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Multistage investment under two sources of uncertainty – a real options approach

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The goals of the study

- Examine sequential investment decisions under two explicit sources of uncertainty
  - The model is based on the one-factor real options model of Majd & Pindyck [1987]
  - Also, McDonald & Siegel [1986] studied a two-factor model where the investment program can be finished instantaneously
  - Examples: R&D projects, new technology adoption
- Particularly study how the inclusion of the second stochastic variable affects the optimal investment policy
- Solve the model numerically

The model

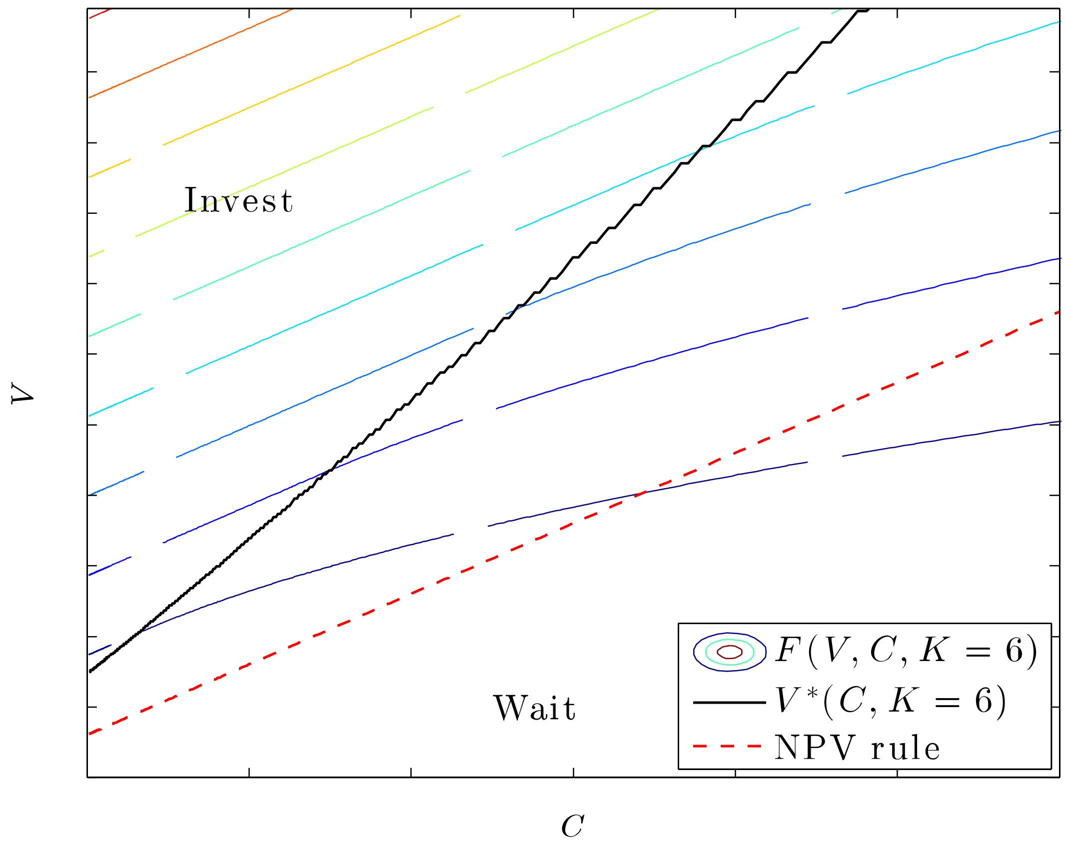
- The sources of uncertainty are modeled by two stochastic variables, i.e., the discounted cash inflows and outflows of the finished project
  - We will denote these by V and C, respectively
  - These are assumed to follow uncorrelated geometric Brownian motions with parameters (α_V,σ_V) and (α_C,σ_C)
- The required rate of return for holding the option is μ
  - We implicitly assume that the investor is risk neutral as we use dynamic programming
- The maximum investment rate is denoted by k and the initial investment left by K
- The investor can choose the investment rate continuously, and the payoff max(V-C,0) is obtained only when $K=0$
  - ➢ How should the investor proceed with the investment program?

