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Integrated public transport and last-mile delivery: Optimizing the delivery vehicle route

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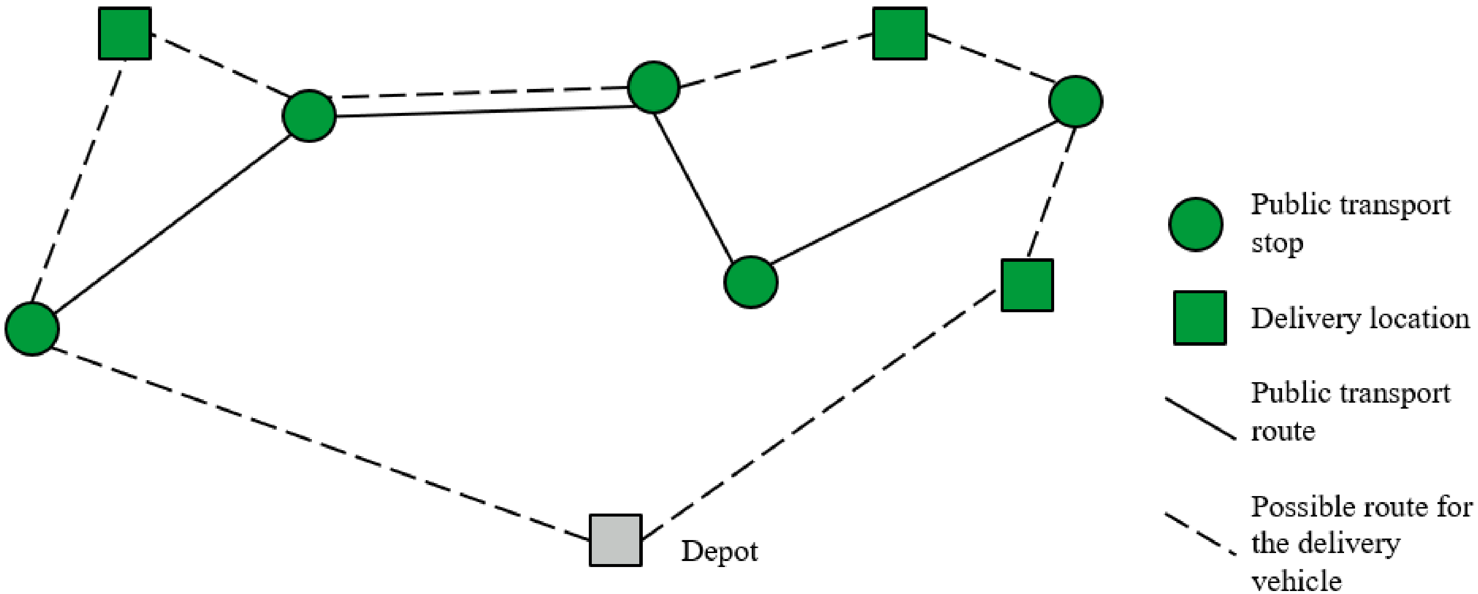
Introduction

- Last-mile delivery is a part of logistics and transportation science, and it focuses on the last part of the supply chain
- Can be modeled with the General Pick-up and Delivery problem
  - Set of pick-up and delivery nodes, goal is to find an optimal route
- Integration with public transport – we use an existing fixed public transport line as a part of the solution
- Constraints from time restrictions, node visits and capacity

Background and motivation

- Demands for delivery services have increased
  - Fast, reliable, affordable, CO_2 emissions, traffic congestion…
- New solutions by combining existing public transport and robots/drones
- Dial-A-Ride problem with a fixed line by Häll et. al. (2009)
- Solution with a truck and a drone presented by Murray and Chu (2015)
- Delivery vehicle using scheduled lines (time windows) by Ghilas et. al. (2016)

