Refining a probabilistic cross-impact methodology for scenario analysis

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15.01.2025

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Background

Aim of the thesis

- Develop and refine the methodology in Roponen & Salo (2024) by interpreting the cross-impact term differently
- The cross-impact for events *a* and *b* :
  - Interpretation in Salo et al. (2021): $C_{ab} := \frac{P(a \mid b)}{P(a)}$ → $P(a \mid b) = C_{ab} P(a)$
  - The new interpretation: $\frac{P(a \mid b)}{1 - P(a \mid b)} = C_{ab} \frac{P(a)}{1 - P(a)}$ → $P(a \mid b) = \frac{C_{ab} P(a)}{1 - P(a) + C_{ab} P(a)}$

Scenario analysis

- Scenarios can be defined as combinations of realizations of uncertainty factors.
- Scenario analysis is widely used to facilitate long-term strategic planning and to identify risks by portraying possible futures as scenarios.

Uncertainty factors and scenarios

**Uncertainty factors are described above and possible outcomes below them.*

Scenario 1

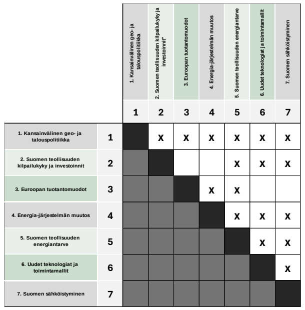
Geopolitics	Economic development	Regulation
Prolonged conflict between Israel and Palestine leads to blocks in world politics.	The EU economy has recovered from high inflation and interest rates.	Moderate regulatory support for green products.
Israel and Palestine agree on peace, which calms the atmosphere and increases international co-operation.	Slow economic growth, with lingering effects of high inflation and interest rates in the EU.	Strong regulatory support for green products, with subsidies and incentives.
Conflict in the Middle-East spreads to neighboring countries, which suppresses global collaboration and freezes global markets.	Moderate economic stability, with the EU maintaining steady but unremarkable growth.	Comprehensive green standards and international agreements promoting sustainable practices.

Illustration of exploring the possibilities of “green” products using scenario analysis.

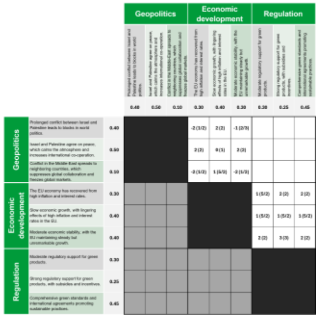
Overview of the method

1: Identifying relevant uncertainty factors

2: Order and dependency structure



3: Cross-impact and marginal probability analysis



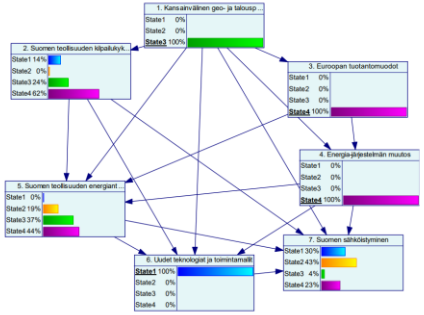
4: Iterative calculation of the joint probability distribution

1st iteration		-
1. Geopolitics	$p(s^{11})$	0.40
	$p(s^{12})$	0.50
	$p(s^{13})$	0.10

2nd iteration		1. Geopolitics		
		s^{11}	s^{12}	s^{13}
2. Economic development	$p(s^{21} \mid s^{11})$	0.21	0.37	0.29
	$p(s^{22} \mid s^{12})$	0.55	0.27	0.49
	$p(s^{23} \mid s^{13})$	0.24	0.36	0.22

