Aalto University
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Simulating artillery fire in forest environment

Master's thesis
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Abstract of master's thesis

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Abstract

Indirect fire provided by artillery and mortars is one of the most powerful weapons available to Finnish Army. Effective use of indirect fire requires as accurate as possible information about its effects on different targets. Military modeling is one possible way of obtaining information that can be used to support decision making. Large percentage of Finland is covered in forest. Forest also affects significantly on the effectiveness of indirect fire. At the moment Finnish Defense Forces do not have a simulation model that could accurately estimate the effect that the forest covering has. The purpose of this thesis is to produce a mathematical model that can estimate the height distribution of air bursts when indirect fire is used against a target that is inside forest. When the probability distribution of airburst locations is known, it can be used to improve the accuracy of the indirect fire model of the operations analysis tool Sandis.

This thesis presents a physics based mathematical model that can be used to estimate the probability distribution of air burst locations in different forest environments. Also presented is how the parameters required by the model can be derived from publicly available data offered by Metla. Because the forest data covers whole Finland, it is easy to use the model for calculating the effects of artillery fire in any known location within the country. However, the mathematical model itself is not depending on the forest data offered by Metla. Thus, it can be extended to handle different types of forest data or entirely different types of forests or jungles.

To validate the mathematical model a test program was created. It was used to calculate damage caused by artillery and mortar strikes to prone soldier targets in a typical Finnish forest environment. The results were then compared to field test data found in literature. The testing revealed that the model's results seem similar to those produced by artillery field tests. The model also produces more accurate results than simply ignoring the forest cover. The benefits of using the model were greatest when the angle of fall of artillery shells was low. On very low angles of fall the difference in casualties sustained by the soldier targets was as much as 50% higher when the forest cover was taken into account. The model presented in this thesis seems to work as intended, and it can be used to significantly improve the accuracy of damage estimations of indirect fire in forest environment.

Keywords military modelling, indirect fire, artillery, mortars, forest, damage estimation

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Tiivistelmä

Tykistön ja kranaatinheittimistön epäsuora tuli on voimakkaimpia Suomen maavoimien käytössä olevia aseita. Epäsuoran tulen tehokas käyttö vaatii mahdollisimman tarkkaa informaatiota tulen tehosta erilaisia kohteita vastaan. Taistelumallinnus on yksi tapa saada tietoa päätöksenteon tueksi. Suuri osa Suomen pinta-alasta on metsän peitossa. Metsällä on myös merkittävä vaikutus epäsuoran tulen tehoon. Tällä hetkellä Puolustusvoimilla ei kuitenkaan ole käytössään taistelumallia, joka pystyisi huomioimaan puuston vaikutuksen epäsuoraan tuleen. Tämän työn tarkoituksena on kehittää matemaattinen malli, jolla ennustaa tykistön ja heittimistön kranaattien räjähdyskorkeuksia metsämaastossa. Kun räjähdyskorkeuksien jakauma on tunnettu, voidaan sitä käyttää parantamaan operaatioanalyysityökalu Sandiksen epäsuoran tulen vaikutuslaskennan tarkuutta Suomalaisissa metsäolosuhteissa.

Työssä esitellään fysikaalinen matemaattinen malli, jolla voidaan estimoida kranaattien räjähdyskorkeuksien jakaumaa erilaisissa metsissä. Työssä myös esitellään kuinka metsäkohtainen laskentaan voidaan suorittaa käyttäen ainoastaan parametreja, jotka ovat julkisesti saatavilla Metlan metsädatatietokannasta tai suoraan johdettavissa sieltä löytyvistä parametreista. Koska metsätietokanta kattaa koko Suomen, on mallia mahdollista käyttää helposti tykistön tulen vaikutuksen laskentaan millä tahansa etukäteen tiedossa olevalla alueella Suomen alueella. Itse matemaattinen malli ei kuitenkaan ole mitenkään sidottu metsätietokannan parametreihin, joten se on myös helposti sovellettavissa myös käytettäväksi tilanteissa, joissa saatavilla on erimuotoista metsädataa tai tutkittavana on jopa täysin suomalaisista metsistä eroava metsä tai viidakko.

Matemaattisen mallin validointia varten tuotettiin testiohjelma, jolla laskettiin, millaista vahinkoa tykistö- tai heitinisku tekee jalkaväkimaaleihin erilaisissa Suomelle tyypillisissä metsissä. Laskennan tuloksia verrattiin kirjallisista lähteistä löytyviin koeammunan tuloksiin. Nämä testilaskennat osoittivat, että mallin tuottamat tulokset näyttävät vastaavan koeammuntojen tuloksia. Malli myös tuottaa tarkempia tuloksia kuin puuston vaikutuksen jättäminen kokonaan huomioimatta. Kaikkein suurimmat erot syntyivät matalilla ammuksen tulokulmilla, jolloin maaliin koituvat tappiot olivat jopa puolitoistakertaiset, kun puuston vaikutus otettiin huomioon. Täten malli näyttää toimivan, ja sillä voidaan saavuttaa huomattavia parannuksia epäsuoran tulen vaikutuslaskennan tarkkuuteen metsämaastossa.

Avainsanat taistelumallinnus, epäsuora tuli, tykistö, kranaatinheittimistö, metsä, vaikutuslaskenta

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Finally, I would like to thank my family and my mother especially for all their support during this project and throughout the years.