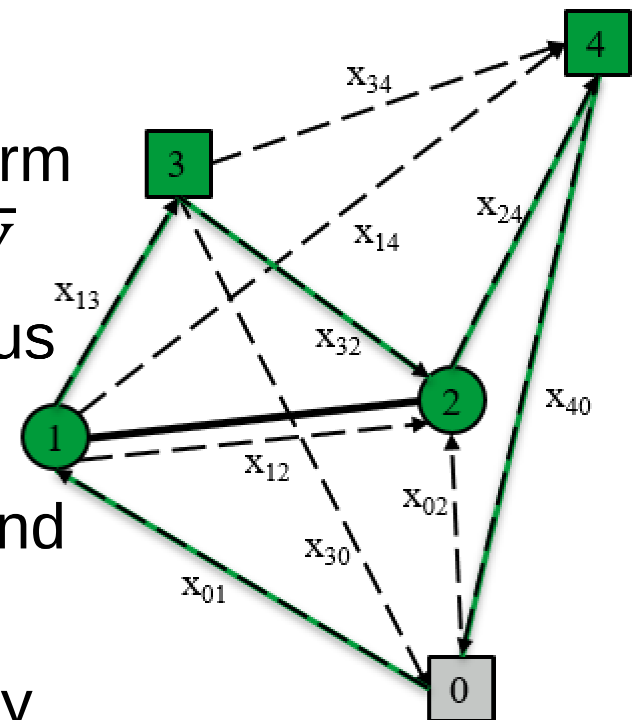
General case



Definition: Given a complete graph including the delivery locations and a bus route the integrated public transport and delivery problem is to find a feasible route for the delivery vehicle minimizing the travel time. A route is feasible if all the delivery nodes are visited, and the capacity of the delivery vehicle is not exceeded.

Model A: General case

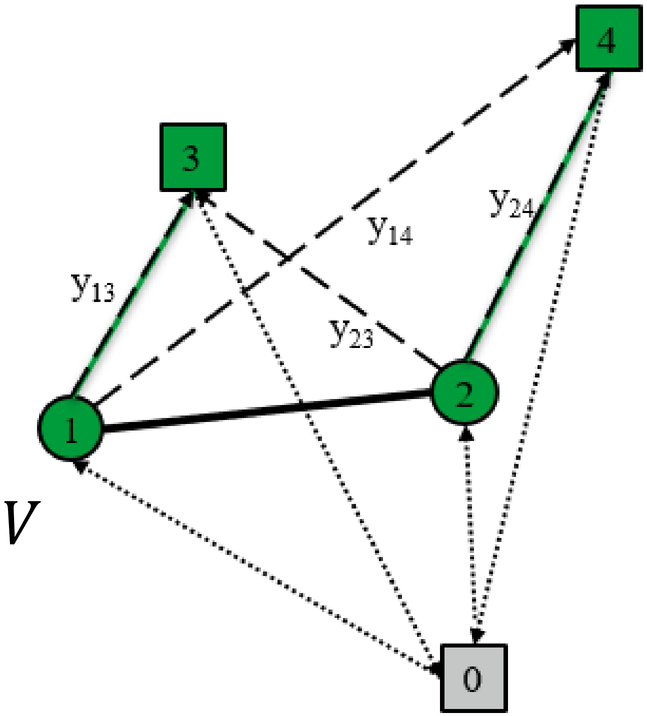
- Bus stops $\overline{V}$ with time windows $[l_i, u_i], i \in \overline{V}$ and delivery nodes V form a complete graph W with nodes $V \cup \overline{V}$
- Vehicle travel time $d_{ij} \in \mathbb{N}, i, j \in W$, bus travel time $t_{i,i+1}, i \in \overline{V}$
- Binary variable $x_{ij} \in \{0,1\}, \forall i, j \in W$ and integer variable $\beta_i \in \mathbb{N}, \forall i \in W$
- These types of problems are generally NP-hard, thus we need to consider a more special case



Note that only some of the x variables are depicted in the figure

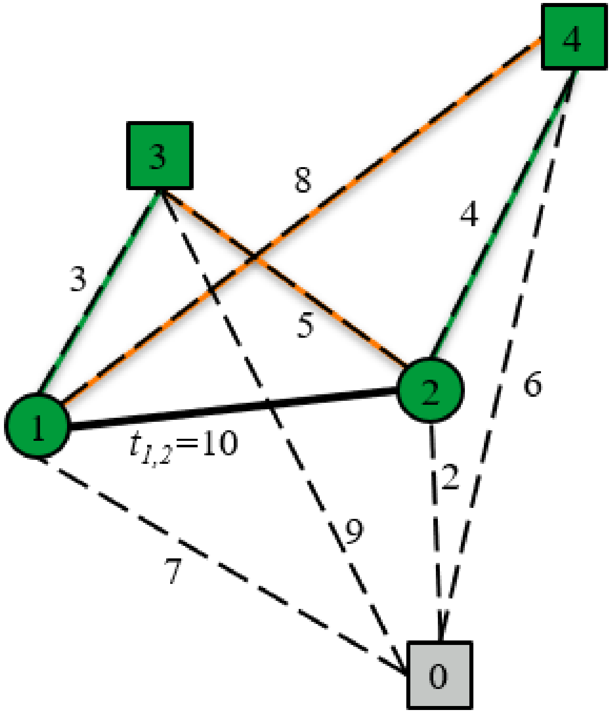
Model B: Matching with additional constraints