3rd iteration		1. Geopolitics								
		s^{11}			s^{12}			s^{13}		
		2. Economic development								
		s^{21}	s^{22}	s^{23}	s^{21}	s^{22}	s^{23}	s^{21}	s^{22}	s^{23}
s_{D3}		(s^{11}, s^{21})	(s^{11}, s^{22})	(s^{11}, s^{23})	(s^{12}, s^{21})	(s^{12}, s^{22})	(s^{12}, s^{23})	(s^{13}, s^{21})	(s^{13}, s^{22})	(s^{13}, s^{23})
3. Regulation	$p(s^{31} \mid s_{D3})$	0.47	0.41	0.38	0.32	0.21	0.23	0.07	0.06	0.08
	$p(s^{32} \mid s_{D3})$	0.24	0.21	0.33	0.17	0.22	0.27	0.43	0.41	0.45
	$p(s^{33} \mid s_{D3})$	0.29	0.38	0.29	0.51	0.57	0.5	0.5	0.53	0.47

5: What if analysis



Cross-impacts

$$C_{ij} = \sqrt{2}^{V_{ij}},$$

Where C_{ij} is the cross-impact multiplier and V_{ij} is a statement ranging from [-3, 3] (Roponen & Salo 2024)

			Geopolitics			Economic development			Regulation		
			Prolonged conflict between Israel and Palestine leads to blocks in world politics.	Israel and Palestine agree on peace, which calms the atmosphere and increases international co-operation.	Conflict in the Middle-East spreads to neighboring countries, which suppresses global collaboration and freezes global markets.	The EU economy has recovered from high inflation and interest rates.	Slow economic growth, with lingering effects of high inflation and interest rates in the EU.	Moderate economic stability, with the EU maintaining steady but unremarkable growth.	Moderate regulatory support for green products.	Strong regulatory support for green products, with subsidies and incentives.	Comprehensive green standards and international agreements promoting sustainable practices.
			0.40	0.50	0.10	0.30	0.40	0.30	0.30	0.25	0.45
Geopolitics	Prolonged conflict between Israel and Palestine leads to blocks in world politics.	0.40				-2 (1/2)	2 (2)	-1 (2/3)			
	Israel and Palestine agree on peace, which calms the atmosphere and increases international co-operation.	0.50				2 (2)	0 (1)	2 (2)			
	Conflict in the Middle-East spreads to neighboring countries, which suppresses global collaboration and freezes global markets.	0.10				-2 (1/2)	1 (5/2)	-2 (1/2)			
Economic development	The EU economy has recovered from high inflation and interest rates.	0.30							1 (5/2)	2 (2)	2 (2)
	Slow economic growth, with lingering effects of high inflation and interest rates in the EU.	0.40							1 (5/2)	1 (5/2)	1 (5/2)
	Moderate economic stability, with the EU maintaining steady but unremarkable growth.	0.30							2 (2)	3 (3)	2 (2)
Regulation	Moderate regulatory support for green products.	0.30									
	Strong regulatory support for green products, with subsidies and incentives.	0.25									
	Comprehensive green standards and international agreements promoting sustainable practices.	0.45									

Iterative computation of the joint probability distribution

- Iterate over all uncertainty factors sequentially and calculate the joint probability distribution using cross-impact multipliers.
- Limiting the number of iterations:
  - Conditioning the realizations of uncertainty factors on the partial scenarios defined by preceding uncertainty factors, conditional independence and directed acyclic graphs.
  - The sequential least squares optimization problem incorporates these properties.

Adjusted least squares optimization problem

$$\min_{q(k|\mathbf{s}_{1:i-1})} \sum_{j=1}^{i-1} \sum_{(k,l)\in R_{ij}} \left[\left(\sum_{\{\mathbf{s}\in S_{1:i-1} | s_j = l\}} q(k \mid \mathbf{s}) q(\mathbf{s}) \right) - \frac{\hat{C}^{ij}_{kl} \hat{p}^i_k \hat{p}^j_l}{1 - \hat{p}^i_k + \hat{C}^{ij}_{kl} \hat{p}^i_k} \right]^2$$

