Aalto-yliopisto
Perustieteiden korkeakoulu

# Symmetric Optimum Requirement Graphs for Grids (Results of the thesis)

*Inka Hirvinen*

*23.05.2024*

Advisor: *Philine Schiewe*
Supervisor: *Philine Schiewe*



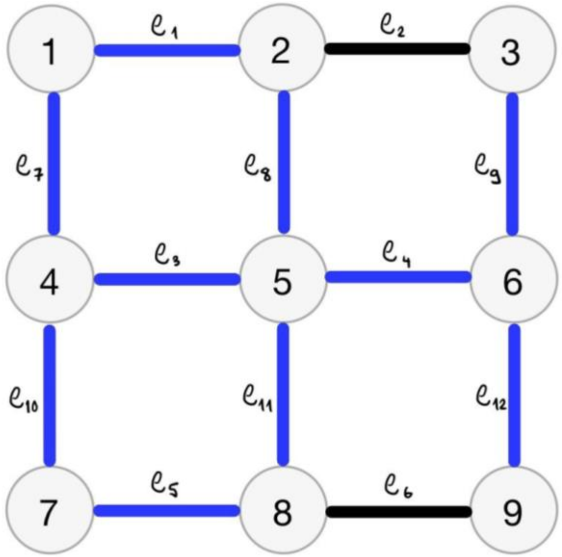
Systeemianalyysin laboratorio

# Background: Passenger assignment

- Usually the first step in traditional sequential public transport planning
- Gathers demand to certain routes
- Resulting traffic loads can then be used for the next steps in the public transport planning
- The GORG formulation developed by Heinrich et al. (2023) and the symmetric optimum requirement graph formulation can be used for passenger assignment

# Background: Generalized optimum requirement graphs

- Spanning graph




- Routing cost:

$$r(H) := \sum_{\{u,v\} \in \binom{V(G)}{2}} a_{u,v} d_H(u,v)$$

- Building cost:

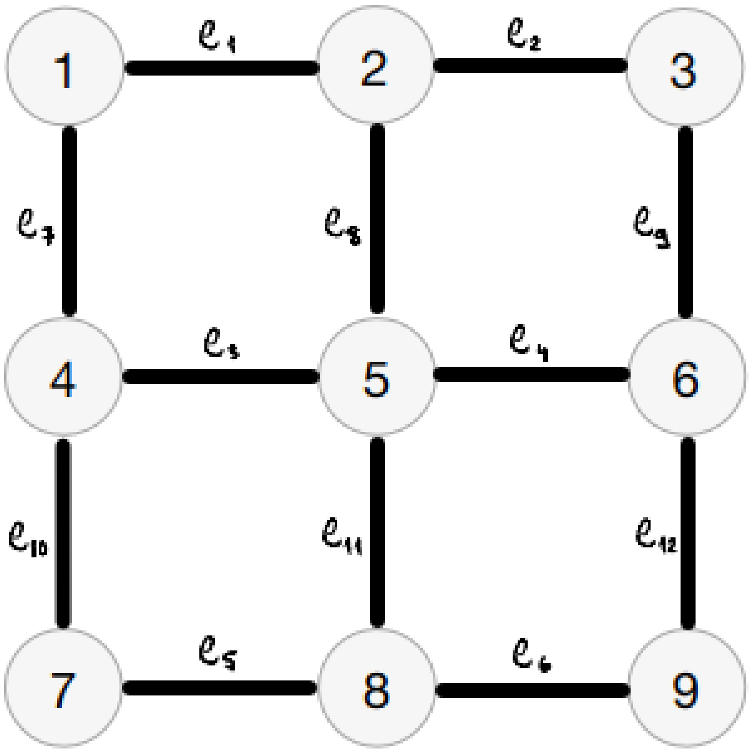
$$c(H) := \sum_{e \in E(H)} c(e)$$

# Background: GORG IP formulation by Heinrich et al. (2023)

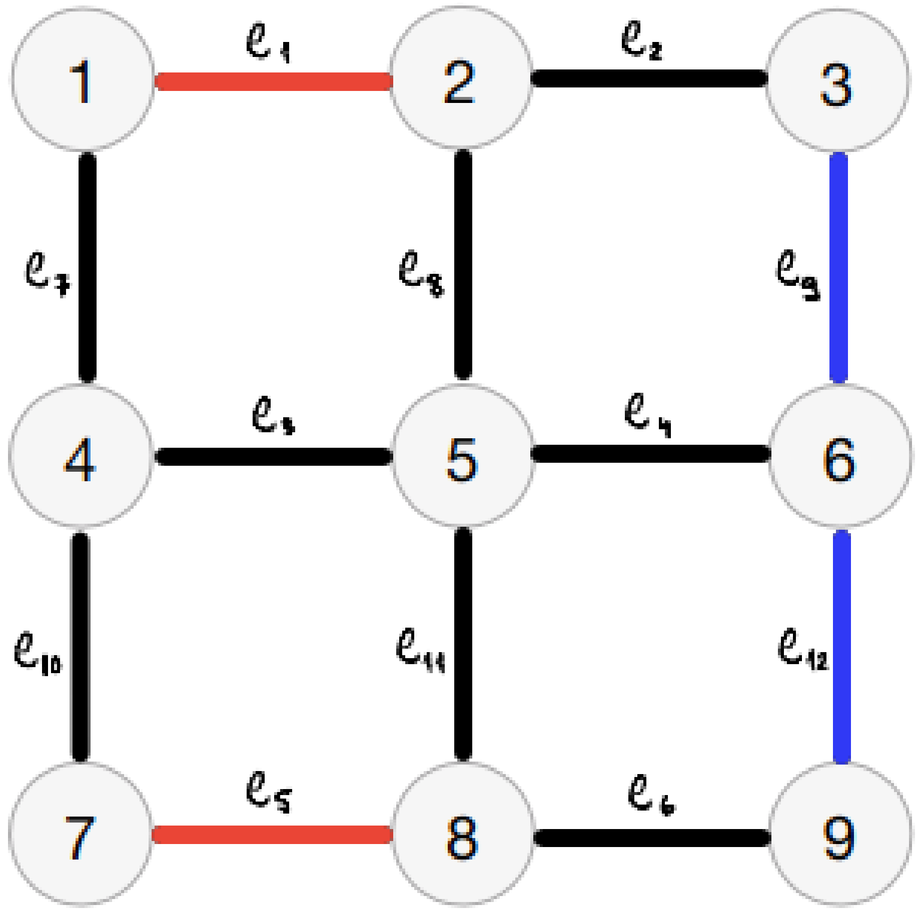
$$
\begin{aligned}
\min \quad & \sum_{s<t\in V}\sum_{uv\in E} a_{u,v}c(uv)(y^{st}_{uv}+y^{st}_{vu}) \\
\text{s.t.} \quad & \sum_{e\in E} c(e)x_e \le K \\
& \sum_{w\in V\,:\,wu\in E}(y^{st}_{wu}+y^{st}_{uw}) \\
& \quad - \sum_{w\in V\,:\,uw\in E}(y^{st}_{uw}+y^{st}_{wu}) = \begin{cases} -1, & \text{if } u=s \\ 1, & \text{if } u=t \\ 0, & \text{otherwise} \end{cases} && s<t\in V, u\in V \\
& y^{st}_{uv} \le x_{uv} && s<t\in V, uv\in E \\
& y^{st}_{vu} \le x_{uv} && s<t\in V, uv\in E \\
& x_e \in \{0,1\} && e\in E \\
& y^{st}_{uv}, y^{st}_{vu} \in \{0,1\} && s<t\in V, uv\in E
\end{aligned}
$$

# Objectives of the thesis

- Study the trade-off between building and routing costs when finding a symmetric optimal passenger assignment on a grid graph
- Analyze the price of symmetry



