Valuation of Asian Quanto-Basket Options

(Final Presentation)

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21.11.2011

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Asian Quanto-Basket Option (1/2)

- Underlying asset is a basket of instruments whose returns are correlated
- Final payoff depends on the average value of the basket over some discrete set of observation points
- One or more of the basket components quoted in different currency than the option payoff
- No closed-form pricing formulas

Asian Quanto-Basket Option (2/2)

$$n = number\ of\ observation\ points$$

$$m = number\ of\ underlying\ assets\ in\ basket$$

$$K_{AQB} = strike\ price$$

$$A = average\ basket\ value$$

$$A = \frac{1}{n}\sum_{\tau=1}^{n}\sum_{i=1}^{m} w_i S_i(0) e^{\left(r_i - \delta_i - \rho_{i,i}\sigma_{X_i}\sigma_i - \frac{1}{2}\sigma_i{}^2\right)\tau + \sigma_i W_i(\tau)}$$

$$Payoff = \max(A - K_{AQB}, 0)$$

Literature review

Implementation

Moment Matching (1/2)

- Calculate the moments of the real distribution A

- Approximate the real unknown distribution with a known distribution such as log-normal utilizing the calculated moments

Moment Matching (2/2)

$$m_{\ 1} = \textit{the first moment of the real distribution}$$
$$m_{\ 2} = \textit{the second moment of the real distribution}$$

$$m_{\ 1} = E[A] = \frac{1}{n}\sum_{\tau=1}^{n}\sum_{i=1}^{m} w_i S_i(0) e^{(r_i - \delta_i - \rho_i \sigma_X \sigma_i) t_\tau}$$

$$m_{\ 2} = E[A^2] = \sum_{\tau_1,\tau_2=1}^{n}\sum_{i_1,i_2=1}^{m} w_{i_1} w_{i_2} S_{i_1} S_{i_2} \exp[(r_{i_1} - \delta_{i_1} - \sigma_X \sigma_{i_1} \rho_{i_1} + \sigma_{i_1} \sigma_{i_2} \Sigma_{i_1,i_2}) * \min(t_{\tau_1}, t_{\tau_2}) + \left(r_{i_2} - \delta_{i_2} - \sigma_X \sigma_{i_2} \rho_{i_2}\right) * \max(t_{\tau_1}, t_{\tau_2})]$$

Levy's approximation (1/2)

- Calculate m_1 and m_2 on the fly
- Approximate the real distribution with a log-normal distribution by setting the two first non-centered moments equal

$$\begin{cases} \sigma^2 = \ln m_1 - \ln m_2 \\ \mu = 2 \ln m_1 - \frac{1}{2} \ln m_2 \end{cases}$$

Levy’s approximation (2/2)

- Approximate risk-neutral call option price within the Black-Scholes framework is

$$C_{Levy} \approx e^{-rT} \int_{K_{AQB}}^{\infty} P_{log}(x)(A - K_{AQB})dx$$

$$= e^{-rT}\left[m_1 N(d_1) - K_{AQB} N(d_2)\right]$$

- N is the cumulative normal distribution function

Reciprocal gamma approximation (1/2)

- Calculate m_1 and m_2 on the fly
- Approximate the real distribution with a reciprocal gamma distribution by setting the first two moments equal

$$\begin{cases} \alpha = \dfrac{2m_2 - m_1^2}{m_2 - m_1^2} \\ \beta = \dfrac{m_2 - m_1^2}{m_1 m_2} \end{cases}$$

Reciprocal gamma approximation (2/2)

- Approximate risk-neutral call option price within the Black-Scholes framework is

$$C_{gam} \approx e^{-rT} \int_{K_{AQB}}^{\infty} P_{gam}(x)(A - K_{AQB})dx$$

$$= e^{-rT} \left[m_1 G\left(\frac{1}{K_{AQB}} \middle| \alpha - 1, \beta \right) - K_{AQB} G\left(\frac{1}{K_{AQB}} \middle| \alpha, \beta \right) \right]$$

- G is the gamma distribution function

Vorst's approximation

- Approximate the arithmetic average A with a geometric average
- Works well for Asian-style options that depend on the geometric average of some underlying
- Inferior performance for arithmetic average options

Edgeworth expansion around log-normal distribution

- The two first moments of the real and approximate distribution are set equal
- The third and fourth moments of the real distribution are used to calculate correction terms

$$C = C_{Levy} - e^{-rT}\frac{l_3 - l_3^{log}}{3!}\frac{dP_{log}(K_{AQB})}{dx} + e^{-rT}\frac{l_4 - l_4^{log}}{4!}\frac{d^2P_{log}(K_{AQB})}{dx^2}$$