A few words on how the results were obtained

- We used the dynamic programming approach to real options valuation
  - The solution is a “bang-bang” one: it is optimal to either wait or invest at the maximum rate
- This combined with the assumptions led to a two-PDE free boundary problem with three independent variables, i.e., *V, C* and *K*
  - McDonald & Siegel [1986] provided an analytical solution that is linear homogenous in *V* and *C*
    - However, this is not the case for the problem here because of the time-to-build issue
- The problem was then solved using an explicit finite difference method
  - ➢ The option value function *F(V,C,K)* and the investment threshold $V^*(C,K)$
  - Then, the effects of the parameters on the results were studied using the method of comparative statics

The base case

($\alpha_V=\alpha_C=0.04$, $\sigma_V=\sigma_C=0.14$, $\mu=0.08$, $k=1$)



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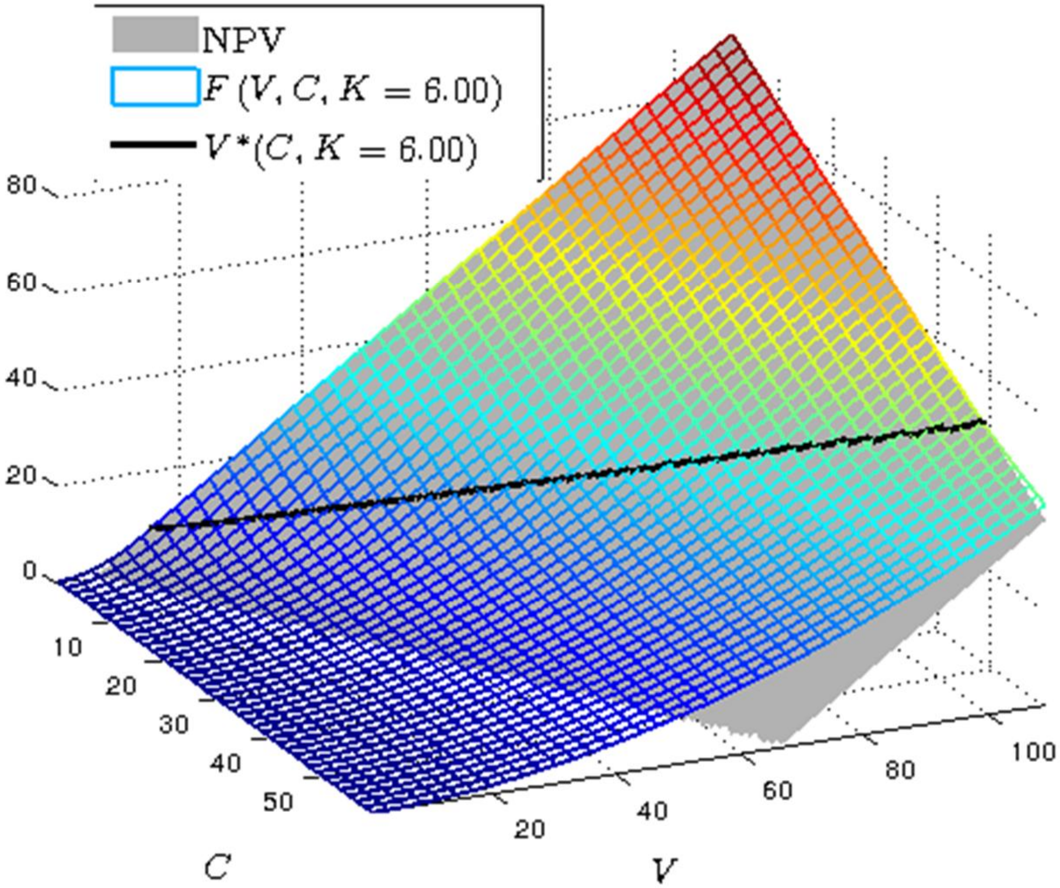

