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Cost-efficient portfolios of reinforcement actions to secure the performance of transportation networks (presentation of results)

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Background

- External hazards and malfunctions may cause disruptions
- The decision maker (DM) can minimize their impacts

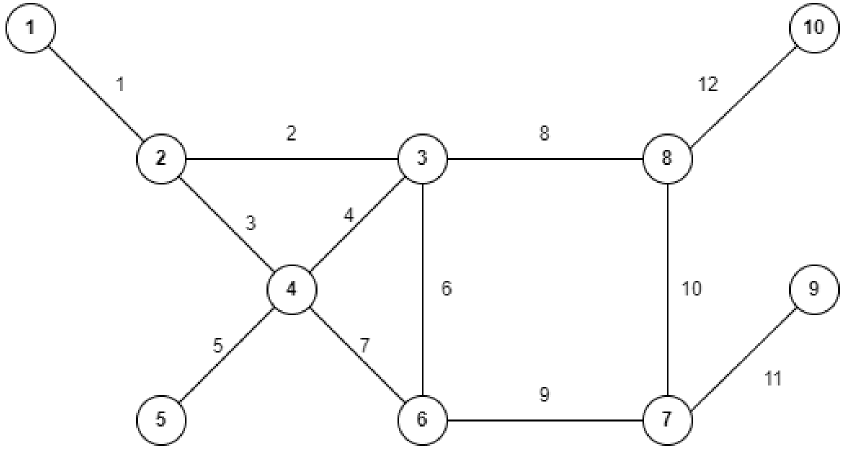


Figure 1: Example of a transportation network adapted from Ip. W. H, Wang. D. (2011)

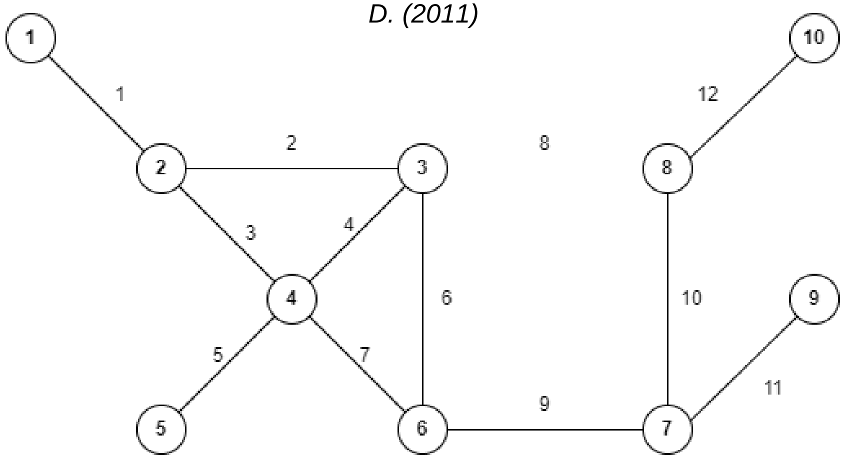


Figure 2: Disruption at edge number 8

Measuring performance (1/3)

1. Average number of independent passageways (IP)

$$IP = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} k(i,j)$$

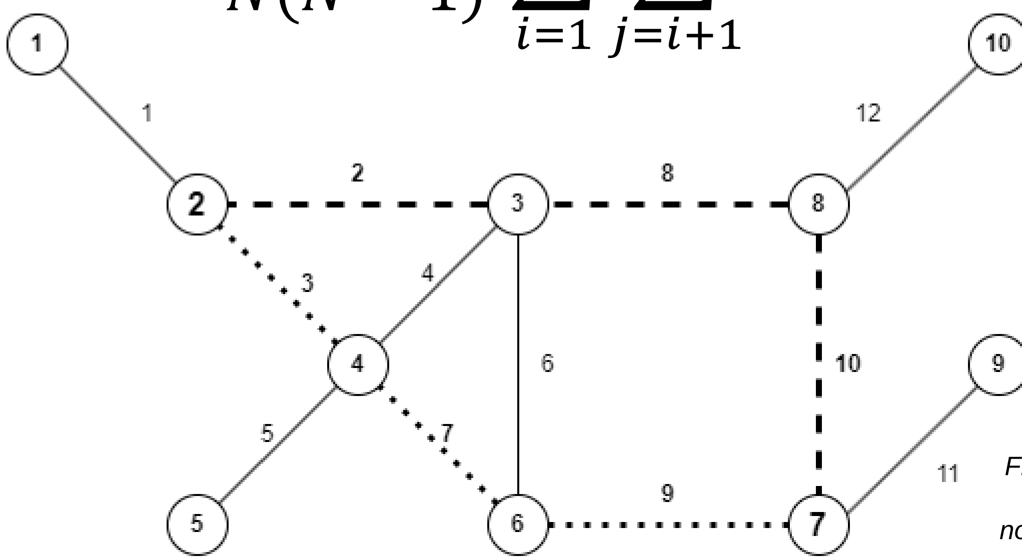


Figure 3: ***Two*** *independent passageways between nodes 2 and 7* $\Rightarrow k(2,7) = \mathbf{2}$

