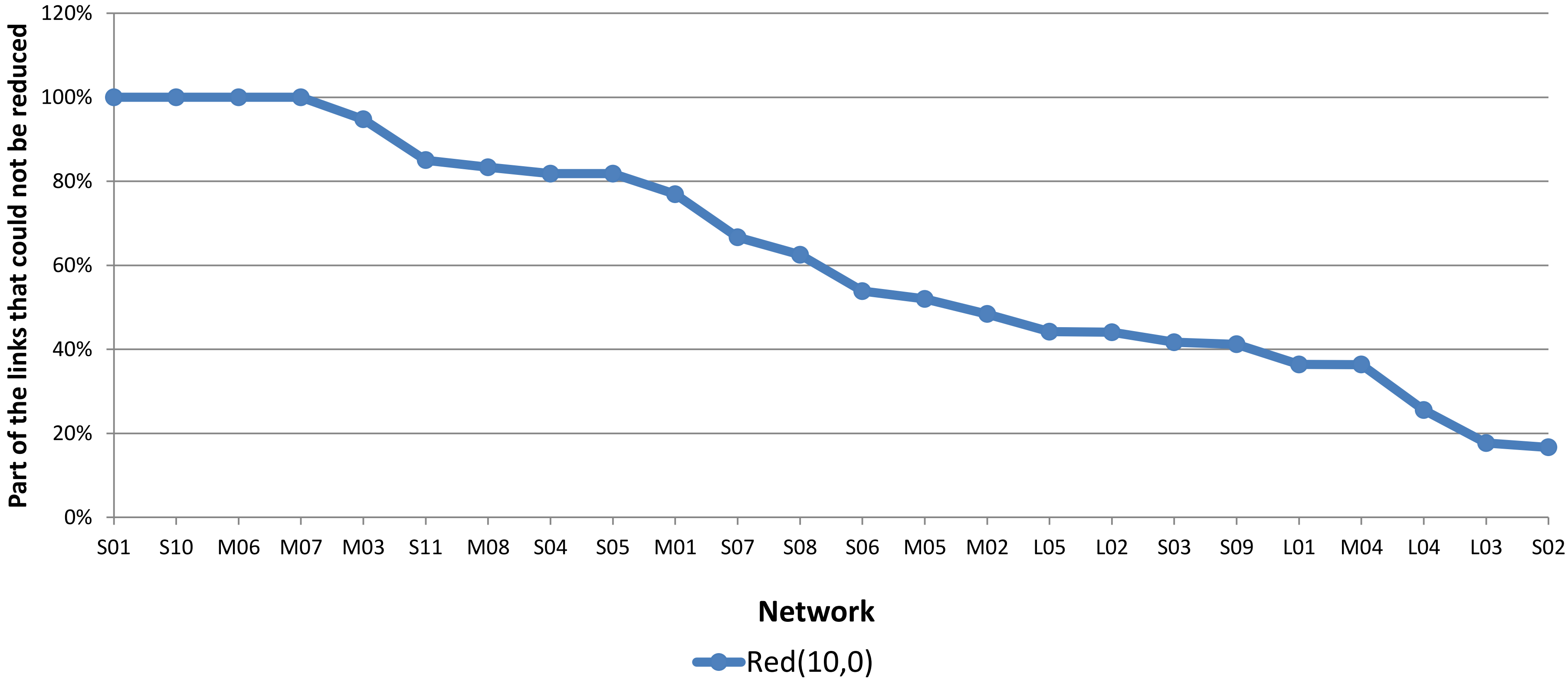


Cf. [Burgschweiger et al., 2009], [Maschler et al., 1999]