# Horizontal symmetry on a grid graph



# Horizontal symmetry on a grid graph

Edge sets:

$$E^1 = \{e_1, e_2, e_7, e_8, e_9\}$$

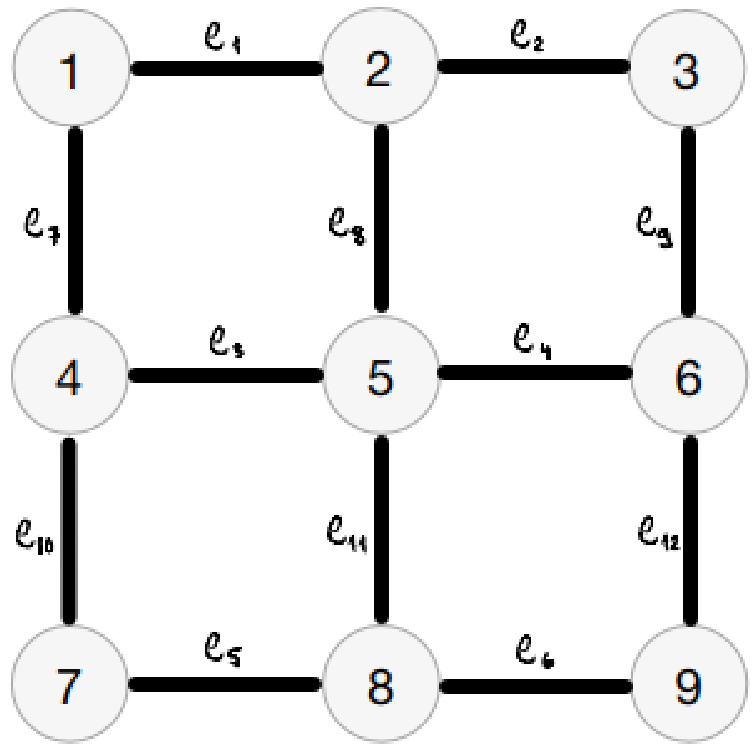
$$E^2 = \{e_3, e_4\}$$

$$E^3 = \{e_5, e_6, e_{10}, e_{11}, e_{12}\}$$

Node sets:

$$V^1 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$$V^2 = \{v_7, v_8, v_9\}$$



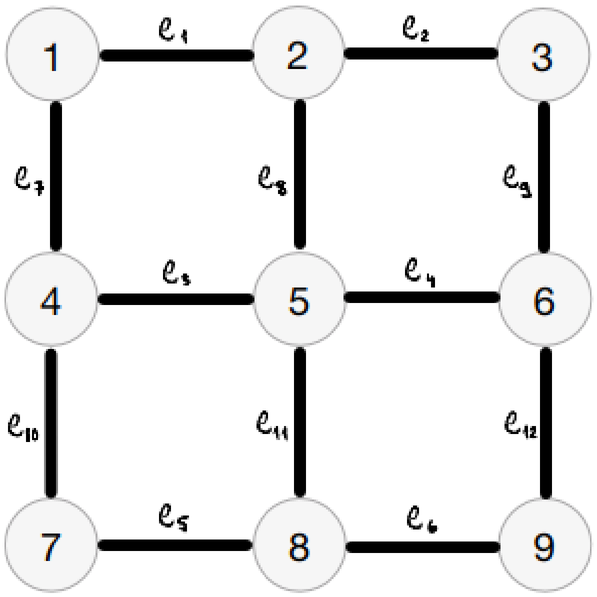
# Horizontal symmetry on a grid graph

Symmetry function:

$$\sigma(uv) = \begin{cases} uv, & \text{if } uv \in E^1 \text{ or } E^2 \\ u'v', & \text{if } uv \in E^3 \end{cases}$$