Measuring performance (2/3)

2. Global Efficiency (GE)

$$GE = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{1}{d(i,j)}$$

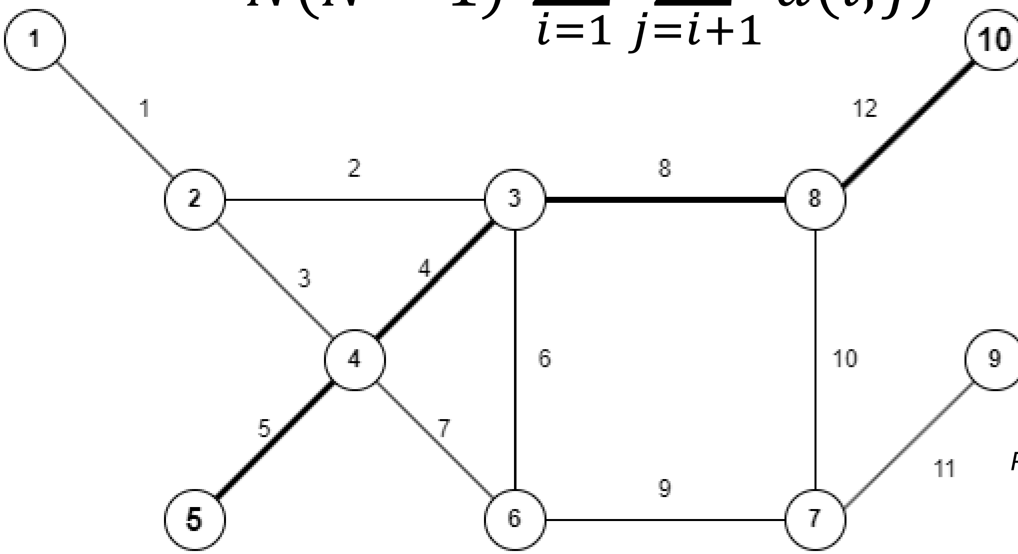


*Figure 4: Distance between nodes 5 and 10 is **4** ⇒* $d(5,10) = \mathbf{4}$

Measuring performance (3/3)

- Linear combination of chosen metrics:

$$V(w,x) = w_1\, IP(x) + w_2\, GE(x)\,, \qquad w \in S$$

- Expected value over all possible states of the network:

$$\mathbb{E}[V(w,x)] = \sum_{x \in \{0,1\}^M} \mathbb{P}(x)\, V(w,x)$$

Reinforcement actions

- Consider r alternative reinforcement actions
  - Type 1: Reinforce existing edges
  - Type 2: Add new edges to the graph

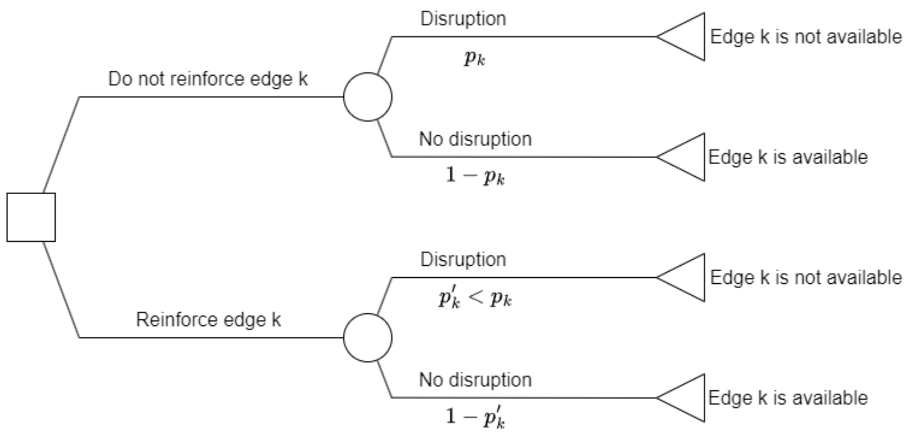


Figure 5: Decision tree for reinforcing edge k in the network

Portfolios of reinforcement actions

$$q = [q_1, \dots, q_r] \in \{0,1\}^r$$

$$q_m = \begin{cases} 1, & \text{if action m is implemented} \\ 0, & \text{otherwise} \end{cases}, m = 1, \dots, r$$

- A portfolio q^k dominates portfolio q^j, denoted $q^k \succ q^j$, in S if and only if

$$\begin{cases} \mathbb{E}[V(w,x) \mid q^k] \geq \mathbb{E}\left[V(w,x) \mid q^j\right] \text{ for all } w \in S \\ \mathbb{E}[V(w,x) \mid q^k] > \mathbb{E}\left[V(w,x) \mid q^j\right] \text{ for some } w \in S \end{cases}$$

