MIP-Based Optimization of Transfer Stations in Multi-Modal Transport Networks (presentation of results)

Kaarle Mäkelä

31.10.2025

Advisor: *Philine Schiewe*

Supervisor: *Philine Schiewe*

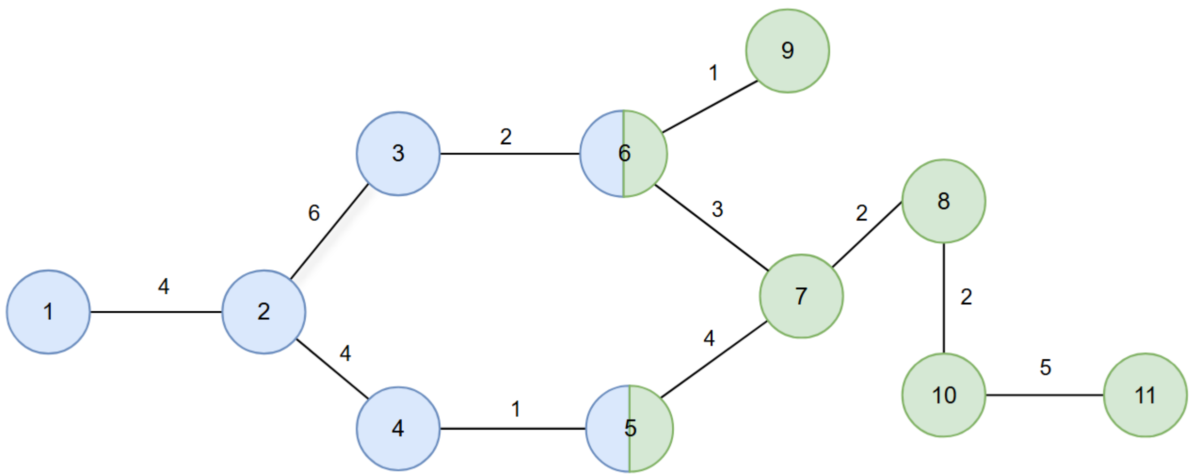
Aalto-yliopisto
Perustieteiden
korkeakoulu

Public Transport Network

- Public transport networks (PTNs) are graphs where the nodes represent traffic junctions, and the edges represent routes between the junctions.
- Most often PTNs are weighted graphs, where the weights may represent many things (operation cost, travel time, etc.)
- PTNs are accompanied by OD-matrices, which contain information about the number of passengers wanting to travel from one node to another. This information can be considered the demand on the PTN.

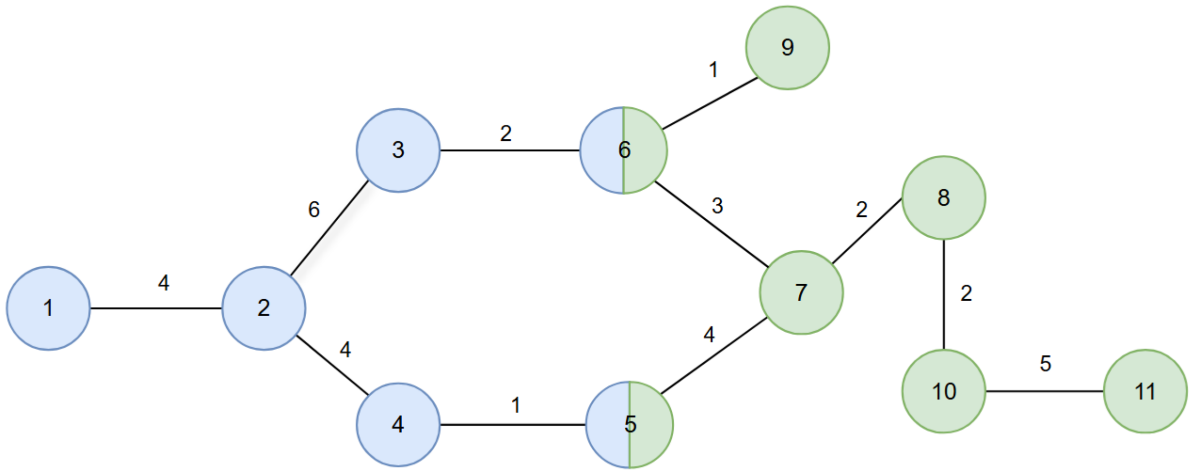
Multi-Modal Transport Networks

- PTNs that contain multiple modes of transport (walking, bicycle, bus, tram, etc.)
- Possible to transfer between modes of transport.



