Aalto-yliopisto
Perustieteiden korkeakoulu

An atomic routing game with hard vehicle capacities (valmiin työn esittely)

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Thesis objectives

- Mathematical formulation of optimal path for individual in EAN with hard capacity constraints and path replacement
  - Discovery of optimal path for individual
  - Experimental analysis of equilibrium states
    - Repeating optimal paths leads to equilibrium
  - How optimal routes and solutions change on depending to
    - Capacity
    - Passenger order

Problem Motivation

- Shortest Path model
  - No capacity constraints
- Multi-commodity model [1]
  - Finds system optimal
    - Doesn't allow selfishness of people
- Dynamic passenger routing model
  - Equilibrium as constraint [2]
    - Can't be implemented for atomic passengers or scheduled flow

Model Inputs

- Event Activity Network (EAN)
  - Directed graph structure with events as nodes and activities as edges
  - Events: arrival and departure from station/stops
  - Activities: transportation, wait and transfer
- Timetable
  - Contains time each event happens
- Origin-destination (OD) matrix
  - OD matrix contains the amount passengers travelling from one node to another
- Capacity Constraints
  - Limit of passengers using an edge

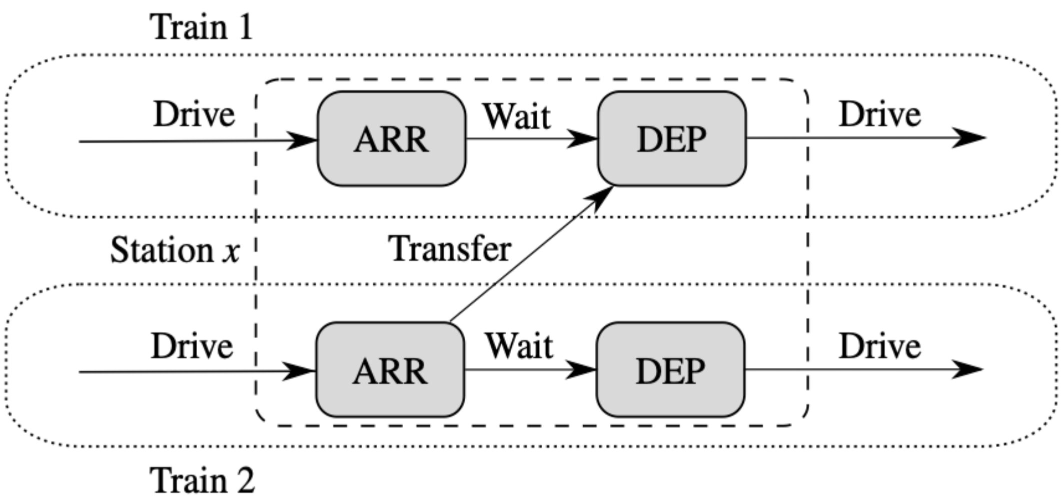


Figure: Event Activity Network

Small portion of an EAN illustrating how events and activities are structured.

Source [3]

Preprocessing of data

- Individualization of passengers

  OD matrix is coverted into list containing each passenger in random order.
- Conversion of origin and destination data
  - Orgin and destination data is given for PTN where nodes are stops or stations
  - To convert this data for the EAN, stops are mapped to corresponding arrival and departure events by expanding the EAN
- Time spent traveling an edge
  - Periodic timetable gives time event happens in each period
  - Cost to travel from i to j: $(\pi_j - \pi_i - L_{i,j})\ Mod\ T + L_{i,j}$

Mathematical Model

- Objective: Minimize travel time
- Constraints:
  - Flow conservation
  - Capacity constraint
  - Blocking constraints
    - Passenger can be blocked if new passenger has priority to the edge
  - Blocking limit
    - New passenger can block only one passenger per edge
  - Blocking monotonicity
- Iterating for multiple passenger
  - Amount of previous travelers increases

Data used

- TimPassLib dataset Hamburg [source]
  - EAN representing S-Bahn Hamburg
    - 508 Events
    - 823 Activities
  - OD matrix based on automatic passenger counts in trains
  - Timetable optimized for fixed passenger routes

Line plan of S-Bahn Hamburg

Public transport network with line plan showing the network EAN was done for.