Example of dominance

- Portfolio x^1 dominates x^3
- Portfolios x^1 and x^2 are non-dominated

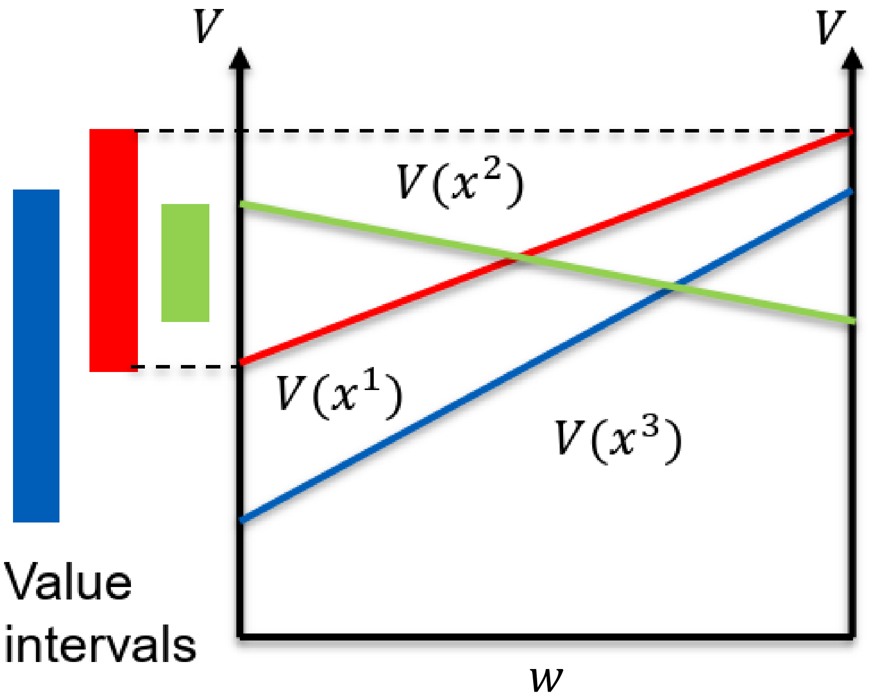


Figure 8: Example of dominance from the course MS-E2135 Decision Analysis D 2023, Lecture 6b by Ahti Salo

Cost-efficient portfolios

- Two portfolios q^k and q^j are equal w.r.t. to efficiency denoted by $q^j \sim q^k$ if and only if

$$\mathbb{E}[V(w,x) \mid q^k] = \mathbb{E}\left[V(w,x) \mid q^j\right] \text{ for all } w \in S$$

- A portfolio q^k is cost-efficient if and only if

$$\begin{cases} \nexists q^j\colon\ q^j \succ q^k \wedge C\left(q^j\right) \leq C(q^k) \\ \nexists q^j\colon\ q^j \sim q^k \wedge C\left(q^j\right) < C(q^k) \end{cases}$$

Objectives

- Evaluate portfolios of reinforcement actions with a linear combination of the two performance metrics
- Implement an algorithm to determine the cost-efficient portfolios of reinforcement actions
- Improve the performance of the network cost-efficiently

Methodology

- Inputs
  - Transportation network
  - Estimates of the disruption probabilities
  - Reinforcement actions and their costs and effects
  - Preference statements regarding metrics
- Incomplete information
  - Disruption probabilities
  - Weights of performance metrics
- Outputs
  - Cost-efficient portfolios of reinforcement actions

Example network

- Type 1 actions:
  - Cost: 1
  - Disruption probability 0.2 → 0.1
- Type 2 actions:
  - Cost: 2
  - Disruption probability 0.3
- Over 250 000 feasible portfolios

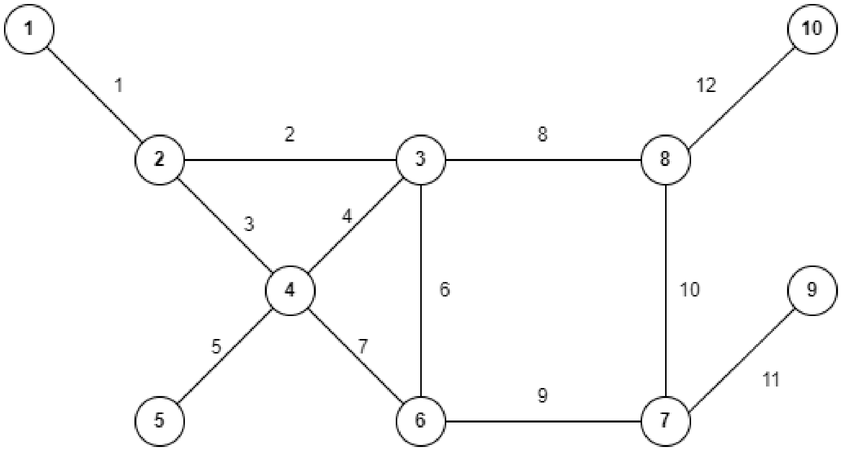


Figure 6: Example network, where all edges can be reinforced (Type 1 actions)