Contents

Chapter 1

Introduction

Artillery has played a major role in land warfare in almost every major war since World War II [33] and also in many smaller conflicts. [42] While its effectiveness is diminished against guerrilla type warfare, it still maintains a great importance in larger battles. The biggest problem in using artillery is not finding suitable targets, but the fact that there are too many good targets. [5] Finnish Army had a total of 678 field guns, field howitzers, rocket artillery and heavy mortars in 2012, which is a large number by European standards. [16] According to estimates, in the conflicts of 20th century 50 to 80 percent of casualties were caused by artillery. [6] Field artillery thus remains one of the most powerful tools at Finnish Army's disposal in stopping a large scale invasion. Should there be a large scale invasion, it is highly unlikely that the opposing force would not also be using field artillery. [48]

Indirect fire is defined by NSA [38] as "Fire delivered at a target which cannot be seen by the aimer." Weapon systems that are most commonly used deliver indirect fire include artillery weapons such as howitzers, mortars, field guns and rocket artillery. In this thesis we focus mainly on cannon type artillery that fires projectiles that do not include propulsion systems of their own, although some of the results might also be applicable to rocket artillery. A projectile is an object projected by an applied exterior force and continuing in motion by virtue of its own inertia. Projectiles fired by artillery are also often called shells while those fired by mortars are often called rounds. The primary projectile of any cannon type artillery weapon system is the HE (High Explosive) projectile that is designed to inflict casualties through fragmentation or damage through impact with the target. In addition to HE projectiles there are different types of carrier projectiles that are designed disperse some sort of payload on the target. [9]

Every type of projectile apart from a solid shot has a fuze that is designed to cause the projectile function as wanted. Impact fuzes function when the projectile strikes an object. They can be further divided into SQ (Superquick) fuzes, that will act right after the point of the shell is crushed by an impact, graze fuzes, that will act even after a glancing or grazing impact, and delay fuzes that will act only after some time has passed after the impact. Time fuzes act after a pre-set time of flight and are used to achieve an air burst or to expel contents of a carrier projectile at a point along the trajectory. Proximity fuzes are designed to function near the target before hitting it. [9]

Using artillery effectively requires that there exists accurate information on the effectiveness of the artillery fire. [25] An accurate estimate of the effectiveness of artillery fire helps to answer questions such as: “How much ammunition is required to achieve, with a certain probability, the desired effect on the target?”, “Where should artillery be deployed?”, “Against which targets should artillery be used?”, “Which weapon systems perform best or most cost efficiently?”, “What kind of risks for collateral damage are there?”, and “How should artillery be modelled in training simulations and exercises?” To answer questions like these new mathematical methods and simulation models are developed and a large number of field tests are conducted to estimate effectiveness of new and older weapon systems and personnel.

Large portion of Finland’s land surface is covered by forest, but at the moment Finnish Defence Forces do not have at their disposal a simulation model that can accurately estimate forest's effect on indirect fire. A literature review suggests that there has been hardly any research into taking foliage into account when modelling artillery fire, or at the very least, it has not been published or is very difficult to find. This thesis's purpose is to develop a mathematical model that is capable of estimating locations of air bursts in forest terrain. This model will make it possible to expand Finnish Defence Research Agency's indirect fire simulation model with a new feature.

1.1 Estimating the effectiveness of artillery fire

The simplest method for estimating the effect of artillery fire is using data tables or graphs. They can be found field manuals and comparable publications. [10, 8] The tables list the amount of ordnance required for a desired effect under certain fixed conditions. Often these tables are based on data gained from field tests instead of elaborate mathematical models. [18] These types of tables are not very trustworthy or useful outside the specific situation they were made for. The tables would require adjustment for example when the intended target takes measures to protect itself. [15] Tables are useful when making rough estimates about how much ammunition should be used against a target, but are not of much use beyond that, which is why more accurate information from more complicated models is usually desired to support decisions.

Another way to estimate the impact of artillery units on a battle is using Lanchester's equations [26] and other deterministic combat models. [See for example [7]] Methods based on differential equations, while being computationally relatively simple, are also limited in their uses. [13] They can be used with some success, for example, in modelling a duel between two artillery forces. While they do have their uses, deterministic combat models are not helpful should one want to choose the best target for the artillery or find out the best ammunition to use in a certain situation. An even bigger problem is that deterministic combat models usually fail to describe the different possible scenarios that can take place during a battle. They could be said describe the expected average result of combat, but reality rarely matches the expected average. On a very large scale different random events start to even each other out, but that is not the case when examining a single artillery battery for example. [19, 4]

One commonly used deterministic method is estimating artillery's effect on target by using area of effect estimations for ammunition. [2] For example, the amount of ammunition required for the desired effect on the target is usually estimated using the formula

$$P_t = 1 - e^{\frac{-(A_E}{A_T} ntp)}, \tag{1.1}$$

where P_t is loss percent during time t, A_E the area of effect of the projectile, A_T the area occupied by the target, n the number of shots fired per time unit, t the time spent firing and p the probability of a single projectile hitting the target area. [25]

The third approach to estimating the effectiveness of artillery fire is the use of probability distribution based methods. These models can be divided roughly to two different categories: Methods that utilize Monte Carlo simulation to estimate the possible distributions of the interesting variables, and analytical methods that handle the randomness of the combat situation by representing variables as probability distributions. [19] For examining the effect artillery fire has on the target the analytical methods can represent unit strengths as a Markov process or a renewal process. [4, 28] Another possibility is that the method represents the damage done by artillery strike as a probability distribution. [18] Probability distribution based models usually rely heavily on computer models and simulation, because the interesting situations that actually require use of combat modelling are most of the time so complicated that finding analytical solutions is impractical or impossible.

Different types of stochastic combat models can be used to estimate the effectiveness of artillery fire. The randomness involved in impact locations of projectiles fired by both mortars and field artillery is well known and the basic equations remain nearly the same across different weapon systems. [2, 10] As a result, there are very few differences between different combat models with regard to calculating impact locations of shells in an open terrain. The biggest differences between combat models can be found in how they estimate the damage caused by artillery shells.

Most artillery models still use very simple methods for damage estimation such as mass of shells per area tables, a cookie cutter model, or exponential decay or test based tables. [28] Models using tables to calculate damage face almost all the same problems as estimating the effectiveness of artillery fire based only on these tables. Models that use cookie cutter or exponential decay functions assume that damage to targets depends only on distance between the impact location and the target. [24] This makes most of these models difficult to use in different environments, because damage caused by high explosive shells varies greatly based on terrain. [33] It is practically impossible to create separate equations for different terrain types, because the problem cannot be fixed with a simple terrain coefficient. [30]

Finnish Defence Forces Technical Research Centre has done research on calculating damage caused by artillery shells based on a physical model that takes into account how shell fragments spread out from the impact location and what kind of damage can they inflict on the target. [12, 27] The downside to this type of approach is that while it produces the most accurate results [28] the calculations require much more computing power.