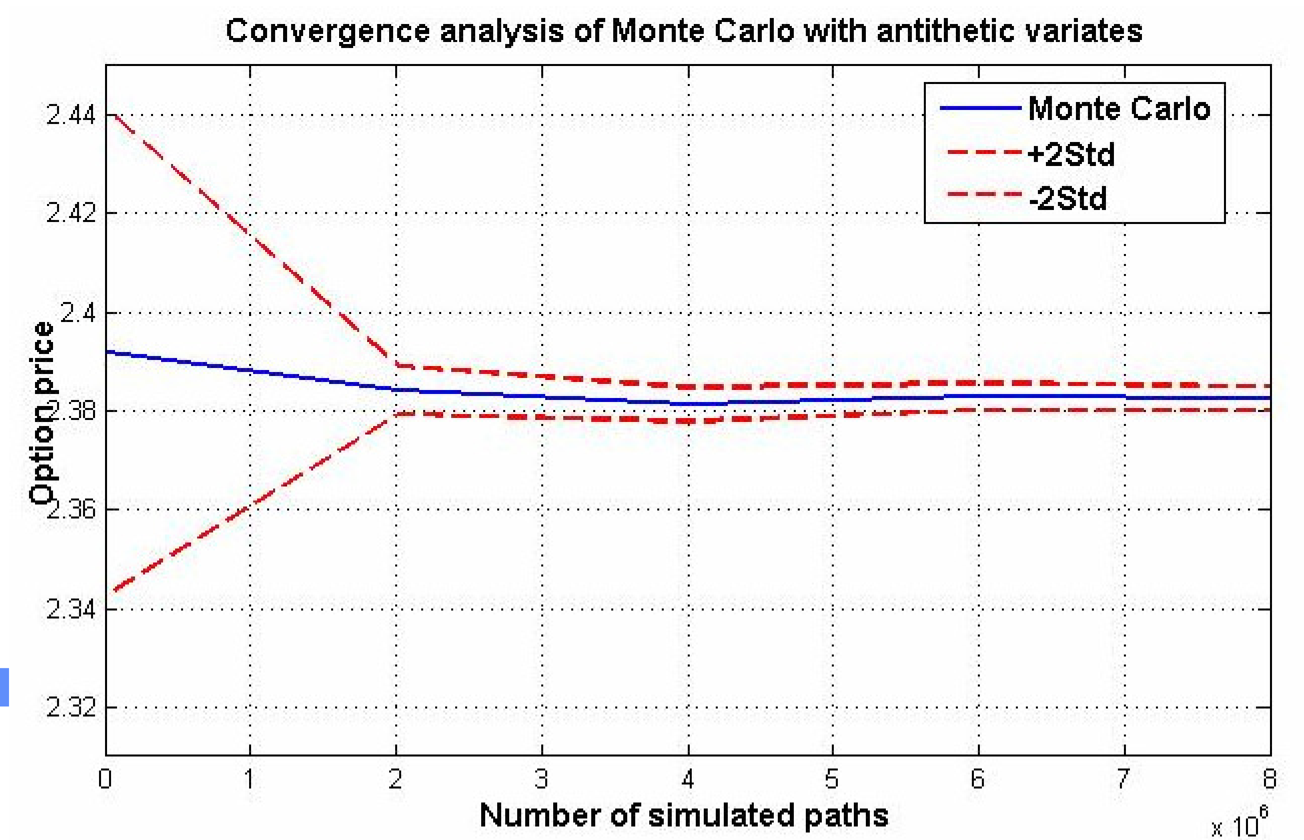
- Has been criticized because Edgeworth series does not always converge