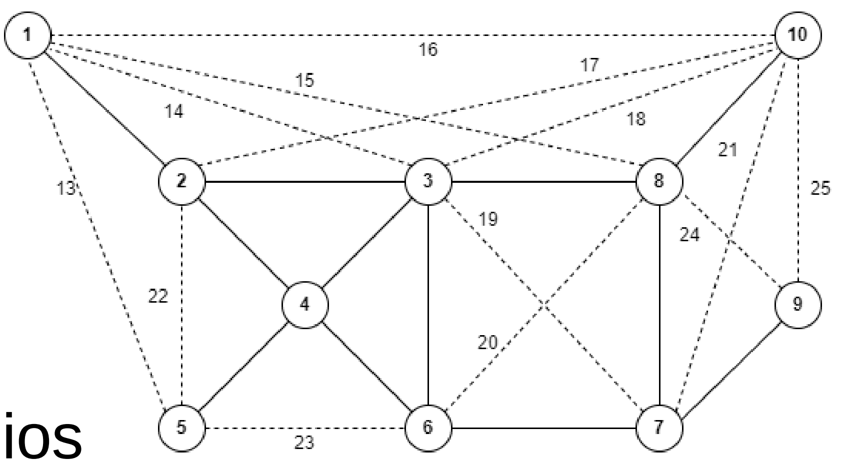
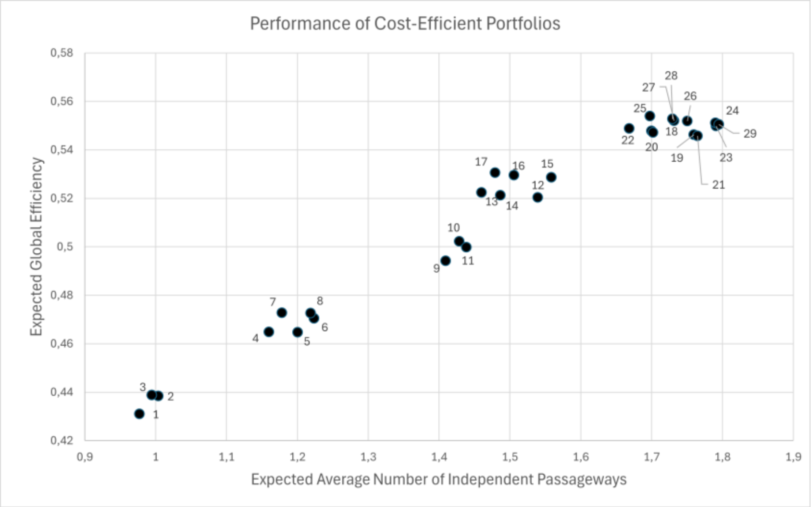
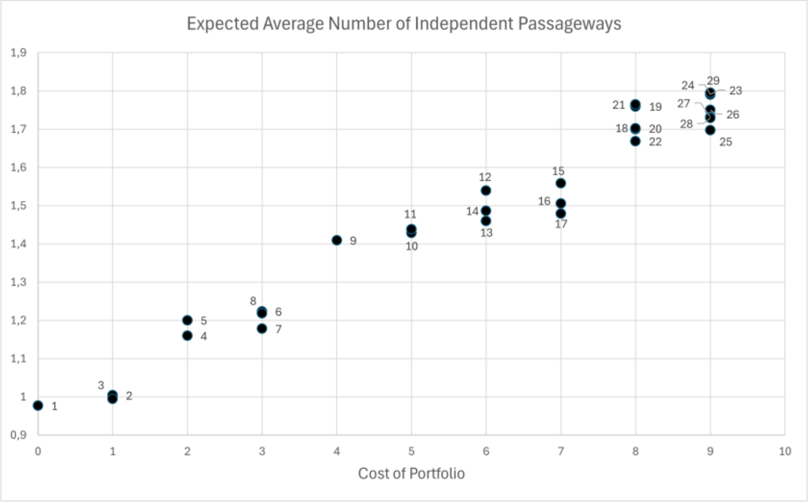


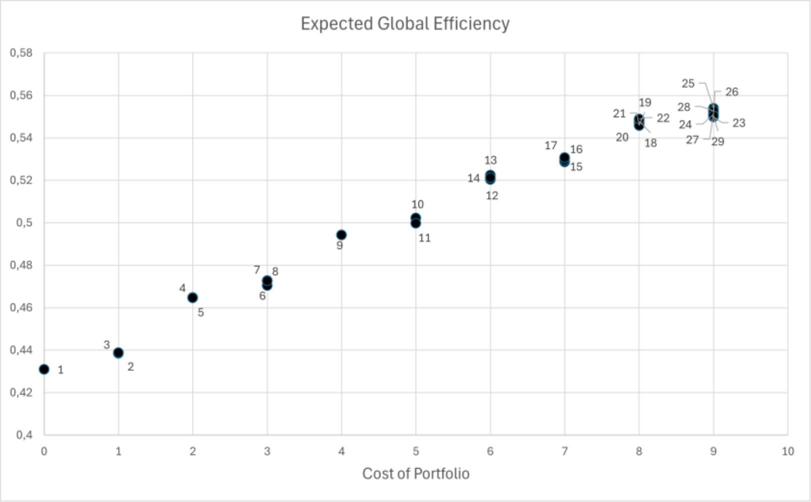
Figure 7: Possible new edges to add to the network (Type 2 actions)

Results (1/2)