Network Reduction

Results of network reduction

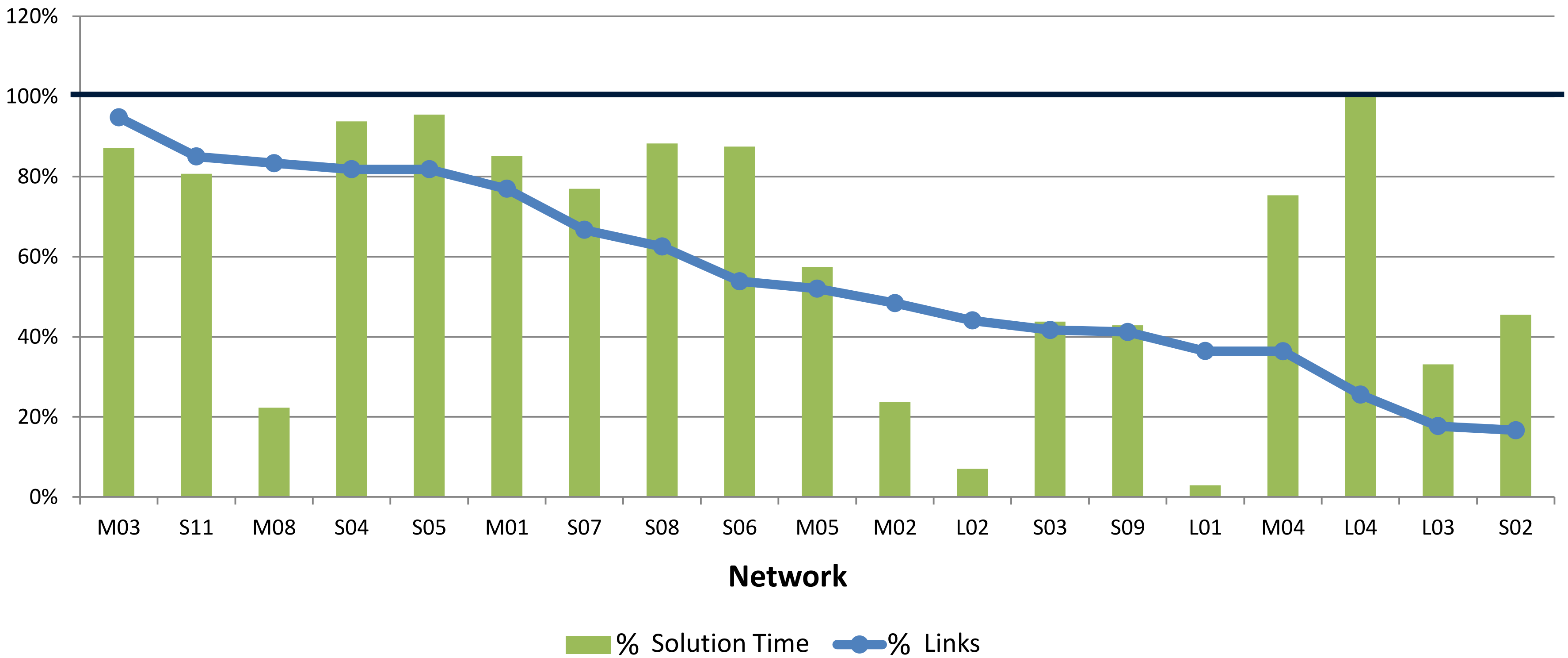
Part of the links that could not be reduced after applying all network reduction techniques



Network Reduction

Results of network reduction in combined with optimization

Solution time of the reduced network models in comparison to the solution time of the original network models