Literature review

Implementation

Testing Levy's method against the reciprocal gamma approximation

- ❑Use Monte Carlo with antithetic variance reduction as benchmark



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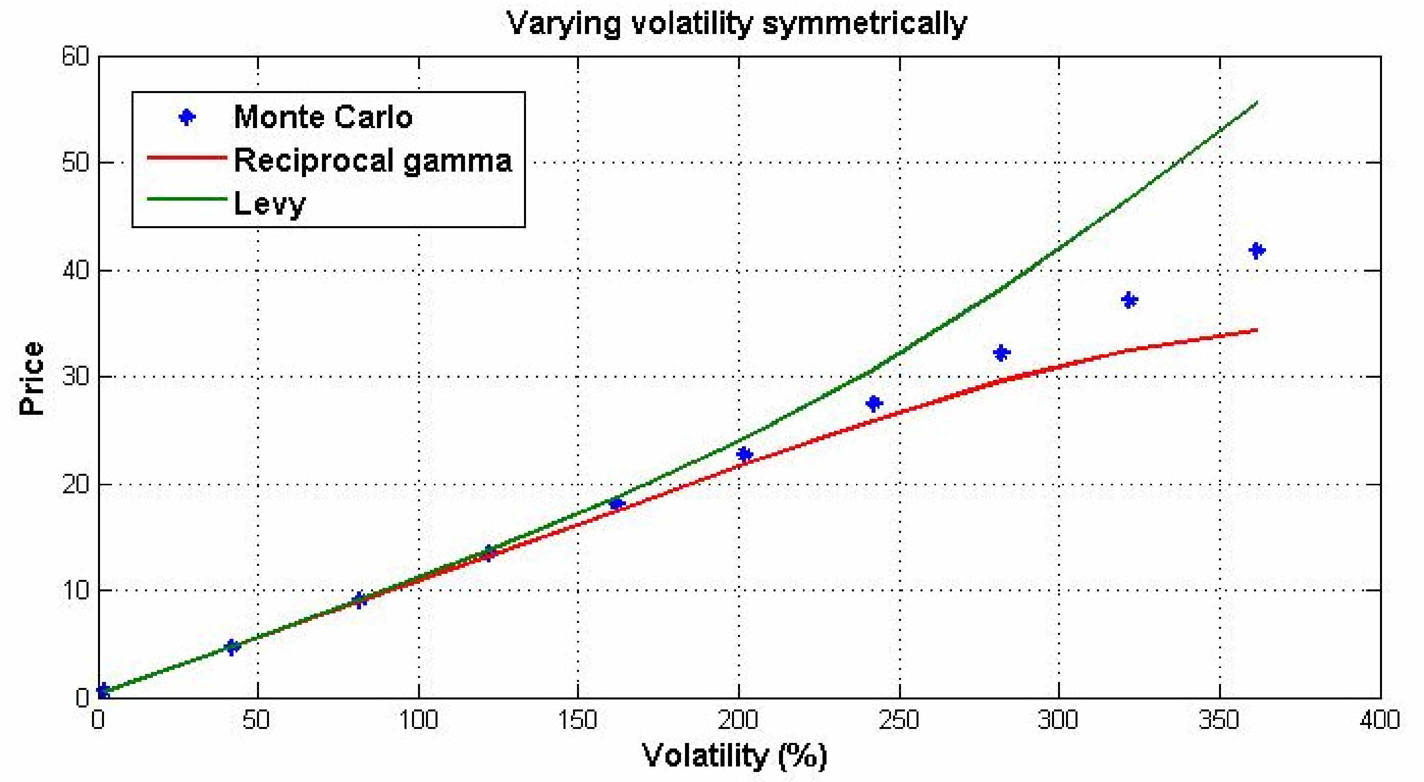
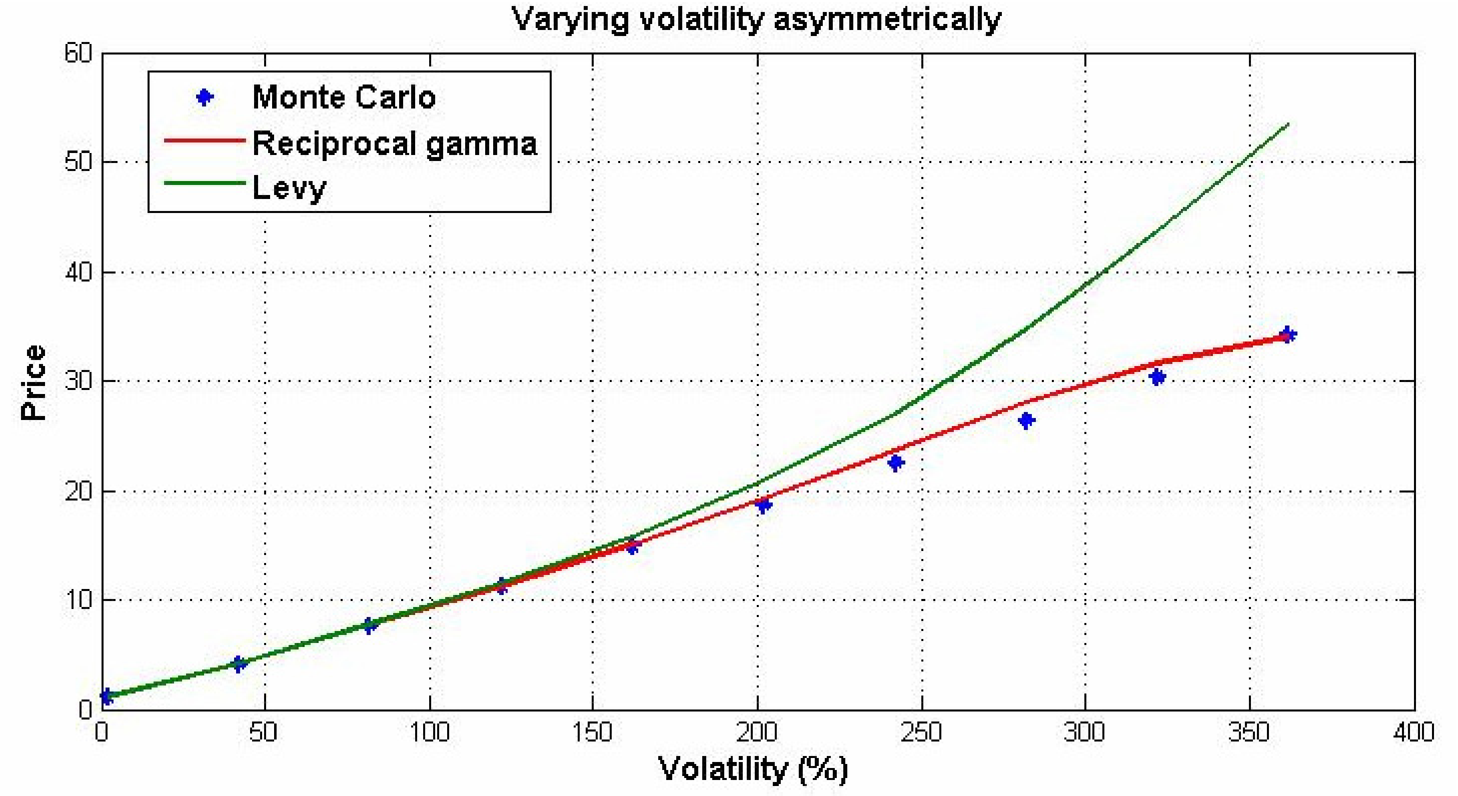
Test 1: Varying Volatilities (1/2)

