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Public transport revenue optimization under no-elongation and no-stopover restrictions (Results of the Thesis)

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13.6.2025

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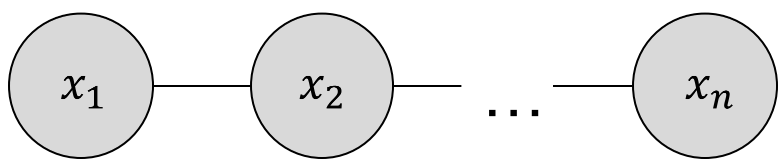
Background (1/5): Motivation

- Public transportation providers aim to maximize revenue
- Providers determine optimal prices for connections based on passengers' willingness-to-pay
- Special interest in no-elongation and no-stopover properties
- They produce a more consistent fare system and reduce the possibility for exploitation

Background (2/5): PTN

- Public transport network (PTN) modeled as a path graph (V, E)
- Set of stations $V = \{x_1, x_2, \dots, x_n\}$
- Set of direct connections (paths) E

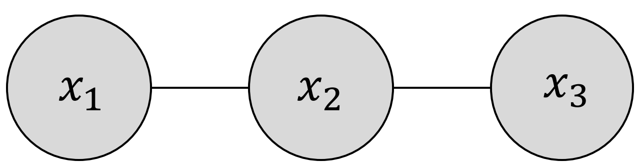


Background (3/5): Ticket

- Tickets for path $W = [x_i, x_j]$
  - Standard ticket $T(W)$
  - Elongated ticket $T(H)$, where $W \subseteq H$
  - Compound ticket $T(W_1, W_2)$, where $W_1 + W_2 = W$
- Price of a ticket $T = (H_1, \ldots, H_t)$ denoted $p_T = \sum_{j=1}^{t} p_{H_j}$

Background (4/5): No-elongation constraint

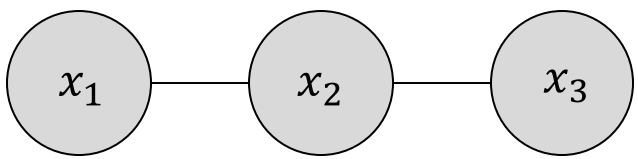
- It cannot be cheaper for a passenger to buy a ticket for a longer path and use only a part of it.



$$p_{[x_1, x_2]} \leq p_{[x_1, x_3]}$$

Background (5/5): No-stopover constraint

- It cannot be cheaper for a passenger to split a path into multiple sections and buy separate tickets for each subpath



$$p_{[x_1, x_3]} \leq p_{[x_1, x_2]} + p_{[x_2, x_3]}$$

Objectives

- Build a model that solves the optimal prices for tickets to maximize the provider's revenue

- Illustrate how the no-elongation and no-stopover properties affect the optimal solution

Methodology (1/2): Assumptions

- Each path $[x_i, x_j]$ has some demand $s_{[x_i,x_j]}$ (number of passengers)
- Demand is distributed into groups with different valuations (willingess-to-pay) for the path
- Distributions
  - Equal (all groups have equal number of passengers)
  - Decreasing (groups with high valuations have less passengers)
  - Random (demand is randomly split into groups)

Methodology (2/2): Model

- Formulate models
  - CPM: no-elongation and no-stopover
  - UPM: prices have no constraints
  - EPM: passengers exploit the prices of UPM
- Compare the optimal solutions between different models
- Compare the optimal solutions between different distributions of valuations

Model (1/3): Constrained Pricing Model (CPM)

$$\max \sum_{\substack{x_i,x_j \in V \\ i \neq j}} \sum_{g=1}^{G} \Gamma^g_{[x_i,x_j]} \cdot s^g_{[x_i,x_j]}$$ **(Maximize the revenue)**

$$\text{s.t.} \sum_{\substack{x_i,x_j \in V \\ i \neq j}} \sum_{g=1}^{G} s^g_{[x_i,x_j]} \cdot \delta^g_{[x_i,x_j]} \geq B$$ **(Minimum quota for tickets sold)**

$$\Gamma^g_{[x_i,x_j]} \leq p_{[x_i,x_j]} \qquad \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1,...,G\}$$

$$\Gamma^g_{[x_i,x_j]} \leq M \cdot \delta^g_{[x_i,x_j]} \qquad \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1,...,G\}$$

$$p_{[x_i,x_j]} \leq v^g_{[x_i,x_j]} + M \cdot (1 - \delta^g_{[x_i,x_j]}) \qquad \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1,...,G\}$$