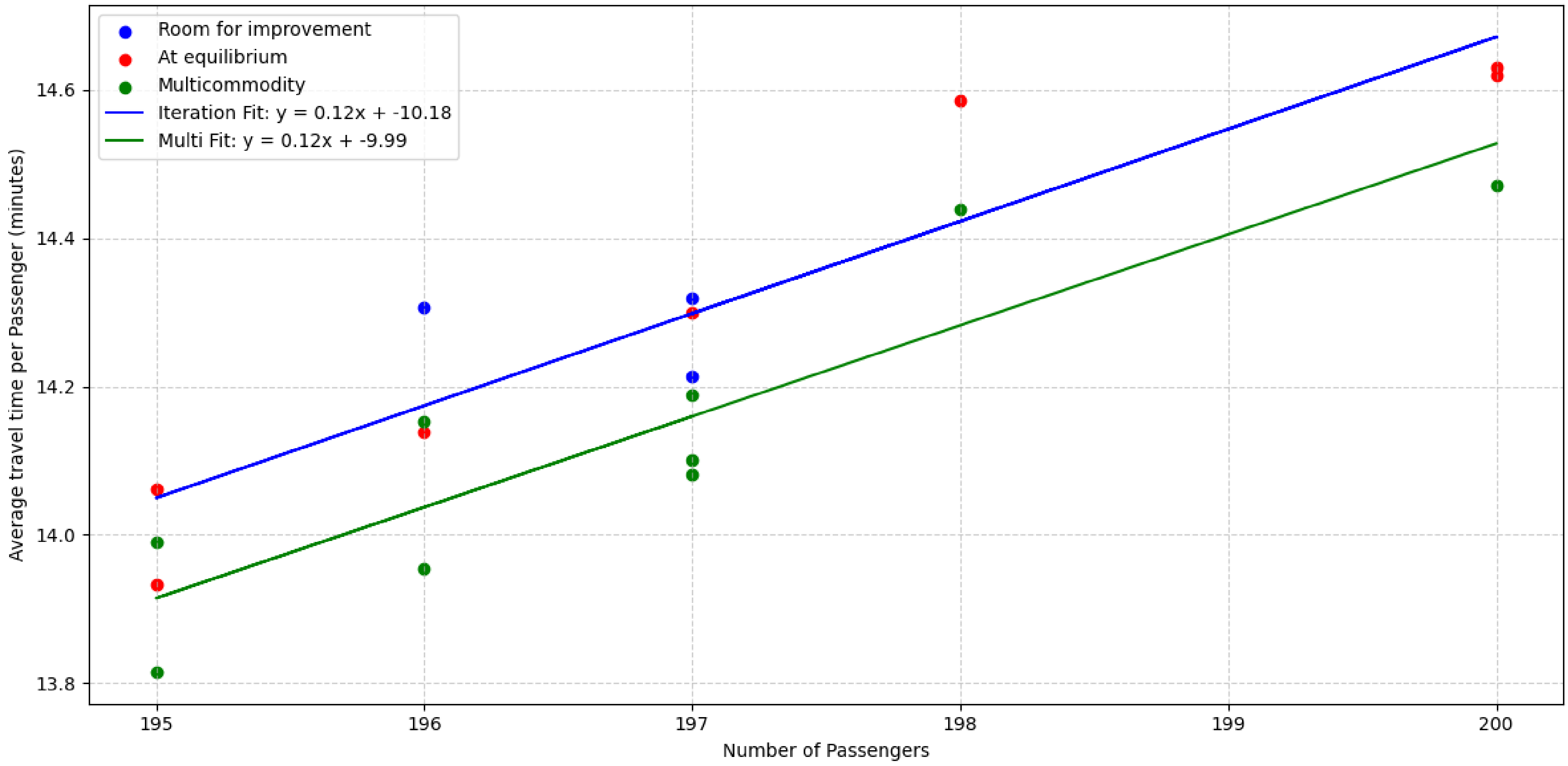
Source [Wiki]

Parameters used

- With Rerouting of blocked passengers
  - Number of assigned passengers = 200
    - Resulted in reasonable run time
  - Capacity of each edge = 11
    - Lowest capacity that had feasible solution for all passengers in multi-commodity model
    - Testing of nearby parameters was included
    - 10 rerouting rounds
- Without rereouting
  - Number of assigned passengers = 1000
    - Resulted in reasonable run time
  - Capacity of each edge = 5
    - Encourages blocking as edges are filled quickly
    - Allows enough options to make blocking beneficial