- Bus stops $\overline{V}$ with time windows $[l_i, u_i], i \in \overline{V}$ and delivery nodes V, both sets of equal sizes
- Vehicle travel time $d_{ij} \in \mathbb{N}, i, j \in W$, bus travel time $t_{i,i+1}, i \in \overline{V}$
- Binary variable $y_{ij} \in \{0,1\}, i \in \overline{V}, j \in V$ and integer variable $\beta_i \in \mathbb{N}, \forall i \in W$
- Matching in a bipartite graph
- Models A and B were proven to be equivalent



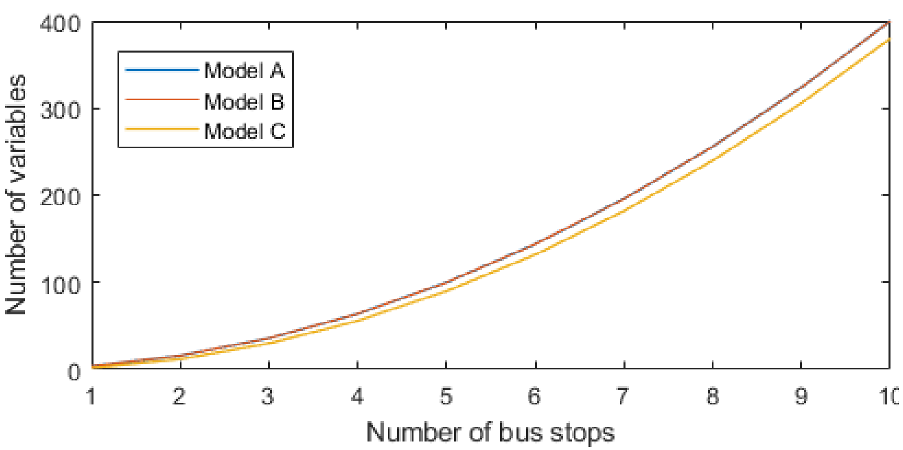
Model C: Perfect matching

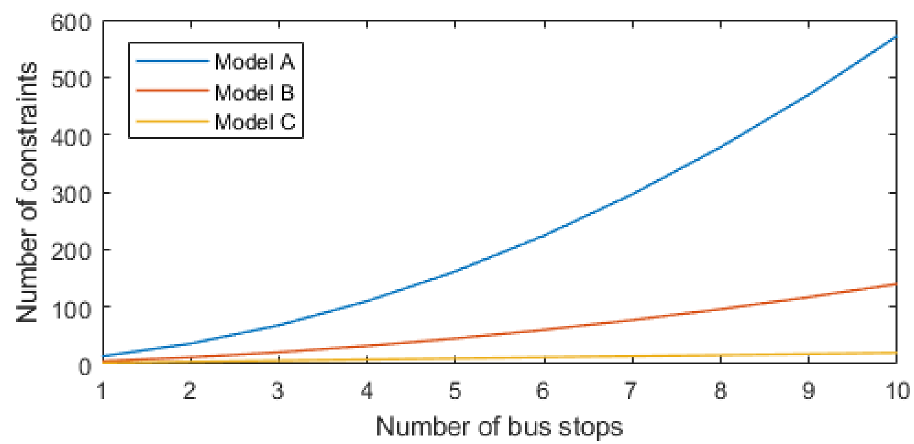
- Instead of calculating departure times for each node, we check the feasible connections beforehand
- Connection (i,j) is feasible if $d_{ij} + d_{ji+1} \leq t_{i,i+1}$, here $t_{i,i+1} = l_{i+1} - l_i$
- Set of feasible connections is $E = \{(i,j): d_{ij} + d_{ji+1} \leq t_{i,i+1}\},$
  $$\forall i \in \overline{V}, j \in \mathrm{V}$$
- Binary variable $y_{ij} \in \{0,1\}, \forall (i,j) \in \mathrm{E}$
- Can be solved in polynomial time



Comparison of the three models

- Comparing the number of variables and constraints
- Small difference in number of variables
  - note that for model C values are upper limits
- Significant difference in number of constraints