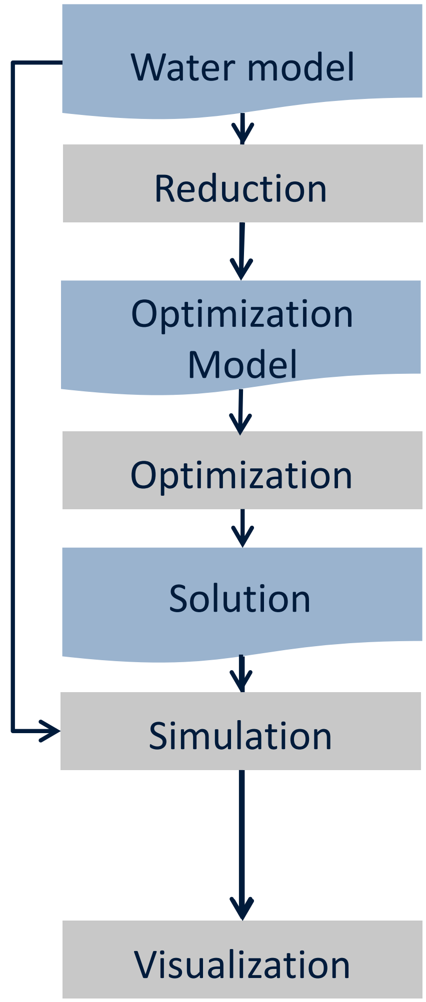
Computed with: Intel Core i7-3370 CPU, 3,4 GHz, 32 GB, Windows 8.1 Pro
Optimization Software: Gurobi 5.02

Network Reduction

Short Summary

- Significant reduction of nodes and links in the network models
- Therefore, significant reduction of the solution time of the corresponding optimization model
- There may occur differences in the hydraulics of the reduced and the original network model
  - Bound for accuracy of the reduction
- Remaining question: Is the solution found by optimizing the reduced network model feasible if transferred to the original network model?