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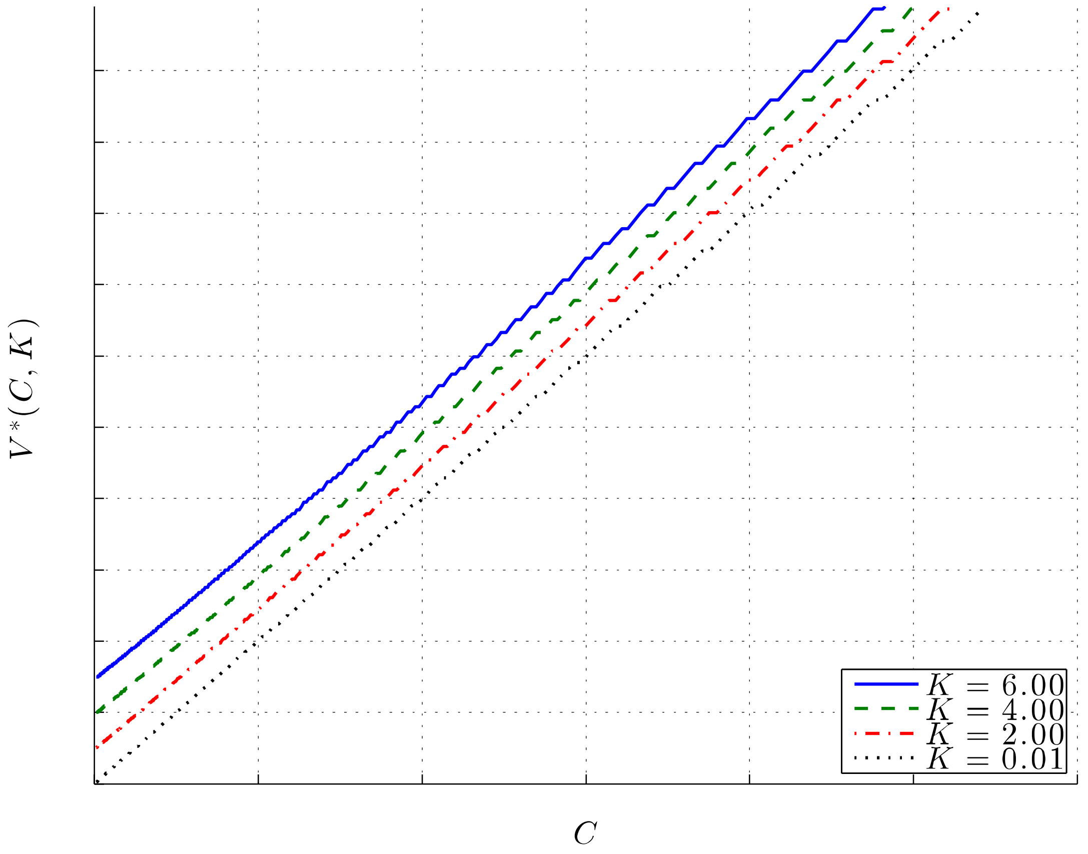
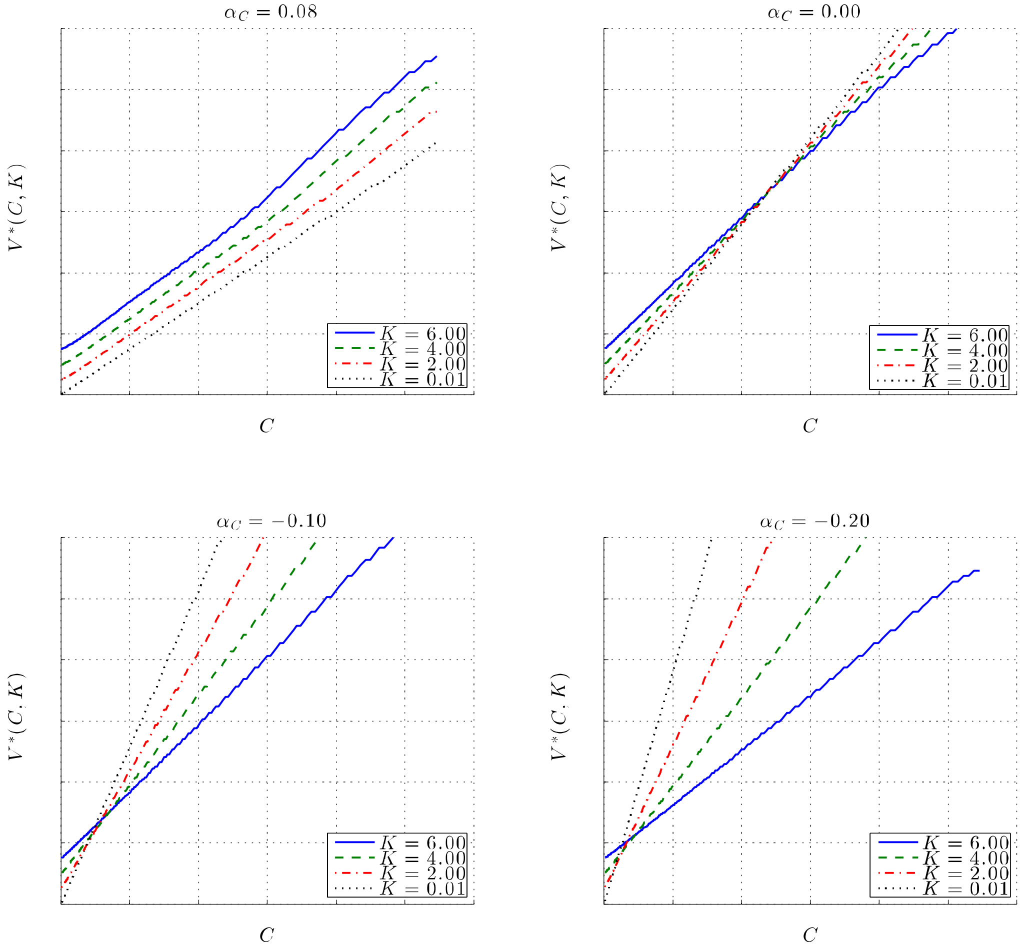