1.2 Artillery fire in forest terrain

Using artillery fire effectively in forests and jungles is not easy. Using heavy artillery is especially difficult because moving and supporting heavy artillery is practically impossible without roads, enemy can attack the artillery positions more easily because forest provides the attackers cover. [36] Possibly the biggest challenge that heavy artillery faces is that they often fire using lower angles than mortars, which means that the projectiles are more likely to hit trees and when they do they are much further away from the target than with higher firing angles. For these reasons mortars are often seen as superior choice in heavy forest and jungle area.[40] However even when using mortars there still remains the challenge of accurately locating a target hiding in the forest.

Forest environment also provides its own challenges for calculating the effectiveness of artillery fire. Artillery shells with different types of fuzes also work quite differently in forests. “In dense jungle or forest, proximity fuzes detonate too early and have little effect. Impact fuzes achieve air bursts in dense forests, and delay fuzes allow rounds to penetrate beneath the heavy canopy before exploding.” [11] That means that explosions will happen on varying heights depending on the specific forest the target is taking cover in. Field test have shown that when firing in a forest about 40% of mortar rounds will explode when hitting trees, when firing into an average strong Finnish forest. [20] That cannot easily be represented using fixed damage equations or tables. One way to derive relatively good estimates is to calculate the damage assuming that a fixed percentage of shells will reach the ground and calculate the rest as air bursts at some fixed height. In reality, however, the height of the air bursts caused by shells hitting trees is not fixed, and the percentage of shells reaching ground depends on the shells’ angle of fall.

The forest environment alters the effectiveness of artillery shells significantly compared to open terrain. It is possible to use graze or time delayed impact fuzes fired at high angles to make a maximal percentage of shells to reach the ground level through the canopy if the target is better fortified. If the shells are fired in low angles without taking thick forest or jungle into account, they will likely explode too far from the intended target to cause any real damage. [40] On the other hand the increased chance of hitting the trees using low angles can also be used as an advantage to achieve even more air bursts. [8] Air bursts from HE-shells are 2-10 times more effective than surface explosions against personnel depending on terrain and other factors. [14, 30, 37] Rough terrain favors air bursts even more than flat terrain, which means that in a typical Finnish forest terrain, they should have a very significant impact, but data from actual field tests in forest terrain is reported in very few publications.[14, 20, 2]

Most artillery models handle forest terrain by just adding a forest coefficient to area of effect of the shells [25], multiply the ammunition consumption for desired effect to take forest into account, or just ignore terrain altogether. For example, in the US Army Field Manual 7-90 [11] ammunition consumption is estimated about 2.5 times greater against a platoon-size target for desired effect when it is in a dense forest compared to open terrain. It is also possible that some other methods for handling forest terrain exist, but cannot be easily found in public sources. It is known that the US Military has its own classified data tables about the effectiveness of surface to surface weapons [11], and it is likely that other military organizations have similar classified data at their disposal also.

The shell fragmentation model developed by Defence Force Technical Research Centre makes it possible to take into account different possible burst heights and how that affects different types of targets. In some earlier research conducted by Defence Force Technical Research Centre, forest terrain has been simulated by making part of the shells explode before hitting ground. [For example [30]] The physical shell fragmentation model has already been proved to be very effective for damage calculations in situations

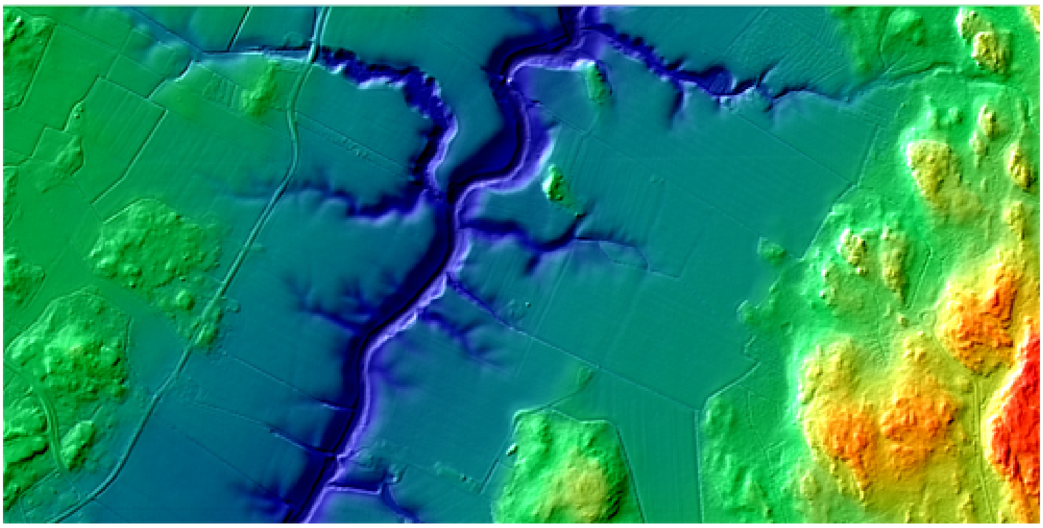

Figure 1.1: A Picture produced from the laser scanning data provided by National Land Survey of Finland. The data gained from laser scanning is so accurate that even small details can be seen.[31]

in which terrain elevations are taken into account. [30]

Because the aim of this thesis is to provide improvements upon existing combat models when the target is in a forest environment and computing power is getting cheaper day by day, Defence Force Technical Research Centre's shell fragmentation model was deemed the best choice for calculating the damage. Using the same model that was used for calculations with the elevation model as basis for this thesis leaves the possibility for future research to combine the forest model represented in this thesis with a model that can handle terrain elevations. The model in described in more detail in section 2.1.