Formulas

Model A

$$\min \sum_{i\in W}\sum_{j\in W} d_{ij}x_{ij} + d_{01}$$

$$\sum_{i\in W} x_{ij} = 1, \forall j \in V$$

$$\sum_{i\in W} x_{ij} \leq 1, \forall j \in W$$

$$\sum_{j\in V} x_{1j} = 1$$

$$\sum_{i\in W} x_{i0} = 1$$

$$\sum_{j\in W} x_{ij} - \sum_{j\in W} x_{ji} = 0, \forall i \in W$$

$$x_{ij} = 0, \forall i,j \in V : i \neq j$$

$$x_{ii} = 0, \forall i \in W$$

$$\beta_j \geq \beta_i + d_{ij} - (1 - x_{ij})M, \forall i,j \in W$$

$$l_i \leq \beta_i \leq u_i, \forall i \in W$$

$$\beta_{i+1} \geq \beta_i + t_{i,i+1}, \forall i \in \bar{V}$$

$$x_{ij} \in \{0,1\}, \forall i,j \in W$$

$$\beta_i \in \mathbb{N}, \forall i \in W$$

Model B

$$\min \sum_{i\in \bar{V}}\sum_{j\in V} y_{ij}(d_{ij} + d_{ji+1}) + d_{01}$$

$$\sum_{i\in \bar{V}} y_{ij} = 1, \forall j \in V$$

$$\sum_{j\in V} y_{ij} = 1, \forall i \in \bar{V}$$

$$\beta_{i+1} \geq \beta_i + d_{ij} + d_{ji+1} - (1 - y_{ij})M, \forall i \in \bar{V}, \forall j \in V$$

$$l_i \leq \beta_i \leq u_i, \forall i \in \bar{V}$$

$$\beta_{i+1} \geq \beta_i + t_{i,i+1}, \forall i \in \bar{V}$$

$$y_{ij} \in \{0,1\}, (i,j) \in W$$

$$\beta_i \in \mathbb{N}$$

Model C

$$\min \sum_{i\in \bar{V}}\sum_{j\in V:(i,j)\in E} y_{ij}(d_{ij} + d_{ji+1}) + d_{01}$$

$$\sum_{i\in \bar{V}:(i,j)\in E} y_{ij} = 1, \forall j \in V$$

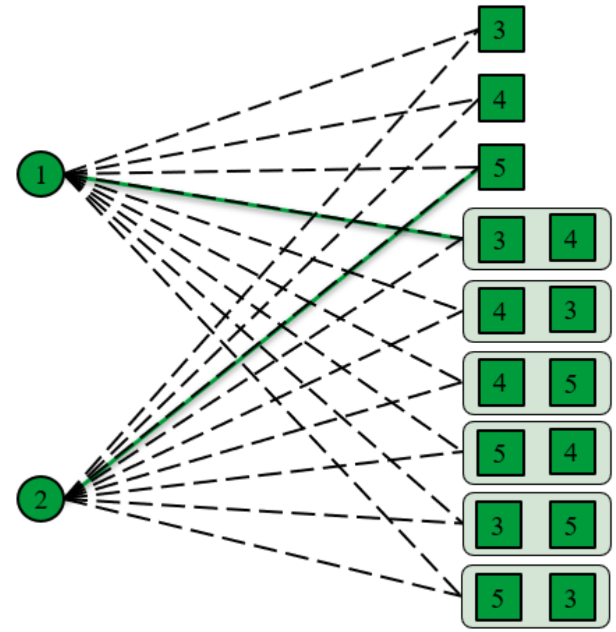
$$\sum_{j\in V:(i,j)\in E} y_{ij} = 1, \forall i \in \bar{V}$$

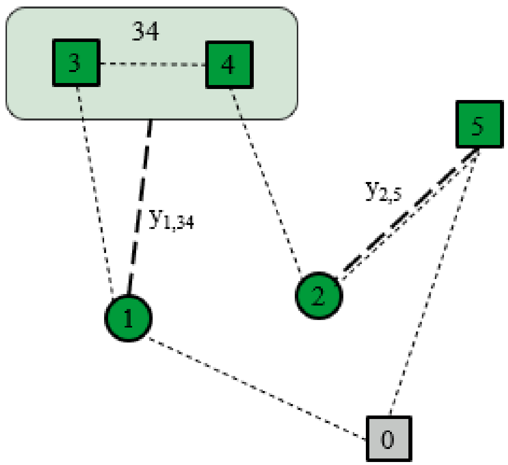
$$y_{ij} \in \{0,1\}, (i,j) \in E$$

$$E = \{(i,j) : d_{ij} + d_{ji+1} \leq t_{i,i+1}\}, \forall i \in \bar{V}, \forall j \in V$$