Goal: Solution Process to support the Planning of Water Tanks

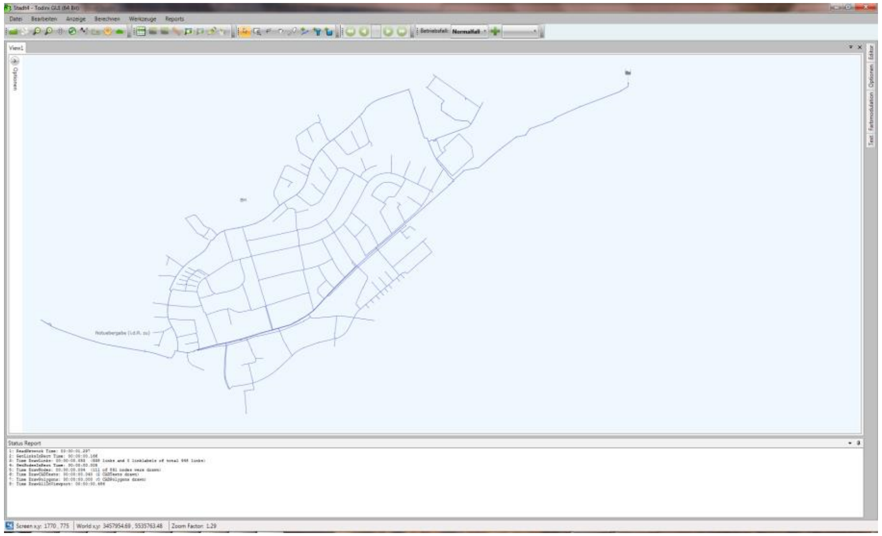


Simulation

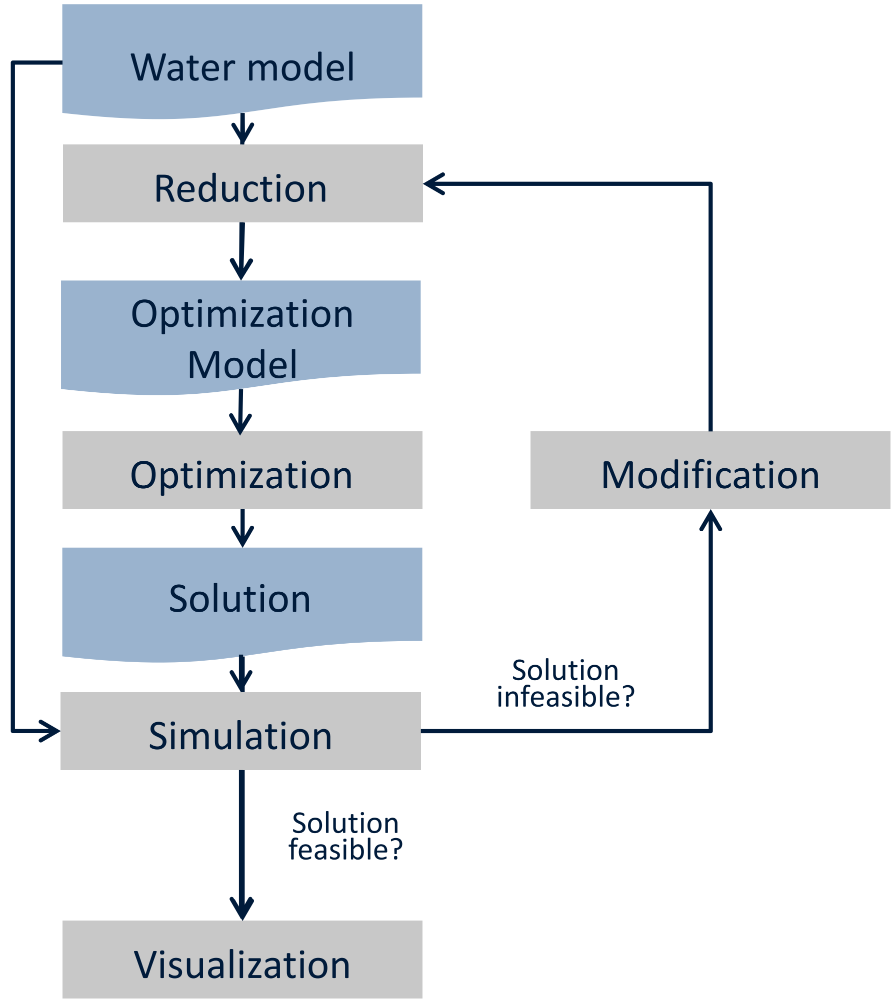
Simulation Tool Roka3

Evaluation of the solution obtained during optimizing the reduced network model

- Use the hydraulic simulation tool Roka3
- Developed by RZVN Wehr GmbH
- Calculates:
  - Hydraulics
  - Tanks
  - Pumps and valves