1.3 Digital forest data

Combat modelling in a forest environment is of special interest in Finland because most of the country is covered in forests. This also means that eco-

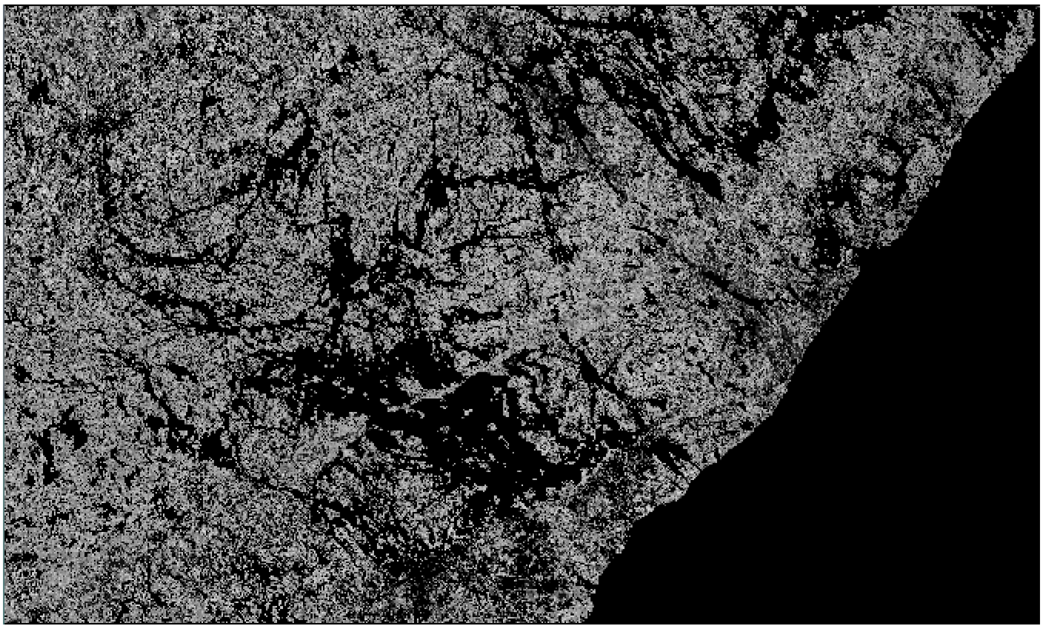

Figure 1.2: Map showing the average height of trees in Saimaa area

nomic significance of those very same forests is also high, which means that there is plenty of data available about Finnish forests that might not be so easily available elsewhere. National Land Survey of Finland (Maanmittauslaitos) has laser scanning data for a part of Finland available [32] and the whole country should be scanned by year 2019. The data gained from laser scanning is so accurate that even smallest details on the ground can be seen. [31] NLS uses the data to produce a new national elevation model, which in itself can prove to be very useful for calculating the effectiveness of indirect fire.

More interesting from the point of view of this thesis is what Finnish Forest Centre (Metsäkeskus) and Finnish Forest Research Institute (Metla) are using the laser scanning data for. They are using the data to gather information about Finnish forests.[35] The information is used to update the Multi-source National Forest Inventory (MS-NFI) that has been gathered from field measurements, remote sensed data and other digital data such as land-use maps, elevation models, and satellite images. This data contains information on how many trees there are in the forests, their average heights and widths, volume, and biomass. The geometric resolution of the resulted maps is 25m. [34] Figure 1.2 shows an example of such a map. The area shown is the utm200 map sheet M5 from southern part of Saimaa. Such information is invaluable when trying to form accurate estimates on probabilities of projectiles hitting trees. It is to be expected that the data available will only get more accurate when the laser scanning has been finished and incorporated to the MS-NFI database.

Chapter 2

The Indirect Fire Model

2.1 The physical model for fragmenting ammunition

The physical model for fragmenting ammunition described here was developed by Defence Technical Research Centre. [12, 29] It is currently in use in the Sandis military analysis tool [27] that was developed by Defence Force Technical Research Centre. It is currently used, owned and maintained by Finnish Defence Research Agency. All information presented in this section is from the earlier work by Lappi et al. [29] unless mentioned otherwise.

In the indirect fire model used in Sandis, both the locations of targets and projectile impact points have probability distributions. While both distributions are continuous in theory, the actual numeric calculations are discrete. To limit the computing load the target unit is currently divided into 7 different calculation points as shown in figure 2.1 with the centre point having larger weight than others. Kill probability for each calculation point is then calculated from the probability of an artillery shell impact location and probability of a shell fragment hitting the target from that location as seen

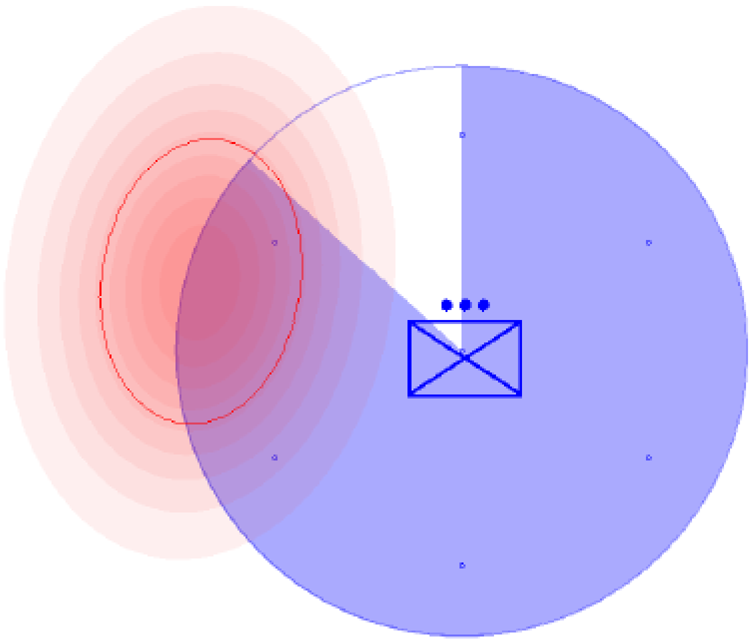

Figure 2.1: A Sandis screen shot from a simple example with a platoon being targeted, calculation points set visible. [29]