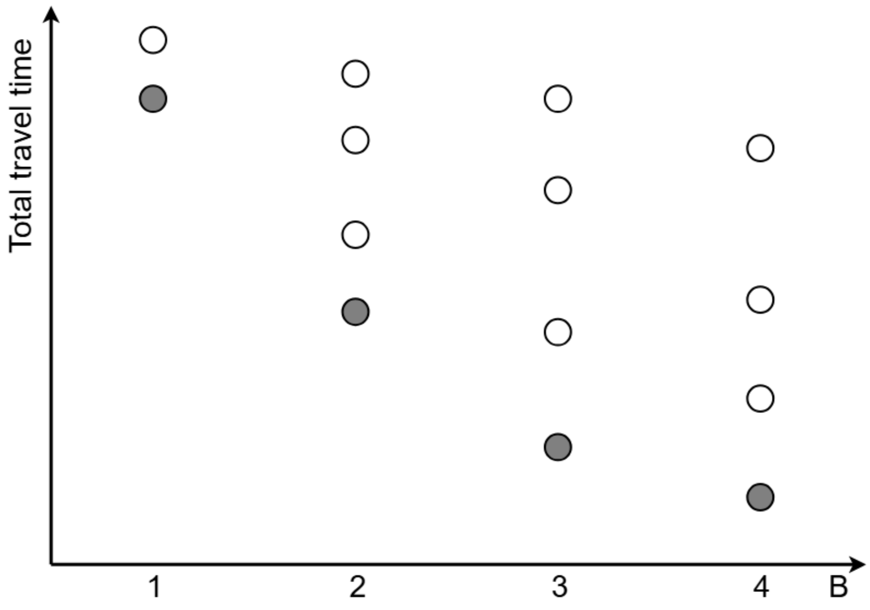
The Problem

- Given multiple PTNs for different modes of transport and a maximum number of transfer stations B, what is the optimal set of transfer stations that minimizes the travel time of all passengers?
  - If B=1, do we select node 5 or node 6 as the transfer station?



Main Objectives

- Formulate the problem as a mixed-integer programming problem.
- Implement the model and compute Pareto front.

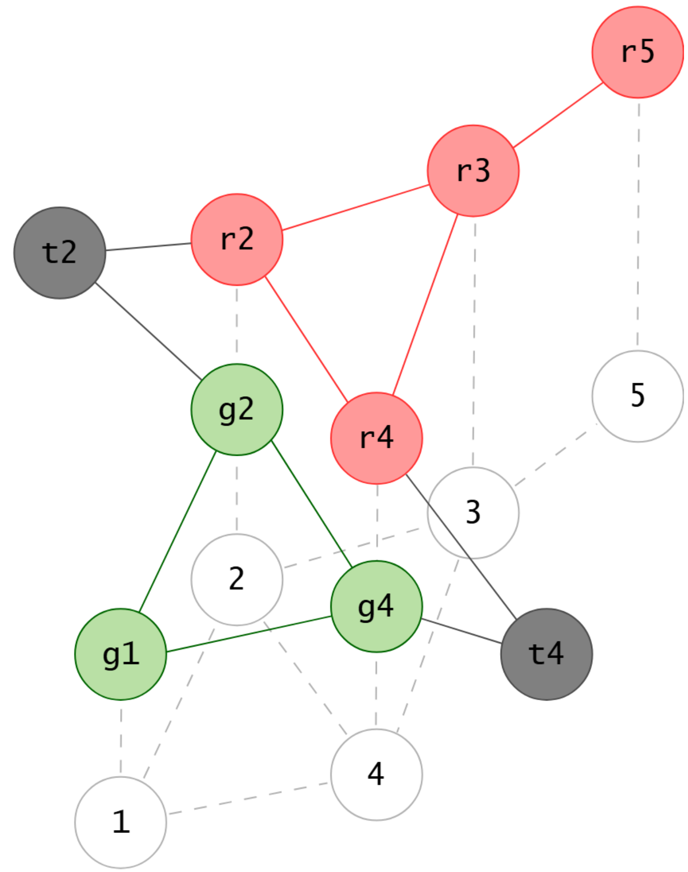


Scope

- Each edge has a set travel time for each modality.
- The underlying OD-matrix is known.
- Transfers can only occur within shared stops.
- Rational of transfers is not considered.