$$\sum_{\mathbf{s}\in S_{1:i-1}} q(k \mid \mathbf{s}) q(\mathbf{s}) = \hat{p}^i_k, \quad \forall k \in \{1, 2, \ldots, n_i\}$$

$$\sum_{k=1}^{n_i} q(k \mid \mathbf{s}_{1:i-1}) = 1, \quad \forall \mathbf{s}_{1:i-1} \in S_{1:i-1}$$

$$q(k \mid \mathbf{s}_{1:i-1}) \geqslant 0. \quad \forall k \in \{1, 2, \ldots, n_i\}, \mathbf{s}_{1:i-1} \in S_{1:i-1}$$

Iterative computation of the joint probability distribution

1st iteration		-
1. Geopolitics	$p(s^{11})$	0.40
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3rd iteration		1. Geopolitics								
		s^{11}			s^{12}			s^{13}		
		2. Economic development								
		s^{21}	s^{22}	s^{23}	s^{21}	s^{22}	s^{23}	s^{21}	s^{22}	s^{23}
s_{D3}		(s^{11}, s^{21})	(s^{11}, s^{22})	(s^{11}, s^{23})	(s^{12}, s^{21})	(s^{12}, s^{22})	(s^{12}, s^{23})	(s^{13}, s^{21})	(s^{13}, s^{22})	(s^{13}, s^{23})
3. Regulation	$p(s^{31} \mid s_{D3})$	0.47	0.41	0.38	0.32	0.21	0.23	0.07	0.06	0.08
	$p(s^{32} \mid s_{D3})$	0.24	0.21	0.33	0.17	0.22	0.27	0.43	0.41	0.45
	$p(s^{33} \mid s_{D3})$	0.29	0.38	0.29	0.51	0.57	0.5	0.5	0.53	0.47