- Let the volatility parameter σ increase symmetrically for all basket components



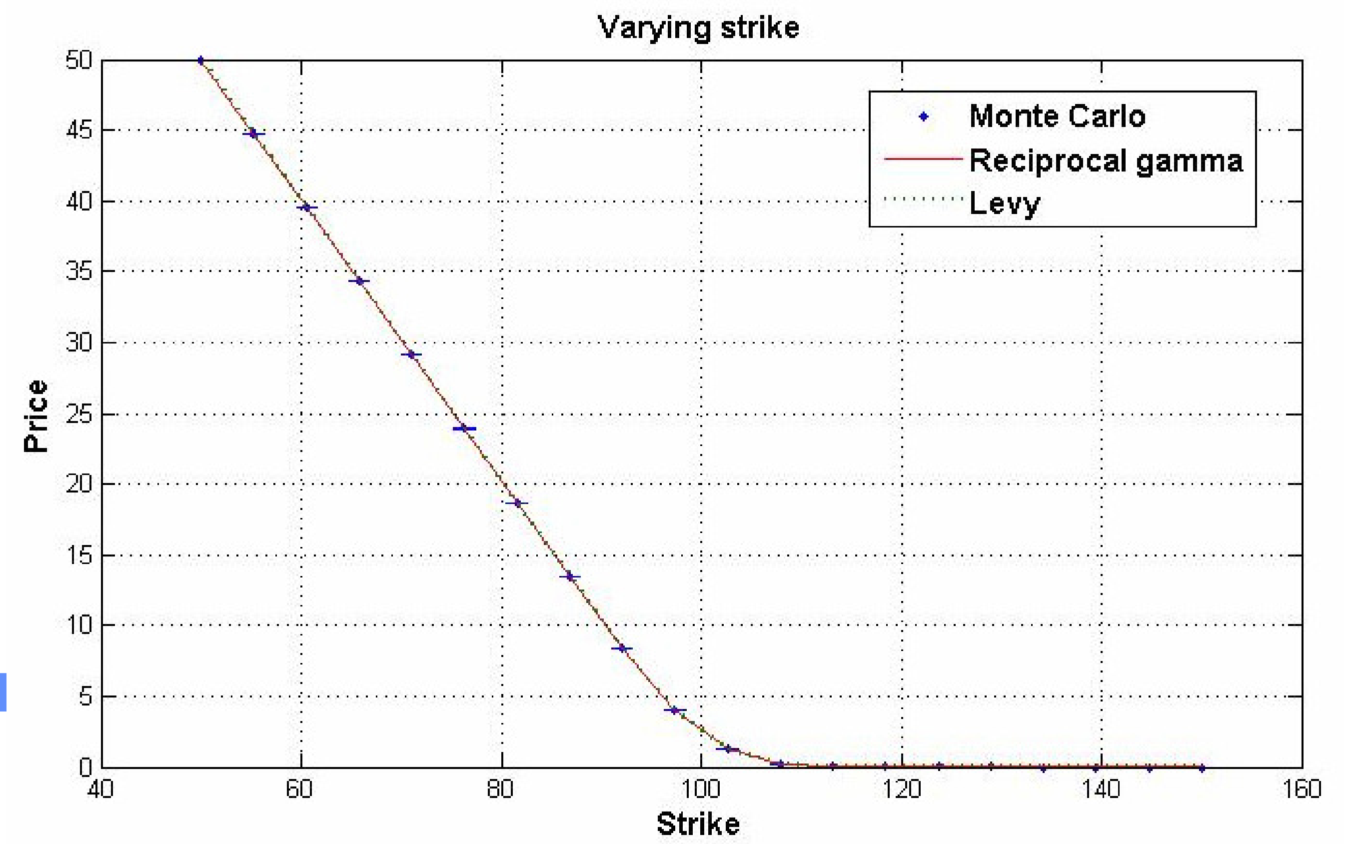
Test 1: Varying Volatilities (2/2)