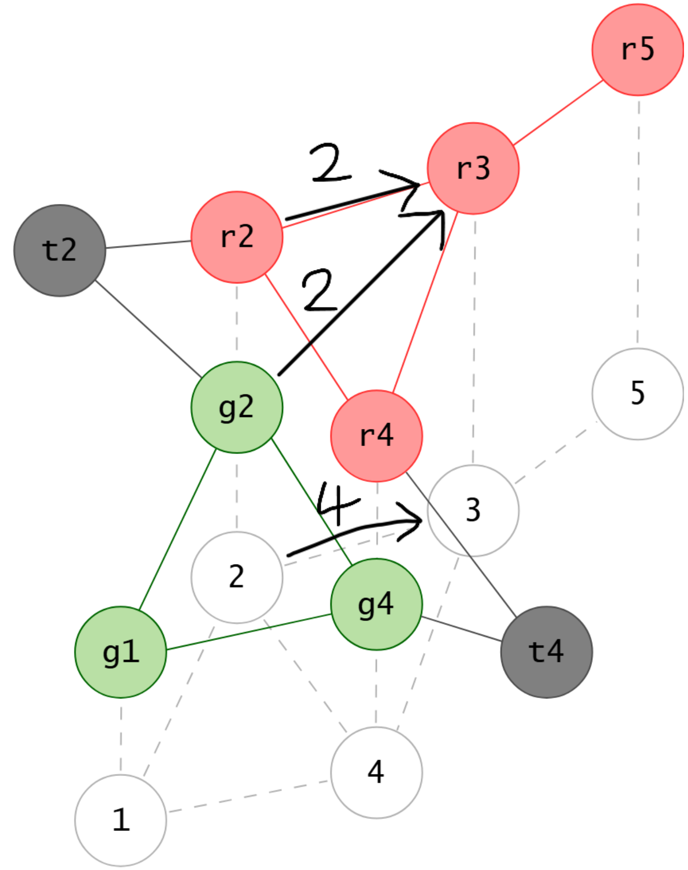
Structure of Multi-Modal PTN

- Multi-Modal PTN is created from mode specific PTNs.
- Underlying PTN represented in gray.
- Transfer nodes and edges added between nodes that share an underlying node.
- Appropriate weight of transfer edges (penalty) up to interpretation.



Mapping Demand to the Multi-Modal PTN

- We only know the demand of the underlying PTN.
- We need to map the demand onto the multi-modal PTN.
- We use a heuristic approach: Demand is split evenly among node pairs of the multi-modal PTN that correspond to the given node pair of the underlying.
  - Creates O x D new OD-pairs for every original OD-pair.
  - Example: Demand of 4 on the underlying PTN from 2 to 3 is split even into demand of 2 on the multi-modal PTN from g2 to r3 and from r2 to r3.



GORG MIP-Formulation (Heinrich et al., 2023)

$$
\begin{aligned}
\min \quad & \sum_{s\in V}\sum_{uw\in E} c(uw)(y^s_{uw} + y^s_{wu}) & \\
\text{s.t.} \quad & \sum_{e\in E} c(e)x_e \le K & \\
& \sum_{w\in V: wu\in E} (y^s_{wu} - y^s_{uw}) + \sum_{w\in V: uw\in E} (y^s_{wu} - y^s_{uw}) & \\
& = \begin{cases} -\sum_{t>s} a_{s,t}, & \text{if } \ u = s \\ a_{s,u}, & \text{if } \ u > s \\ 0, & \text{otherwise} \end{cases} & s\in V, u\in V \\
& y^s_{uw} \le x_{uw} \sum_{t>s} a_{s,t} & uw\in E, s\in V \\
& y^s_{wu} \le x_{uw} \sum_{t>s} a_{s,t} & wu\in E, s\in V \\
& x_e \in \{0,1\} & e\in E \\
& y^s_{uw}, y^s_{wu} \ge 0 & s\in V, uw\in E
\end{aligned}
$$

Modified Implementation

$$
\begin{aligned}
\min \quad & \sum_{s\in V}\sum_{uw\in E} c(uw)(y^s_{uw} + y^s_{wu}) & \\
\text{s.t.} \quad & \sum_{t\in T} x_t \le B & \\
& \sum_{w\in V: wu\in E} (y^s_{wu} - y^s_{uw}) + \sum_{w\in V: uw\in E} (y^s_{wu} - y^s_{uw}) & \\
& = \begin{cases} -\sum_{i>s}(a_{s,i} + a_{i,s}), & \text{if } \ u = s \\ a_{s,u} + a_{u,s}, & \text{if } \ u > s \\ 0, & \text{otherwise} \end{cases} & s \in V, u \in V \\
& y^s_{ut} \le x_t \sum_{i>s}(a_{s,i} + a_{i,s}) & t \in T : ut \in E, s \in V \\
& y^s_{tu} \le x_t \sum_{i>s}(a_{s,i} + a_{i,s}) & t \in T : tu \in E, s \in V \\
& x_t \in \{0,1\} & t \in T \\
& y^s_{uw}, y^s_{wu} \ge 0 & s \in V, uw \in E
\end{aligned}
$$

