Aalto-yliopisto
Perustieteiden
korkeakoulu

A Study on Decision Making Using Fuzzy Decision Trees

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päivämäärä muodossa 20.4.2015

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Työn saa tallentaa ja julkistaa Aalto-yliopiston avoimilla verkkosivuilla. Muilta osin kaikki oikeudet pidätetään.

Systeemianalyysin
laboratorio

Objectives of the Thesis

- Study extensions of decision trees using fuzzy arithmetics
- Examine methods to capture imprecision in estimated quantities
- Consider formation of fuzzy decision rules

Characteristic and membership functions

- Fuzzy set theory often viewed as an extension of classical (crisp) set theory
- Membership function as an extension of the characteristic function

Characteristic function $A : X \rightarrow \{0,1\}$

Membership function $A : X \rightarrow [0,1]$

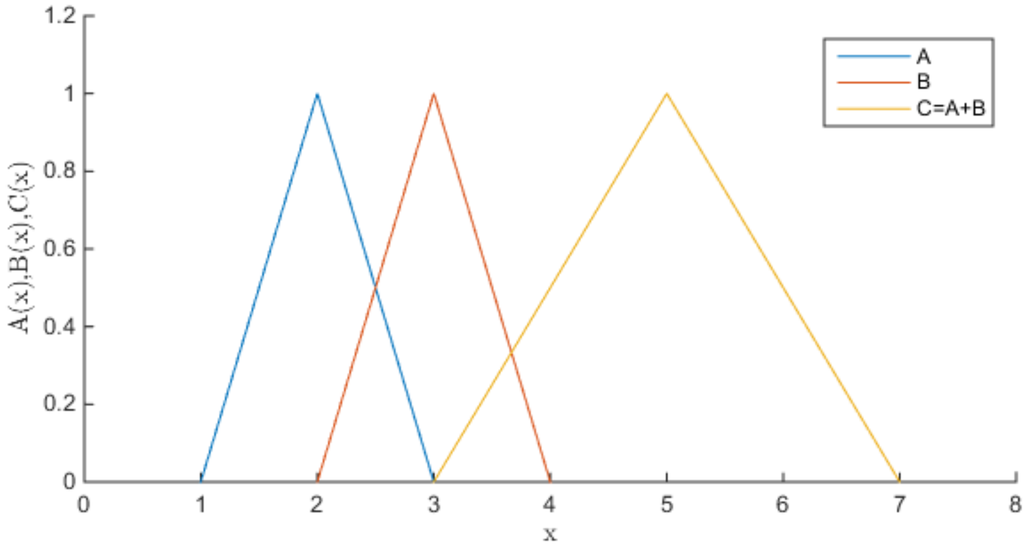
Alpha cuts and the decomposition theorem

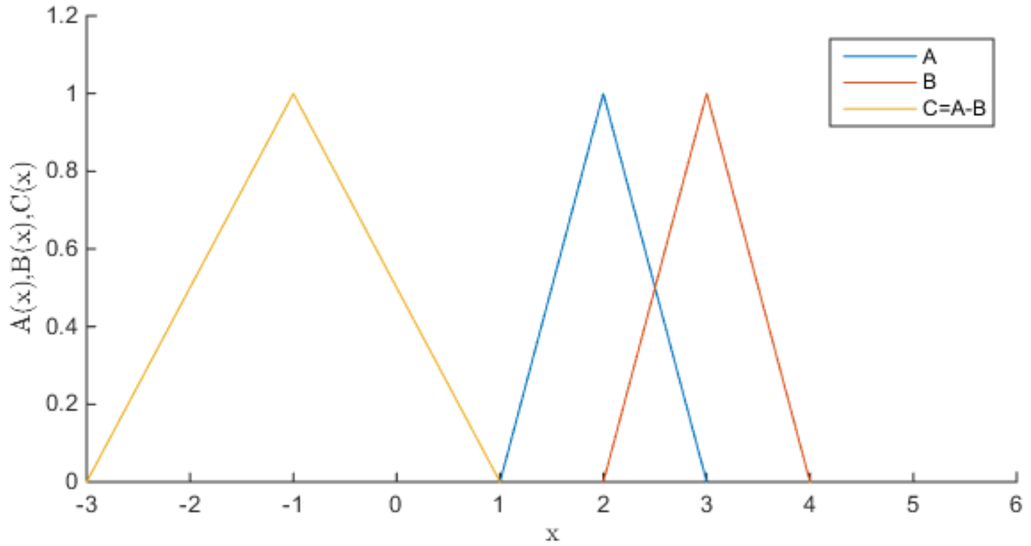
$$ {}^{\alpha}A = \{x \mid A(x) \geq \alpha\} $$

