Aalto-yliopisto
Perustieteiden korkeakoulu

Travel-time Optimal Line Plans On Trees (Results of the Thesis)

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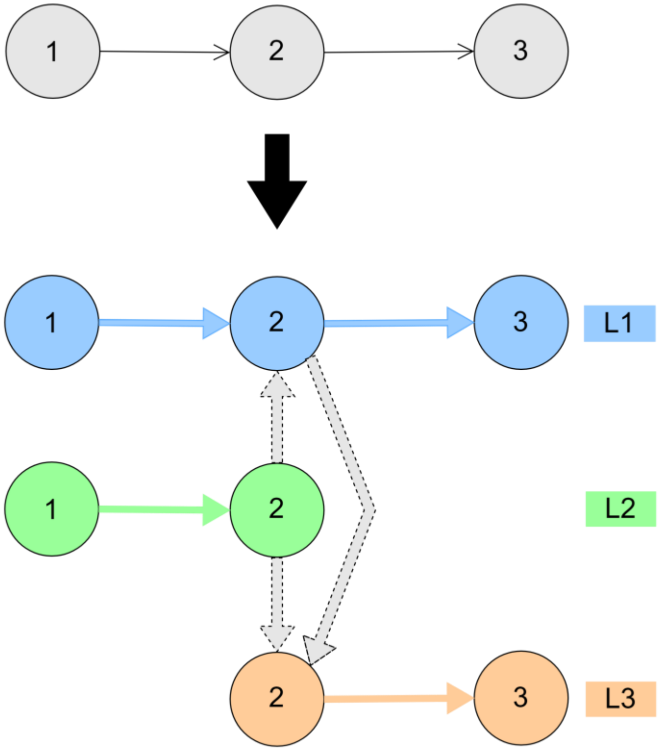
Työn saa tallentaa ja julkistaa Aalto-yliopiston avoimilla verkkosivuilla. Muilta osin kaikki oikeudet pidätetään.

Systeemianalyysin laboratorio

Background: Solving Time-Optimal LPPs*

- We aim to minimize the total travel time of all passengers.
- The line plan is modelled as a change & go –graph
- Either binary or integer frequencies are used



*LPP = Line Planning Problem

Background: LPMT1 by Schöbel & Scholl

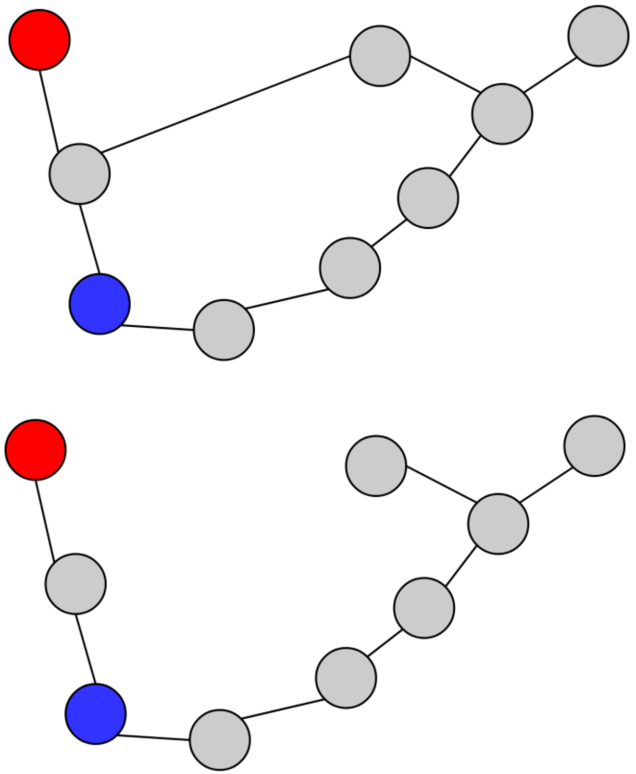
The standard IP formulation for binary frequency:

$$
\begin{aligned}
\min \quad & \sum_{(s,t)\in\mathcal{R}} \sum_{e\in\mathcal{E}} w_{st}\ c_e\ x_{st}^e & \\
s.t. \quad & \sum_{(s,t)\in\mathcal{R}} \sum_{e\in\mathcal{E}^l} x_{st}^e \leq |\mathcal{R}||\mathcal{E}^l|y_l & \forall\ l \in \mathcal{L} \\
& \theta x_{st} = b_{st} & \forall\ (s,t) \in \mathcal{R} \\
& \sum_{l\in\mathcal{L}} C_l y_l \leq B & \\
& x_{st}^e, y_l \in \{0,1\} & \forall\ (s,t) \in \mathcal{R}, e \in \mathcal{E}, l \in \mathcal{L}
\end{aligned}
$$

Schöbel and Scholl, 2006

Background: Advantages of trees

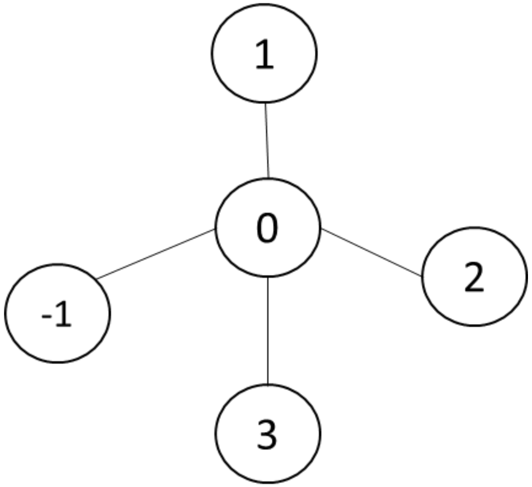
If the network is limited to trees, the travel time is minimized, when tranfers are minimized.