Used Data

- 5x5 grid.
- 25 nodes.
- 25 transfer nodes.
- 3 modes of transport.
- 567 non-zero OD-entries.

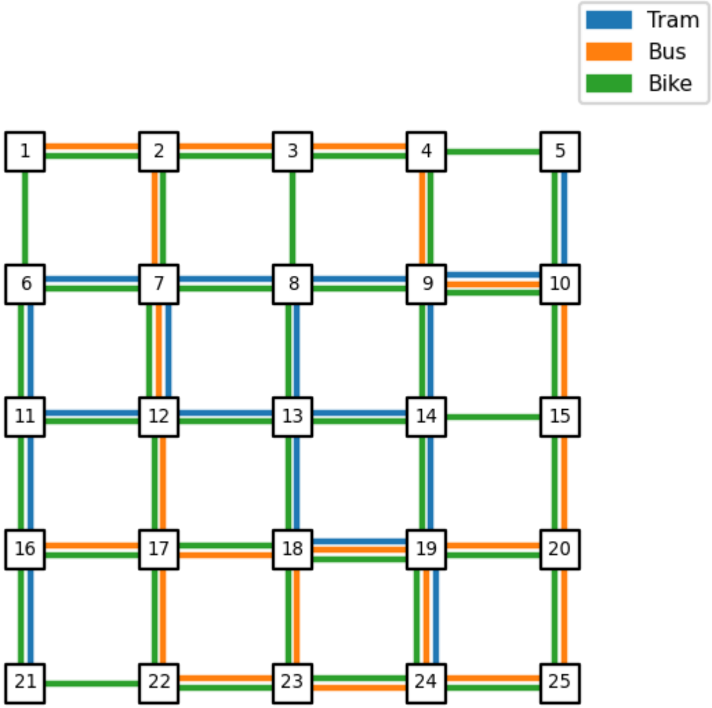
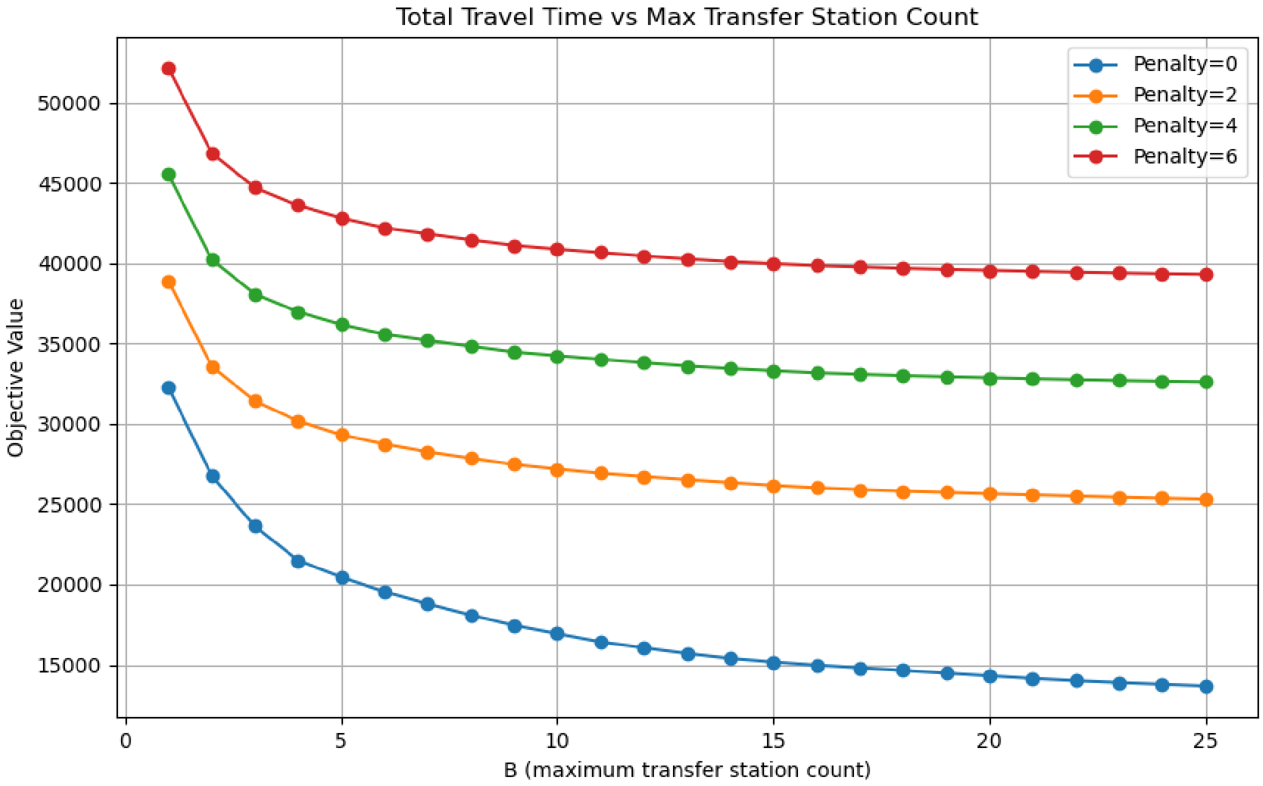
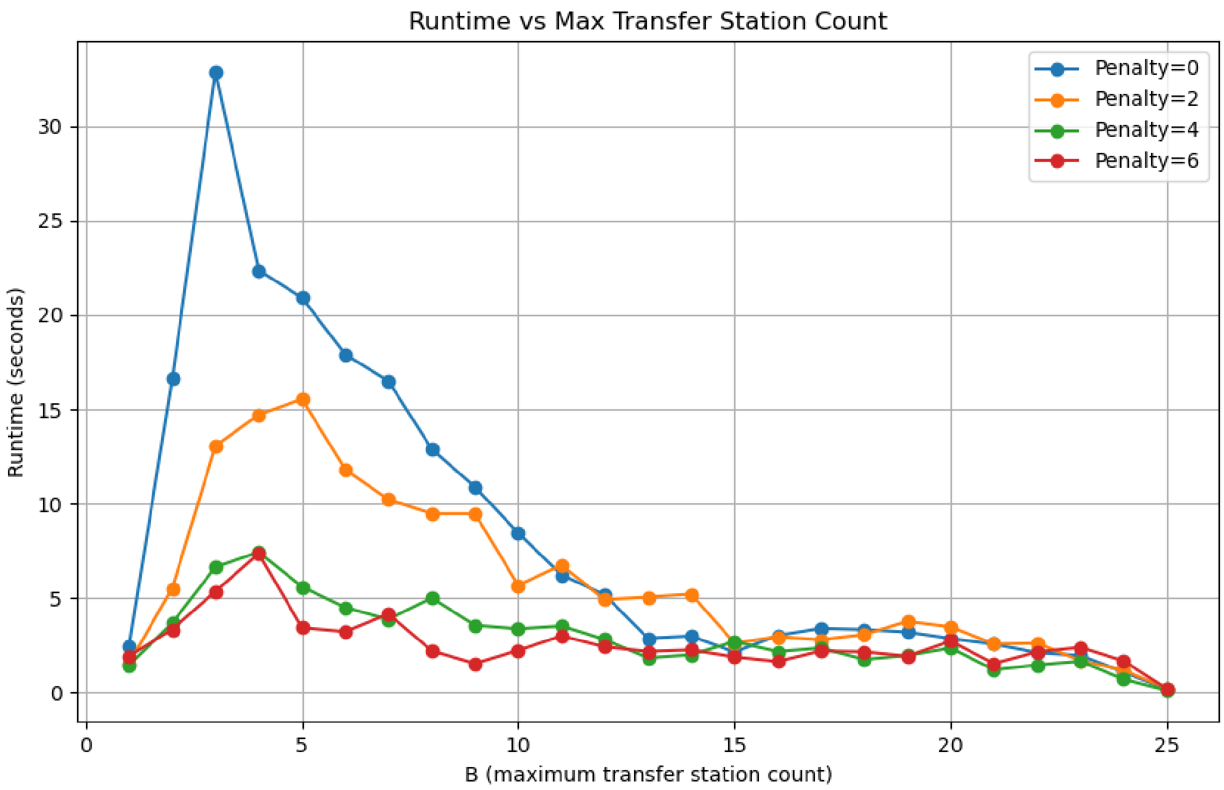