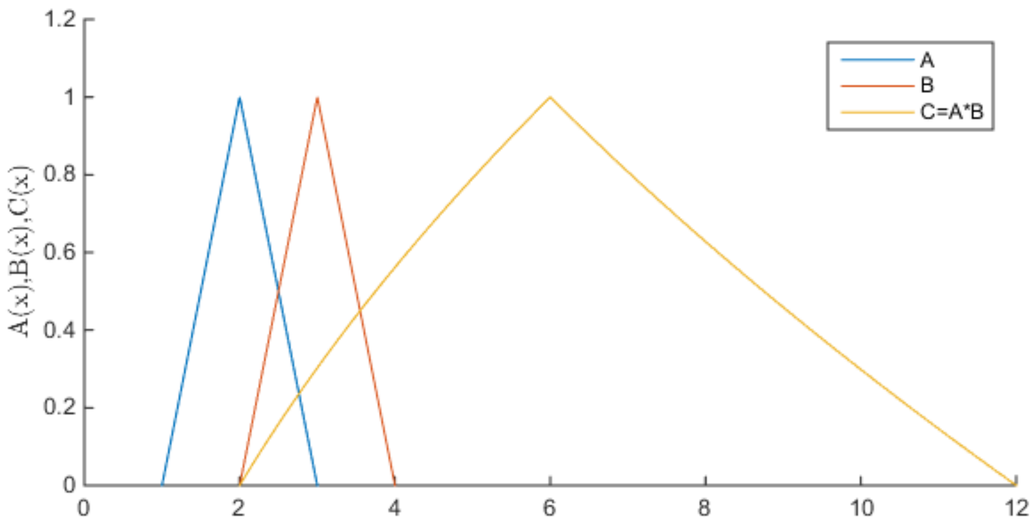
$$ A = \bigcup_{\alpha \in [0,1]} \alpha \cdot {}^{\alpha}A $$

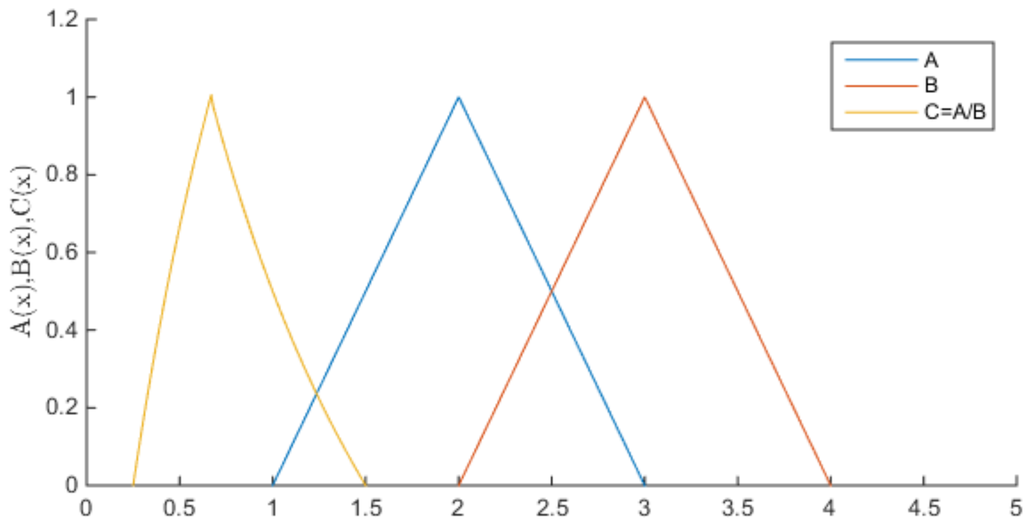
$$ A * B = \bigcup_{\alpha \in [0,1]} \alpha \cdot ({}^{\alpha}A * {}^{\alpha}B) $$