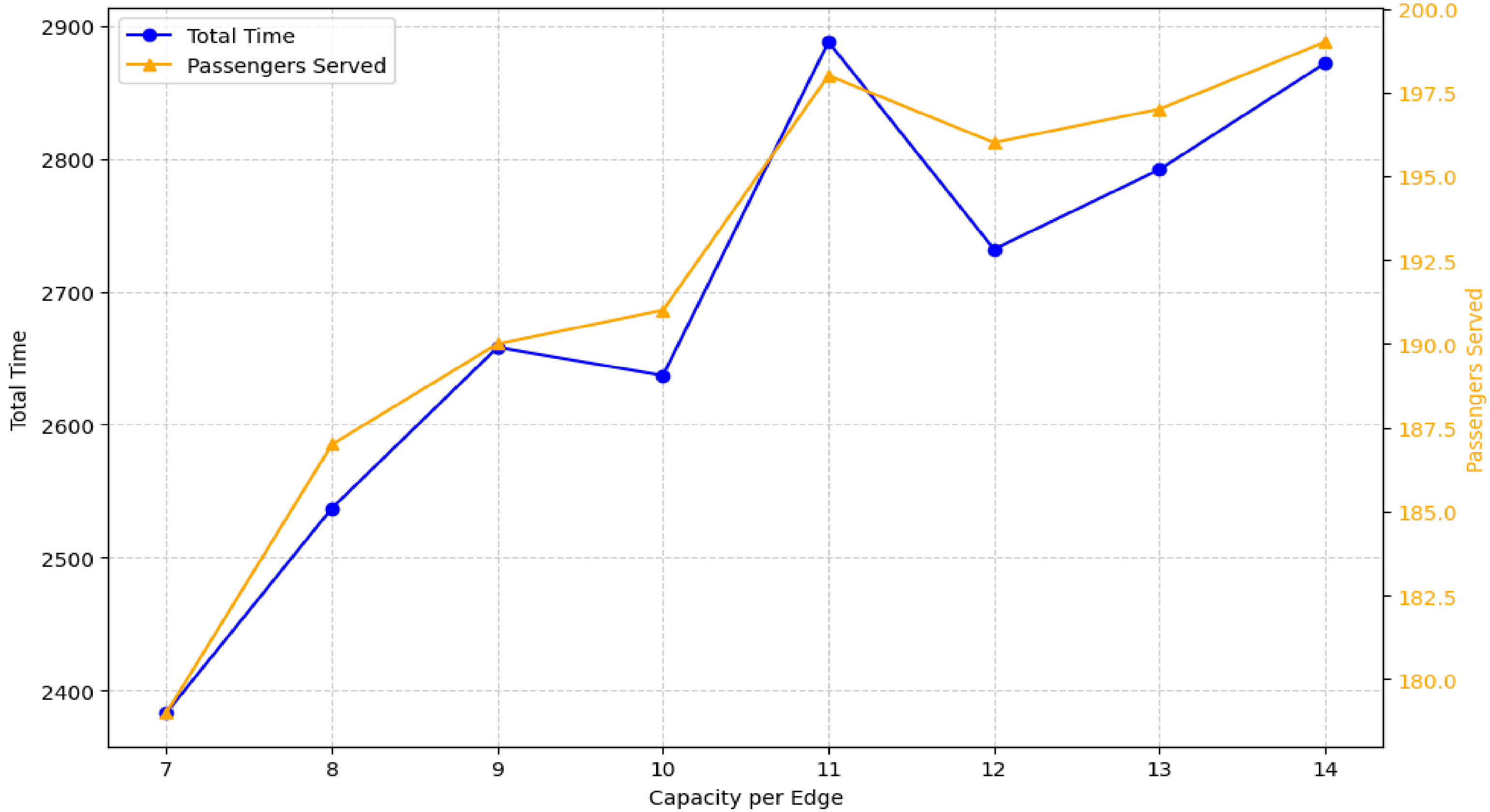
Results

- Using different orders (n=10)
  - Total Travel Time (minutes)
    - Mean: 2821.00
    - Std Dev: 68.14
    - Median: 2810.50
  - Average Travel Time per Passenger (minutes)
    - Mean: 14.31
    - Std Dev: 0.23
    - Median: 14.30
  - Equilibrium at end: 70%
  - Lower Bound for Price of Anarchy: 1.01



Effect of passenger order

Average travel times in the end routing with rerouted passenger model.
Each result is from same set of passengers with different order.