  - Different time steps
  - Network based rules
  - Etc.



Goal: Solution Process[2] to support the Planning of Water Tanks



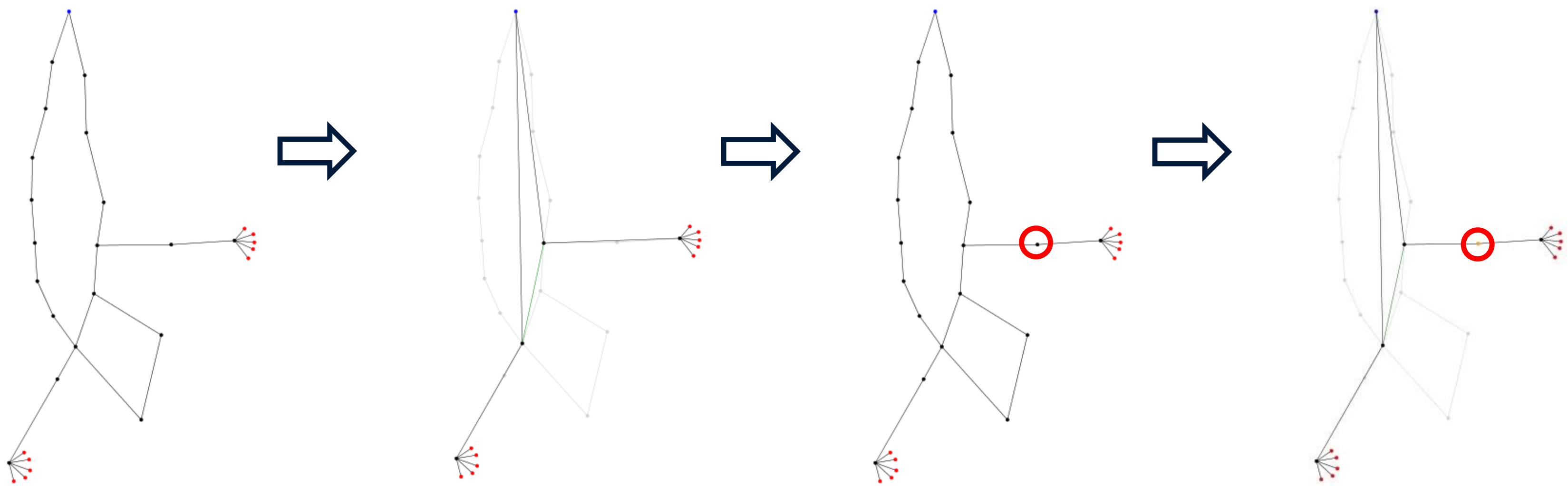
[2] Hallmann C., Suhl L.: *Optimizing water tanks in water distribution systems by combining network reduction, mathematical optimization and hydraulic simulation*, OR Spectrum, 38(3), pp. 577-595, DOI: 10.1007/s00291-015-0403-1