Arithmetic Operations

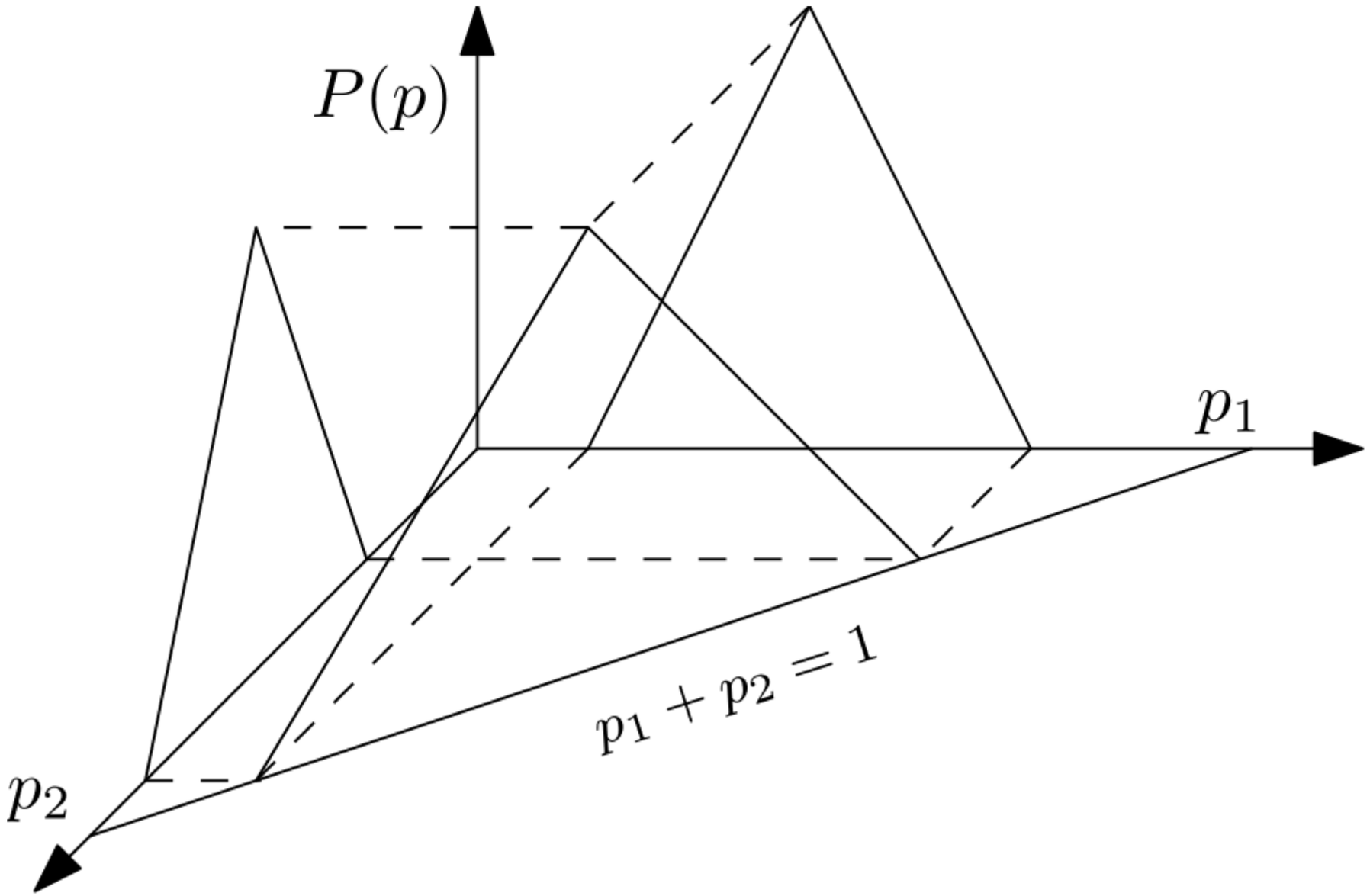






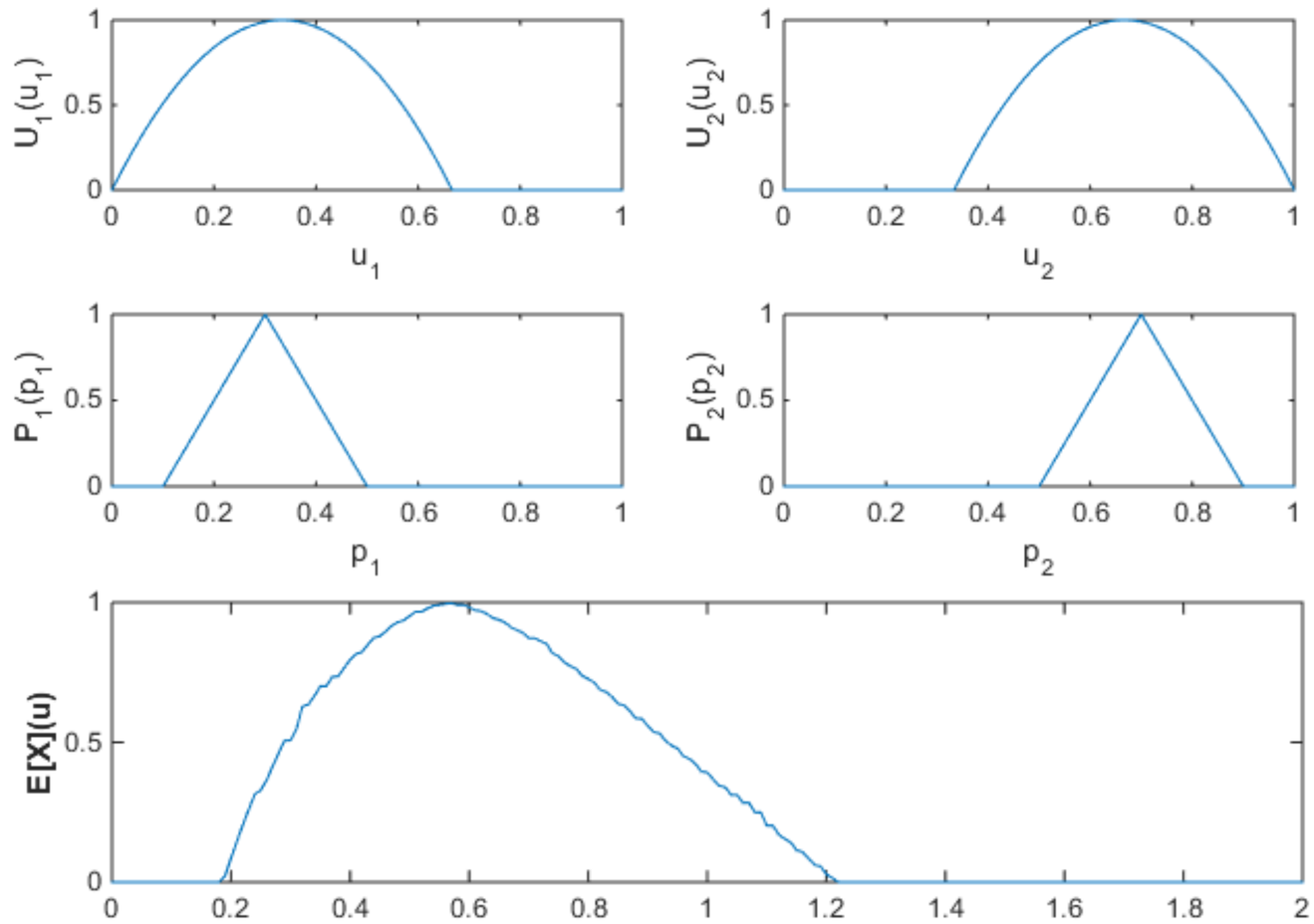


Normalization

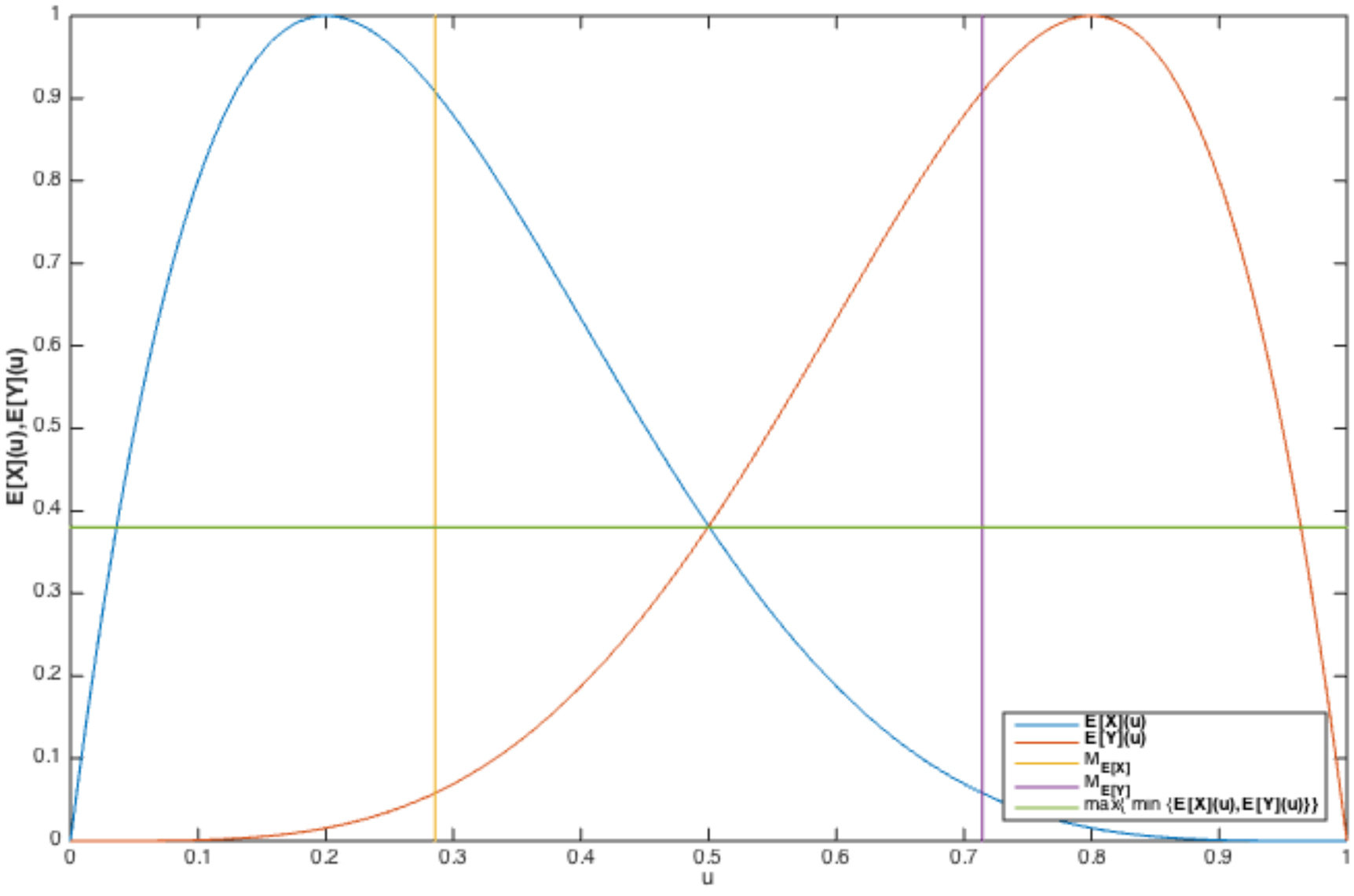


Expectation

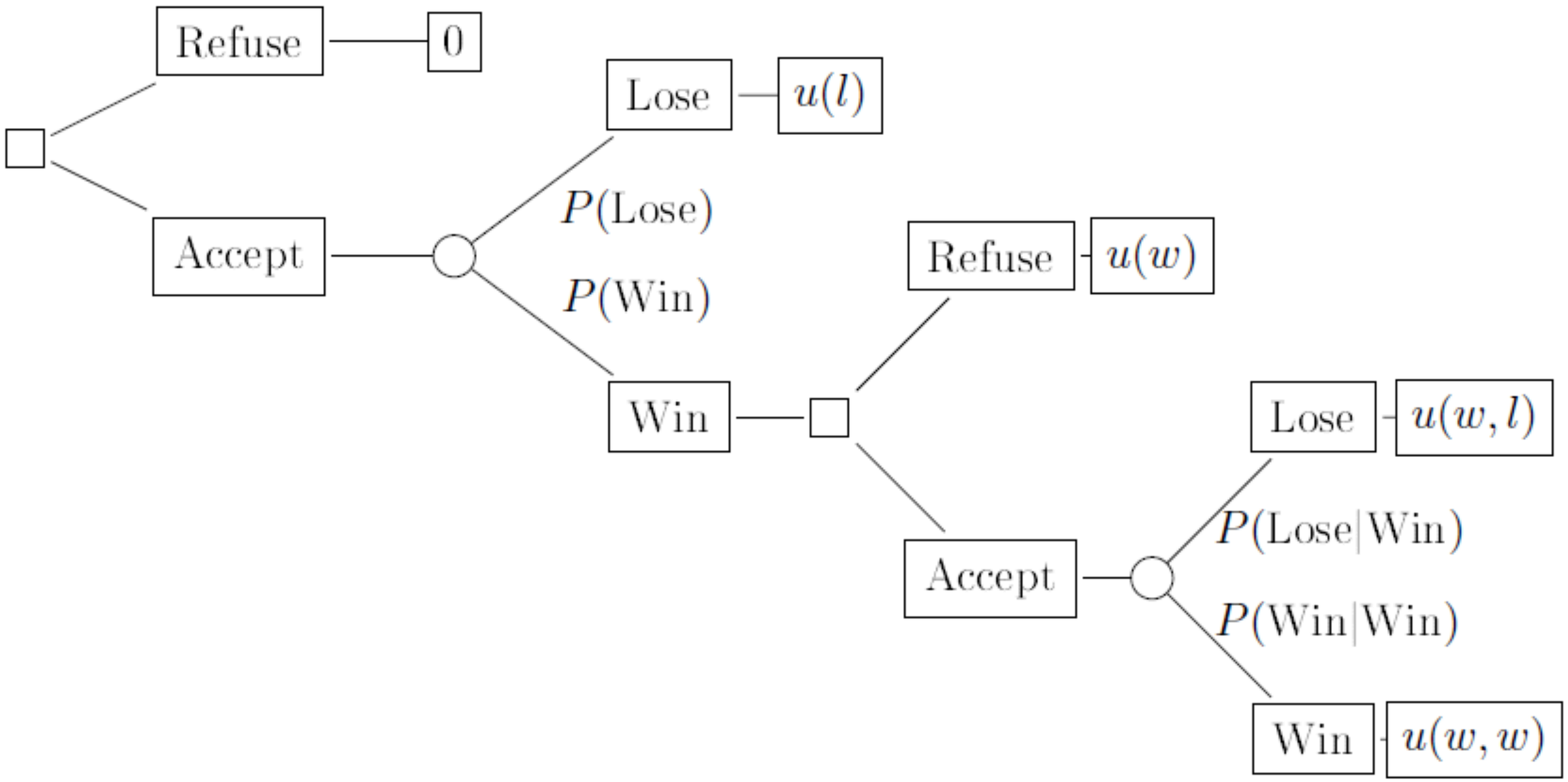
$$\mathbb{E}[X] = \sum_i p_i x_i$$

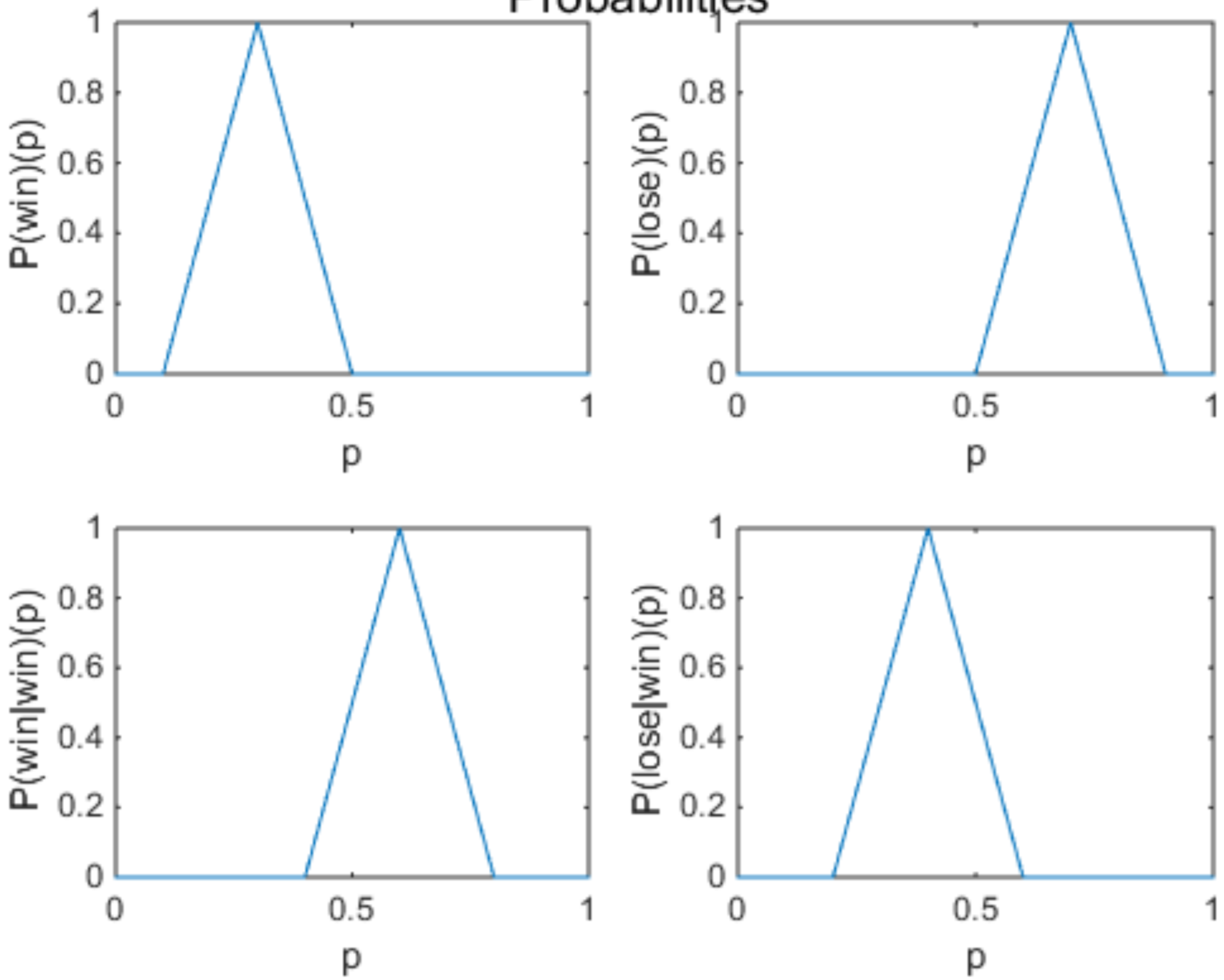