Sensitivity with respect to α_C

(The other parameters are the same as in the base case)

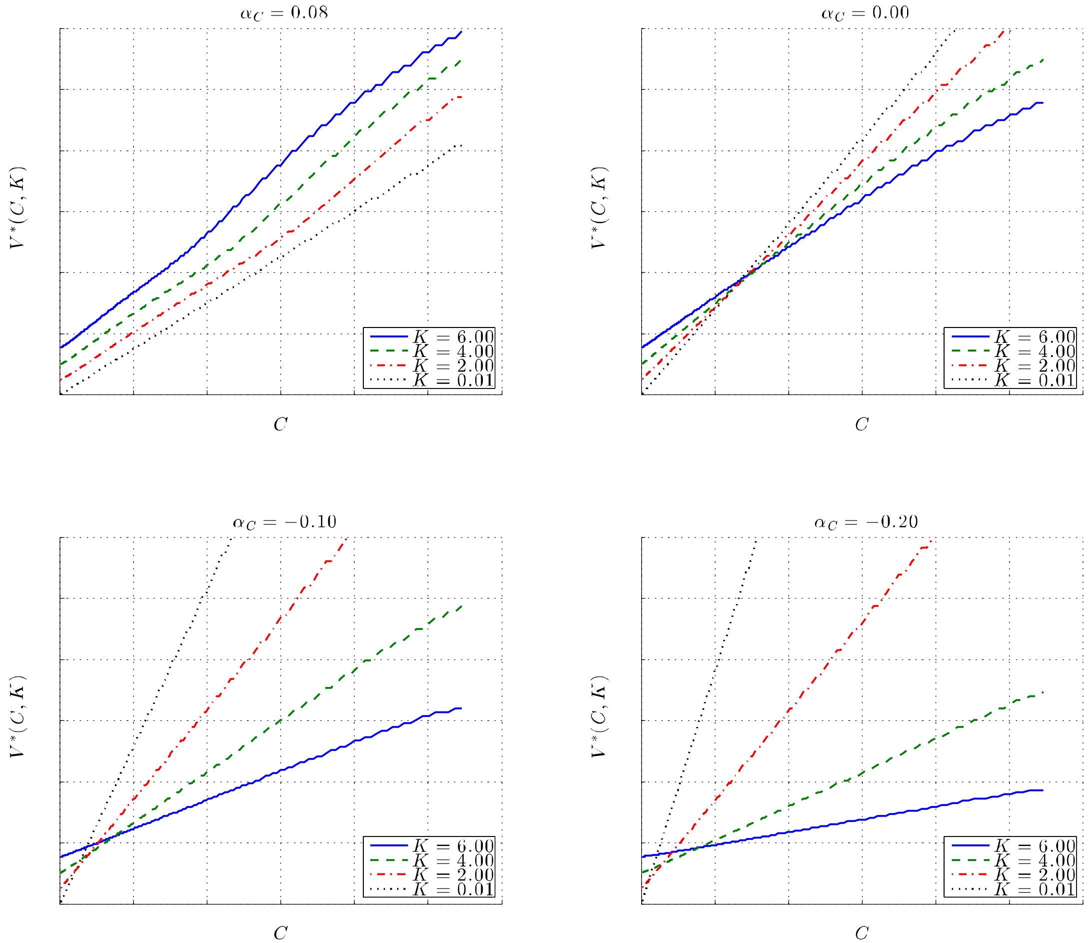


The explanation

- When $K<<1$, a decrease (increase) in α_C increases (decreases) the incentives of waiting [McDonald & Siegel, 1986]
  - ➢ The threshold shifts up (down)
- At larger values of K, the optimal investment policy can be explained by the principle of dynamic programming
  - – The investor knows the optimal investment policy for smaller values of K
  - – Both the payoff and the initial investment outflows are discounted
  - ➢ It is optimal to invest so that the investment program can be completed without pauses in most cases
- Shouldn't this imply that the effect of α_C on the investment threshold is amplified when k is smaller and, thus, the minimum construction time is longer?

Sensitivity with respect to α_c, when $k=0.5$

(The other parameters are the same as in the base case)



Sensitivity with respect to other parameters

- The logic behind the effect of α_V on the results is the same as above
  - However, $V^*(C,K)$ grows without bounds as $\alpha_V \rightarrow \mu$
- As μ represent the cost of waiting, an increase (decrease) in its value shifts the investment threshold down (up)
- An increase (decrease) in either of the volatilities increases (decreases) the value of waiting and therefore shifts the investment threshold up (down)
- If the increments of the stochastic variables were positively (negatively) correlated, the volatility of the process that the payoff follows would decrease (increase) shifting the investment threshold down (up)

Summary

- The investor's problem was solved numerically yielding both the option value and the investment threshold
- Comparative statics was used to analyze the impacts of the different parameters on the optimal investment policy
  - The effects of the drift terms were explained in the framework of dynamic programming
- The model is general and can be applied in situations that meet the underlying assumptions by modifying the boundary conditions

References

- *Time to build, option value, and investment decisions*, Majd & Pindyck, 1994
- *The value of waiting to invest*, McDonald & Siegel, 1986