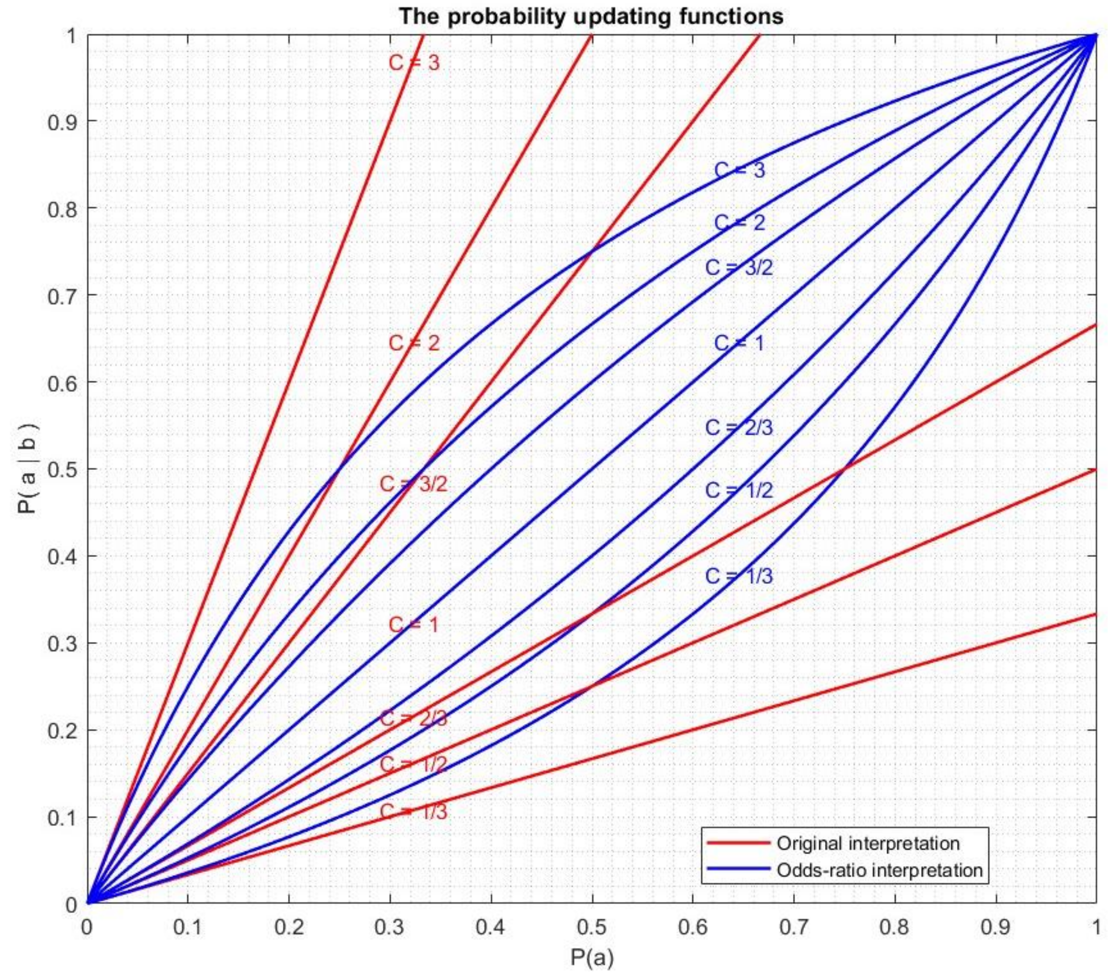
Comparison of both methods

- Visual tests
- Statistical tests
