Aalto University
School of Science
Degree programme in Engineering Physics and Mathematics

Optimization of artillery fire based on aerial reconnaissance

Bachelor's Thesis
10.10.2017

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ABSTRACT OF THE
BACHELOR'S THESIS

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Title: Optimization of artillery fire based on aerial reconnaissance

Date: 10.10.2017 Language: English Number of pages: 4+34

Degree programme: Engineering physics and mathematics

Main Subject: Mathematics and systems sciences

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The objective of this thesis is to examine Attacker's effective use of artillery when the amount of available information on the opposite units varies. The information is expressed as a probability of the Attacker to locate the Defender's unit. By studying the effects of the used artillery, it is possible to optimize the way artillery is used corresponding to the Attacker's objectives. These results provide well-founded decision support, which can, for instance, benefit the acquisition of weapon systems, which often require significant investments.

In this thesis the effective use of artillery is studied by using a computational combat model. This builds on a data farming process for generating the data, which can be analyzed to determine the optimal use of artillery. The analyzes are based on evolutionary algorithms to carry out multi-objective optimizations, in which the inflicted losses is maximized and the total number of used ammunition is minimized. Furthermore, the results of the optimizations can be extended to consider the utility of the Attacker and to consider the effects of imperfect information.

Based on the results the effective tactic for use of artillery can be obtained when the amount of available information varies. This information can be applied to support decision making and to carry out future studies, such as studies based on the concept of adversarial risk analysis.

Keywords: Data farming, computational combat models,
evolutionary algorithm, multi-objective optimization,
adversarial risk analysis

Aalto-yliopisto
Perustieteiden
korkeakoulu

KANDIDAATINTYÖN
TIIVISTELMÄ

Tekijä: Tuukka Stewen

Työn nimi: Lentotiedustelutietoon perustuva tykistön tulenkäytön optimointi

Päivämäärä: 10.10.2017 Kieli: Englanti Sivumäärä: 4+34

Koulutusohjelma: Teknillinen fysiikka ja matematiikka

Pääaine: Matematiikka ja systeemitieteet

Vastuuopettaja: Prof. Ahti Salo

Työn ohjaaja: DI Juho Roponen

Työn tavoitteena on tutkia optimaalista tykistön tulenkäyttöä hyökkääjälle, jolla on erilaisia tiedustelutiedon määriä. Tiedustelutieto esitetään todennäköisyyksinä, joilla hyökkääjä löytää puolustajan kohteet. Tutkimalla tykistötulen vaikutuksia hyökkääjä voi optimoida tykistön tulenkäytön tavoitteitaan vastaavasti. Optimointituloksia voidaan hyödyntää päätöksenteossa, kuten asejärjestelmien hankinnoissa, jotka edellyttävät merkittäviä panostuksia.

Työssä tutkitaan tykistön tulenkäyttöä hyödyntäen taistelumallinnusta, joka tuottaa tietoa dataa viljelemällä. Tämän tiedon analysointi tukee optimaalisen tulenkäytön määrittämistä. Analyysit perustuvat evoluutioalgoritmeilla ratkaistaviin monitavoitteisiin optimointeihin, joissa hyökkäyksen tuottamia tappioita maksimoidaan ja käytettyjen ammusten määrää minimoidaan. Saadut optimiratkaisut ovat yleistettävissä ottamalla huomioon myös hyökkääjälle olevan informaation määrä sekä hyöty, jonka hyökkääjä hyökkäyksestään saa.

Tulosten perusteella tykistön tehokas tulenkäyttötaktiikka voidaan määrittää erilaisille tiedustelutiedoille. Sitä voidaan myöhemmin käyttää päätöksenteossa ja erilaisissa tulevaisuuden tutkimuksissa, kuten vastakkainasettelullisessa riskianalyysissä.

Avainsanat: Datan viljely, taistelumallinnus,
evoluutioalgoritmi, monitavoitteinen optimointi,
vastakkainasettelullinen riskianalyysi

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1 Introduction

Artillery has played a major role in warfare ever since the beginning of World War I (Marshall, 1978). According to some estimates 50 to 80% of war casualties were caused by artillery in the 20th century (Bellamy and Zajtchuk, 1991). Even though artillery fire had been used for great effect in the previous wars, its usage had not been studied as extensively as in the recent decades (Lappi, 2012). The main reason for this was the lack of capability to efficiently and cost-effectively study the performance of different indirect weapon systems. However, in the past couple of decades computational models have improved, which has made it possible to study the effects of artillery fire more thoroughly. At the same time, these models have helped optimize the use of artillery and provide more information in support of decision making in the field of modern warfare.

Many recent military studies have focused on calculating and comparing the possible outcomes of different types of artillery. These calculations are often done by estimating the damages to the adversary by running different kinds of simulations on varying scenarios (see e.g. Kangaspunta et al., 2008). Even though most of these simulations involve a probabilistic approach, the studies often lack the aspect of uncertainty of information on the opposite units. In most of the previous studies, the adversaries usually have full knowledge on how the troops on the opposite side are located and what is the strength of these units. However, this is not the situation in reality, because usually adversaries try to limit available information (Vansén, 1938). Occasionally, the information can also be incorrect, which complicates matters even further. All in all, uncertainty of information is ever-present in warfare, which makes it an important aspect to include in the studies.

To our knowledge, there is no published research that takes into account the uncertainty of information of the opposite units when investigating the effects of artillery fire. Therefore, this thesis focuses on combining these aspects. The objective is to examine how the information gained by aerial reconnaissance can affect the optimal use of artillery. Furthermore, the attack can also be optimized from the cost-effectiveness point of view, which provides well-founded decision support. This can, for instance, benefit the acquisition of weapon systems, which require multi-million euro investments annually (see e.g. Dunne et al., 2002).

2 Background of the work

The complexity of combat situations has made systematic military studies increasingly important in the modern warfare. Even though the recent development of technology has made more sophisticated studies possible, the effects of war have been studied for centuries. Based on historical findings Sabin (2012) indicates that simulation has been used as a tool in war for over 2000 years. He reports how games were used to understand different strategies, but also to create new information from which new more complex strategies could be built. Since then, the technology and warfare have changed drastically, which has led to more complex decision making.

During the last 50 years, computer-aided simulations, simulators and other computational methods have become important for decision making, which has led to the development of various kinds of new combat models (see e.g. Lappi, 2012). The development of these models has further led to more informed and thorough decision support. Combat models have been used, for instance, in different types of weapon system portfolio analyses (see e.g. Puhakka, 2016, Kangaspunta et al., 2008) and to investigate historical events (see e.g. Lappi et al., 2015). Furthermore, the recent development has made even more sophisticated tactical analyses possible, because they provide information on some aspects and parameters that were earlier unknown.

One approach in the military studies enabled by computational combat analysis is adversarial risk analysis (ARA). ARA is a decision-theoretic approach to games. It builds on statistical risk analysis and game theory to provide appropriate methods for analyzing decision making situations involving one or more intelligent adversaries who make decisions with uncertain outcomes (see e.g. Rios Insua et al., 2009).

Traditional statistical risk analysis was developed to estimate and mitigate risks in contexts where the loss is governed by chance or nature. Usually traditional statistical risk analysis can be used, for making decisions in a variety of different contexts including finance, insurance, transport and health-care (Jensen, 2002). In addition to risks caused by chance events, ARA seeks to capture risks caused by the malicious or otherwise self-interested actions of one or more intelligent adversaries. Consequently, modelling the decision making behavior of these actors is central to ARA. The models of decision making behavior can be based, for instance, on game theory (Myerson, 2013) or on psychological discretions (Frederick, 2005).

However, using game theory as an approach for describing and predicting human behavior is not usually considered an ideal tool. Summarizing on a large empirical work Camerer (2003) and Gintis (2009) have criticized minimax solutions, in which each adversary seeks to minimize his expected losses across all the actions that are available to his opponents. Minimax and related solutions can lead to sub-optimal decisions, because in reality opponents rarely follow the minimax rationality principle. In addition, minimax solutions are often difficult to compute in real situations and often require strong and unreasonable assumptions about the common knowledge the adversaries share (see e.g. Kadane and Larkey, 1982). Furthermore, the solutions based on minimax may be excessively too pessimistic, because mitigating the worst conceivable scenario, which may in some cases have an extremely low probability, can cause the adversaries to make decisions that no human would realistically make.

