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Solving the Generalized Assignment Problem using Lagrangian Decomposition based subgradient optimization

Mika Koponen

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Supervisor: Fabricio Oliveira

Advisor: *Juho Andelmin*

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Background

- Generalized assignment problem, or GAP, is a NP-hard optimization problem
- Used in many real life appliances, for example assigning jobs to workers or scheduling variable length commercials to time slots
- There are several existing approaches to the problem with exact and approximation algorithms

Generalized assignment problem (GAP)

$$\text{(GAP)} \quad \text{Min} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \tag{0}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} x_{ij} \leq b_i, \quad i = 1, \ldots, m, \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, \ldots, m, \tag{2}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \tag{3}$$

Reformulated GAP

$$\text{(EGAP)} \quad \text{Min} \quad \alpha \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \beta \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} y_{ij}, \tag{0}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} x_{ij} \le b_i = 1, \ldots, m, \tag{1}$$

$$\sum_{i=1}^{m} y_{ij} = 1, \quad j = 1, \ldots, n, \tag{2}$$

$$x_{ij} = y_{ij}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \tag{3}$$

$$x_{ij}, y_{ij} \in \{0, 1\}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \tag{4}$$

- The thesis will mostly focus on the new Lagrangian relaxation method introduced by Jörnsten and Näsberg (1985)

Objectives

- Create an algorithm which uses Lagrangian decomposition based subgradient optimization to solve GAP
- Compare the effectiveness of this method compared to other known methods
- Compare the convergence with different step size rules, for example CFM and Polyak

Tools

- Julia
  - Actual implementation of the Lagrangian decomposition based subgradient solver
  - Measuring CPU time
  - Solvers to obtain exact solutions
- Python3
  - Result visualisation

Key dates

- Presentation of the topic 17.03.2021
- Implementation of algorithms 4/2021
- Writing thesis and obtaining results 5/2021
- Thesis ready 6/2021

Literature and references

- G. Ross and Richard Soland. A branch and bound algorithm for the generalized assignment problem Mathematical Programming, 8:91–103, 1975
- Kurt Jörnsten and Mikael Näsberg. A new Lagrangian relaxation approach to the generalized assignment problem. European Journal of Operational Research, 27:313–323, 1986
- Martello S., Toth P. Generalized assignment problems. In: Ibaraki T., Inagaki Y., Iwama K., Nishizeki T., Yamashita M. (eds) Algorithms and Computation. ISAAC 1992. Lecture Notes in Computer Science, vol 650. Springer, Berlin, Heidelberg, 1992