- Let the volatility parameter σ increase for all but one of the basket components



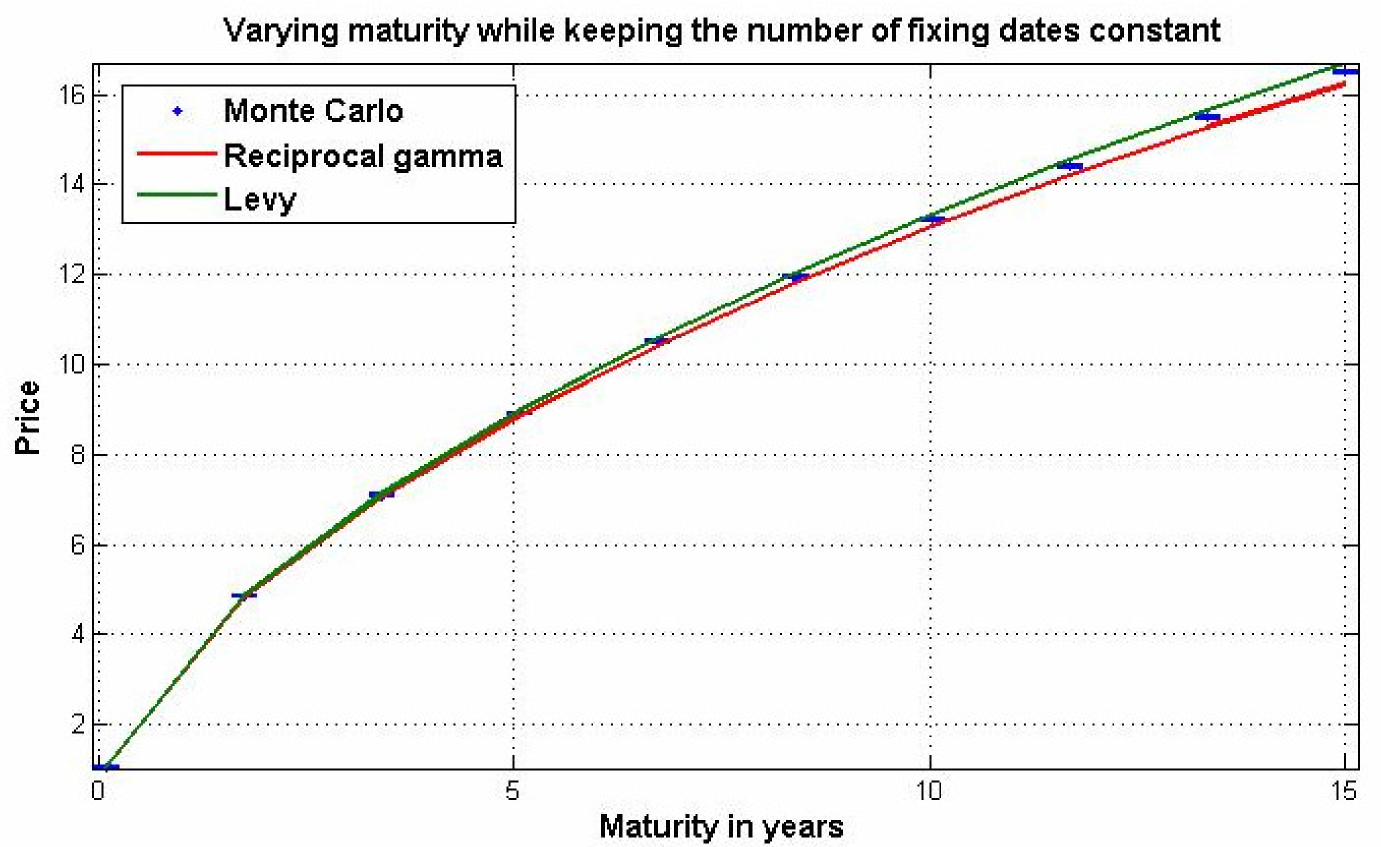
Test 2: Varying Strike Prices

- Increase the strike price of a call option while holding other parameters constant