ARA, however, does not collapse in this kind of reasoning, because in ARA the adversaries act towards maximizing their expected utility under some kind of subjective beliefs about the probabilities of the choices of their opponents. Much of the recent ARA literature has focused on counter terrorism and corporate competition. Rios and Rios Insua (2012), for instance, apply the concept of ARA to devise strategies for the allocation of resources against terrorist threats. In addition, Zhuang and Bier (2007) study the resource allocation between the protection of intentional attack and natural disasters.

ARA has many obvious uses in military organizations and many of the existing ARA approaches can be applied to support military decision making. For instance, ARA methods can be used to guide the allocation of resources between strategically important targets as well as the investment planning of military equipment and projects. Moreover, uses of ARA in finance and acquisition are also relevant, because military organizations acquire products and services from external contractors.

However, in the context of military combat modelling, ARA has not been used systematically. One of the studies that has used ARA in the context of military combat modelling was performed by Roponen and Salo (2015). In this study, the relevance of ARA to military combat modelling was discussed and an example in which ARA was combined with stochastic combat modelling was presented. This example demonstrated that with ARA approach, it is possible to generate information that can benefit decision making. Furthermore, the study showed that there is much potential in using the concept of ARA to tackle realistic problems in the context of stochastic combat modelling.

This thesis links to the undergoing research performed within a research project funded by the Finnish Scientific Advisory Board for Defence. The research extends ARA for determining efficient portfolios of countermeasures, which consist of (i) antiaircraft weapon system deployment and (ii) camouflage as alternative options for defending against aerial reconnaissance performed with unmanned aerial vehicles (UAVs). In addition, the study introduces the concepts of ranking intervals and dominance relations of portfolios in the context of ARA. Specifically, these concepts help determine which portfolios outperform others in view of all relevant uncertainties.

The illustrative case study of the research considers a scenario in which a supply company is threatened by UAV reconnaissance and possibly by artillery fire as well. This case combines the results from two different simulation models of which the first determines the success probabilities of the reconnaissance while the second calculates the losses due to the artillery fire in keeping with the information gained by the Attacker. In this case ARA is used (i) to characterize the input parameters needed in both simulators, (ii) to combine the computational results from them both, and (iii) to produce a tentative ranking of alternative portfolios of countermeasures.

This thesis focuses on studying the results of the second simulation model, which calculates the losses due to the artillery fire in keeping with the information gained by the Attacker. Based on the simulation results, it is possible to determine the optimal use of artillery. This can be further applied in the ARA concept to support decision making when the decisions of an intelligent adversary are also taken into account.

3 Research problem

The objective of this thesis is to investigate the effective use of artillery when only imperfect information of the opposite troops is available. This information is provided by the aerial reconnaissance carried out by UAVs. By studying the effects of the used artillery, it is possible to optimize the amount of used ammunition to inflict the most casualties for the Defender or to achieve some other objective. Naturally, the more shells or rockets are fired the more losses the Defender will suffer. However, ammunition are not free and may come in limited supply. Therefore, the situation is also examined from the perspective of cost-effectiveness.

3.1 Scenario

We examine a scenario, in which a supply company is attacked. The scenario consists of both the decisions of the Attacker and the Defender, but also of random events. An influence diagram of the scenario is shown in Figure 1. First the supply company deploys its troops in a specific area. Simultaneously, it chooses the anti-UAV portfolio, which consists of different methods that they use in order to defend against the reconnaissance. These methods can include air defense, camouflage or dummy equipment.

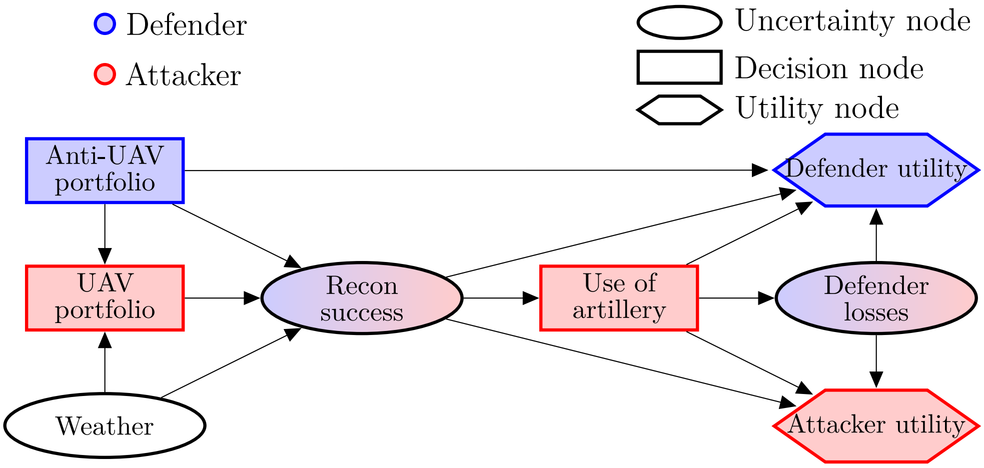

Figure 1: An influence diagram of the whole scenario.

After the Defender's choices, the attacking side chooses the UAV portfolio, which consists of the types and the number of UAVs that will be used. However, when the attacking side is making the decision, they have limited knowledge of the anti-UAV portfolio in the defending side. Therefore, the anti-UAV portfolio together with weather, as a random node, affect the UAVs they will choose. The weather mostly affects the visibility, so that it has to be taken into consideration when choosing the UAV portfolio, because some UAVs can perform only well when the visibility is good, whereas others might be suitable for more challenging conditions.

Depending on the choices made in choosing the anti-UAV and UAV portfolios alongside with the weather conditions, the Attacker will gain information about the Defender and its units. This information is recorded as recon(naissance) success and it consists of locations in which Defender's units or equipment have been identified. Then, the Attacker makes decisions regarding the use of artillery, taking into account the information gained by the UAV reconnaissance. The use of artillery can be modified by changing the number and type of ammunition and the locations in which the artillery is fired. The use of artillery fire then causes damage to the supply company.

The losses sustained by the supply company together with the reconnaissance success and use of artillery affect the utilities both the Attacker and the Defender experience. For the Attacker, the utility consists mostly of the damages done to the Defender. Still, during the combat, the usage of ammunition and possibility of losing UAVs have a negative effect on the utility. The Defender's utility, instead, increases if the Attacker has to use more ammunition or multiple UAVs in order to inflict losses to the Defender. However, the combat losses and the investments in the anti-UAV portfolio negatively affect the utility gained.

Although the whole scenario consists of both the decisions of the Defender and the decisions of the Attacker, the objective of this thesis is to examine how the decisions regarding the use of artillery affect the consequences to the Attacker and the Defender. Therefore, the focus is on the second decision node of the Attacker which specifies the use of artillery. By examining the parameters of the artillery fire affecting the Defender's losses, it is possible to determine the best attack in different situations based on the information gained from the UAV reconnaissance. Consequently, the Attacker and the Defender can make informed decisions when allocating the resources, with the aim of obtaining the most desired outcomes, respectively.

3.2 Formation of the supply company

In the scenario, the supply company has located its troops in the village of Tarttila. The company consist of 23 separate units, such as a loading group, a signal group and evacuation patrols. All the units have a fixed location and each include certain amount of infantry and other equipment. The equipment consists mostly of different types of vehicles. The detailed information about the strengths of the units and the number and type of equipment is shown in Table 7 of Appendix A. Figure 2 shows the fixed location of each unit on a map. Next to each unit is a number which identifies the units.