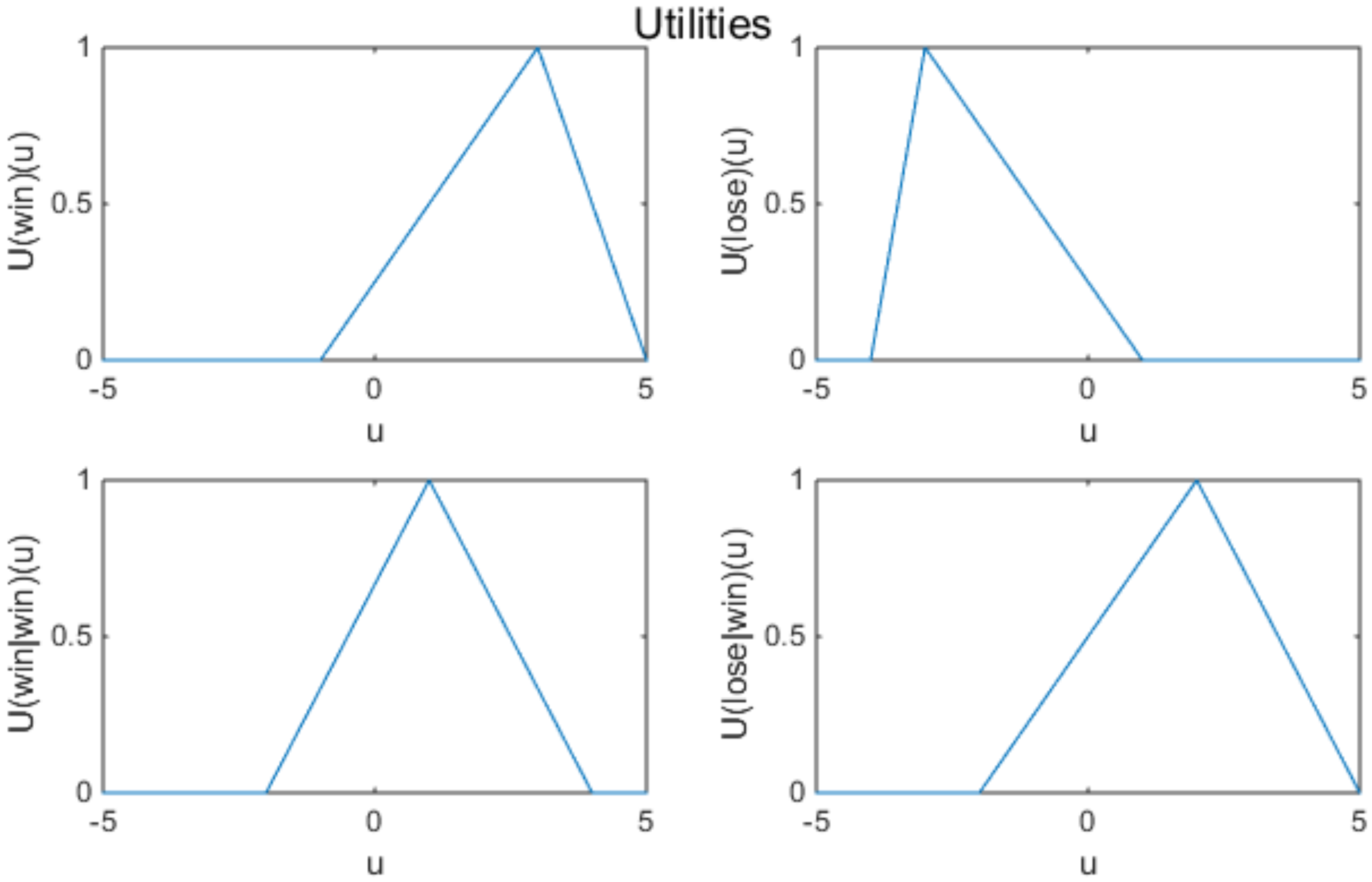
Crisp and Fuzzy Preference

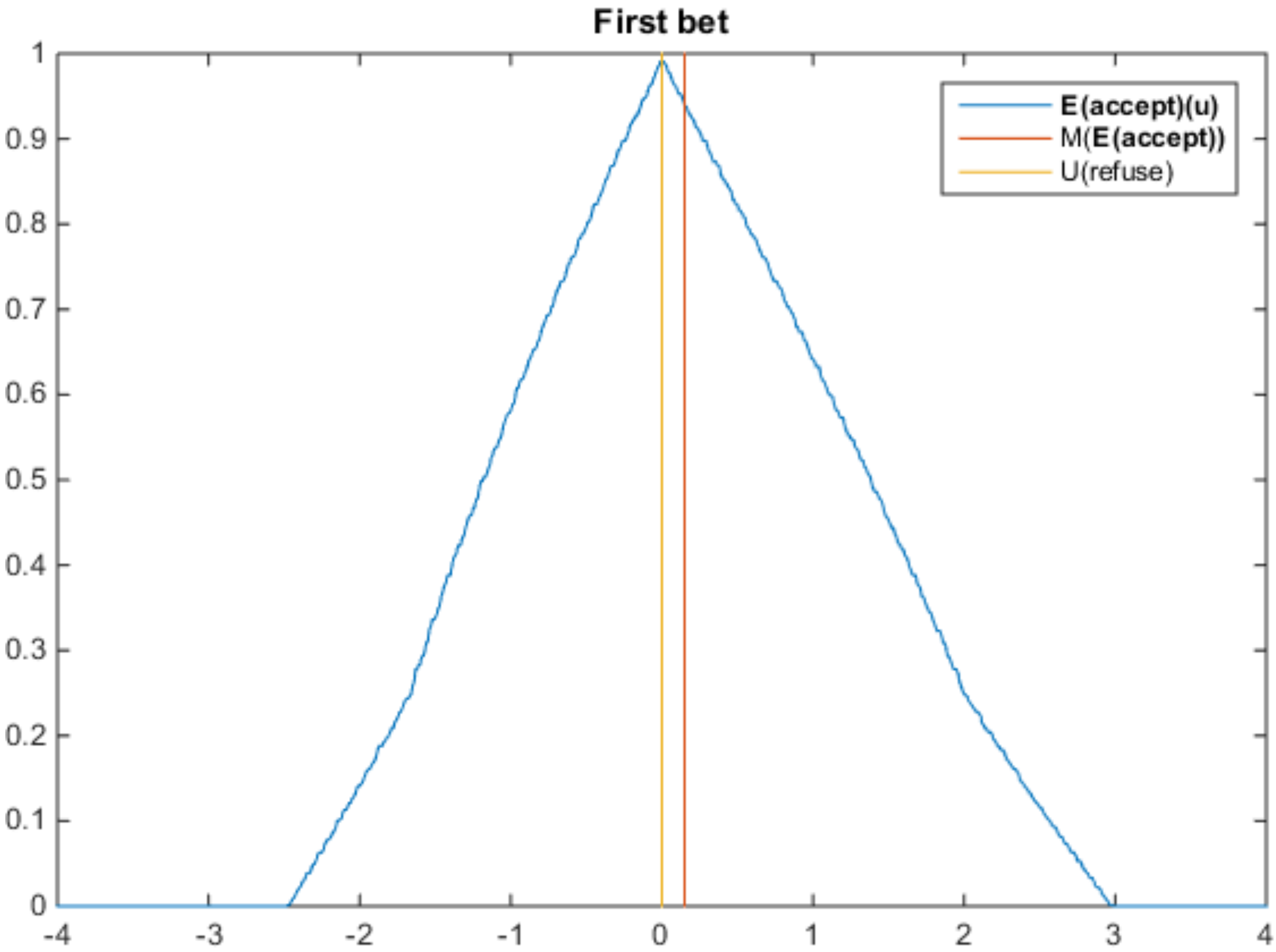


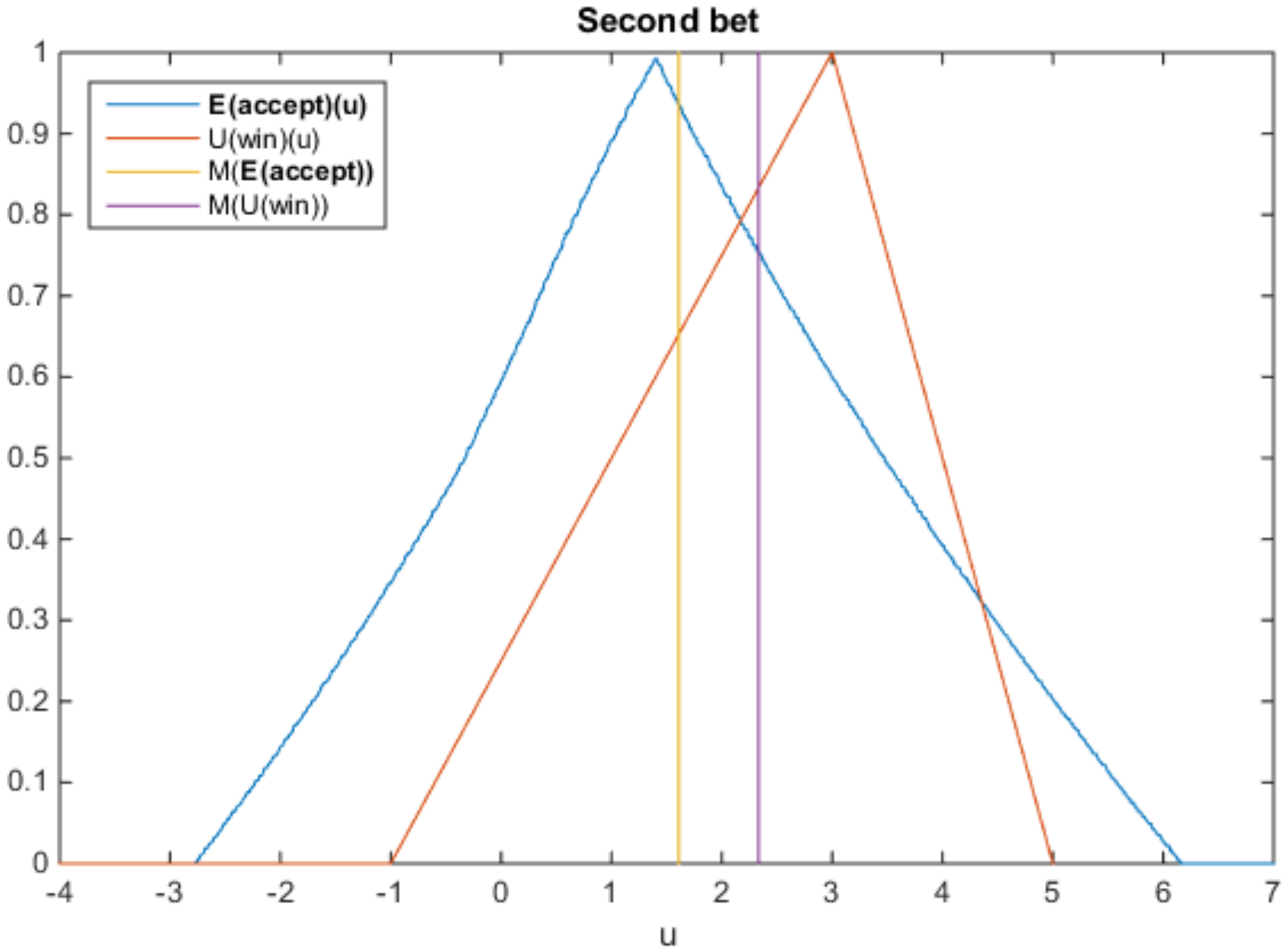
The Decision Tree




Probabilities
P(win)(p)
P(lose)(p)
P(win|win)(p)
P(lose|win)(p)
p
0
0.2
0.4
0.6
0.8
1
0.5


Utilities
U(win)(u)
U(lose)(u)
U(win|win)(u)
U(lose|win)(u)
1
0.5
0
-5
0
5
u


First bet
E(accept)(u)
M(E(accept))
U(refuse)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4


Second bet
E(accept)(u)
U(win)(u)
M(E(accept))
M(U(win))
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
5
6
7
u

Conclusions

- Especially the fuzzy preference relation provides insight to the degree of belief
- Whether sensitivity to parameters is reduced remains unanswered
- Validation needed of improvement over crisp decision tree analysis

“...that adding fuzzy sets to an existing method improves it in a significant way -- needs to be articulated by convincing arguments, based on sufficient knowledge of state-of-the-art existing techniques.”, Dubois [2011]

Literature

- G. Klir and B. Yuan. *Fuzzy Sets and Fuzzy Logic*, volume 4. Prentice Hall New Jersey, 1995.
- S. Watson, J. Weiss and M. Donnell. Fuzzy Decision Analysis. *IEEE Transactions on Systems, Man and Cybernetics,* 9(1):1-9, 1979
- D. Dubois. The role of fuzzy sets in decision sciences: Old techniques and new directions. *Fuzzy Sets and Systems*, 184(1):3-28, 2011.