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Modeling space elasticity of demand to support retail replenishment planning (presentation of complete work)

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Background

- The demand of a product is affected by the amount of shelf space allocated for it
- This dependency is called space elasticity
- The magnitude of the space elasticity depends on multiple product and store specific attributes

Objectives

1. Study the literature and find the attributes of a product that determine its space elasticity
2. Determine ways to estimate the space elasticity of product or product group
3. Figure out how to use the space elasticity information to support retail replenishment planning
4. Develop guidelines for estimating and utilizing the space elasticity information for a large group of products in multiple stores

Space elasticity

- The space elasticity varies between product categories [Desmet and Renaudin, 1998]
- Also, the store was found to have a significant effect in the dependency between demand and shelf space [Van Dijk et al., 2004]
- These product group and store specific variables couldn't be determined specifically

Space elasticity needs to be estimated separately for each product or product category in each store

Estimating the space elasticity 1/2

- The dependency between shelf space and demand was most often modeled as:

$$D = as^{\alpha}$$

- where D is the part of the demand that depends of the shelf space, a is a scaling factor, s represents shelf space and α is the space elasticity
- The obtained space elasticity values most often averaged between 0.15 and 0.20
- **Using this formula, space elasticity can be estimated with regression**

Estimating the space elasticity 2/2

- The effect of shelf space is minor compared to other factors in demand, such as:
  - seasonality
  - campaigns
  - price changes
  - products' position in the shelf [Dreze et al.,1994]

These factors need to be controlled in the estimation, either by correcting the sales data or using additional regressors in the model

Optimizing the shelf space allocation 1/3

- Multiple models that use space elasticity data to optimize shelf space allocation exists [Corstjens and Doyle, 1981] [Hansen and Heinsbroek, 1979]

$$\max \sum_{i=1}^{K} w_i \left[\alpha_i (s_i)^{\beta} \prod_{j=1, j \neq i}^{K} s_j^{\delta_{ij}} \right] - \sum_{i=1}^{K} \gamma_i \left[\alpha_i^{\tau_i} s_i^{\beta_i \tau_i} \prod_{j=1, j \neq i}^{K} s_j^{\delta_{ij} \tau_i} \right]$$

s.t

$$\sum_{i=1}^{K} s_i \leq S^*$$

$$\alpha_i s_i^{\beta_i} \prod_{j=1, j \neq i}^{K} s_j^{\delta_{ij}} \leq Q_i^* \qquad i = 1, ..., K,$$

$$s_i^L \leq s_i \leq s_i^U \qquad i = 1, ..., K$$

$$s_i \geq 0, \qquad i = 1, ..., K$$

$$\max \quad \sum_{i=1}^{n} \pi_i - f(N, L)$$

s.t

$$\sum_{i=1}^{n} r_i \leq R$$

$$r_i \geq r_i^m y_i, \qquad i = 1, 2, ..., n$$

$$r_i \leq R y_i, \qquad i = 1, 2, ..., n$$

$$y_i \in \{0, 1\}, \qquad i = 1, 2, ..., n$$

$$\frac{r_i}{l_i} \in N^+, \qquad i = 1, 2, ..., n$$

Optimizing the shelf space allocation 2/3

- However, these models contain components and assumptions that complicate their application to practical use in large retail chains

  - **Simplified model is needed**

- The position of the products should be taken into account in the optimization model

Optimizing the shelf space allocation 3/3

- The proposed approach: Define optimal way to allocate shelf space between product categories in each store so that:
  - Profit related to the allocation is maximized
  - Constraints ensuring a realistic allocation are followed
- The profit related to the shelf space can be obtained using the formula from slide 4
- The position can be taken into account by dividing the shelf space into different blocks and adding the blocks into the model as a decision variable

Conclusions

- Space elasticity needs to be estimated separately for each product category in each store
- Shelf space independent variation in sales needs to be controlled
- Space elasticity information can be used to optimize profit related to shelf space allocation
- In large retail chains, this can be done using simplified model while still taking product's position into account

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