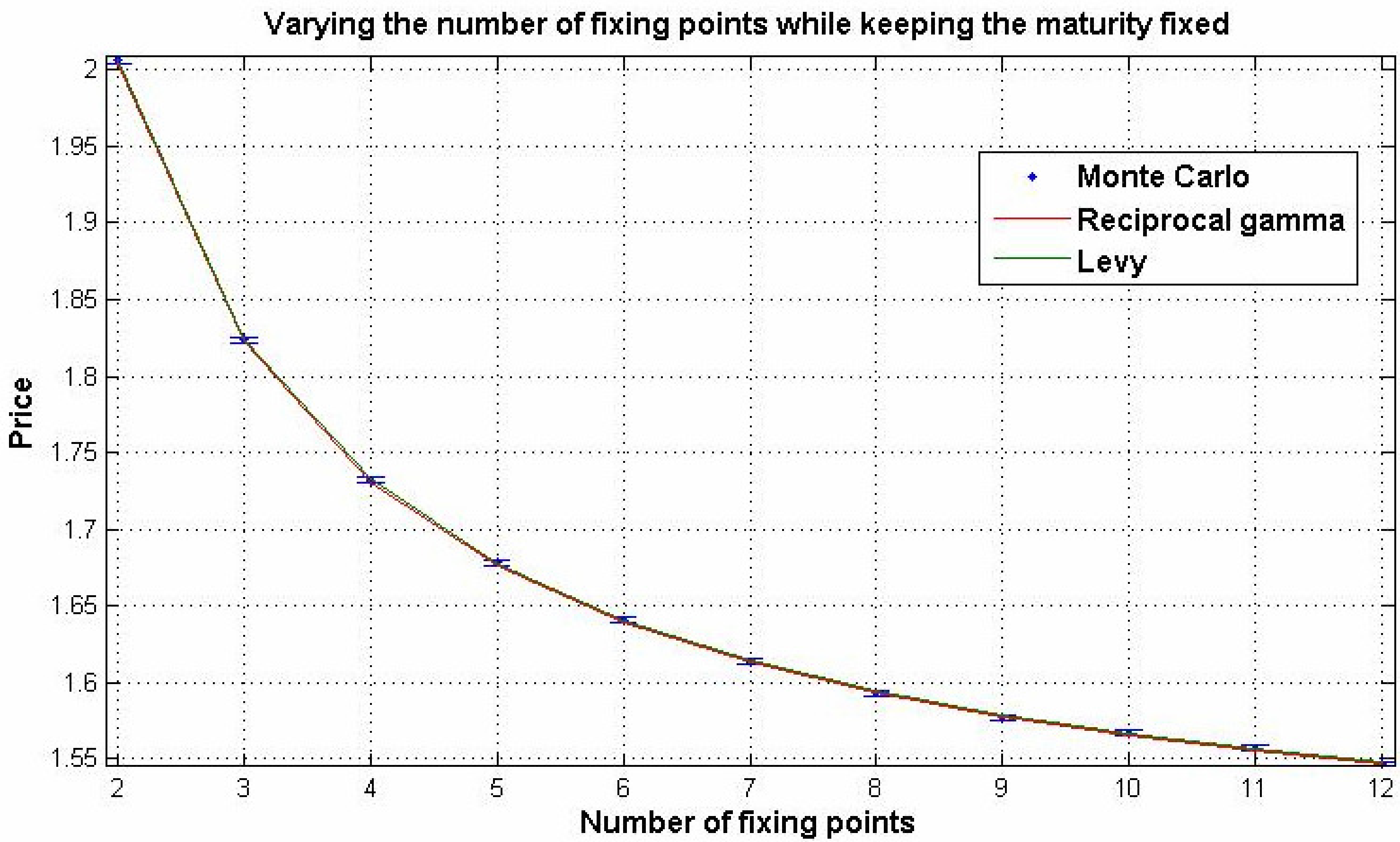
Test 3: Varying Maturities

- Increase the maturity of a call option from a few months to 15 years