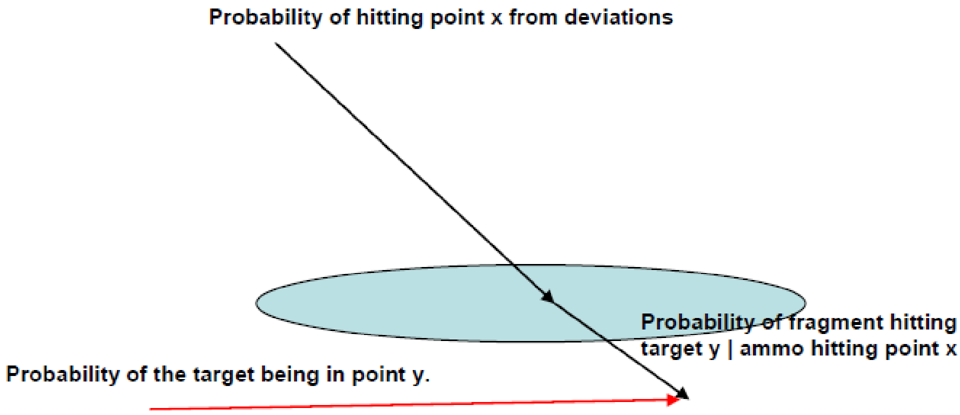

Figure 2.2: The basics of hit probability calculation. [29]

in figure 2.2. The probability of a target in certain calculation point being killed by fragments from a single projectile is

$$P_{kill} = \iint\limits_{A} P_{impact}(x, y) \cdot P_{kill|impact}(x, y)\ dxdy, \tag{2.1}$$

where $P_{impact}(x, y)$ is the probability that the projectile lands at (x, y) and $P_{kill|impact}(x, y)$ is the probability that the target is killed if a projectile lands at (x, y). [30]

The impact points of projectiles are generally assumed to follow a bivariate normal distribution around the aim point. The variances for different firing distances and weapon systems are well known and can usually be found in firing tables. Calculating the probabilities for different impact points is thus quite straight forward. What sets the physical model for fragmenting ammunition apart from other indirect fire models is, as the name suggests, how it handles damage caused by fragmenting ammunition.

The perforation capability of a fragment is according to Rilbe's formula

$$g = qvm^{\frac{1}{3}} \tag{2.2}$$

where q is a coefficient that depends on materials of the target and the fragment, v the fragment's velocity, and m its mass.

The fragment is slowed by drag. Its velocity at distance s is

$$v(s) = (v_0 + v_2)e^{\frac{-1}{c_1}(\frac{m_{ref}}{m})^{1/3}s} - v_2, \tag{2.3}$$

where v_0 is the initial velocity, v_2 and c_1 are constants describing the deceleration ($c_1 = 17,51m,\ v_2 = 17m/s$ by default) and m_{ref} is the mass of the reference particle ($m_{ref} = 0.4g$).

The mass distribution for a naturally fragmenting shell follows Mott's distri-

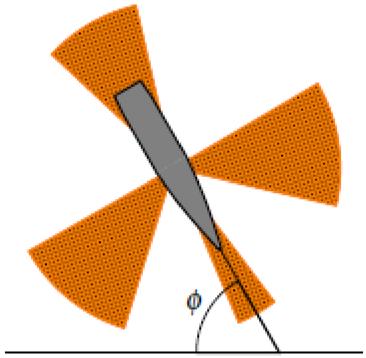

Figure 2.3: A schematic of the fragment fans of an exploding shell. The angle of fall is denoted by ϕ. [30]

bution [12]

$$N_m = N_0 e^{\sqrt{\frac{2m}{m_{avg}}}}, \tag{2.4}$$

where N_m is the number of fragments with a mass of at least m, N_0 the total number of fragments, and m_{avg} the average mass of the fragments.

By combining (2.2), (2.3), and (2.4) we can then calculate the largest effective range for shells' fragments

$$s = c \left(\frac{m_{ref}}{m_{max}} \right)^{\frac{1}{3}} ln \left(\frac{v_0 + v_2}{\frac{g}{q m_{max}^{\frac{1}{3}}} + v_2} \right) \tag{2.5}$$

and the number of effective fragments at certain distance from impact point. More detailed information is given in the article Lappi et al. [29].

HE-projectiles are designed to fragment in specific ways. Figure 2.3 shows an example of a typical fragmentation pattern. Fragmentation arena tests provide experimental data on fragmentation patterns, number of fragments, and masses of fragments. When the fragmentation pattern is known so that the probabilities for different flight directions of fragments are also known, the kill probability from (2.1) can be calculated using (2.2), (2.3), and (2.4). The Sandis military analysis tool does this by using adaptive integration over

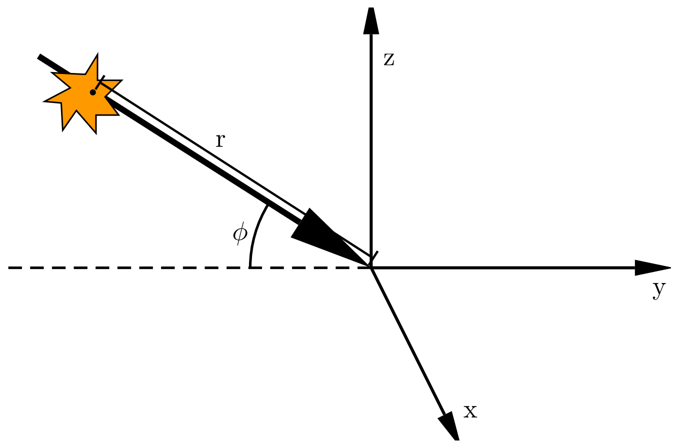

Figure 2.4: Location of an air burst. The big arrow represents the trajectory of the projectile. The origin of xyz-coordinates is the point where the trajectory intersects ground level. The distance of air burst location from origin is denoted by r. The angle of fall is denoted by ϕ.

the impact point. Other methods like for example Monte Carlo could also be used.