(N-S) $$p_{[x_i,x_j]} \leq p_{[x_i,x_k]} + p_{[x_k,x_j]} \qquad \forall x_i, x_k, x_j \in V,\ i < k < j \text{ or } i > k > j$$

(N-E) $$p_{[x_i,x_j]} \leq p_{[x_k,x_l]} \qquad \forall x_i, x_j, x_k, x_l \in V,\ k \leq i < j \leq l \text{ or } k \geq i > j \geq l$$

$$\delta^g_{[x_i,x_j]} \in \{0,1\} \qquad \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1,...,G\}$$

$$p_{[x_i,x_j]} \in \mathbb{R}_+ \qquad \forall x_i, x_j \in V,\ i \neq j$$

$$\Gamma^g_{[x_i,x_j]} \in \mathbb{R}_+ \qquad \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1,...,G\} \tag{11}$$

Model (2/3): Unconstrained Pricing Model (UPM)

(N-E) and (N-S) are removed

$$
\begin{aligned}
\max \quad & \sum_{\substack{x_i,x_j \in V \\ i \neq j}} \sum_{g=1}^{G} \Gamma^g_{[x_i,x_j]} \cdot s^g_{[x_i,x_j]} \\
\text{s.t.} \quad & \sum_{\substack{x_i,x_j \in V \\ i \neq j}} \sum_{g=1}^{G} s^g_{[x_i,x_j]} \cdot \delta^g_{[x_i,x_j]} \geq B \\
& \Gamma^g_{[x_i,x_j]} \leq p_{[x_i,x_j]} && \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1, ..., G\} \\
& \Gamma^g_{[x_i,x_j]} \leq M \cdot \delta^g_{[x_i,x_j]} && \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1, ..., G\} \\
& p_{[x_i,x_j]} \leq v^g_{[x_i,x_j]} + M \cdot (1 - \delta^g_{[x_i,x_j]}) && \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1, ..., G\} \\
& \delta^g_{[x_i,x_j]} \in \{0, 1\} && \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1, ..., G\} \\
& p_{[x_i,x_j]} \in \mathbb{R}_+ && \forall x_i, x_j \in V,\ i \neq j \\
& \Gamma^g_{[x_i,x_j]} \in \mathbb{R}_+ && \forall x_i, x_j \in V,\ i \neq j,\ \forall g \in \{1, ..., G\}
\end{aligned}
\tag{12}
$$

Model (3/3): Exploited Pricing Model (EPM)