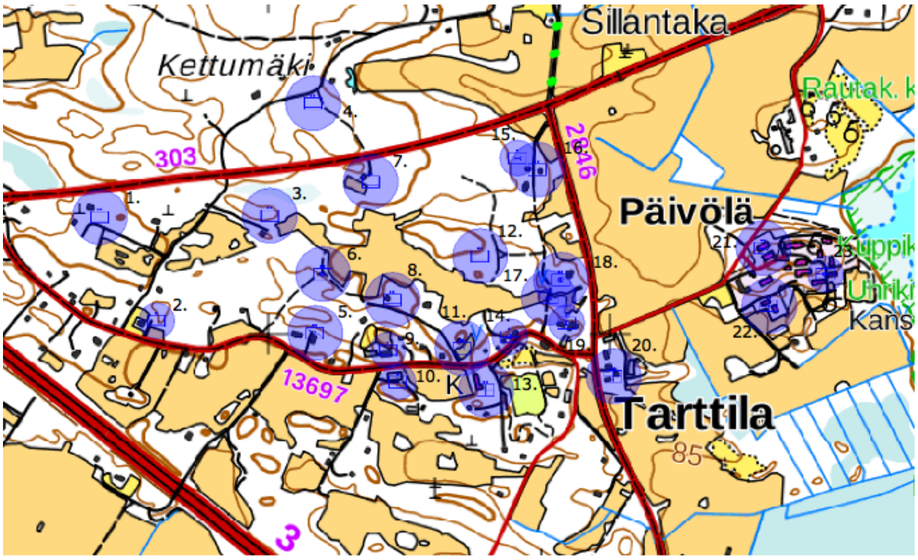

Figure 2: Positions of the supply company. Each unit is marked with a blue circle and next to each location is a number which identifies the unit in the location. Information on the units can be found in Table 7 in Appendix A. The background map is from the National Land Survey of Finland (2017).

4 Methods

Studying this scenario with varying information on Defender's units and with different parameters of artillery fire is quite complex. The most crucial part in this respect is choosing the aiming locations as well as the quantity and type of ammunition used for each of these locations. When examining the effects of artillery fire, many different aspects affect the losses inflicted. Because each calculation case is different, earlier examined scenarios can not be used in order to obtain reliable results. Therefore, different calculations must be performed separately and earlier calculations should not be relied on.

Often, one of the biggest challenges in decision making is the lack of data from each of the different situations, especially when outcomes depend on multiple variables. In order to fight this drawback, the data can be self-made. One of the widely used methods to "grow" data is the process called data farming (see e.g. Brandstein and Horne, 1998, Horne et al., 2014). With data farming, it is possible to generate a sufficient amount of data, which can then be further analyzed in order to optimize the use of artillery in the attack. This thesis uses operational analysis tool Sandis for data farming.

4.1 Sandis

Sandis is an operational analysis tool widely used for data farming in military studies in Finland (see e.g. Bruun et al., 2010, Åkesson et al., 2012). Since previous studies have proved it to be reliable in data farming processes, this thesis also uses Sandis for data farming (Lappi et al., 2015).

Sandis is a computer program for comparative combat analysis. With Sandis it is easy to simulate different scenarios, since the parameters of each unit and artillery can be easily adjusted (Lappi, 2008). In addition, the human operator is only responsible for the tactical decisions during the scenario building phase, but during the calculations no decisions need to be made (Bruun et al., 2010). These calculations provide many different results, but in this thesis only combat losses for each unit and unit element are considered.

The core of the calculations of Sandis are based on Markovian combat modelling and state machines (Kangas and Lappi, 2004). Thus, the result of the calculations is not just one outcome, but the probabilities of all the outcomes (Lappi, 2008). Based on these probabilities, Sandis calculates the expected value of the losses of each unit and unit element and further creates different types of charts and tables but most importantly a killer-victim scoreboard, which can be further analyzed.

The most vital feature in Sandis from our point of view is the model for indirect fire, because it allows to examine the use of artillery (Lappi, 2012). The model takes into account the qualities of ammunition, including mass, amount of fragments and amount of explosives, but also considers the firing distance and the angle, in which the artillery fire hits the ground (Lappi, 2008). Based on these parameters Sandis numerically calculates the probability of fragments to hit a particular target. Furthermore, the losses are calculated based on physical equations recommended by NATO (Lappi, 2008).

4.2 Data farming

Data farming is a process of using designed computational experiments to "grow" data. This data can then be analyzed to support decision making and answer questions which face a lot of complexity and uncertainty. In practice, data farming is performed using computational models to simulate a certain scenario with varying initial conditions to understand the possible outliers and trends as well as the distribution of the results (Horne and Meyer, 2010). In this thesis, the data farming process is performed using similar principles as those introduced by the NATO modelling and simulation task group (Horne et al., 2014).

Usually the data farming process starts with the development of the model which will be used to create the data. After this multiple simulations are performed using different sets of input variables. These simulations generate information of which the results are derived. However, since the computational model used to farm data has already been developed, the main focus of this thesis is designing of the data farming process and implementing it.

The design of the data farming with Sandis begins with specifying the base scenario that will be examined, whereas the creation of the base scenario starts with specifying the units of the supply company. The creation of the units is done using Sandis' troops editor. With the troops editor, it is possible to define the equipment and other weapons in the units according to the information shown in Table 7 in Appendix A. When the creation of the units is complete, each unit is located on a map using Sandis' scenario editor. The editor makes it possible to specify the exact location of each of the units and the radius in which the units have spread. For larger groups, the default radius is set to 83,3 m, while for commands and patrols the radius is 55,6 m. The specific location of each unit is shown in Figure 2.

When the creation of the base scenario is finished, the process of data farming can be further planned. The most crucial decisions are the choice of (i) the parameters of ammunition and (ii) the methods to investigate scenarios, including imperfect information in a way which is computationally possible. The challenge is that the time of the data farming process can become too long, if the number of parameters is varied too excessively. In order to perform the computation, the effects of imperfect information will be examined afterwards. This is possible, because Sandis calculates the combat losses of each unit separately. Therefore, the number of known units can be easily adjusted after the simulations have been performed. This makes it possible to examine a scenario with complete information of each of the units.

After that the parameters of artillery must be chosen. The first decision is to choose the ammunition types to use. In this thesis, the use of two different types of ammunition is examined, rockets with high-explosive (HE) fragmenting warheads and 155 mm artillery (HE) shells. The former is used by heavy multiple rocket launchers, whereas the latter is used in field artillery. In order to examine the effects of these two ammunition types, two different computation batches are executed. In batch I, the optimal use of heavy rocket launchers is examined and in batch II the optimal use of 155 mm artillery. The simulation parameters used for these weapon systems are shown in Table 1 and the information of the fragmentations of each ammunition is shown in Table 2. In Table 1 the explosive mass is the TNT equivalent, which is used to describe the blast effect in the explosion.

Table 1: Parameter values for the two artillery types.

	Artillery	
Parameters	Heavy rocket launcher	155 mm artillery
Explosive mass	26.8	5.36
Fragmentation velocity	1200	1200

Table 2: Parameter values for the fragmentations of each of the ammunitions.

	Fragmentations			
	Heavy rocket launcher		155 mm artillery	
Angle of fan	Avg mass	Amount	Avg mass	Amount
0°-5°	0.030	5	0.030	1
0°-10°	0.00163	24995	0.00163	4999
65°-115°	0.00163	133350	0.00163	26670
170°-180°	0.00163	8330	0.00163	1666

After the examined ammunition types are chosen, the dispersion of ammunition and the angle of fire hitting the ground are defined. For rockets with HE-fragmenting warheads the dispersion is set to a radius of 125 meters and the angle of fire to hit the ground to 75°. These values are based on a firing distance of about 90 kilometers (Mäkinen, 2006).

However, for the 155 mm artillery shell, the dispersion pattern of the ammunition is not a circle. For field artillery the longitude (d_o) and latitude (d_a) for the dispersion can be calculated with equations

$$\begin{aligned} d_o &= p_o \cdot d_f \\ d_a &= p_a \cdot d_f, \end{aligned}$$

where p_o is the standard deviation of ammunition from the aim point in the longitudinal direction with respect to the firing distance, p_a is the standard deviation of ammunition from the aim point in the latitudinal direction with respect to the firing distance and d_f is the firing distance. The value for the standard deviation to the longitudinal direction is 0,45% of the firing distance and to latitudinal direction 0,10% (Pääesikunta, 1984). Therefore, the values for the dispersion are 180 meters and 40 meters, when the firing distance, d_f, is set to 40 kilometers. When firing from this distance the angle of fire to hit a ground is set to 30°.