Modification

Modification process

If simulation discovers an infeasible solution, the modification is applied:

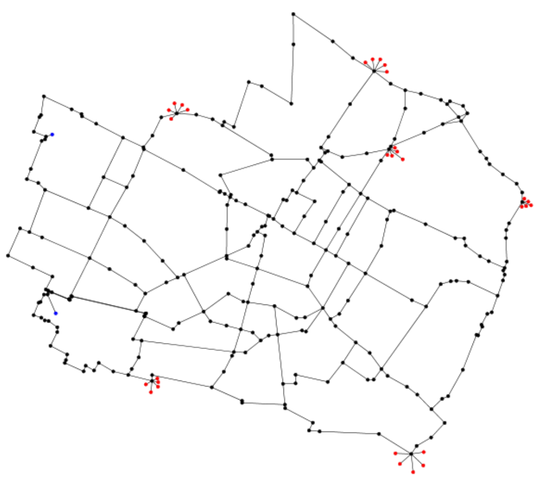
- Localize the infeasibilities in the network model.
- Mark and lock the corresponding nodes for a next reduction step.
- Reduce the original network model with some locked nodes.
- Optimize this modified reduced model and evaluate solution.



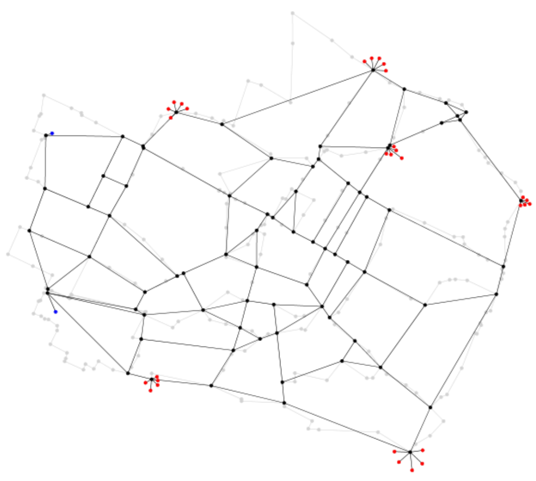
Modification

Results – Net L02

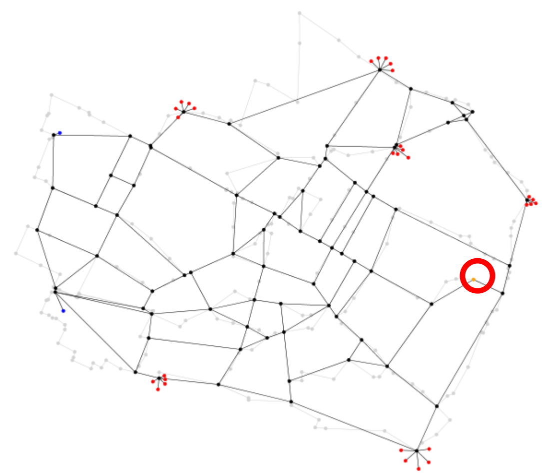
	# Nodes	# Links	Objective	Warn.	Diff	Time (s)
L02-Ori	300	345	380973.1757	-	-	86400.00
L02-Red	107	152	373235.5552	12	4.2511	6063.34
L02-Red-Modi	108	153	378961.2780	0	-	4250.92
L02-SCIP-UB	300	345	41203988.2812	-	-	86400.00
L02-SCIP-LB	300	345	367365.8627	-	-	86400.00



L02-Ori



L02-Red(10,0)



L02-Red(10,0)-Modi

Solution Process - Complete

Results

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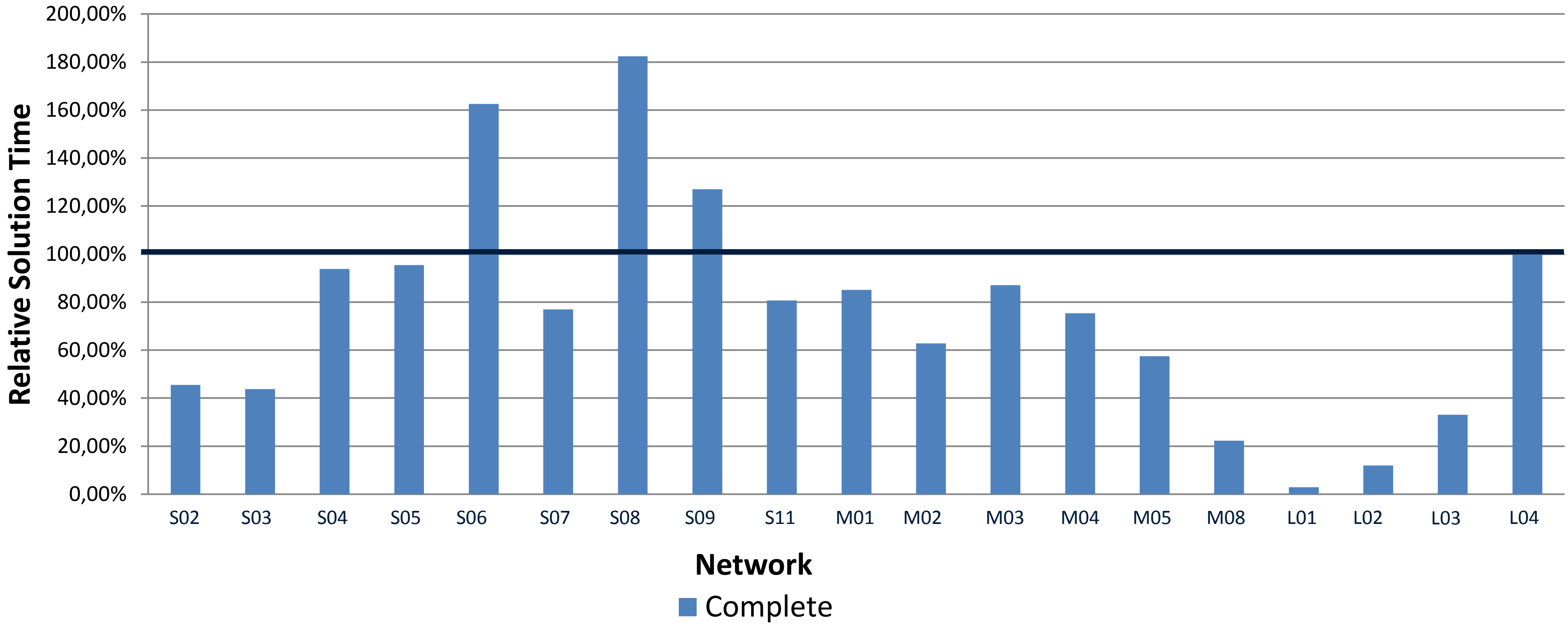
Solution time of complete solution process compared to solution time of solving the original network model



