Solving energy capacity expansion problem with SDDP

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Agenda

1. Background
    1. Problem formulation & goal
    2. Decision types
2. Implementation
    1. Uncertainty
    2. Models
3. Data

4. Timetable

Goal: Increase solution efficiency of capacity problems of the form

$$\underset{x_t(\tilde{\xi}_{t-1}),\, y_t(\tilde{\xi}_t)}{\text{Minimize}} \; E_{\tilde{\xi}}\left\{ \sum_{t=1}^{T} \left[\tilde{C}_t^{I^\top} x_t\left(\tilde{\xi}_{t-1}\right) + C_t^{O^\top} y_t\left(\tilde{\xi}_t\right) \right] \right\}$$

Investment costs + Operating costs

Capacity built × Annualized cost × Operational years

Decision variable | Random variable | Constant (per stage)

Served demand cost + Unserved demand cost

Determined stage-wise by own optimization problem.
Function of demand, operating points and available capacity

1 Stage

Both decision types have their own determinants

Operational decisions

Variables:

- Power generated in each unit
- Energy transmitted by each line

Constraints:

- Transmission line throughput
- **Built capacity**
- Capacity availability

$$\sum_{op=1}^{OP}\left(d_{unserved}+\sum_{g=1}^{G}price_g\cdot gen_{g,op}\cdot h_{op}\right)$$

Investment costs & decisions

Variables:

- Capacity built at current stage

Constraints:

- Max capacity that can be built

Note:

Decision variable is multiplied by random variable in objective function

$$\sum_{g=1}^{G}c_{built,g}\cdot\widehat{\mathrm{cost}}_g\left(t\right)$$

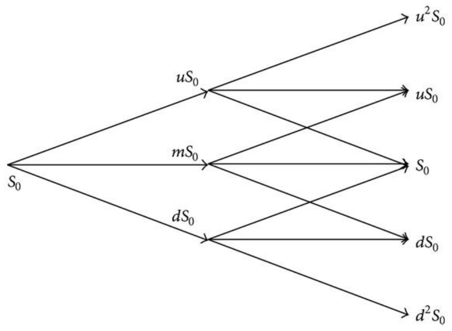
Uncertainty modeled with scenario lattice to overcome SDDP limitations

Demand growth

Trinomial tree

N(t) = t

N_tot= T(T-1)/2

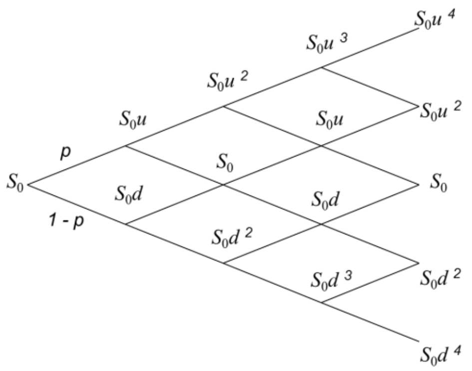


Price reduction

Binomial tree

N(t) = 2t-1

N_tot = T^2



Node characteristics:

Total nodes per stage = 2t^2-t = O(t^2)
Total nodes = T^3(T-1)/2 = O(T^4)
Scenarios = 6^T

Randomness Characterization

$$D_{s,n,o} = D_{n_0} c_s d_o$$

$$c_s\,(n,t) = (1+u)^{\,n\cdot T}\, g_e^t$$

Motivation & Results:

- Number of nodes reduced from 6^T to O(T^4)
- Cuts can now be modeled for each node separately
- **No stage-independent randomness** is included in the model
- Only way to model exponential trends in SDDP
- Way of overcoming problem with product of decision variable and random variable

3 types of models examined

Model 1

Operating points per investment decision: **1**

Issues:

- Operating costs not comparable between different stage runs
- Investment costs and amounts heavily influenced by decommissioning schedule

Model 2

Operating points per investment decision: **Arbitrary**

Issues:

- Investment costs and amounts heavily influenced by arbitrary decommissioning schedule

Model 3

Operating points per investment decision: **Arbitrary**

At each investment stage, capacity to be built for each of following operating stages is defined

Issues:

- None of the aforementioned

Difference between submodels solely dependent on decreasing uncertainty w.r.t stochastic parameters

Model 2 splits stages into operational and investment

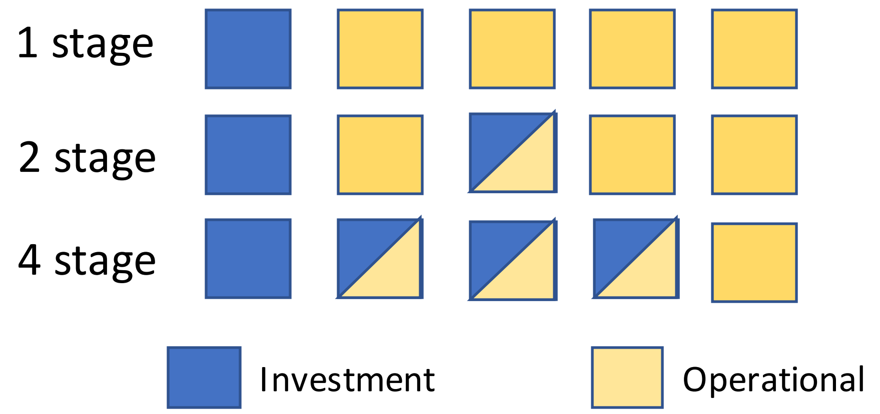
Details

- Stages come in 3 types:
  1. Investment (first stage)
  2. Operational
  3. Investment + operational
- One explicit and 2 implicit state variables:
  1. current production capacity (explicit)
  2. Investment cost (implicit through scenario)
  3. Demand (implicit through scenario)

Motivation

- Allows for implicitly enforcing non-anticipativity constraints
- Multiple operating stages per decision stage allows for *comparable* operating costs

Comparison to source article



Model 2: Example with 2 operational stages per investment decision

Stage 1

State variables:
- Existing capacity

Random variables
- Demand growth
- Investment cost decline

Key actions
- Build capacity for next stage

Stage objective
Investment costs

Capacity
Demand level
Investment cost

Stage 2

State variables:
- Existing capacity

Random variables
- Demand growth
- Investment cost decline

Key actions
- Optimize operational performance

Stage objective
Operational costs

Capacity
Demand level
Investment cost

Stage 3

State variables:
- Existing capacity

Random variables
- Demand growth
- Investment cost decline

Key actions
- Optimize operational performance
- Build capacity for next round

Stage objective
Investment costs + Operational costs

Note: All operational points for stage solved simultaneously resulting in bad scaling

Model 3 adds a state variable indicating capacity to be built per stage until next investment decision

Stage 1

State variables:

- Existing capacity
- **Capacity built during each operating stage before next investment decisions**

Random variables

- Demand growth
- Investment cost decline

Key actions

- Build capacity for next stage

Stage objective

Investment costs

Capacity
Building plans
Demand level
Investment cost

Stage 2

State variables:

- Existing capacity

Random variables

- Demand growth
- Investment cost decline

Key actions

- Optimize operational performance
- Build capacity planned in stage 1

Stage objective

Operational costs +
Investment costs

Capacity
Demand level
Investment cost

Stage 3

State variables:

- Existing capacity

Random variables

- Demand growth
- Investment cost decline

Key actions

- Optimize operational performance
- Build capacity for next round

Stage objective

Investment costs +
Operational costs

Note: All operational points for stage solved simultaneously resulting in bad scaling

Data

- Open-source data

- IEEE standard electricity grid

- Electricity data from literature

Timetable

31. 1 All code written

2.2 Begin work on thesis

10.2 Finalize all but results

20.2 Finalize results & Conclusions

31.2 Thesis submitted