After the parameters of artillery are defined, the aiming locations of the artillery fire and the examined amounts of ammunition are specified. Based on the locations of the units, it is determined that ten different aiming locations of artillery fire is sufficient to examine this type of scenario. These locations are kept constant in both of the computation batches, which helps determine the differences in casualties inflicted by the two ammunition types. The base scenario, with the aiming locations, is shown in Figure 3. Next to each aiming location is a number, which identifies said aiming location.

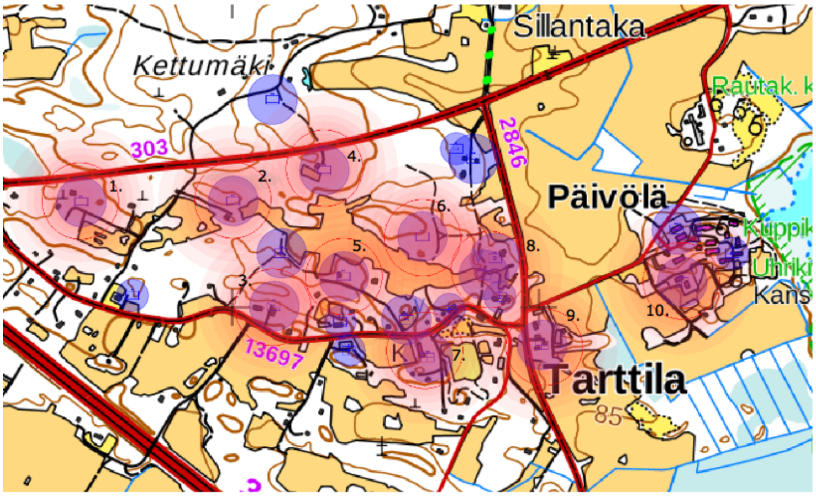

Figure 3: The aiming locations, shown as red circles, and the locations of units in the supply company. Next to each aiming location is a number, which identifies the aiming location. The circular dispersion patterns indicate that the use of heavy rocket launchers is examined. The background map is from the National Land Survey of Finland (2017).

After the locations are chosen, the ammunition amounts must be decided. By rapid tests, the sufficient amount of ammunition for the heavy rocket launchers is between zero and 25 with a interval of five, whereas for the 155 mm artillery it is from zero to 200 with a interval of 25. These magnitudes and intervals are chosen to obtain conclusive enough results to analyze the optimal use of artillery, but also so the two computation batches could be analyzed in a more reasonable time. Specifically, if no intervals are used, the number of executed simulations would be 26^{10} and 201^{10} without changes to other parameters. Calculating these would take millions of years. In addition, defining the number of ammunition so precisely is not necessary, because whole artillery units are normally used to fire at the same location. At the same time, the optimal uses of ammunition between the examined ammunition amounts could be determined by interpolating the obtained results.

Additionally, in batch II, where the effects of 155 mm artillery are investigated, also the optimal air burst height of the shell is studied. Therefore, in batch II this parameter value is changed between zero and ten meters, with a two meter interval. All the constant parameters and the changed variables are shown in Table 3.

Table 3: Parameter values in the two computation batches.

Parameter	Batch I	Batch II
Artillery type	Heavy rocket launcher	Field artillery
Ammunition type	HE-fragmenting warhead	155 mm HE-shell
Ammunition per location	0-25 (intvl of 5)	0-200 (intvl of 25)
Locations	10	10
Firing distance	90 km	40 km
Angle	75°	30°
Exploding height	0 m	0/2/4/6/8/10 m
Dispersion (long/lat)	125 m /125 m	180 m /40 m

4.2.1 Implementation of the data farming process

Based on the choices made in the parameters, the number of combinations in the batches are 6^{10} and $6 \cdot 9^{10}$. Because running one simulation takes about 20 seconds using one processor, the time of running only the combinations in batch I would take a little less than 40 years. With a high performance computer this time could be reduced to a couple of years, which is still too long.

Because going through all the combinations is impossible, the batches must be modified. One way to circumvent time limitations is to examine each unit separately. By carrying out twenty different simulations in which rockets or shells are only aimed in one of the ten locations, it is possible to specify which of the aiming locations affect which units. By examining the results of these simulations the number of required combinations in the batches can be decreased, because those aiming locations which have zero effect to the unit do not need to be examined. For instance, if a unit experiences damages from 4 different aiming locations, when the use of HE-fragmenting warheads is studied, the combinations to examine all the possible outcomes of this unit is 6^4, since the used ammunition in the other six aiming locations can be ignored.

This process was performed for all units using 10 rockets in batch I and 50 artillery shells in batch II. Since the killer-victim scoreboards of Sandis are based on expected values, the scoreboards contain multiple values close to zero. Including those aiming locations, that inflict such low losses for the units slows down the computation, because the number of combinations increases. However, these values are insignificant when examining the total losses suffered. Thus, they are assumed to be zero. The level when deciding, if the combat losses in a certain location are significant enough is calculated with equation

$$\alpha = 0,01 \cdot S_{loc}, \tag{1}$$

where α is the significance level and S_{loc} is the strength of the unit in this location. The strength of the unit is the number of infantry combined with the number of equipment.

The tables showing the effective aiming locations of each unit, determined with equation (1), are shown in Appendix B. The locations of batch I are shown in Table 8 and the locations of batch II in Table 9. The tables show that most of the combat losses to an unit are due to only three or four aiming locations. Therefore, the proposed approach significantly reduces the time required for the computations as the number of examined combinations can be decreased, without significantly affecting the accuracy. Furthermore, changing the height of the air burst in batch II made no difference on whether the damages to certain unit were significant. Therefore, the tables would be similar with all the examined heights.

Once all the effective aiming locations have been determined for both ammunition types, the next computations can be made. At this time all the combinations of artillery parameters for single units are examined according to the effective aiming locations, shown in Tables 8 and 9 of Appendix B.

4.3 Data analysis

After completing the data farming, the data must be analyzed. Because, there are 23 units in different locations, the process of data farming produces 23 different files for each of the computation batches which all focus on one specific unit and its combat losses for infantry and other equipment. Furthermore, one file includes all the possible combinations of changed parameters that have an effect on the examined unit. The data these files contain can be analyzed to obtain the optimal use of artillery.

4.3.1 Formulation of the optimization problem

The objective of this thesis is to examine how the available information of the opposite units can affect the number of identified firing locations and the optimal use of artillery. In this thesis, the Attacker's objective is to inflict the maximal losses for the Defender and simultaneously minimize the amount of used ammunition. Therefore, the problem can be considered as a multi-objective optimization problem. Once this problem has been solved, it is possible to define the Pareto optimal solutions, which are solutions that can not be improved with regard to any criterion without degrading at least one other criterion. Furthermore, it is possible to determine the utility of the attack based on these solutions.

When optimizing the artillery parameters a von Neumann-Morgenstern utility theorem is applied (Von Neumann and Morgenstern, 1944). This approach makes it possible to define the preferences of the Attacker by scaling the losses of each equipment according to their expected values. For instance, damaging a container that stores valuable electronics should be more desirable than destroying one off-road vehicle (ORV). Simultaneously, the values of expended ammunition need to be considered in order to include the perspective of cost-efficiency in the studies. The used values for each ammunition type are shown in Table 4 and the values for each unit element are shown in Table 5.

Table 4: The values used for the ammunition types.

A	Ammunition type	Value
1	HE-fragmenting warhead	400
2	155 mm HE-shell	10

Table 5: The values used to describe the utility of wounding infantry or damaging equipment.

j	Unit element	Value
1	Infantry	250
2	ORV	500
3	Truck	1500
4	Container	3000
5	Tractor	600
6	Van	700
7	Ambulance	1000

In this study, the calculations are executed in the expected utility context, because the data farming process produces numbers of expected losses. The expected losses of the Defender consist of the expected losses of each unit. Furthermore, the expected losses of each unit consist of the expected losses of the unit elements and the values of these elements. Therefore, the expected value of the Defender's losses $E[L_D]$ can be calculated using equation

$$E[L_D] = \sum_{n=1}^{23} \sum_{j=1}^{7} E[L_{n_j}] \cdot V_j, \tag{2}$$

where $E[L_{n_j}]$ is the expected losses of unit n element j and V_j is the value of element j.