while keeping the number of fixing points constant



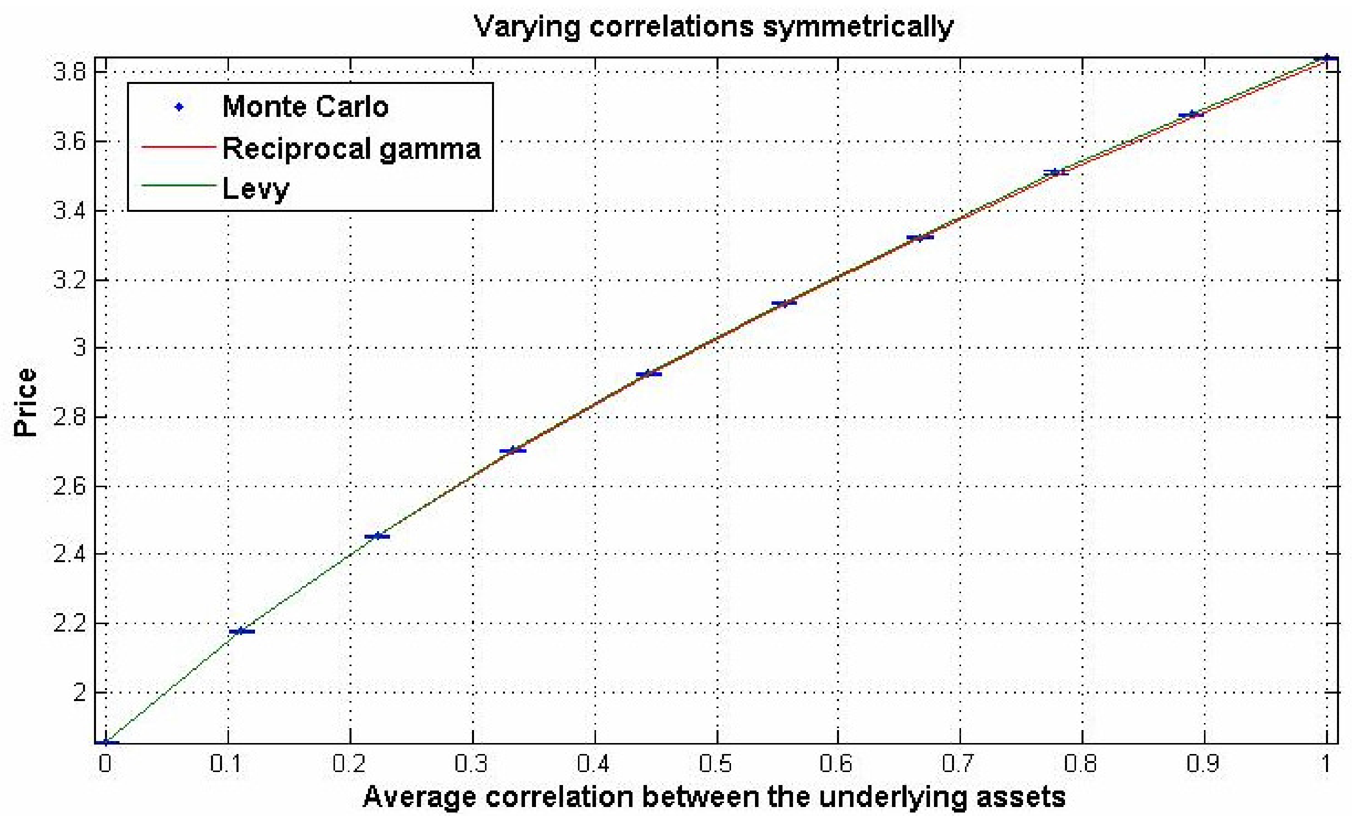
Test 4: Varying the Number of Fixing Dates