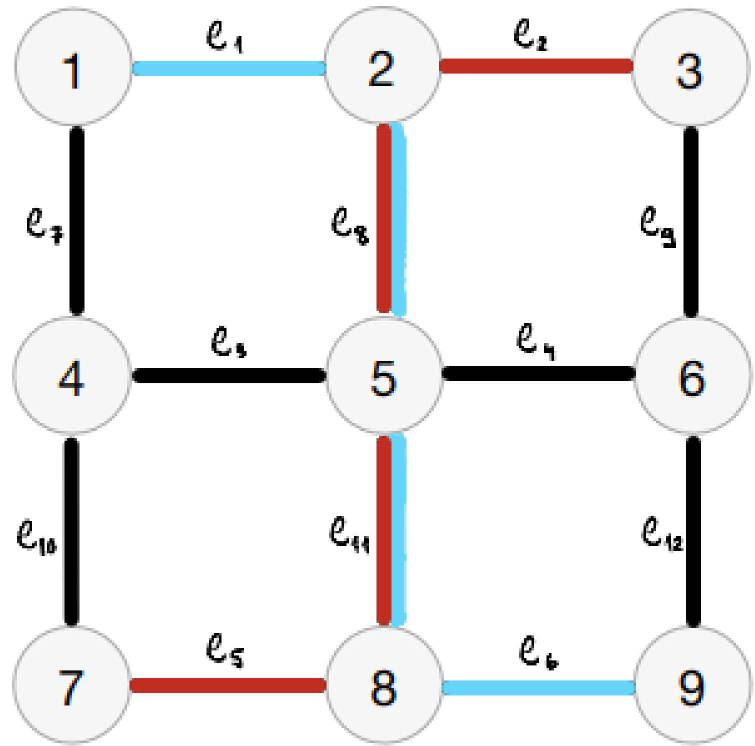
Symmetric node:

$$s'_{ij} = s_{(n-i+1)j}$$



# Reducing amount of flow constraints

The shortest u − v paths are equivalent to the shortest u′ − v′ paths if u, u′ and v, v′ are symmetric nodes.



# Resulting symmetric optimum requirement graph IP formulation

$$\min \sum_{s<t\in V} \sum_{uv\in E} a_{u,v} c(uv)(y^{st}_{uv} + y^{st}_{vu})$$

$$\text{s.t.} \qquad \sum_{e\in E^1} 2c(e)x_e + \sum_{e\in E^2} c(e)x_e \le K$$

$$\sum_{w\in V\,:\,wu\in E} y^{st}_{wu} + y^{st}_{uw}$$

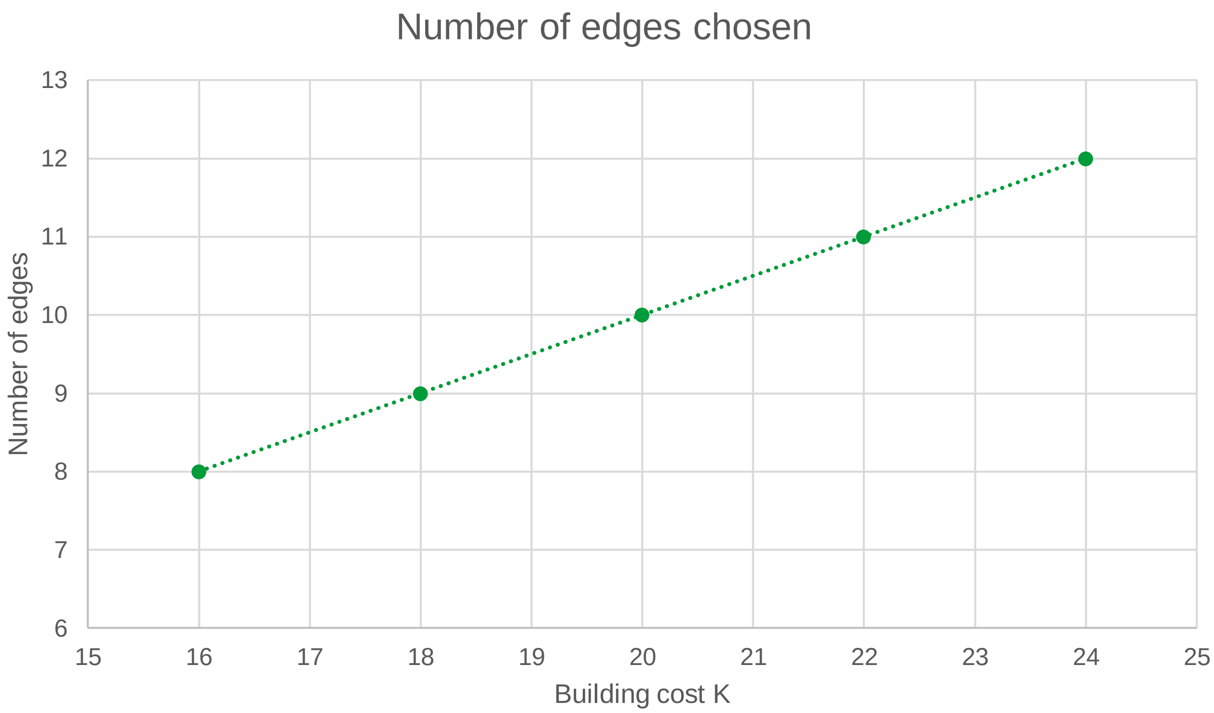
$$-\sum_{w\in V\,:\,uw\in E} y^{st}_{uw} + y^{st}_{wu} = \begin{cases} -1, & \text{if} \quad u=s \\ 1, & \text{if} \quad u=t \\ 0, & \text{otherwise} \end{cases} \qquad s<t\in V, u\in V, s\in V^1 \vee V^2$$