- 32 cost-efficient portfolios whose cost are very different

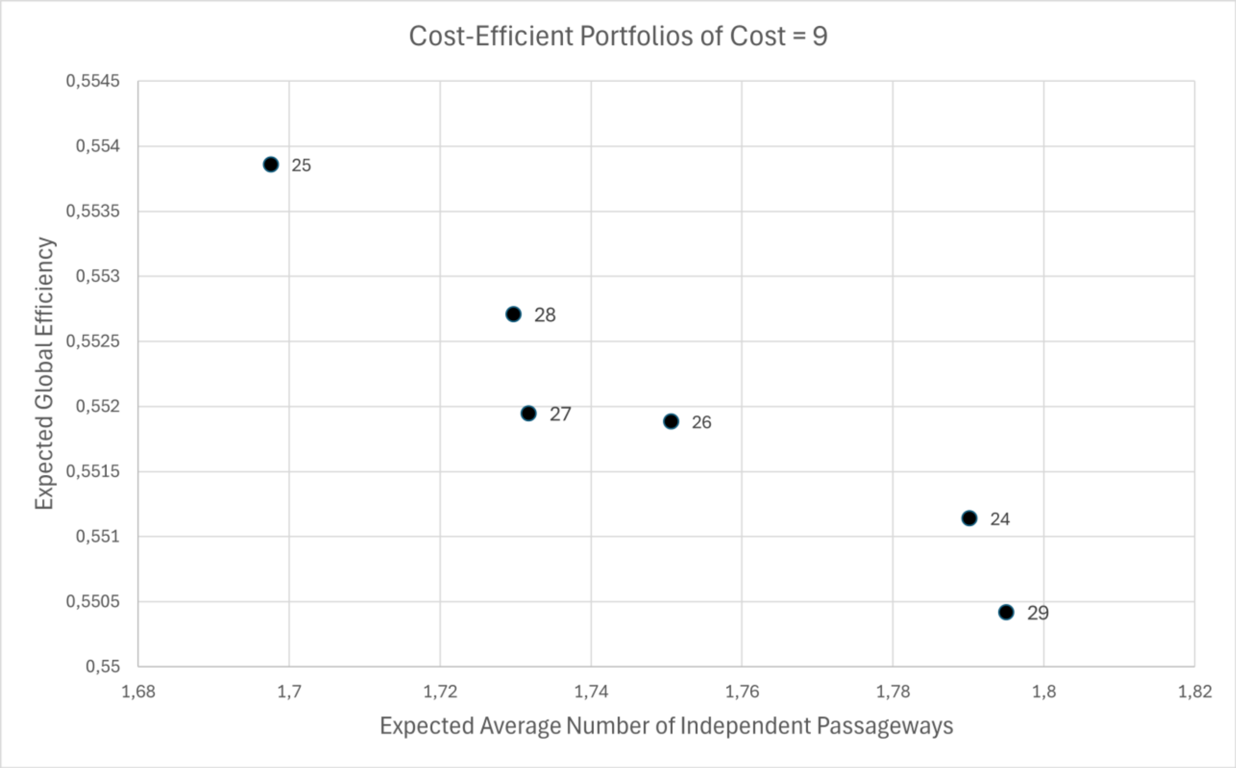






Results (2/2)

- Cost-efficient portfolios with a total cost of 9



Example portfolios

- The algorithm recommends mostly Type 2 actions
- Isolated and weakly connected nodes 1, 5, 9 and 10 were identified