Effect of Edge Capacity

Effect of edge capacity on final outcomes of the rerouting process over ten rounds. Total travel time (left axis, blue) reflects overall system efficiency. Passengers served (right axis, orange) indicates system throughput.

Results

- End state of model without rerouting
  - 291 passengers with route
  - 383 feasible routes found
  - 92 overtakes
  - 3 non-optimal routes
    - Fixing those lead to equilibrium
- Sum of travel durations for each passenger at end state
  - This model: 2391 minutes
  - Closest equilibrium: 2379 minutes
  - Multi-commodity model: 2344 minutes
  - Without capacity constraint: 2313 minutes

$$\min \sum_{(i,j)\in A} x_{i,j} c_{i,j} \tag{5}$$

$$\sum_{(i,j)\in\delta-(i)} x_{i,j} - \sum_{(i,j)\in\delta+(i)} x_{j,i} = \begin{cases} 1 & i = s \\ -1 & i = t \\ 0 & else \end{cases} \quad \forall i \in V \tag{6}$$

$$\sum_{(i,j)\in\delta^+(i)} x_{i,j} \leq 1 \quad \forall i \in V \tag{7}$$

$$x_{i,j} + \sum_{h\in H_{i,j}} y_{i,j,h} \leq K_{i,j} \quad \forall (i,j) \in A_H \tag{8}$$

$$\sum_{h\in H_{i,j}} y_{prev(i,j,h)} - y_{i,j,h} \leq 1 \quad \forall (i,j) \in A_H \tag{9}$$

$$1 - \sum_{(l,k)\in\delta^+(j))} x_{l,k} \leq y_{i,j,h} \quad \forall (i,j) \in A_h \cap A_{transfer}, \quad \forall h \in H \tag{10}$$

$$x_{i,j} + \sum_{h\in H_{i,j}} y_{i,j,h} \geq K_{i,j} \cdot \sum_{(i',j')\in\delta^+(i)} \sum_{h\in H_{i',j'}} y_{prev(i',j',h)} - y_{i',j',h} \quad \forall (i,j) \in A_{drive}, \tag{11}$$

$$y_{i,j,h} = 1 \quad \forall (i,j) \in A_{start} \tag{12}$$

$$y_{prev(i,j,h)} = y_{i,j,h} \quad \forall (i,j) \in A_h \backslash A_{transfer}, \quad \forall h \in H \tag{13}$$

$$y_{prev(i,j,h)} \geq y_{i,j,h} \quad \forall (i,j) \in A_h \cap A_{transfer}, \quad \forall h \in H \tag{14}$$

$$x_{i,j}, y_{i,j,h} \in \{0,1\} \tag{15}$$

(i,j)=a: arc between nodes
A = all arc
A_H all arcs that have been used by previous travelers
A_h ordered list of actions of individual h
$\delta^+(i)$: coming arcs to node i
$\delta^-(i)$: leaving arcs from node i
H: Previous travelers
$H_{i,j}$: Previous travelers with path (i,j)
$y_{a,h}$: indicator if traveler h uses arc a
A_{wait}: List of actions with description "wait"
prev(i,j,h): Gives the previous arc of traveler h.

Mathematical Model

References

[1] Khodakaram Salimifard and Sara Bigharaz. The multicommodity network flow problem: state of the art classification, applications, and solution methods. Operational Research, 22(1):1–47, 2022.

[2] Valentina Trozzi, Guido Gentile, Michael Bell, and Ioannis Kaparias. Dynamic user equilibrium in public transport networks with passenger congestion and hyperpaths. Procedia-Social and Behavioral Sciences, 80:427–454, 2013.

[3] Marc Goerigk, Marie Schmidt, Anita Schöbel, Martin Knoth, and Matthias Müller-Hannemann. The price of strict and light robustness in timetable information. Transportation Science, 48(2):225–242, 2014

[4] NordNordWest. Karte der s-bahn hamburg 2023. https://de.wikipedia.org/wiki/Datei:Karte_der_S-Bahn_Hamburg_2023.svg, 2022. Based on a map by Arbalete. Licensed under CC BY-SA 4.0