2.2 The hit point probability in three dimensions

No matter which model is used for calculating the damage caused by a high explosive shell, we need to know where the shell will explode. We have chosen to represent the projectile's possible impact locations with a probability assigned for each this location. This way the model can be most readily utilized with existing models for calculating the effect of the high explosive shells.

We first need to calculate the probability that a shell's trajectory intersects with the point. For simplicity's sake, we will assume that shells' velocities are so high that their trajectories are straight lines near the impact point. On flat terrain the impact points are generally assumed to follow a bivariate normal distribution around the aim point. [2] We can thus get the probability that the shell's trajectory intersects certain point as seen in figure 2.4.

$$P(\text{“Itersects point (x,y,z)”}) = P(\text{“Would hit point (0,0)”}), \text{ where} \tag{2.6}$$

$$x = 0, \quad y = -\cos(\phi)r, \quad z = \sin(\phi)r, \tag{2.7}$$

where α is the angle of fall of the projectile as seen in figure 2.4.

Now we need to calculate the probability that the projectile explodes in that exact point of its trajectory. To make the calculations simpler we will do the calculations only using r and ϕ as seen in figure 2.4

If we were trying to calculate the location of the explosion in an environment where we know what exactly will cause the shell to explode, it would be simple to calculate the exact location of explosion on each trajectory. Yet, in forest environment we usually do not have information on the location of every tree; and even if we did, we most definitely will not have information on where each and every branch is. To address this problem we adapt a probabilistic approach. The probability of a shell exploding at a specific point $\hat{r}$ of its trajectory in a forest environment can be represented as follows:

$$\begin{aligned} P(\hat{r} = r_0) &= \frac{P(\text{“Hits a branch at } r_0\text{”} \vee \text{“Hits a trunk at } r_0\text{”})}{P(\text{“Has not hit anything before } r_0\text{”})} \\ &= \frac{P(\text{“Hits a branch at } r_0\text{”} \vee \text{“Hits a trunk at } r_0\text{”})}{1 - \int_{r_0}^{\infty} P(\text{“Hits a branch at } r\text{”} \vee \text{“Hits a trunk at } r\text{”})\, \mathrm{d}r} \end{aligned} \tag{2.8}$$

By “hitting a trunk or a branch” we mean in this context a hit that will cause the shell to explode. Thus the probabilities will differ based on the type of fuze used in the projectiles. A graze fuze most likely will not cause a shell to explode just from hitting a branch while a SQ (superquick) fuze or other sensitive impact fuze might. The exact methods for evaluating the probabilities of hitting tree trunks or branches are further discussed in subsection 2.3 starting on page 17.

Because the exact placement of trees and the dispersion of projectiles' trajectories are totally independent, we can calculate the probability of a projectile exploding at point (x,y,z) simply by multiplying the associated probabilities. We thus get

$$P(\text{“Explodes (x,y,z)”}) = P(\text{“Would hit point (0,0)”})P(\text{“Explodes at distance r”}), \quad (2.9)$$

where $x = 0, \quad y = -\cos(\phi)r, \quad z = \sin(\phi)r.$

2.3 Estimating the probability of hitting a tree with an artillery shell

2.3.1 Hitting a tree trunk

We will start by estimating the probability of hitting a tree with an artillery shell by examining a shell moving in a straight line in a forest parallel to the ground. By "hitting a tree" we once again mean a hit that causes the shell to explode. [9] The ground is assumed to be flat. Trees are all assumed to have the same diameter that is equal to the average diameter for the trees in the area and trees' placement is assumed random. These assumptions mean that the probability of hitting a tree remains constant over distance moved. That means that probability of hitting a tree follows exponential distribution.

$$P(\text{“Hitting a tree”}) = 1 - e^{-\lambda_t d}, \quad (2.10)$$

where d is the distance moved. This just leaves us with the problem of estimating λ_t.

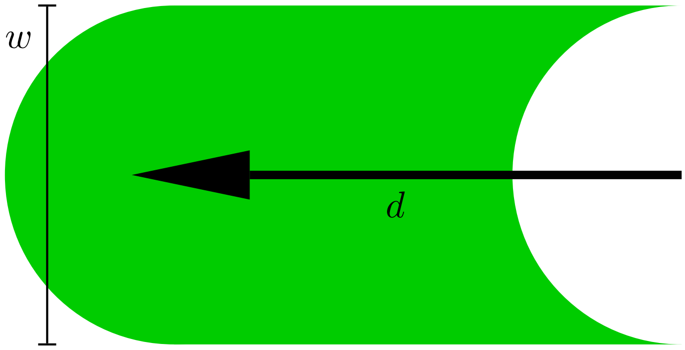

Figure 2.5: The green area shows the area in which the center point of a tree with width w must be in so that a shell moving a distance d on the trajectory shown by the black arrow will hit it.

If an artillery shell has a graze fuze or another nonsensitive impact fuze, it will take a direct hit to a tree trunk to set it off. That means that there must be a tree trunk in an area shown by figure 2.5. The size of the area is $A = wd$, where w is the width of the tree and d distance moved by the shell. The expected number of trees inside that area is

$$E(\text{“Number of trees in A”}) = nwd, \tag{2.11}$$

where n is the number of trees per square metre. If we choose $d = 1$ meter, we can get a good approximation for λ_t.

$$\lambda_t \approx nw, \tag{2.12}$$

which when combined with (2.10) gives us

$$P(\text{“Hitting a tree”}) = 1 - e^{-nwd} \tag{2.13}$$

as a result the probability density function for the shell exploding is

$$f(d) = nwe^{-nwd}. \tag{2.14}$$

The calculations get more complicated when the projectile moves in three dimensions instead of parallel to the ground. We get a workable estimate for most numerical calculations by using (2.13) and (2.14), but it cannot be used in all situations. Let us assume that tree trunks are shaped like cones. That means that the width of the trees varies as the shell falls down. The width can be represented as

$$w(r) = \begin{cases} w_0 \frac{h - \sin(\phi) r}{h}, & \text{if } r \leq \frac{h}{\sin(\phi)} \\ 0, & \text{if } r > \frac{h}{\sin(\phi)}, \end{cases} \tag{2.15}$$

where h is the height of the trees in the area, w_0 width of the tree trunk at the base, and r is the distance from the origin and ϕ the angle of fall as shown by figure 2.4. If we substitute in $s = \frac{h}{sin(\phi)} - r$, we get

$$w(s) = \frac{\sin(\phi) w_0}{h} s, \quad \text{when } 0 \leq s < \frac{h}{\sin(\phi)}. \tag{2.16}$$

s can be interpreted as the distance moved below the tree tops as seen in figure 2.6.