Based on the optimal use of artillery, the expected utility of the Attacker can also be determined. In this thesis, the utility function of the Attacker is assumed linear in respect to the value of the inflicted losses and used ammunition. Therefore the expected utility of the attack $E[U]$ is

$$E[U] = E[L_D] - L_A, \tag{3}$$

where L_A is the cost of used ammunition. This can be calculated as

$$L_A = \sum_{l=1}^{10} A_l \cdot C_A,$$

where A_l is the amount of ammunition aimed towards location l and C_A is the value of one ammunition of type A.

To examine how the number of firing locations affects the value of inflicted losses and the utility of the attack, all the possible aiming combinations must be examined. The multi-objective optimization for these combinations is performed by maximizing the value of the inflicted losses with equation (2) while simultaneously trying to minimize the amount of ammunition. The amount of ammunition A_A is calculated by the equation

$$A_A = \sum_{l=1}^{10} A_l.$$

The decision variables in the optimization problems are the parameters varied in the simulations. In other words, the used ammunition for each aiming location and the height of the air burst when the use of 155 mm artillery shells, is examined. Because not all possible combinations of decision variables are simulated, the constraints in both optimization problems require that the decision variables must belong to the set of calculated cases. Therefore, in computation batch I the constraint condition is

$$A_l \in A,$$

where $A = \{0, 5, 10, 15, 20, 25\}$ and in computation batch II the constraint conditions are

$$\begin{aligned} A_l &\in B \\ h &\in H, \end{aligned}$$

where $B = \{0, 25, 50, 75, 100, 125, 150, 175, 200\}$, h is the height of the air burst and $H = \{0, 2, 4, 6, 8, 10\}$.

After the optimizations for all the possible combinations have been performed, it is possible to determine the utility of the attack with equation (3). In addition, the effect of the imperfect information can now be considered. In this thesis the available information of the opposite units is described as a probability of identifying the unit $P(n)$. When the imperfect information is taken into account, it is possible to determine the probability of the Attacker considering firing in a certain location $P(F_l)$. This probability is determined by calculating the chance that Defender's units closest to a specific aiming location are identified. The probability of the Attacker considering firing in certain location can now be determined with equation

$$P(F_l) = 1 - (1 - P(n))^q, \tag{4}$$

where q is the number of opposite units closest to the firing location. The value of q can be found in Tables 8 and 9 in Appendix B. Furthermore, with equation (4) it is possible to determine the probability of all the possible firing combinations.

All in all, the multi-objective optimization problem could have been performed using just one objective function and only maximizing the value of the expected losses. However, such an approach would not provide information on how the usage of ammunition affects the losses, because it generates only one solution, i.e., the most optimal solution, for which the values of the expected losses are the largest. Nevertheless, this drawback could have been managed by using the amount of used ammunition as an constraint condition, but still this would have been more time consuming than executing the multi-objective optimization.

4.3.2 Technical implementation of the optimization

The optimization of these constrained multi-objective optimization problems is carried out with Matlab R2017a using the optimization toolbox. The optimizations are performed using evolutionary algorithms and more precisely a genetic algorithm, because the losses are nonlinear and the problems which need to be solved are discontinuous due to the data farming process (see e.g. Deb, 2001, Deb et al., 2002).

In order to perform the optimization for all the possible aiming combinations the decision variables are combined in vector $\mathbf{x}$, the values of lost infantry and equipment in vector $\mathbf{v}$ and the expected losses of each unit in different calculation cases of one computation to 23 separate matrices which focus to one specific unit each. Because the used gamultiobj-function does not allow for integer constraints, a custom mutation function and a custom crossover function are created to generate only integer outputs. The general idea is to take an approach based on a continuous parameter space strategy and make it integer based on well-placed calls to the rounding functions (Mathworks, 2017).

Furthermore, the constraint conditions for the decision variables are implemented in the following way. In batch I the constraint conditions for the examined aiming locations are $0 \leq A_l < 5.5$ and in batch II the constraint conditions for the examined aiming locations are $0 \leq A_l < 8.5$ and for the air burst height are $0 \leq h < 5.5$. In both the batches the aiming locations that are not examined in the combinations are limited to zero by determining the lower and upper bound to be zero. These constraints guarantee that only the generated combinations are examined, because when an integer based approach is applied, and these integer values are multiplied with the used interval, only the simulated combinations are generated.

Based on the integer variables, the right amount of expected combat losses $\mathbf{c_n}$ for each of the units is chosen. Furthermore, the overall value of the expected combat losses to the Defender can now be calculated as

$$E[L_D] = \sum_{n=1}^{23} \mathbf{c_n} \mathbf{v^t}. \tag{5}$$

To maximize the expected losses to the Defender, the equation (5) is multiplied by -1, so that the Attacker's problem becomes a minimization problem. Therefore, the multi-objective optimization problem is

$$\min\ [-E[L_D], A_A].$$

5 Results

In batch I the effects of heavy rocket launcher HE-fragmenting warheads was examined. First the multi-objective optimization was performed for all the possible aiming combinations. After these optimizations, the results could be extrapolated for larger amounts of used ammunition, if necessary. Based on the results, the expected maximal utilities of the attack were calculated. Figure 4 shows how the average value of the expected maximal losses and the average of the expected maximal utilities corresponds to the total number of used ammunition and number of possible aiming locations.

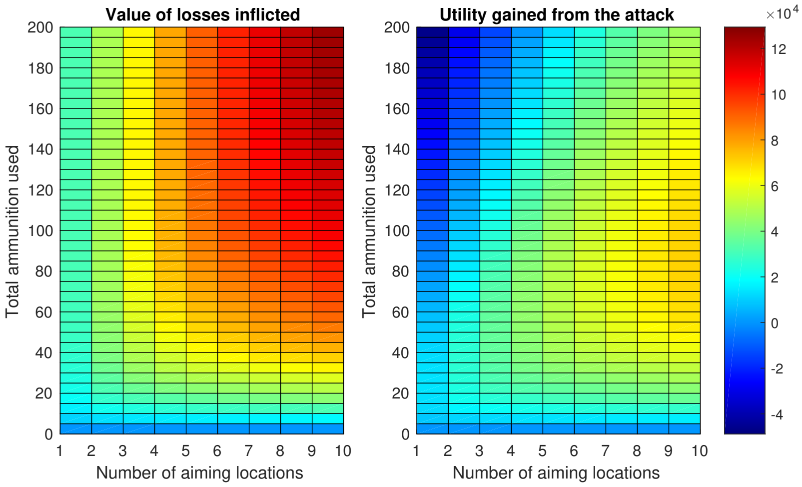

Figure 4: The average value of inflicted losses and the average of gained utility with respect to the number of aiming locations and total ammunition used. In this figure, the optimal use of heavy rocket launchers is examined.

From Figure 4, it can be seen that the possibility to fire at multiple aiming locations increases both the expected maximal value of inflicted losses and the expected maximal utility of the attack. However, the marginal benefit decreases when more ammunition is used, because, like the figure shows, the utility of the attack decreases and simultaneously the effects in the values of the inflicted losses are not as significant. Moreover, the marginal benefit of being able to fire in multiple aiming locations also decreases, when the number of possible locations increases.

Similar optimizations were also performed for the 155 mm artillery shells. Figure 5 reveals how the average of the expected maximal value of the losses and, furthermore, the average of the expected maximal utilities corresponds to the total number of used shells and number of aiming combinations. These results are obtained when the optimizations are performed for all the possible aiming combinations, after which the obtained results are extrapolated, if necessary.

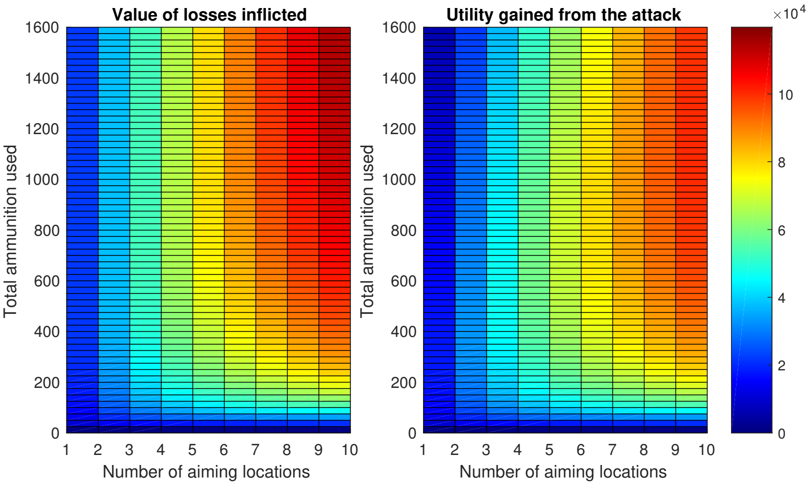

Figure 5: The average value of inflicted losses and the average of gained utility with respect to the number of aiming locations and total ammunition used. In this figure, the optimal use of 155 mm artillery shells is examined.

