Aalto-yliopisto
Perustieteiden
korkeakoulu

Missing preferences in pairwise comparison matrices: a numerical study

Vili Ojala

20.4.2015

Instructor: Ph.D Matteo Brunelli
Supervisor: Prof. Raimo P. Hämäläinen

Systeemianalyysin
laboratorio

Preference

- Our preferences affect our decision making.
  - Effortless: banana or apple? Movie A or movie B?
  - Demanding: job A or job B? Apartment A or Apartment B
- Analytic Hierarchy Process (AHP) is a multi criteria decision making tool.
  - When choosing an apartment one might decide to score the apartments based on three criteria: 1. price, 2. location, 3. size.
  - Weights w_i are values which represent the strenght of one preference over another. $\sum w_i = 1, w_i \in [0,1], i = 1, \dots, n$

Pairwise comparison matrices

- Weights can be estimated from a pairwise comparison matrix **A.**

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1.5 \\ 0.33 & 1 & 0.5 \\ 0.66 & 2 & 1 \end{pmatrix} \approx \begin{pmatrix} w_1/w_1 & w_1/w_2 & w_1/w_3 \\ w_2/w_1 & w_2/w_2 & w_2/w_3 \\ w_3/w_1 & w_3/w_2 & w_3/w_3 \end{pmatrix}$$

- The weights can be estimated using the eigenvector method:
  - $\boldsymbol{wA} = \lambda \boldsymbol{w} \ \rightarrow \boldsymbol{w} = [0.5, \frac{1}{6}, \frac{1}{3}]$

- Consistency: $a_{ik} = a_{ij} a_{jk}, \forall\ i, j, k$
  - $a_{13} = a_{12} a_{23} \quad 1.5 = 3 * 0.5$

Estimating weights from incomplete pairwise matrices

- The first method was developed by Harker.
  - Modify the incomplete matrix -> estimate weights
  - Simple & fast

- The second method was developed by Shiraishi et al.
  - Optimization problem -> complete the matrix -> estimate weights
  - More complex & computationally demanding

Research questions

- How does the order of the pairwise comparison matrix and the amount of missing information affects the results?
- Is there a difference between the performances of the two methods?

Aalto-yliopisto
Perustieteiden
korkeakoulu

Systeemianalyysin
laboratorio

Methodology

- A numerical study made with Wolfram Mathematica

Complete pairwise comparison matrix

$$\boldsymbol{A} \rightarrow \boldsymbol{w} \rightarrow \text{rank}\,(w_m) = 1$$

m is the index of the largest weight estimated from the full comparison matrix **A**