$$y^{st}_{uv} \le x_{\sigma(uv)} \qquad s<t\in V, uv\in E$$

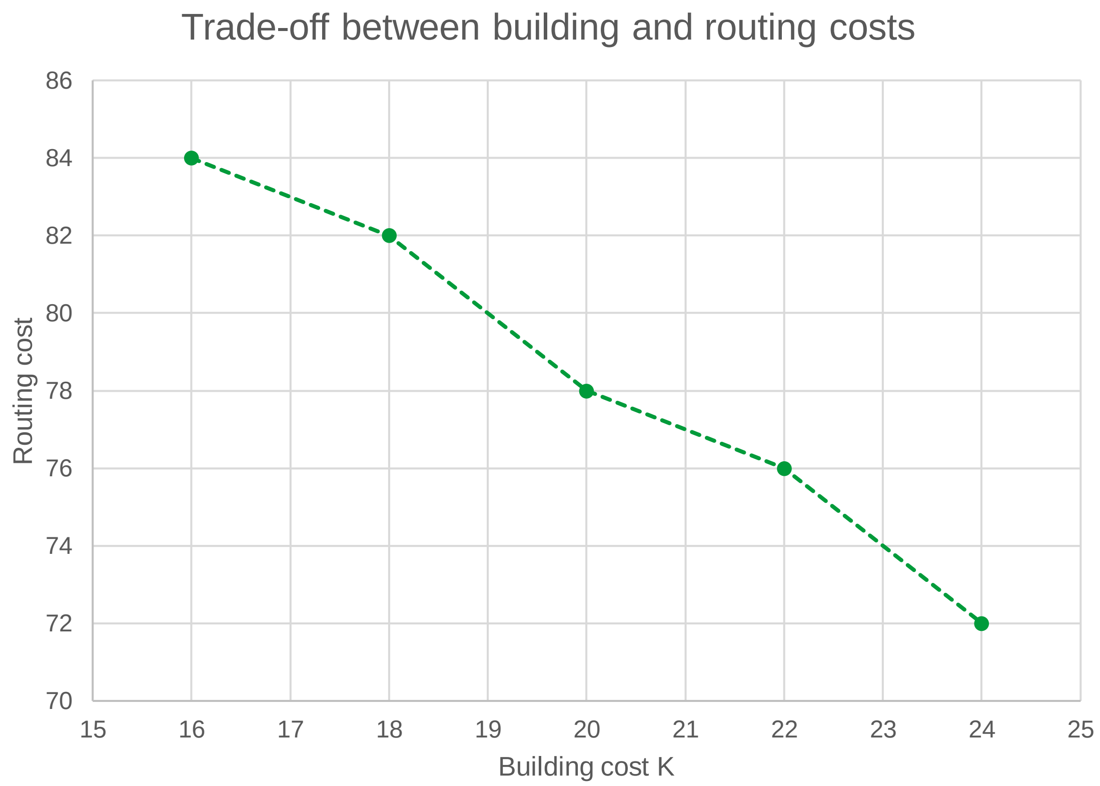
$$x_e \in \{0,1\} \qquad e\in E^1 \vee E^2$$