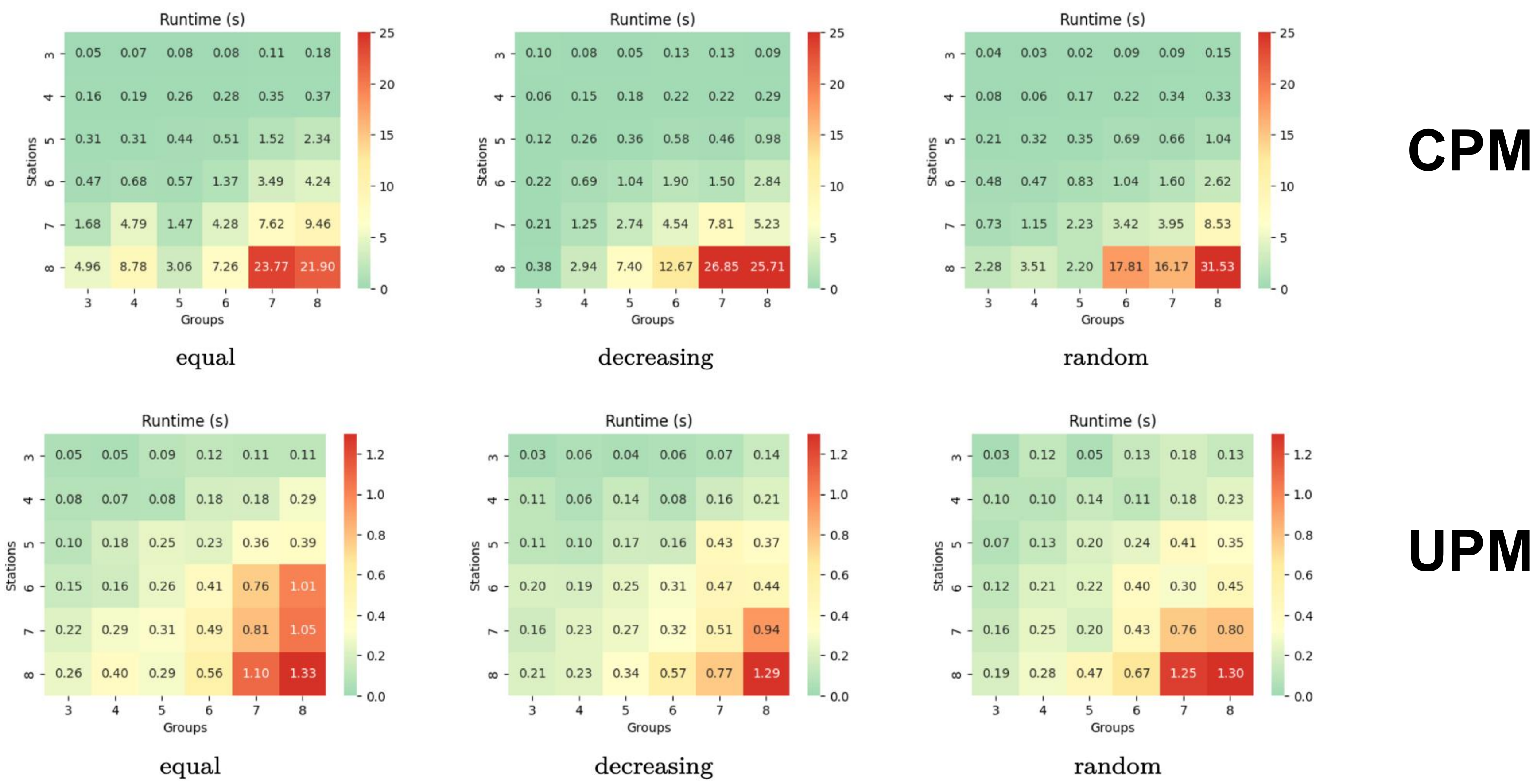
- Passengers exploit the optimal ticket prices given by UPM
- Out of ticket prices given by UPM, manually find the cheapest ticket (standard, elongated, or compound) for each path
  - Elongated: consider all elongated tickets and choose the cheapest one
  - Compound: shortest path algorithm

Results (1/2): Comparing optimal solutions

Distribution of groups	Model	Revenue	Average price	# Tickets sold	# Paid max valuation
Equal	CPM	34444	30.5	1171	368
	UPM	**38212**	34	1169	385
	EPM	26800	20	**1299**	385
Decreasing	CPM	28299	28	1181	405
	UPM	**31157**	29.5	1170	407
	EPM	26800	20	**1299**	433
Random	CPM	41820	34.5	1293	368
	UPM	**51470**	44.5	1293	490
	EPM	38900	30.5	**1350**	287

Number of stations = 5 and number of groups = 5

Results (2/2): Performance analysis



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Conclusions

- Introducing the no-elongation and no-stopover constraints yields higher revenue, assuming passengers would otherwise exploit the system
- Benefits the provider but passengers face higher prices and less tickets are sold
- Future study: complex graphs, bi-objective (e.g. revenue and tickets sold)

References

Anita Schöbel and Reena Urban. The cheapest ticket problem in public transport. Transportation Science, 56(6):1432–1451, 2022.