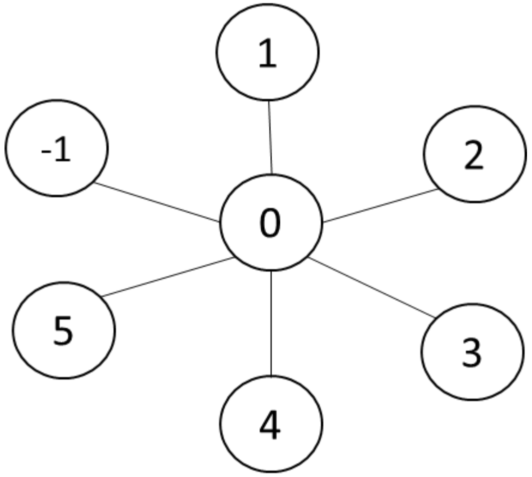


Objective of the Thesis

- Develop an optimization formulation that acts as a simpler alternative to the LPMT1

- The scope is limited to star shaped trees and binary frequencies

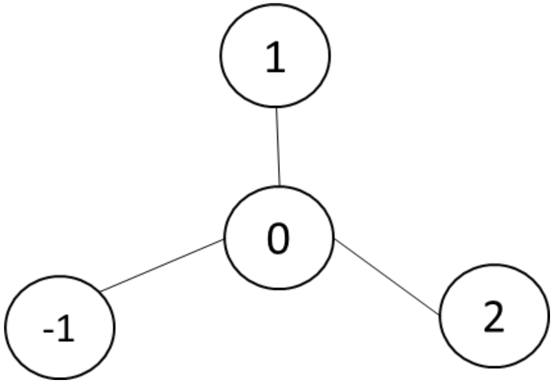




The Assumptions of the Model (1/2)

1. If passengers travel from a leaf node, at least one of them wishes to travel to another leaf node.

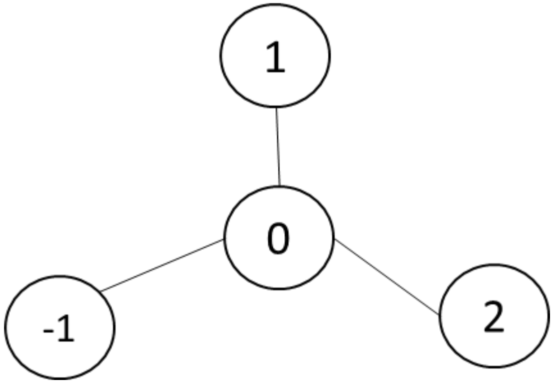
2. If passengers wish to travel to a leaf node, at least one of them is leaving from another leaf node.



Basic Notation of the Model (1/3)

The PTN = undirected graph (S, E):

$$S := \{-1, 0, 1, \dots, n\}$$
$$E := \{(i, j) \mid i, j \in S, i \cdot j = 0, i < j\}$$



The PTN when $n = 2$.

The origin-destination pairs:

$$\mathcal{R} := \{\, (s, t) \in S \times S \mid s \neq t, s \cdot t > 0 \,\}$$

The line pool:

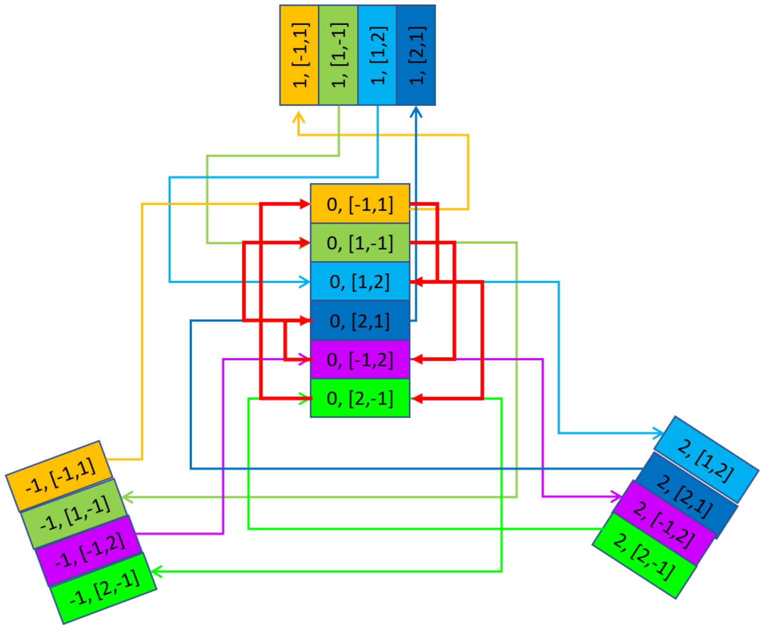
$$\mathcal{L} := \{\, [i, j] \mid i, j \in S \setminus \{0\},\ i \neq j \,\}$$