- Conditional distributions with Bayes-networks

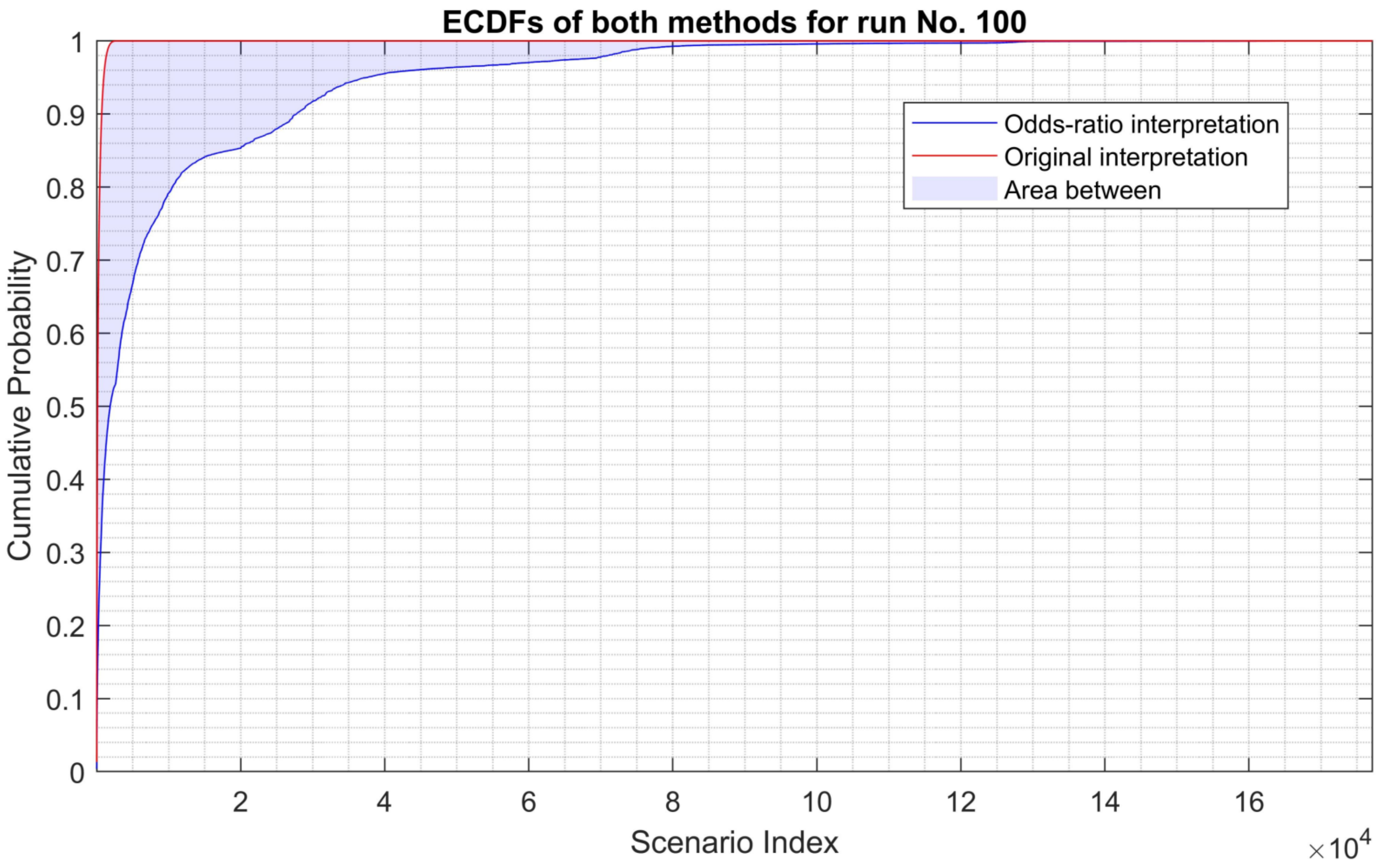
Two interpretations of the cross-impact term