Figure 5 shows that the optimal use of 155 mm artillery shells exhibits similar behavior to the optimal use of heavy rocket launcher rockets. The main difference compared to the heavy rockets is that artillery shells have to be used more to obtain similar expected value of the losses, because the shells are less effective. Furthermore, contrary to the optimal use of rockets, the utility of the attack does not differ from the value of the inflicted losses as significantly as in Figure 4, because the value of using one artillery shell is significantly smaller. As a result, the graphs in the Figure 5 are almost identical.

In addition, in batch II the optimal air burst height was also considered. In Figure 6 the averages of the optimal air burst heights are shown with respect to the number of aiming locations and total ammunition used. From the figure, it can be inferred that when not many shells are used the optimal air burst height is lower than when more artillery shells are used. Mainly, it can be concluded that when the number of aiming locations or the total ammunition used increases, the optimal air burst height increases as well. However, when the total ammunition used is extremely high compared to the number of aiming locations, the optimal air burst height starts to decrease.

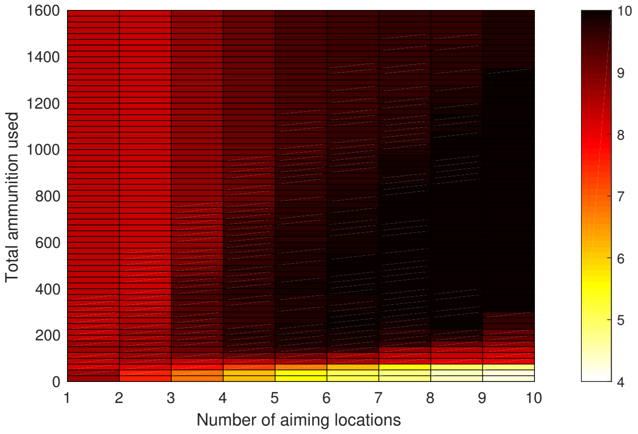

Figure 6: The average of the optimal air burst height when the number of aiming locations and the used total ammunition are considered.

5.1 Results for imperfect information

In Figures 4, 5 and 6, the results of the optimal uses of artillery were calculated when the probabilities for all the possible aiming combinations were the same. However, usually the Attacker does not have same probabilities for all the combinations. In general, this is because perfect information of the opposite units is not available. In addition, some aiming combinations are more desirable than others, because more units can be caught on fire when firing in a single location. As a result the probabilities for different aiming combinations vary. In this thesis, the imperfect information is considered as a probability of identifying the enemy units. Because some aiming locations have more units closer to them, the probabilities of the Attacker considering firing towards these are higher. The information on which units are closest to which aiming location is shown in Appendix B. Based on this information it is possible to determine the probability of all the aiming combinations if the probability of identifying a single unit is defined.

In this thesis, the probability of identifying a single enemy unit using aerial reconnaissance is switched between two values, 50% and 75%. These probabilities are examined since, according to Kosola and Solante (2013) and Lipsonen (2008), the probability of identifying the opposite units using aerial reconnaissance can be as low as 50%. In Table 6, the number of aiming locations is expressed with the overall probability of considering firing in a number of aiming locations when the probability of identifying opposite units is changed. In this table the probabilities for both the ammunition types are expressed. The table shows that the probability of identifying the opposite units affects the probable number of identified aiming locations significantly.

Table 6: The number of aiming locations together with the probability of firing in a number of aiming locations when the probability of identifying a unit is 50% or 75%. In the table probabilities for both ammunition types are examined.

	50%		75%	
Number of aiming locations	Rockets	Artillery Shells	Rockets	Artillery Shells
1	$6.0 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$7.2 \cdot 10^{-12}$	$3.0 \cdot 10^{-11}$
2	$1.2 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	$1.3 \cdot 10^{-9}$	$5.8 \cdot 10^{-9}$
3	0.0014	0.0028	$1.1 \cdot 10^{-7}$	$5.4 \cdot 10^{-7}$
4	0.010	0.018	$5.6 \cdot 10^{-6}$	$2.7 \cdot 10^{-5}$
5	0.047	0.076	$1.6 \cdot 10^{-4}$	$7.4 \cdot 10^{-4}$
6	0.14	0.19	0.0027	0.014
7	0.26	0.29	0.027	0.068
8	0.30	0.26	0.15	0.24
9	0.19	0.13	0.41	0.41
10	0.050	0.026	0.41	0.28

Because the different aiming combinations have different probabilities, the chance of obtaining specific value of losses varies. Furthermore, some Pareto optimal solutions are more probable than others. Based on the probabilities of each combination, it is possible to determine the expected Pareto optimal solutions regarding the number of aiming locations and further the expected

Pareto optimal solutions overall. In Figure 7 the expected Pareto optimal solutions are shown regarding the number of aiming locations when the value of inflicted losses is maximized and the number of fired heavy rockets is minimized.

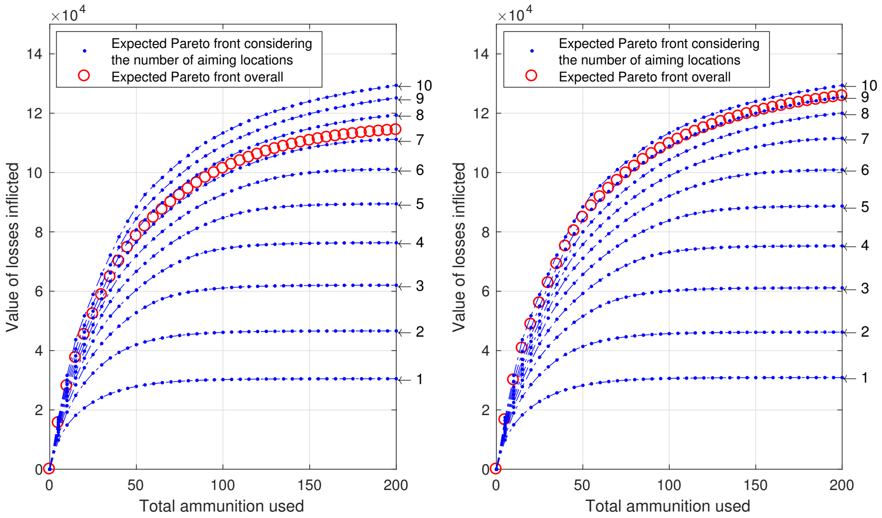

Figure 7: The expected Pareto front for varying number of aiming locations when the probability of identifying units is 50% (left) or 75% (right). The value of expected losses is maximized while simultaneously minimizing the use of heavy rockets.

Figure 7 shows how the expected Pareto front of the value of losses corresponds to the number of aiming locations. In the left pane the probability of identifying the opposite units is 50 % and in the right pane 75 %. From the figure it can be concluded that when the probability of identifying the opposite units increases from 50% to 75%, the expected maximal values of losses inflicted also increase. This is caused by the fact that it is probable that more units are identified. This further increases the number of possible aiming locations.

Naturally, the possibility to aim at multiple aiming locations increases the value of maximal losses that can be obtained by any total ammunition used. Similarly, the more ammunition used, the more losses can be obtained regardless of the number of aiming locations. However, the marginal benefit of being able to fire at more locations or to fire multiple rockets decreases when the number of aiming locations or the number of used rockets increases. Based on the expected maximal values of the inflicted losses the Pareto front for the maximal utility can be further obtained, when imperfect information is considered. These results are shown in Figure 8.