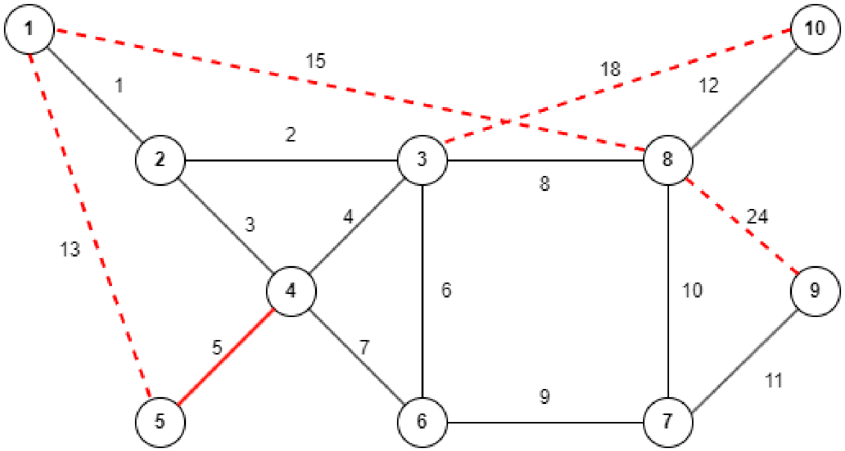


Figure 8: The network when portfolio number 25 is applied

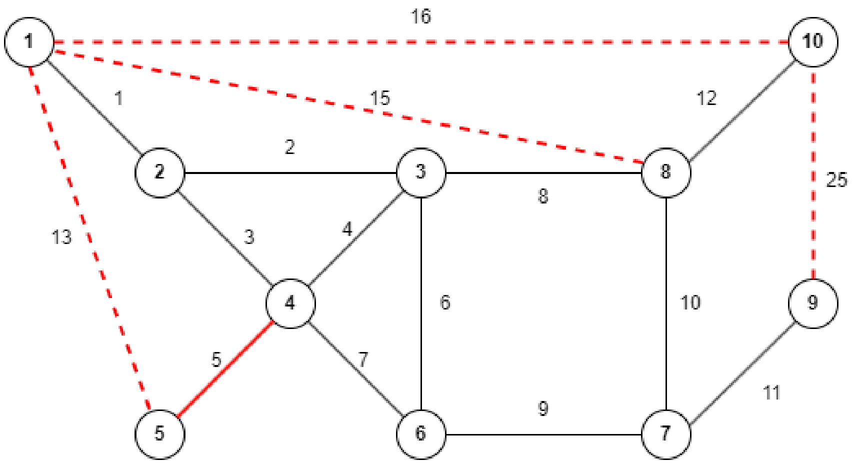


Figure 9: The network when portfolio number 29 is applied

Sensitivity analysis (1/5)

- The performance of the cost-efficient portfolios depends on the parameters
  - Weights
  - Disruption probabilities
  - Impacts and costs of reinforcement actions
- Some actions can still be recommended to the DM
  - Core index: Relative share of actions in the cost-efficient portfolios

$$CI(m) = \frac{|\{q \in Q_{CE} \mid q_m = 1\}|}{|Q_{CE}|}, m = 1, \ldots, r$$

Sensitivity analysis (2/5)

- How do the results depend on the parameters?
  - Scenario 1: Modify the disruption probability of new edges
    - Disruption probability: $[0.2, 0.4]$
  - Scenario 2: Modify the costs of Type 2 actions
    - Cost: $[1.5, 2.5]$