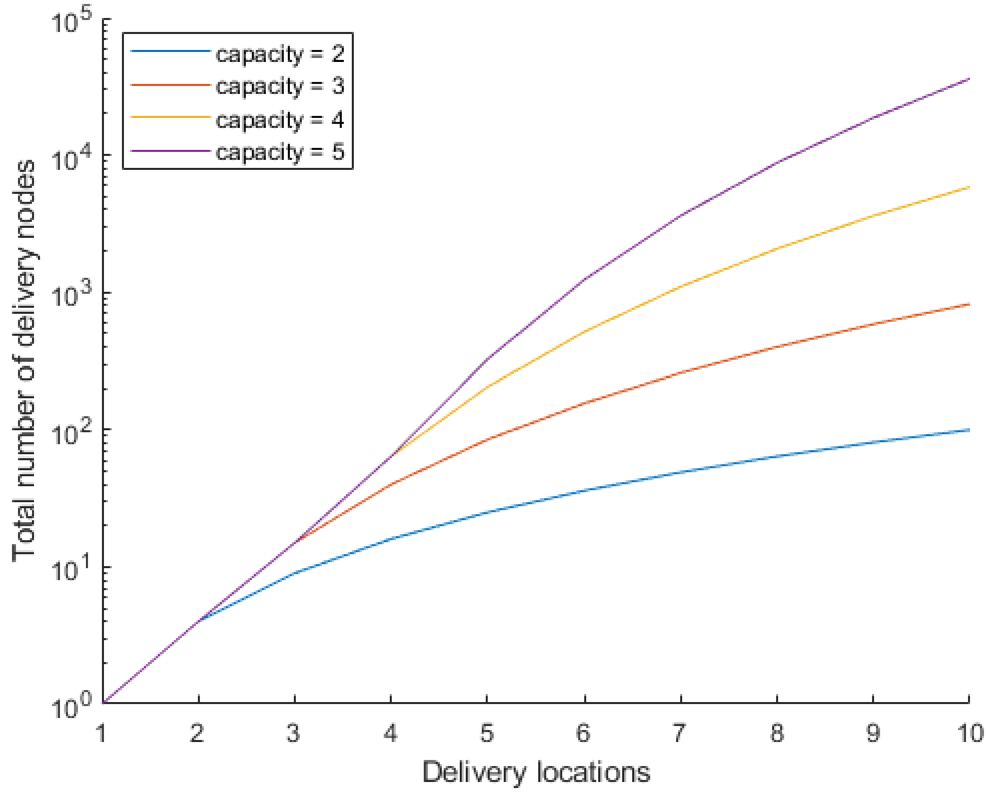
Extension of Model C

- Capacity of the delivery vehicle is more than one
- Idea is to combine single delivery nodes into one combined node
  - Travel times inside the combined nodes must be considered and the order in which the nodes are visited matters
- Total number of nodes in the model increases
- Constraints change a bit from model C





Total number of nodes in the extended model



Conclusions and future research

- Of the three models constructed, A and B possibly NP-hard, but model C can be solved in polynomial time
- Model C has strict restrictions, but the extended version considers a more realistic case
  - However, there are no waiting times in model C
- These models ignore many aspects of the real world and are too simple to model the real world
- In the future we should consider including other into the model

Literature

- M.O. Ball et al. Handbook in OR and MS, Vol. 7. Elsevier Science B.V., 1995.
- G. Berbeglia, J. Cordeau, I. Gribkovskaia, and G. Laporte. Static pickup and delivery problems: a classification scheme and survey. Sociedad de Estadística e Investigación Operativa, 2007.
- E. Demir, A. Syntetos, and T. van Woensel. Last mile logistics: Research trends and needs. IMA Journal of Management Mathematics, 2022.
- V. Ghilas, E. Demir, and T. van Woensel. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows and scheduled lines. Computers Operations Research Volume 72, 2016.
- Carl H Häll, Henrik Andersson, Jan T Lundgren, and Peter Värbrand. The integrated dial-a-ride problem. Public Transport, 1:39–54, 2009.
- Clair E Miller, Albert W Tucker, and Richard A Zemlin. Integer programming formulation of traveling salesman problems. Journal of the ACM (JACM), 7(4): 326–329, 1960.
- Chase C. Murray and Amanda G. Chu. The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery. Transportation Research Part C, 2015.
- M. W. P. Savelsbergh and M. Sol. The general pickup and delivery problem. Transportation Science, 1995.