(Original) $P(a \mid b) = C_{ab}P(a)$

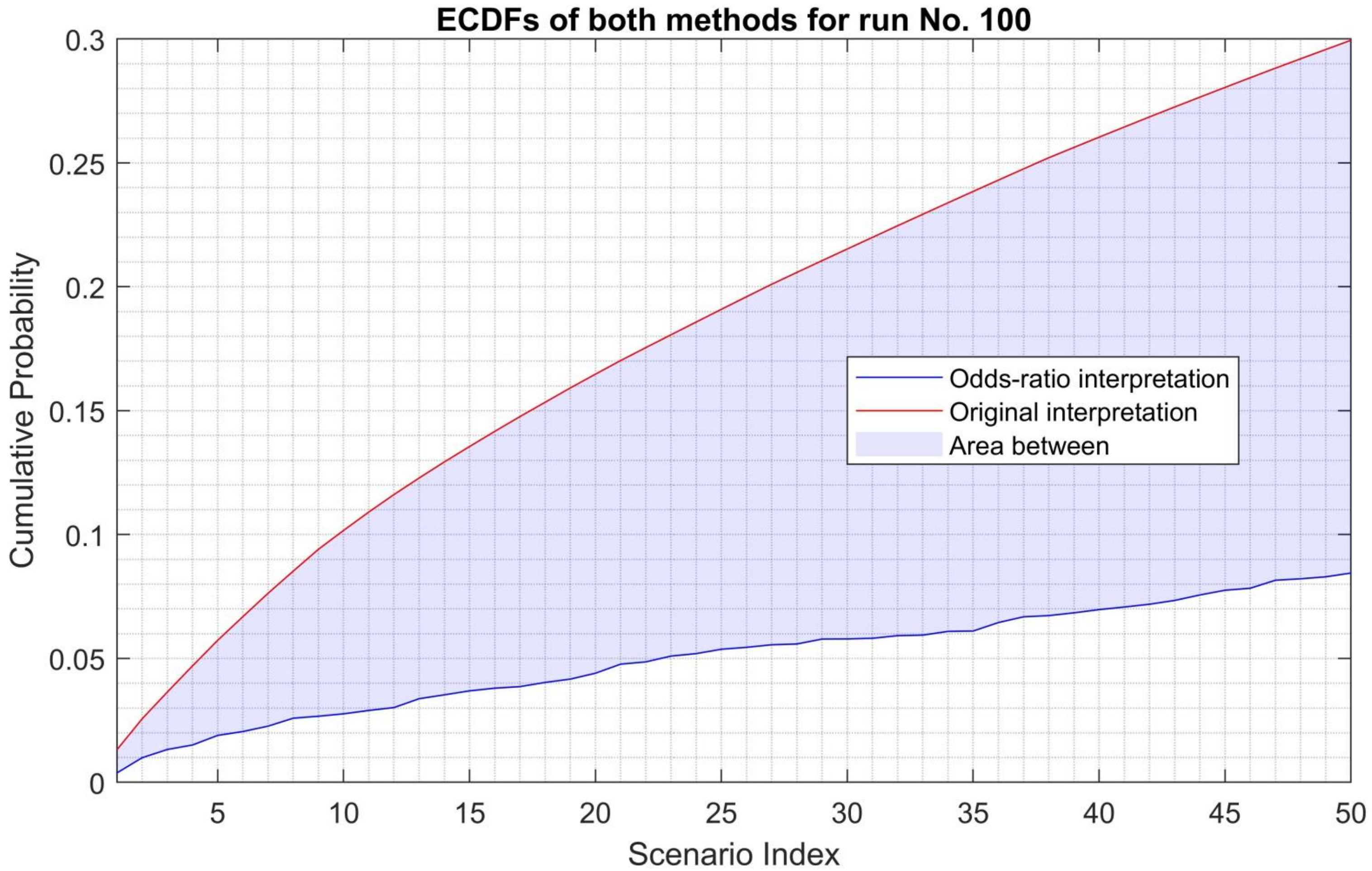
(Odds-ratio) $P(a \mid b) = \dfrac{C_{ab}P(a)}{1 - P(a) + C_{ab}P(a)}$