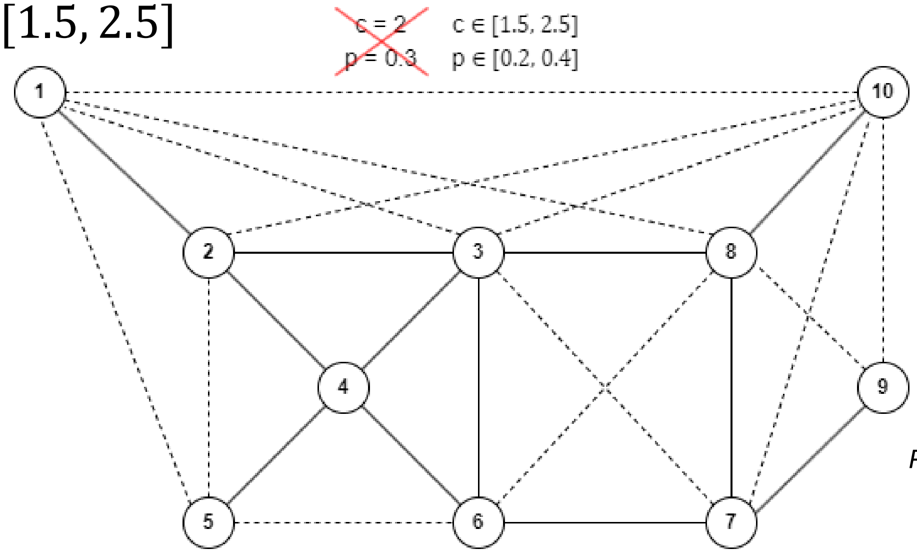
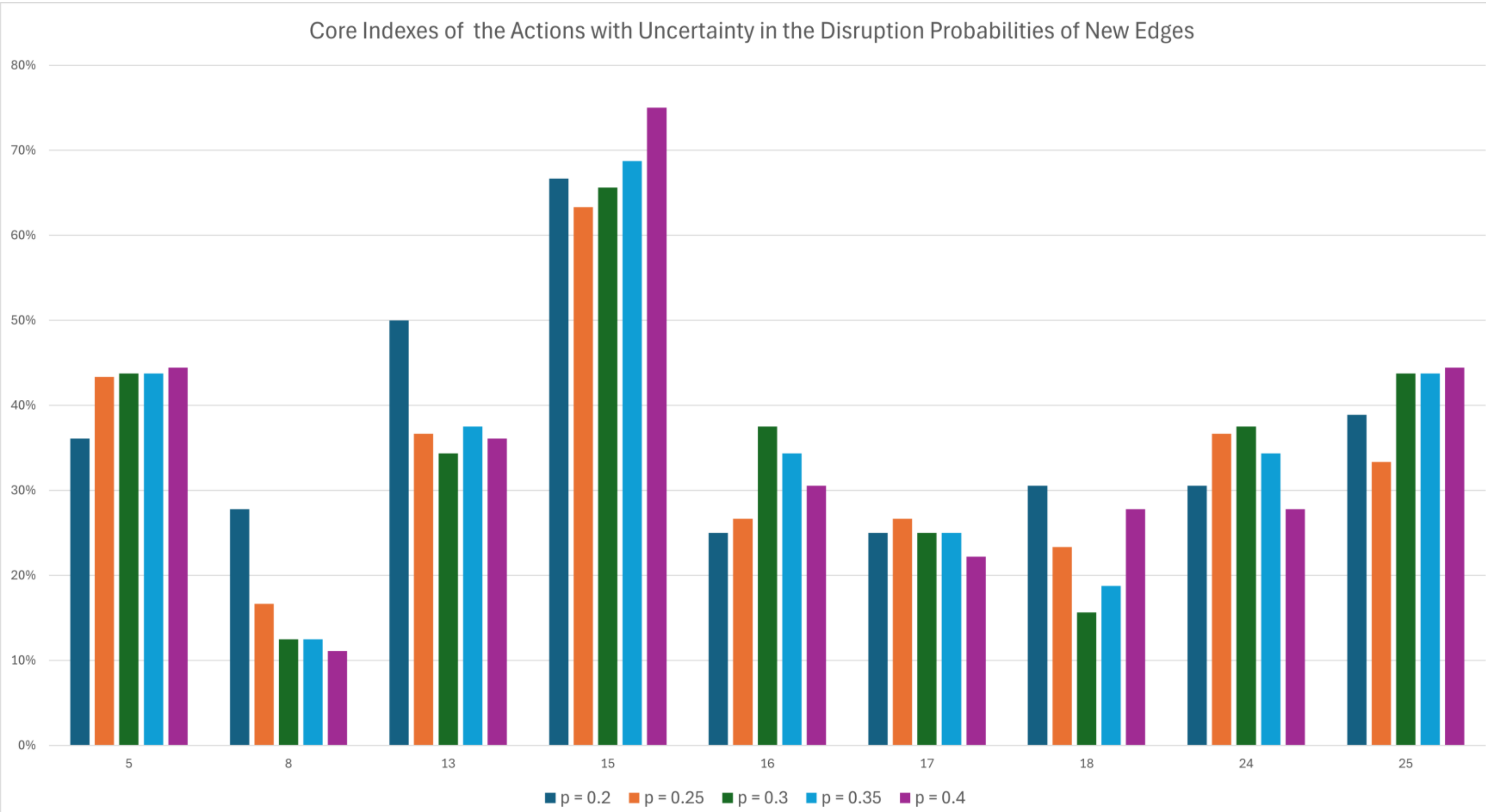
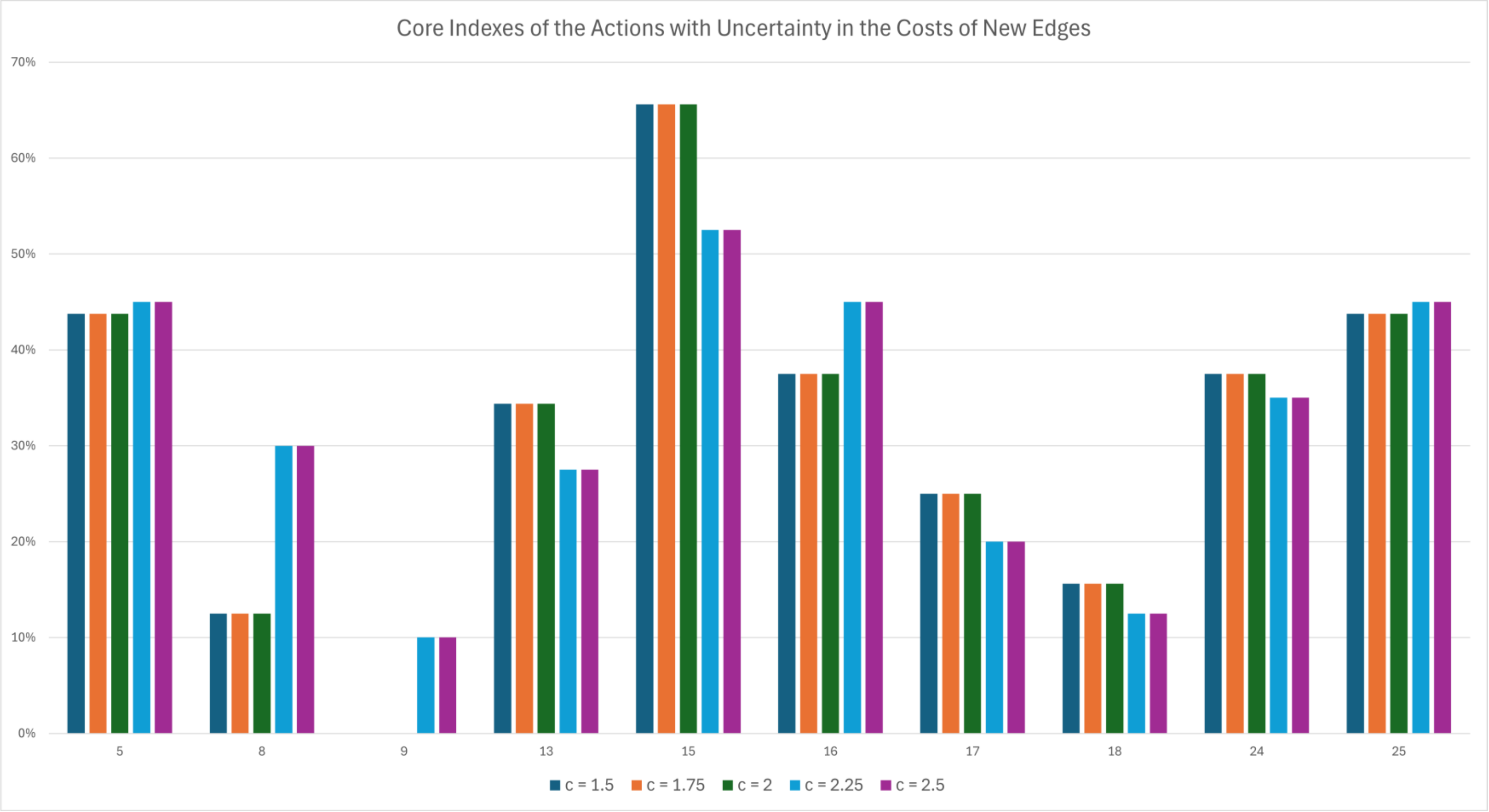