- Increase the number of averaging points



Test 5: Varying Correlations

- Increase symmetrically the average correlation between the basket components

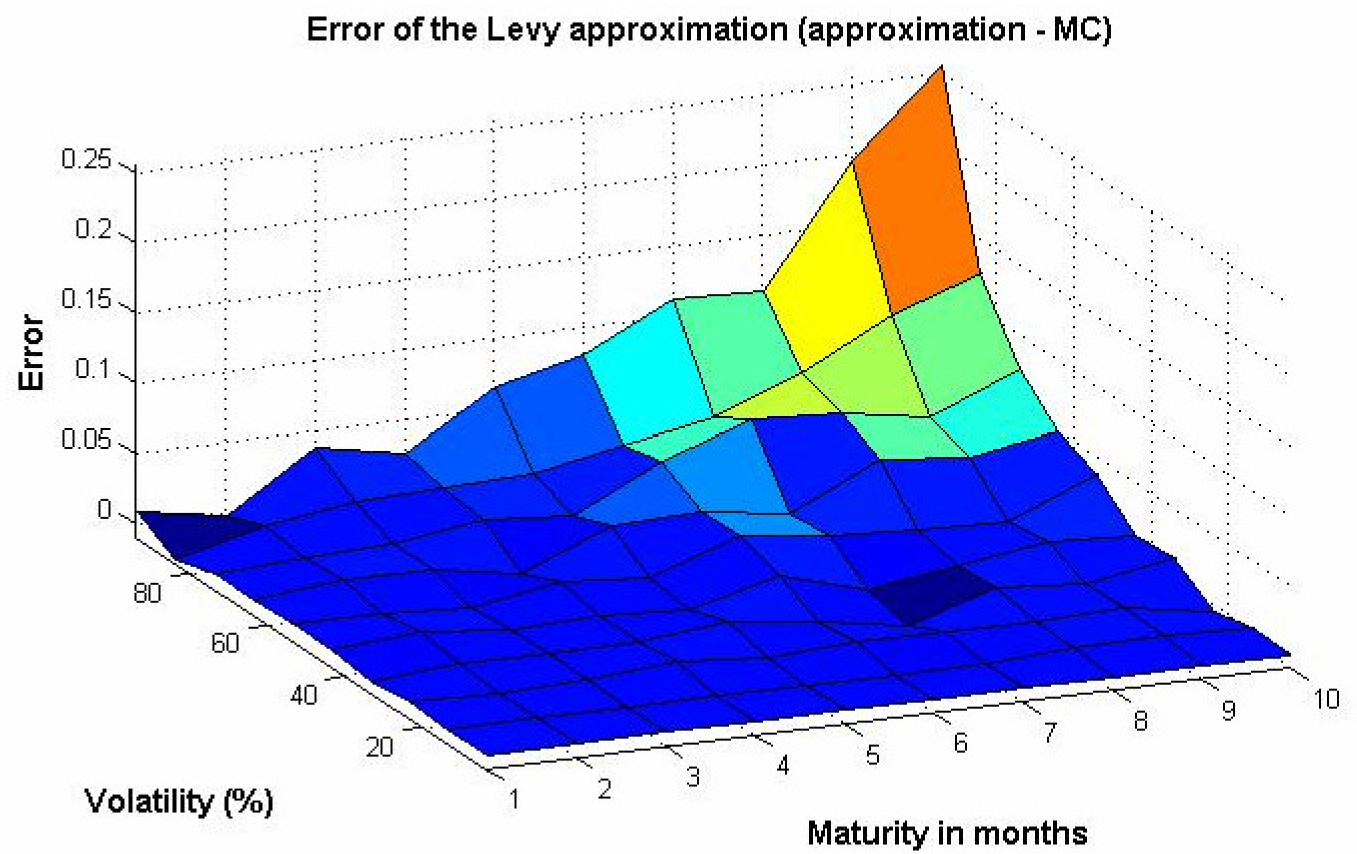


Conclusion (1/2)

- Simple two-moment approximations quick, flexible and easy to implement also outside Matlab
- Levy's approximation yields slightly more accurate pricing than the reciprocal gamma approximation
- Ju's approximation formulas should be derived for Asian basket options and tested

Conclusion (2/2)

- The weak point of Levy's approximation overpricing when volatility high or maturity long



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