Method of Harker

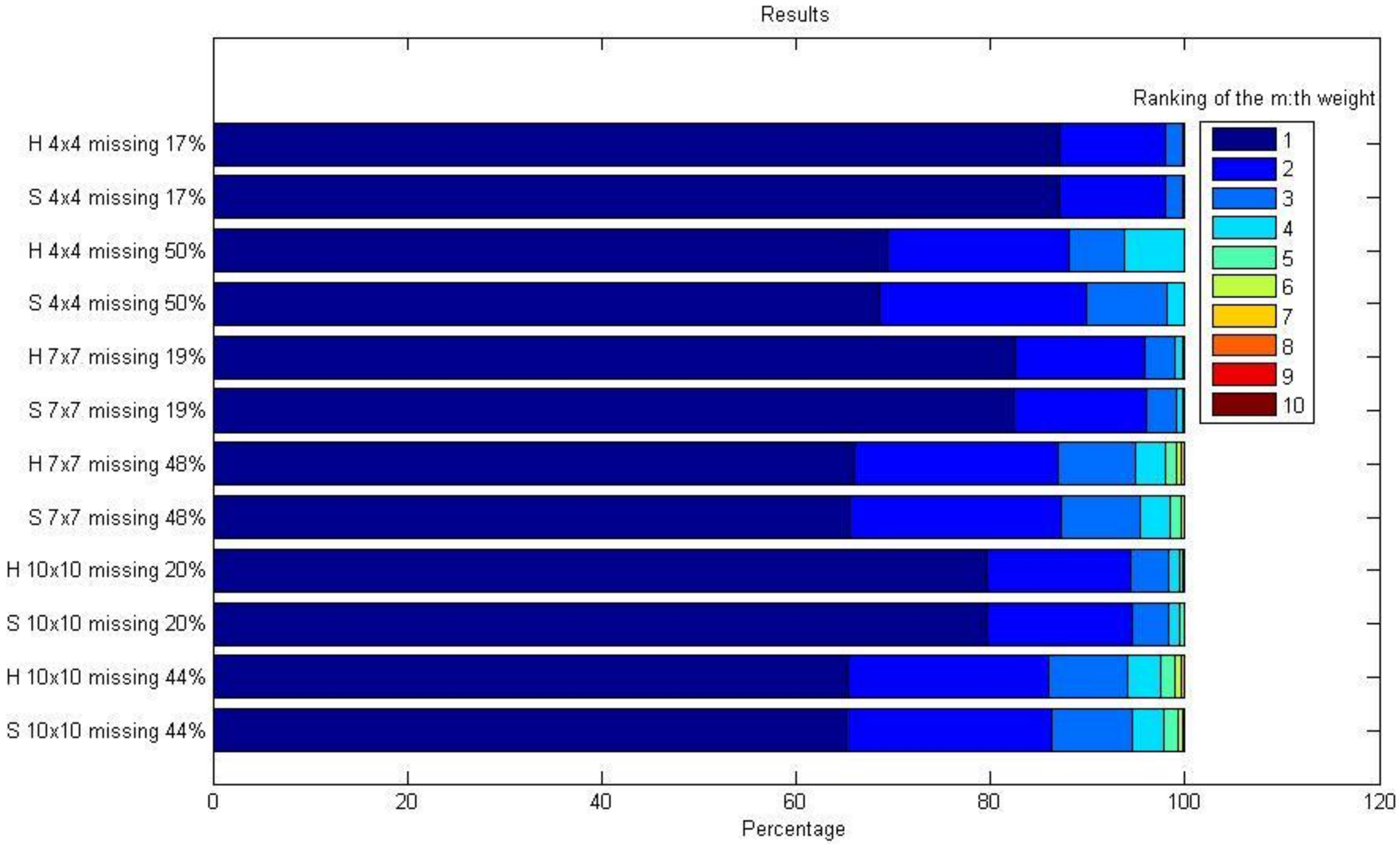
$$\boldsymbol{A} \rightarrow \boldsymbol{A'} \rightarrow \boldsymbol{w} \rightarrow \text{rank}\,(w_m) =?$$

Method of Shiraishi et al.

$$\boldsymbol{A} \rightarrow \boldsymbol{A'} \rightarrow \boldsymbol{w} \rightarrow \text{rank}\,(w_m) =?$$

To what rank do the two methods place the *m*-th weight?

Results



Aalto-yliopisto
Perustieteiden
korkeakoulu

Systeemianalyysin
laboratorio

Results

- Both methods performed similarly, but:
  - Harker's method had slightly more extreme results: The *m*-th weight got the rank one and the last rank more often than the method of Shiraishi et al.
  - Proposed explanation: When all the comparisons are missing from a row -> Harker's method estimates the corresponding weight to be zero.

$$\begin{pmatrix} 1 & x_{12} & x_{13} & x_{14} \\ 1/x_{12} & 1 & 0.86 & 0.68 \\ 1/x_{13} & 1.17 & 1 & 0.46 \\ 1/x_{14} & 1.47 & 2.17 & 1 \end{pmatrix}$$

- Method of Shiraishi et al. took over 90% of the simulation time.
- Even a small percentage of missing information caused errors in the ranking.

Thank you for your attention

Questions?