Figure 10: Uncertainty in the costs and effects of Type 2 reinforcement actions

Sensitivity analysis (3/5)

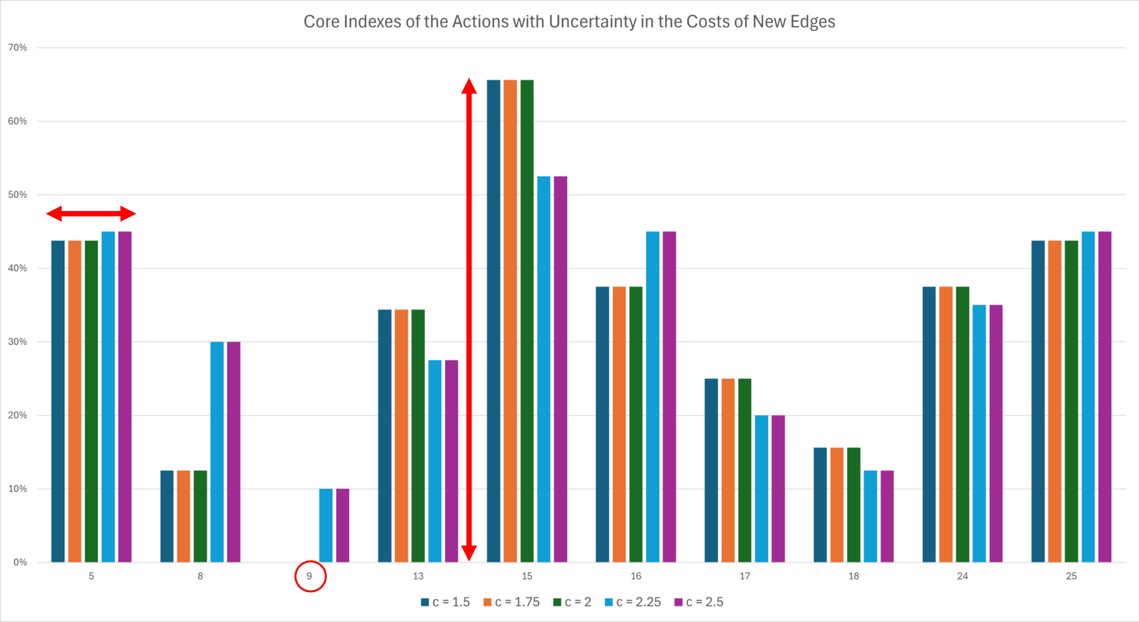


Sensitivity analysis (4/5)



Sensitivity analysis (5/5)

- Reinforcement actions with $CI(m) = 0$ can be discarded
- A reinforcement action with $CI(m) = 1$ is surely cost-efficient



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Conclusions

- Sensitivity analysis helps identify robust reinforcement actions
- Limitations:
  - Only small networks can be used
- Future work:
  - Find a faster way to evaluate performance

Literature and references

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- Ip, W.H., Wang, D. (2011). Resilience and Friability of Transportation Networks: Evaluation, Analysis and Optimization, *IEEE Systems Journal*, 5(2), 189-198
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The algorithm

Algorithm 2 Procedure for finding the cost-efficient portfolios

$Q^0 \leftarrow [0, 0, 0, ..., 0] \in Q_F$
for l = 1, 2, 3, ..., r **do**
    $Q^l \leftarrow \{q^k \in Q_F \mid q^j \in Q^{l-1} : q_l^k = 1 \wedge \forall i \neq l : q_i^k = q_i^j\}$
    for $q^k \in Q^l$ **do**
        $G^k \leftarrow G$
        Apply portfolio q^k to G^k
        Compute $\mathbb{E}[V(w, x) \mid q^k]\ \forall w \in S_{ext}$
    end for
    $Q^l \leftarrow \{q^k \in Q^l \mid \forall q^j \in Q^{l-1} : q^j \not\succ_C q^k\}$
    $Q^l \leftarrow Q^l \cup \{q^j \in Q^{l-1} \mid \forall q^k \in Q^l : q^k \not\succ_C q^j\}$
end for
$Q_{CE} \leftarrow Q^r$
Output the result Q_{CE}