The Assumptions of the Model (2/2)

3. The passengers only transfer at the center node 0.

4. The passengers do not transfer between lines with the same origin or the same terminus.

5. The passengers do not transfer to lines where the terminus is the same station as the origin of the current line.



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Basic Notation of the Model (2/3)

The change&go-nodes:

$$\boldsymbol{\mathcal{V}} := \boldsymbol{\mathcal{V}}_{CG} \cup \boldsymbol{\mathcal{V}}_{OD}$$

where

1. $\mathcal{V}_{OD} := \{(i, OD) \mid (i,j) \in \mathcal{R} \text{ or } (j,i) \in \mathcal{R}\}$

2. $\mathcal{V}_{CG} := \{(k, [i,j]) \mid k \in S,\ \ [i,j] \in \mathcal{L},\ \ \mathrm{k} = 0 \text{ } or \text{ } k = i \text{ } or \text{ } k = j\}$

Basic Notation of the Model (3/3)

The change&go-edges:

$$\mathcal{E} := \mathcal{E}_{go} \cup \mathcal{E}_{change} \cup \mathcal{E}_{OD}$$

where

1. $\mathcal{E}_{l=[i,j]} := \{((p,l),(q,l)) \in \mathcal{V}_{CG} \times \mathcal{V}_{CG} \,|(p = i, q = 0)$ $or \;\; (p = 0, q = j)\}$

2. $\mathcal{E}_{go} := \bigcup_{l \in \mathcal{L}} \; \mathcal{E}_l$

3. $\mathcal{E}_{change} := \{((0,[i,j]),(0,[p,q])) \in \mathcal{V}_{CG} \times \mathcal{V}_{CG} \,|\, p \neq i, q \neq i, q \neq j\,\}$

4. $\mathcal{E}_{OD} := \{((k,OD),(k,[k,j])) \in \mathcal{V}_{OD} \times \mathcal{V}_{CG}, ((k',[i,k']\,),(k',OD\,)) \in$ $\mathcal{V}_{CG} \times \mathcal{V}_{OD} \,|(k,k') \in \mathcal{R}\,\}$

The Resulting Compact Formulation

$$
\begin{aligned}
\min \quad & \sum_{(s,t)\in\mathcal{R}} w_{st} \cdot x_{transfer}^{(s,\,0),\,(0,\,t)} & \\
s.t. \quad & x_{direct}^{(i,\,0),\,(0,\,j)} + x_{transfer}^{(i,\,0),\,(0,\,j)} = 1 & \forall\,(i,j)\in\mathcal{R} \\
& x_{direct}^{(i,\,0),\,(0,\,j)} \le z_{[i,j]} & \forall\,[i,j]\in\mathcal{L} : i\cdot j > 0 \\
& \sum_{k\neq i} z_{[i,k]} \ge 1 & \forall\, i \in S \setminus \{-1,\,0\} : (i,j)\in\mathcal{R} \\
& \sum_{k\neq j} z_{[k,j]} \ge 1 & \forall\, j \in S \setminus \{-1,\,0\} : (i,j)\in\mathcal{R} \\
& x_{transfer}^{(i,\,0),\,(0,\,j)} \le \sum_{i\neq k\neq j} z_{[i,k]} & \forall\,(i,j)\in\mathcal{R} \\
& x_{transfer}^{(i,\,0),\,(0,\,j)} \le \sum_{i\neq k\neq j} z_{[k,j]} & \forall\,(i,j)\in\mathcal{R} \\
& \sum_{l\in\mathcal{L}} C_l \cdot z_l \le B & \\
& x_{direct}^{(i,\,0),\,(0,\,j)},\; x_{transfer}^{(i,\,0),\,(0,\,j)} \in \{0,1\} & \forall\,(i,j)\in\mathcal{R} \\
& z_l \in \{0,1\} & \forall\, l\in\mathcal{L}
\end{aligned}
$$

The Size of the Formulation

The size approximations as a function of the number of leave nodes in the PTN (n):

Size aproximation	Compact Formulation	LPMT1
Number of variables	$\sigma(n^2)$	$\sigma(n^6)$
Number of constraints	$\sigma(n^2)$	$\sigma(n^4)$

The compact formulation is significantly smaller in size

→ Should be easier to solve (needs more research)

References

- A. Schöbel and S. Scholl, ”Line Planning with Minimal Traveling Time” in *5th Workshop on Algorithmic Methods and Models for Optimization of Railways,* Dagstuhl, Germany, 2006. Available: https://drops.dagstuhl.de/opus/volltexte/2006/660/