$$y^{st}_{uv}, y^{st}_{vu} \in \{0,1\} \qquad s<t\in V, uv\in E$$

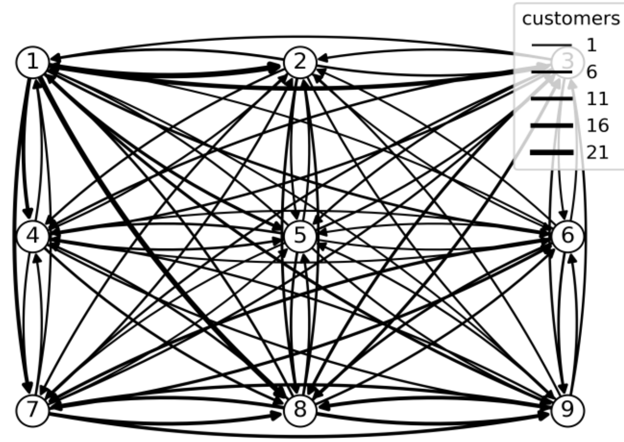
# Effect of the building cost bound K on the resulting spanning graph



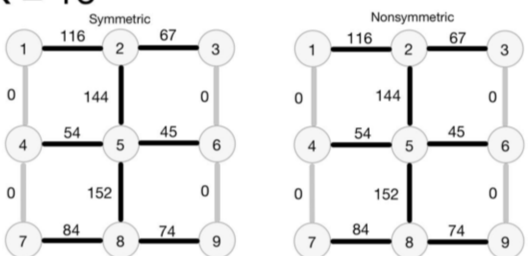
# Trade-off between building and routing costs