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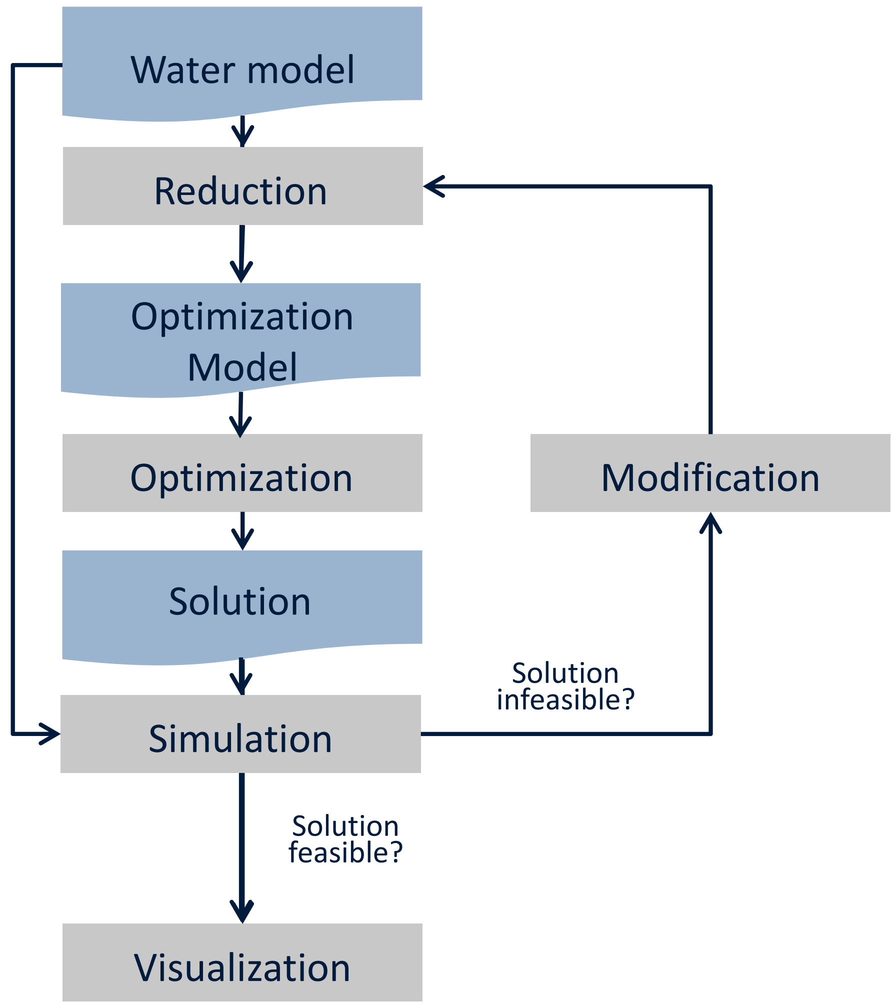

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Summary and Outlook

Summary

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- Solution process generated a feasible solution for all networks within 24 hours
- Even large networks can be solved in reasonable time
- Each step of the solution process is necessary
  - Reduction to reduce the solution time of the optimization
  - Simulation to evaluate the solution
  - Modification to find iteratively a feasible solution
- Solution process very efficient when applied to medium sized and large networks

Summary and Outlook

Outlook

- Extend the optimization model
- Implement other reduction techniques
- Improve the modification process
- Apply approach to other problems and domains

http://www.dsor.de

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Thank you for your attention!