When the width of the tree is not treated as a constant but depends on s, the probability density function no longer follows exponential distribution (2.14). The generalized exponential distribution, known as Weibull distribution, is used in failure analysis to describe processes where the failure rate does not remain constant but is instead proportional to a power of time.[45] The special case where the failure rate increases proportionally to time produces a distribution also known as Rayleigh distribution. Our case is otherwise identical, but instead of time and failure rate we are examining the distance moved by the projectile and hitting trees.

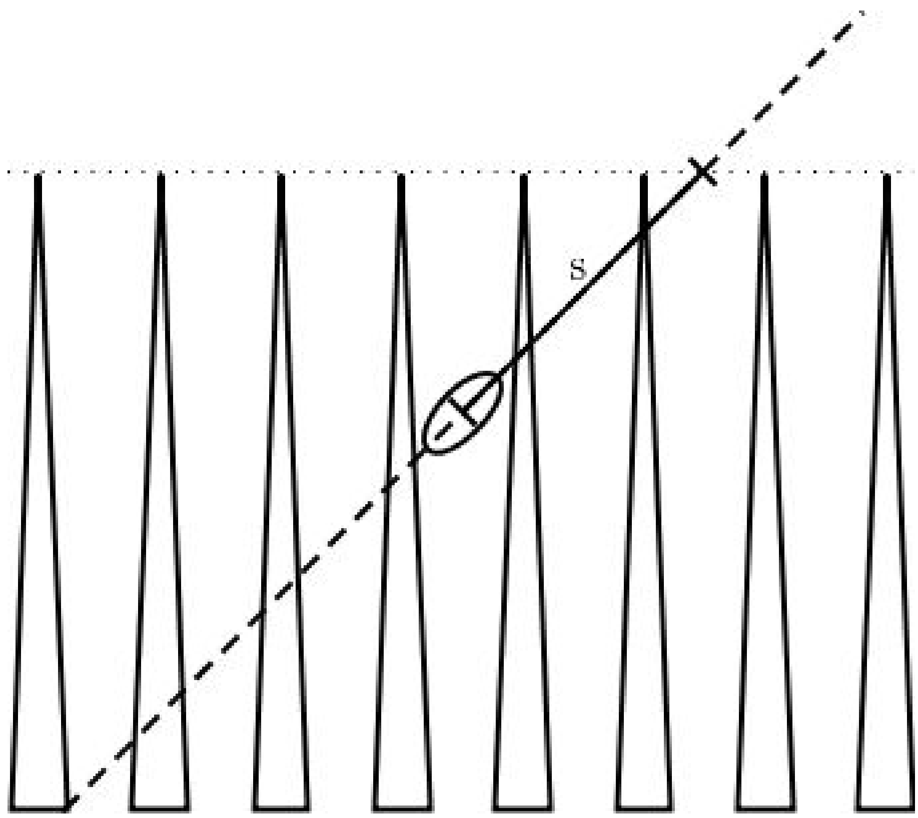

Figure 2.6: The distance the projectile has moved below tree tops is denoted by s.

Rayleigh distribution has the probability density function

$$f_R(x;\sigma) = \frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}, \quad x \geq 0, \tag{2.17}$$

and the cumulative distribution function

$$F_R(x;\sigma) = 1 - e^{-\frac{x^2}{2\sigma^2}}, \quad x \geq 0. \tag{2.18}$$

From (2.13) and (2.16) we get parameter $\sigma = \sqrt{\frac{\tan(\phi)h}{nw_0}}$, which means that the probability density function becomes

$$f_{ts}(s) = \begin{cases} 0, & \text{if } s < 0 \\ \frac{nw_0 s}{\tan(\phi)h}e^{-\frac{nw_0}{2\tan(\phi)h}s^2}, & \text{if } 0 \leq s < \frac{h}{\sin(\phi)}, 0 \end{cases} \tag{2.19}$$

and the cumulative distribution function

$$F_{ts}(s) = \begin{cases} 0, & \text{if } s < 0 \\ 1 - e^{-\frac{nw_0}{2\tan(\phi)h}s^2}, & \text{if } 0 \leq s < \frac{h}{\sin(\phi)} \\ 1, & \text{if } s \geq \frac{h}{\sin(\phi)}, \end{cases} \tag{2.20}$$

because the projectile will explode with probability $p = 1$ when it reaches ground.

When we substitute $r = \frac{h}{sin(\phi)} - s$ into (2.19) and (2.22), we get

$$f_t(r) = \begin{cases} \frac{nw_0}{\tan(\phi)h}(\frac{h}{\sin(\phi)} - r)e^{-\frac{nw_0}{2\tan(\phi)h}(\frac{h}{\sin(\phi)}-r)^2}, & \text{if } 0 < r \leq \frac{h}{\sin(\phi)} \\ 0, & \text{if } r > \frac{h}{\sin(\phi)}, \end{cases} \tag{2.21}$$

and the cumulative distribution function becomes

$$F_t(r) = \begin{cases} 0, & \text{if } r < 0 \\ e^{-\frac{nw_0}{2tan(\phi)h}(\frac{h}{\sin(\phi)}-r)^2}, & \text{if } 0 \leq r < \frac{h}{\sin(\phi)} \\ 1, & \text{if } r \geq \frac{h}{\sin(\phi)}. \end{cases} \tag{2.22}$$