Comparison of the empirical CDFs



Comparison of the empirical CDFs



Statistical tests – First run

- Jensen–Shannon Divergence (JS)
- Total variation distance (TV)
- Kolmogorov–Smirnov test (KS)

the most probable scenarios, which account for 10% of probability mass

The mean values of the test statistics across 100 optimization runs.

Percentage of scenarios	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**	**100%**
JS Divergence	0.017	0.034	0.050	0.067	0.083	0.099	0.115	0.130	0.144	0.241
TV Distance	0.043	0.082	0.121	0.159	0.197	0.233	0.270	0.305	0.341	0.504
KS test p-value	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KS test rejections	95%	100%	100%	100%	100%	100%	100%	100%	100%	100%

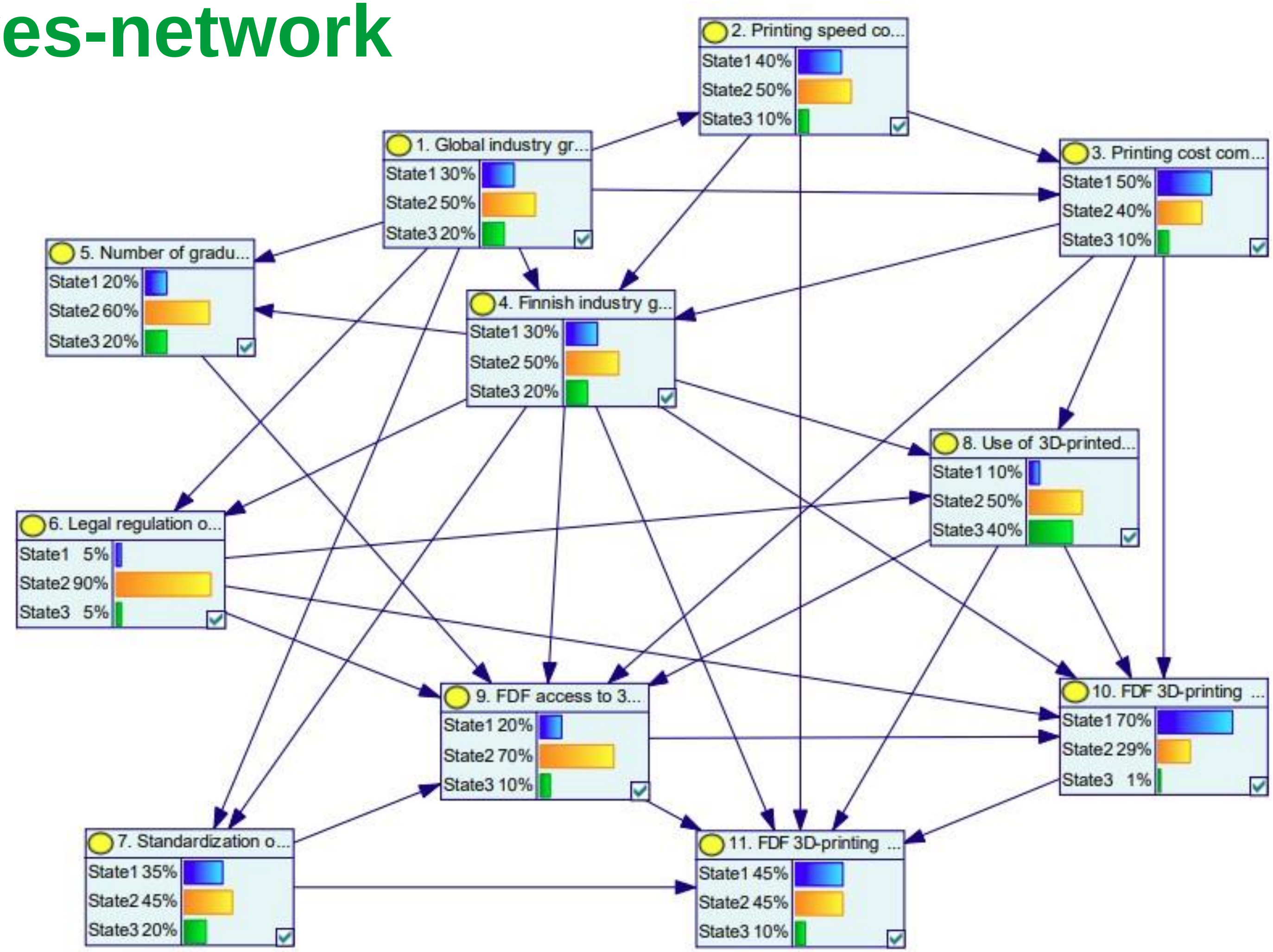
Statistical tests – Second run

- Considering only the most probable scenarios, which account for 10% of probability mass