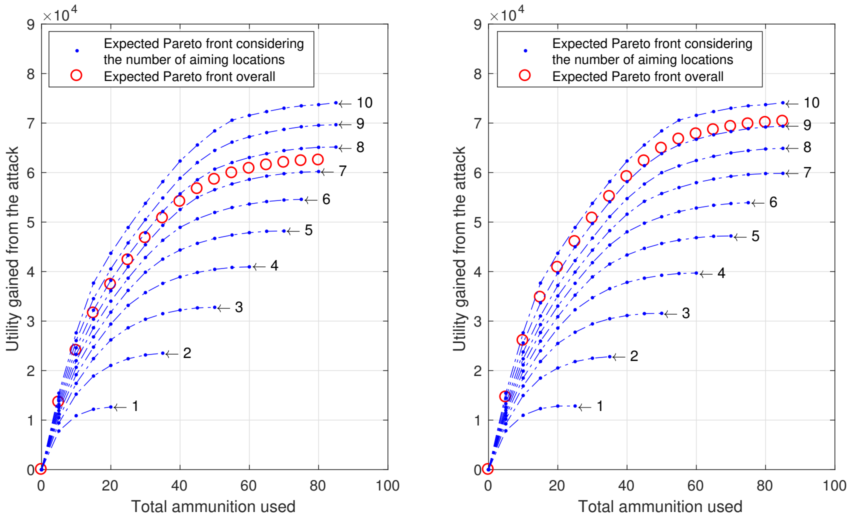

Figure 8: The expected Pareto front for varying number of aiming locations when the probability of identifying units is 50% (left) or 75% (right). The expected utility is maximized while simultaneously minimizing the use of heavy rockets.

Figure 8 suggests that the use of ammunition should be limited in order to achieve the highest expected utility. At the same time it shows that when the amount of possible aiming locations increases, it is in the Attacker's best interest to use slightly more ammunition than when fewer aiming locations are considered. However, when almost all the possible aiming locations are identified, it is not necessary to use more ammunition. In other words, identifying more aiming locations can benefit the resource allocation. Furthermore, one can conclude that expected maximal utilities for different numbers of aiming locations are very similar regardless of the probability of identifying the opposite units.

After examining the use of heavy rockets, the Pareto optimal solutions for the 155 mm artillery shells are also examined. These optimizations are performed similarly as to the heavy rockets. The expected Pareto optimal solutions are determined when the value of inflicted losses is maximized and the number of used shells is minimized. The Pareto optimal solutions when the value of inflicted losses is maximized is shown in Figure 9 and based on these results the Pareto front of the expected utilities is determined. These results are shown in Figure 10. In Figures 9 and 10, the probabilities of identifying the opposite units are the same as when examining the effects of heavy rocket launcher HE-fragmenting warheads.

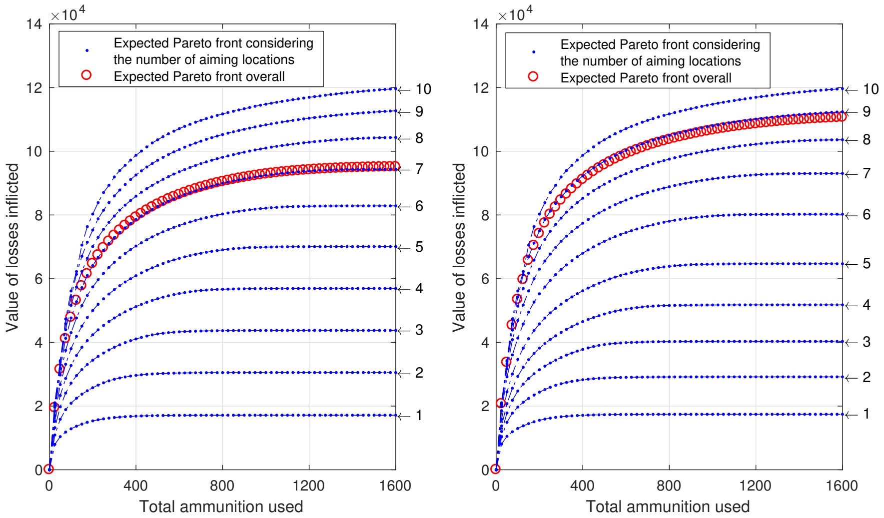

Figure 9: The expected Pareto front for varying number of aiming locations when the probability of identifying units is 50% (left) or 75% (right). The value of expected losses is maximized while simultaneously minimizing the use of 155 mm artillery shells.

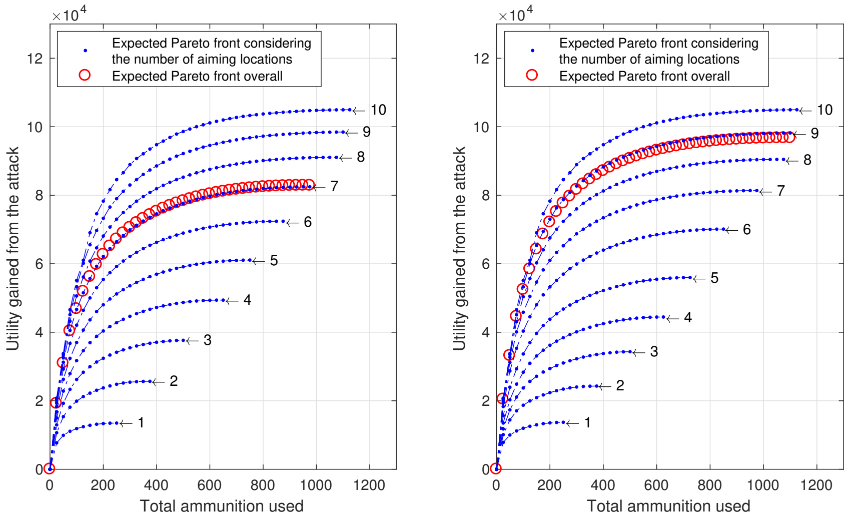

Figure 10: The expected Pareto front for varying number of aiming locations when the probability of identifying units is 50% (left) or 75% (right). The expected utility is maximized while simultaneously minimizing the use of 155 mm artillery shells.

Figures 9 and 10 show that the optimal use of 155 mm artillery shells exhibits similar behavior as the optimal use of heavy rockets. The difference when examining the Pareto optimal solutions of the 155 mm artillery shells is that because the artillery shells are less effective than the heavy rockets, the total amount needed to make similar damages is greater than that of heavy rockets. At the same time, artillery shells are significantly cheaper, which further leads to a higher total ammunition used. As a result, the Pareto fronts of 155 mm artillery shells are not as steep when comparing them to the Pareto fronts of heavy rockets.

5.2 Combining the results

The results of the optimizations are reasonable and provide information, which can be greatly applied by the adversaries. The most obvious information the results provide is how the optimal usage of the two ammunition types varies with different objectives and with varying information and resources available. However, the results can also be exploited in the resource allocation in combat situations and also different tactical decisions can be made when the expected maximal values of losses or the maximal utilities of the possible attacks are examined. The information the results provide is considerably beneficial for the Attacker, but also the Defender can benefit from it, because the information makes it possible to determine the probable attack and further to prepare for the attack.

The optimal usage of the two ammunition types differs significantly. The results show that the optimal uses of artillery shells are reached when a much greater number of shells are fired compared to the uses of heavy rockets. Furthermore, the use of heavy rockets must be more precise than the use of artillery shells. This is quite intuitive, because heavy rockets are considered much more expensive than artillery shells. At the same time, they are also far more effective. However, artillery shells can still generate significant casualties when the amount of used ammunition is large. Based on this information, even slightly excessive usage of artillery shells can be encouraged, whereas the use of heavy rockets should be considered carefully.

The results show that the number of possible aiming locations significantly increases the expected value of the losses inflicted and further the Attacker's expected utility. This suggests that the Defender should try to decrease the number of possible aiming locations by limiting the information available. On the contrary, the Attacker should pursue to maximize the number of possible aiming locations by identifying as many enemy units as possible.

These objectives can be obtained, by allocating more resources to the anti-UAV portfolio or UAV portfolio, respectively. However, the adversaries need to asses if allocating more resources is beneficial, because also the possibility of losing these resources in the combat situations must be considered.