Figure by Schiewe et al. (2024)

Results: Total Travel Time



Results: Runtime



Changes in Optimal Sets

```
Penalty:                         0     2     4     6
Change to optimal set for B=1:  {18} {18} {18} {18}
Change to optimal set for B=2:  {7} {7} {7} {7}
Change to optimal set for B=3:  {13} {9} {9} {9}
Change to optimal set for B=4:  {9} {13} {13} {13}
Change to optimal set for B=5:  {24} {24} {24} {24}
Change to optimal set for B=6:  {1} {16} {12} {12}
Change to optimal set for B=7:  {17, 19, 24} {1} {16} {16}
Change to optimal set for B=8:  {8} {19} {19} {19}
Change to optimal set for B=9:  {25} {12} {2} {2}
Change to optimal set for B=10: {16} {8} {8} {8}
Change to optimal set for B=11: {14} {10} {1, 2, 3} {6}
Change to optimal set for B=12: {12} {3} {6} {10}
Change to optimal set for B=13: {4} {22, 24, 25} {10} {1, 2, 3}
Change to optimal set for B=14: {23} {14} {14} {14}
Change to optimal set for B=15: {2} {24} {25} {22}
Change to optimal set for B=16: {22} {6} {22} {25}
Change to optimal set for B=17: {24} {17} {17} {15}
Change to optimal set for B=18: {3} {15} {15} {17}
Change to optimal set for B=19: {11} {4} {4} {4}
Change to optimal set for B=20: {20} {23} {23} {23}
Change to optimal set for B=21: {6} {5} {21} {21}
Change to optimal set for B=22: {10} {21} {2} {11}
Change to optimal set for B=23: {15} {11} {11} {2}
Change to optimal set for B=24: {5} {2} {5} {5}
Change to optimal set for B=25: {21} {20} {20} {20}
```

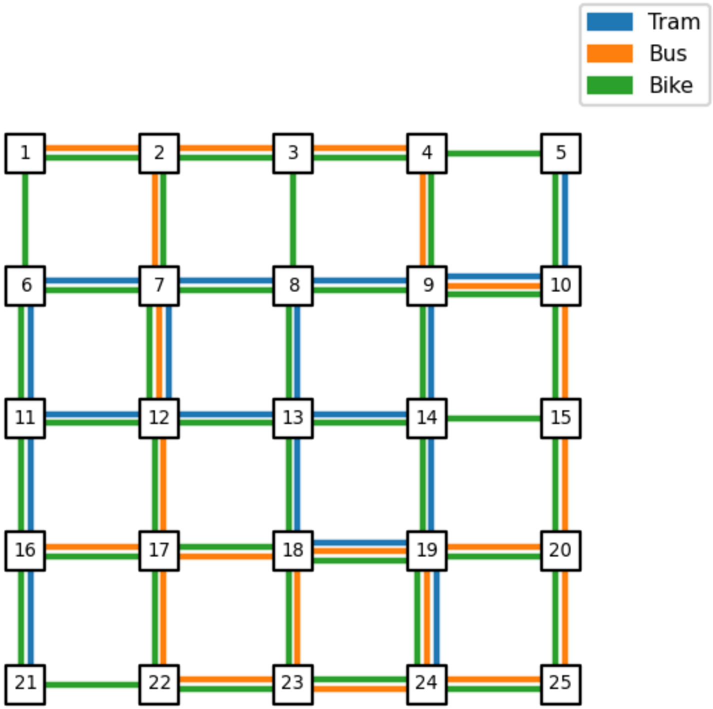


Figure by Schiewe et al. (2024)

Further Research

- Other/better ways of transforming the demand on the underlying PTN onto the multi-modal PTN.
- Other formulations for multi-modal PTNs for the given problem.
  - Connected to transforming the demand.
- Effect of number of OD-entries on runtime.
- Behaviour on larger datasets.

References

- I. Heinrich, O. Herrala, P. Schiewe, and T. Terho. Using light spanning graphsfor passenger assignment in public transport. In *Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems*, pages 1–16.Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2023. doi: 10.4230/OASIcs.ATMOS.2023.2
- P. Schiewe, A. Schöbel, S. Jäger, S. Albert, C. Biedinger, T. Dahlheimer, V. Grafe,O. Herrala, K. Hoffmann, S. Roth, A. Schiewe, M. Stinzendörfer, and R. Urban.Documentation for lintim 2024.12, 2024. URL https://nbn-resolving.de/urn:nbn:de:hbz:386-kluedo-85839.