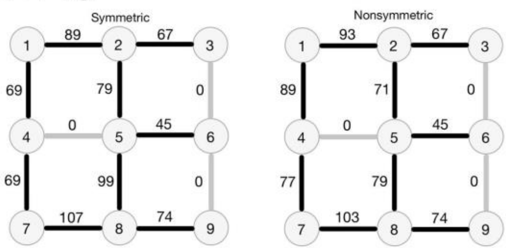
# Price of symmetry



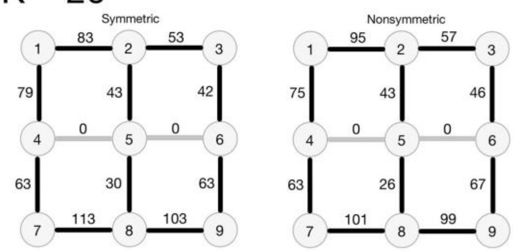

K = 16



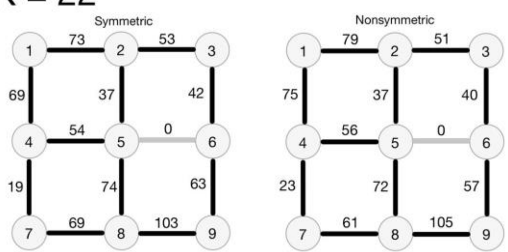

K = 18



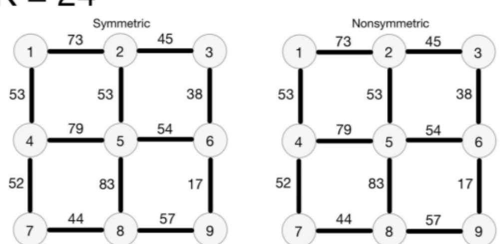

K = 20



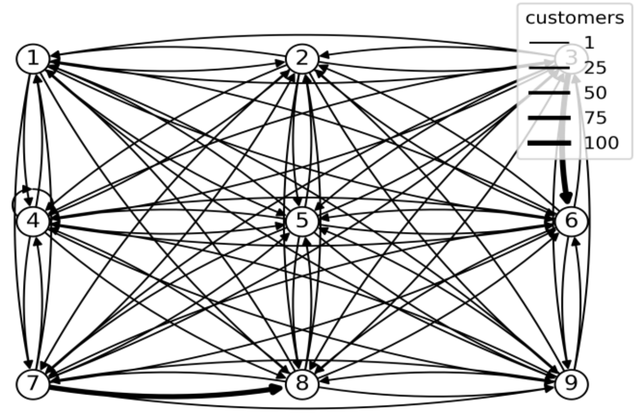

K = 22



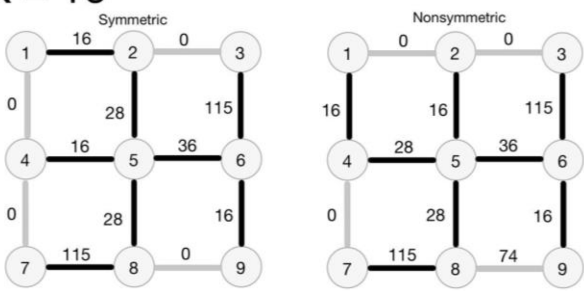

K = 24



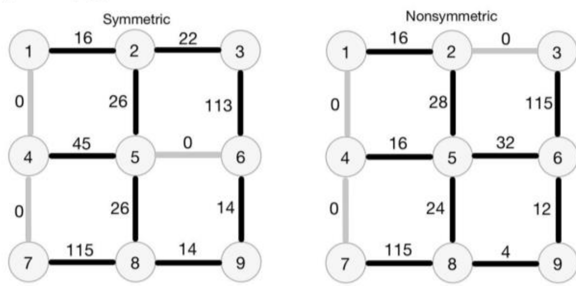
# Price of symmetry – Asymmetrical demand



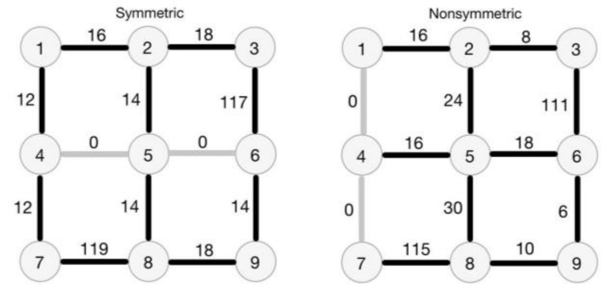

K = 16



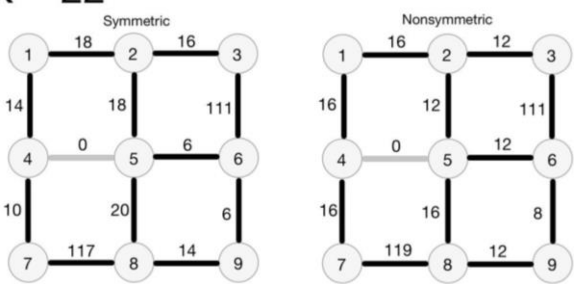

K = 18



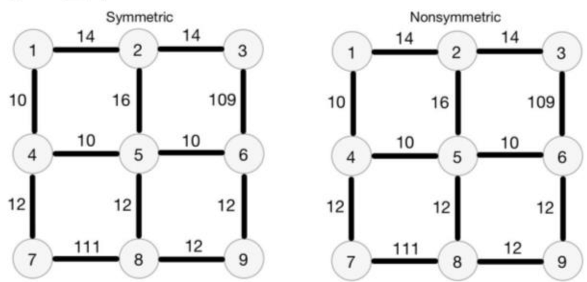

K = 20




K = 22




K = 24



# References


- Irene Heinrich, Olli Herrala, Philine Schiewe, and Topias Terho. Using light spanning graphs for passenger assignment in public transport. In 23rd Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2023). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2023.