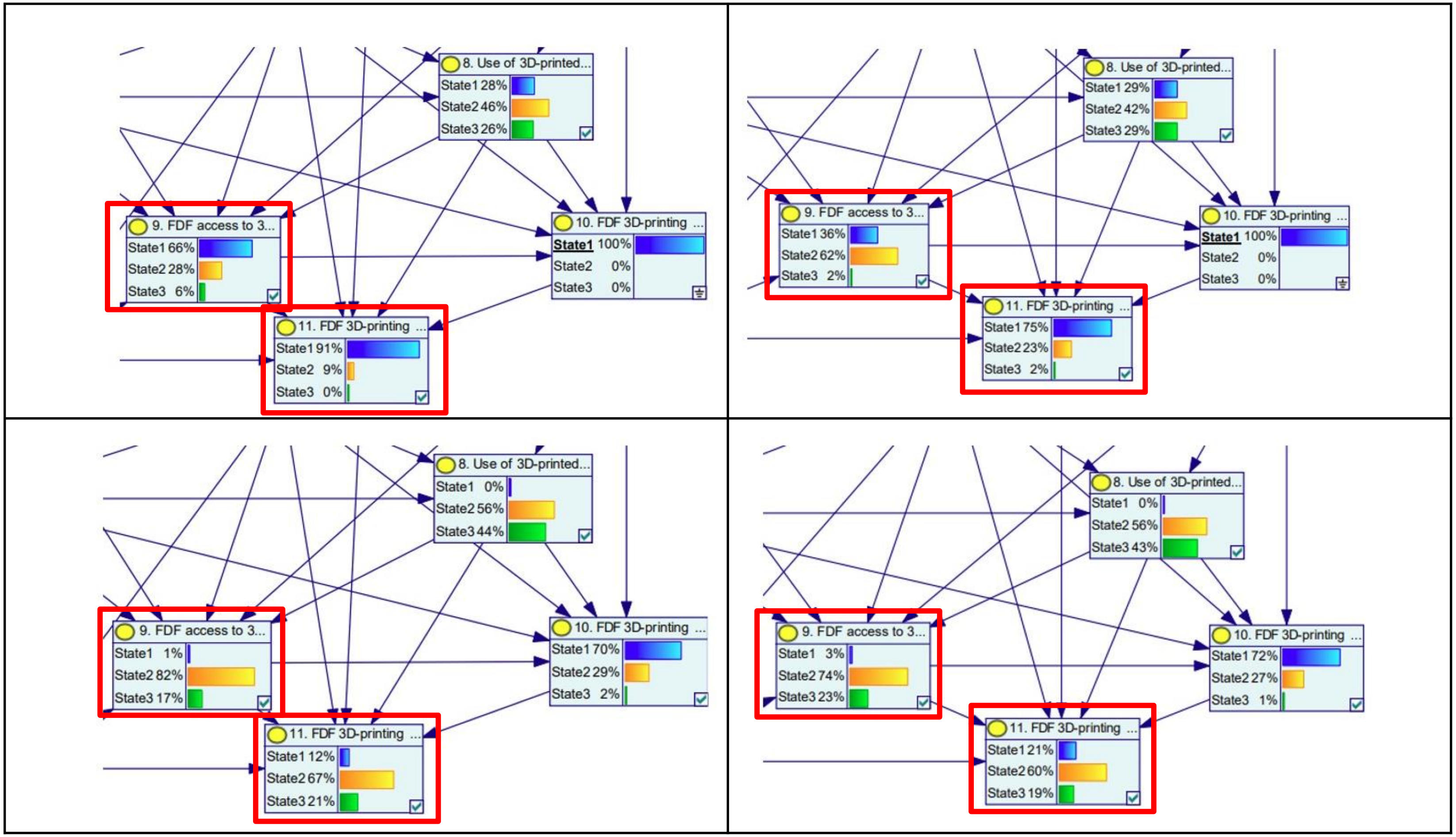
The mean values of the test statistics across 100 optimization runs.

Percentage of scenarios	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
JS Divergence	0.004	0.005	0.006	0.008	0.010	0.011	0.013	0.014	0.016	0.018
TV Distance	0.009	0.012	0.015	0.020	0.023	0.027	0.031	0.035	0.039	0.042
KS test p-value	0.274	0.196	0.127	0.075	0.049	0.031	0.022	0.015	0.008	0.006
KS test rejections	0%	11%	30%	51%	69%	84%	89%	94%	98%	99%

Bayes-network



Comparison of the conditional distributions



Key Results

1. Significant difference in individual scenarios.
2. Similar joint probability distributions for the most probable scenarios.
3. Significantly different joint probability distributions for the remaining scenarios.
4. New interpretation produced similar but more balanced conditional distribution.
5. No practical advantage in the new approach. Both approaches seem viable.

Key references

- Salo, A., Tosoni, E., Roponen, J., & Bunn, D. W. (2021). Using cross-impact analysis for probabilistic risk assessment. Futures & Foresight Science, 4, e2103
- Roponen, J. & Salo, A. (2024). A probabilistic cross-impact methodology for explorative scenario analysis. Futures & Foresight Science, 6, e165