Given that the Defender's resources are usually limited, it is not possible to limit all the available information. In this situation the Defender can pursue to maximize the Attacker's resource consumption. This objective can be fulfilled by locating units far from others. This increases the Attacker's ammunition consumption, because less units are caught by the artillery fire of single firing locations. Nevertheless, if units are forced to locate close to another, the Defender's usage of counter-UAV should be increased in this area to avoid massive losses. Because if some of the units are identified, it is more probable that enemy fires in this area. This can cause casualties for all the units in the impact area.

All in all, both the Attacker and the Defender can benefit from the information the results provide when making tactical decisions. However, an informed decision maker also considers how the information affects the behavior of the opposite side. Therefore, when making tactical decisions also the decisions of the intelligent adversary should be considered. Such an analysis results in a loop where both adversaries try to be one step ahead of the other. In this setting, adversarial risk analysis can be readily applied.

6 Conclusion

In this thesis, the effects of artillery and the effect different parameters of artillery have on the possible consequences were explored by using a computational combat model. By examining the optimal use of artillery it is possible to gain information that can be beneficial in decision making. Based on the information gained, the adversaries can make informed decisions to achieve the most desirable outcomes. The Attacker can decide the type of artillery used and the amount of ammunition expended in each of the aiming locations. Furthermore, the Attacker can choose whether it is profitable to invest in a more expensive UAV portfolio. Similarly, the Defender can make informed decisions about, for instance, the optimal use of camouflage or other counter-UAV.

The numerical results of this thesis do not directly apply in other scenarios due to the complexity of battle situations and because this thesis focuses on examining a specific scenario. Still the methods used can be applied elsewhere in other similar studies. Furthermore, this thesis demonstrated that computational combat analysis, and especially the used approach, can be beneficial in tactical decision making. However, one must consider that the adversaries do not probably share the same beliefs or they can behave maliciously or in other ways self-interestedly. Therefore, especially adversarial risk analysis, in which this thesis links to, can be readily applied in the future studies.

In the future also other approaches could be applied to study the optimal use of artillery. One possibility is to use the methods of this thesis in a different sequence. The optimization of the best possible use of artillery could be initialized by running the genetic optimization on Matlab. This script would then generate a set of possible solutions of which the results could be simulated using, for instance, Sandis. These results would then be imported back to Matlab, to generate new possible solutions based on the previous results, which could be re-examined. This loop could be executed until no better solutions are found. This approach could be faster, because not all possible combinations need to be examined. However, it is possible that this would still take plenty of time and the performance of a normal computer could be insufficient for such an approach.

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A Information on each unit

Table 7: Information on the size of the infantry and the number of equipment of each unit. This information loosely follows the actual strength of a supply company of Finnish military (Kosonen, 2012).

Nro	Name of the unit	Type	Amount
1	Canteen Supply Platoon / Canteen Supply Group	Infantry Truck Container	9 2 2
2	Canteen Supply Platoon / Command	Infantry Off-road vehicle	4 2
3	Canteen Supply Platoon / Supply Group	Infantry Truck Container	9 2 2
4	1. Supply Platoon / Rations Supply Group	Infantry Container Truck	9 2 2
5	Canteen Supply Platoon / Signal Group	Infantry Off-road vehicle	9 2
6	Transport Platoon / Loading Group	Infantry Container Tractor	9 3 3
7	Canteen Supply Platoon / Rations Group	Infantry Container	12 4
8	1. Supply Platoon / 1. Ammo Supply Group	Infantry Container Truck Tractor	9 2 2 1
9	1. Supply Platoon / Command	Infantry Off-road vehicle	4 2
10	Medical Platoon / Command	Infantry Off-road vehicle	4 2
11	Maintenance Platoon / Weapon Maintenance Group	Infantry Off-road vehicle Van	9 2 1
12	1. Supply Platoon / Water and petroleum group	Infantry Truck	9 4
13	Medical Platoon / 1. Medical Group	Infantry Container Off-road vehicle	8 2 1
14	Medical Platoon / Evacuation Group / 2. Evacuation Patrol	Ambulance	3
15	Maintenance Platoon / Command	Infantry Off-road vehicle	2 2
16	Maintenance Platoon / Electronic Maintenance Group	Infantry Off-road vehicle Van	9 2 1
17	Maintenance Platoon / 1. Vehicle Maintenance Group	Infantry Off-road vehicle Van	9 2 1
18	Transport Platoon / 1. Transport Group	Infantry Truck Container	9 3 3

Nro	Name of the unit	Type	Amount
19	Transport Platoon / Command	Infantry Off-road vehicle	4 2
20	Medical Platoon / Surgery Group	Infantry Container Off-road vehicle	5 2 1
21	Command	Infantry Off-road vehicle	5 2
22	Medical Platoon / 2. Medical Group	Infantry Container Off-road vehicle	8 2 1
23	Medical Platoon / Evacuation Group / Evacuation Patrol	Ambulance	3

B Information on the firing locations

Table 8: Each unit with information whether or not a specific aiming location causes damages to the unit. "X" defines which of the aiming location is the closest to the unit. The used ammunition is heavy rocket launcher HE-fragmenting warhead.

		Aiming locations									
Nro	Name of the unit	1	2	3	4	5	6	7	8	9	10
1	Canteen Supply Platoon / Canteen Supply Group	X									
2	Canteen Supply Platoon / Command	X									
3	Canteen Supply Platoon / Supply Group		X	x	x	x					
4	1. Supply Platoon / Rations Supply Group		x		X						
5	Canteen Supply Platoon / Signal Group		x	X		x					
6	Transport Platoon / Loading Group		x	X	x	x					
7	Canteen Supply Platoon / Rations Group		x		X	x	x				
8	1. Supply Platoon / 1. Ammo Supply Group			x	x	X	x	x			
9	1. Supply Platoon / Command			x		X	x	x			
10	Medical Platoon / Command			x		x		X			
11	Maintenance Platoon / Weapon Maintenance Group					x	x	X	x		
12	1. Supply Platoon / Water and petroleum group				x	x	X	x	x		
13	Medical Platoon / 1. Medical Group					x	x	X	x	x	
14	Medical Platoon / Evacuation Group / 2. Evacuation Patrol						x	X	x	x	
15	Maintenance Platoon / Command						X		x		
16	Maintenance Platoon / Electronic Maintenance Group						X		x		
17	Maintenance Platoon / 1. Vehicle Maintenance Group						x	x	X	x	
18	Transport Platoon / 1. Transport Group						x	x	X	x	
19	Transport Platoon / Command						x	x	X	x	
20	Medical Platoon / Surgery Group							x	x	X	
21	Command										X
22	Medical Platoon / 2. Medical Group										X
23	Medical Platoon / Evacuation Group / Evacuation Patrol										X

Table 9: Each unit with information whether or not a specific aiming location causes damages to the unit. “X” defines which of the aiming location is the closest to the unit. The used ammunition is 155 mm artillery shell.

		Aiming locations									
Nro	Name of the unit	1	2	3	4	5	6	7	8	9	10
1	Canteen Supply Platoon / Canteen Supply Group	X	x								
2	Canteen Supply Platoon / Command			X		x					
3	Canteen Supply Platoon / Supply Group	x	X		x						
4	1. Supply Platoon / Rations Supply Group										
5	Canteen Supply Platoon / Signal Group			X		x					
6	Transport Platoon / Loading Group		x	x		X	x				
7	Canteen Supply Platoon / Rations Group		x		X						
8	1. Supply Platoon / 1. Ammo Supply Group			x		X	x		x		
9	1. Supply Platoon / Command			X		x		x			
10	Medical Platoon / Command			x				X		x	
11	Maintenance Platoon / Weapon Maintenance Group			x		x		X	x		
12	1. Supply Platoon / Water and petroleum group		x			x	X		x		
13	Medical Platoon / 1. Medical Group							X		x	
14	Medical Platoon / Evacuation Group / 2. Evacuation Patrol			x		x		X	x		
15	Maintenance Platoon / Command				X						
16	Maintenance Platoon / Electronic Maintenance Group				X						
17	Maintenance Platoon / 1. Vehicle Maintenance Group					x	x		X		x
18	Transport Platoon / 1. Transport Group					x	x		X		x
19	Transport Platoon / Command					x			X		x
20	Medical Platoon / Surgery Group							x		X	
21	Command										X
22	Medical Platoon / 2. Medical Group								x		X
23	Medical Platoon / Evacuation Group / Evacuation Patrol								x		X