It should be noted that because there is practically always a chance for the

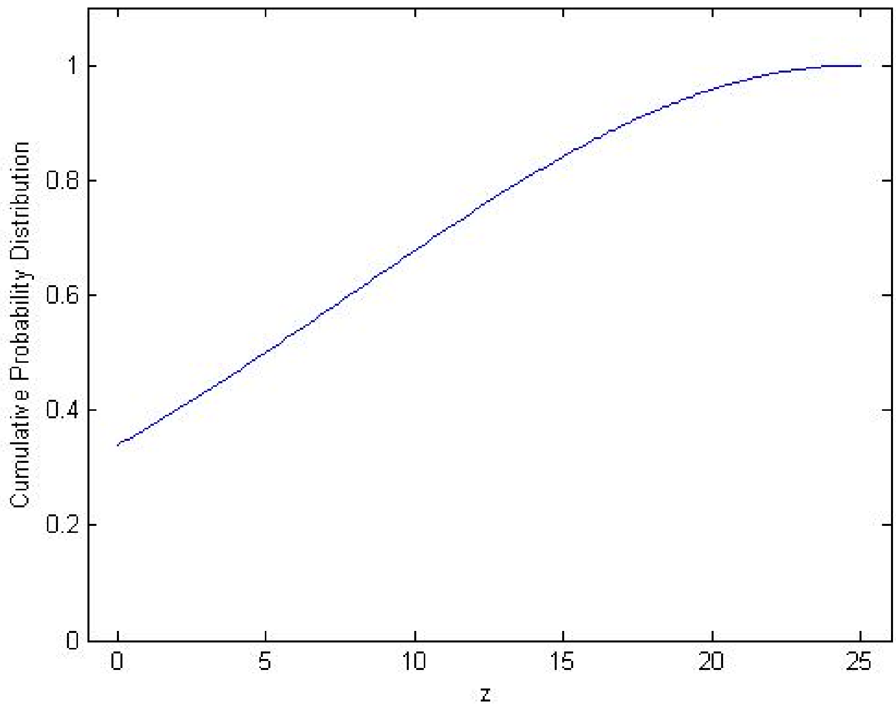

Figure 2.7: The cumulative probability distribution function $F_t(r)$ in a relatively thick forest with $\phi = 30°$.

projectile to reach the ground, $F_t(r)$ has a step at the ground level as seen in figure 2.7.

2.3.2 Hitting a tree branch

If the shell has a superquick fuze instead of a graze fuze, the situation will become more complicated, because hitting a large enough branch will likely cause the shell to explode. The difficulty comes mostly from estimating the amount of large enough branches, and not from the mathematical formulae for hit probabilities themselves. If we make similar assumptions about tree branches as we did about tree trunks for (2.10), that is that all the branches are identical and that their placement is random, we will reach the conclusion that the probability to hit a branch only varies based on the distance travelled. That means that it is also exponentially distributed

$$P(\text{“Hitting a branch”}) = 1 - e^{-\lambda_b d}, \tag{2.23}$$

and again we are faced with the problem of estimating λ_b.

At best we get a rough estimate for λ_b, because in reality trees and their branches differ from each other and the actual probability of the fuze triggering when hitting a branch depends on the specific fuze used and the physical properties of the branch. The statistical forest data from Metla [34] includes estimates for living branches' biomass of pine, spruce and deciduous trees, so we will calculate λ_b based on that.

It makes sense to assume that the frequency of hitting a branch correlates directly with the number of branches. If we also assume that the number of branches correlates with the biomass of the branches we get from the hit probability of shell moving distance $d = 1$ meter

$$\lambda_b \approx c_t m_b, \tag{2.24}$$

where m_b is the biomass of the branches per hectare and c_t is a coefficient that depends at least on the physical and mechanical properties of the tree type and properties of the fuze used. If we assume the trees' branches are randomly located within distance h_b of trees' tops and all trees are the same height as seen in figure 2.8, we get that the probability density function for a projectile on a downward trajectory hitting a tree branch is

$$f_b(r) = \begin{cases} 0, & \text{if } 0 < r < \frac{h-h_b}{\sin(\phi)} \\ c_t m_b e^{-c_t m_b (\frac{h}{\sin(\phi)} - r)}/P, & \text{if } \frac{h-h_b}{\sin(\phi)} \le r \le \frac{h}{\sin(\phi)} \\ 0, & \text{if } r > \frac{h}{sin(\phi)}, \end{cases} \tag{2.25}$$

where h is the height of the trees in the area, r is the distance from the would be impact point of the projectile on the ground level and ϕ is the angle of

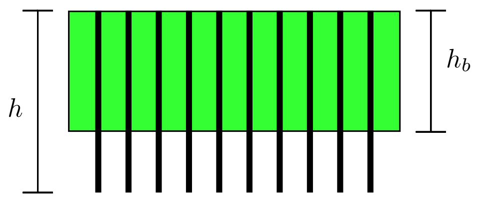

Figure 2.8: The green area represents the area partially covered by tree branches. Here, h denotes the height of the trees and h_b denotes the height of the area from tree tops that has branches, also known as the crown.

fall as shown by figure 2.4 and P is the probability of hitting a branch before reaching the ground

$$P = e^{-c_t m_b \frac{h_b}{\sin(\phi)}}. \tag{2.26}$$

Determining c_t accurately is practically impossible without extensive field tests using the actual ammunition. Even if such tests were conducted, the results would most likely not be made publicly available. It is, however, possible to form a rough estimate by using mostly guesswork and basic physics. Metla file service [34] contains publicly available area data about average biomasses for branches of pine, spruce and deciduous trees. Using the biomass we can calculate the total volume of the branches in one hectare area

$$V_b = \frac{m_b}{\rho_t}, \tag{2.27}$$

where m_b[kg] is the biomass of tree branches in one hectare of forest and ρ_t[kg/m^3] the average density of those branches, which can be calculated when biomasses for different tree species' branches are known. Let us once again examine situation where the projectile travels one metre among those branches. One metre slice of the forest hectare can be estimated to contain $\frac{1}{100}$th of the combined volume of all the branches. If we assume all the branches are cylindrical with diameter d_bT[m], we can calculate the total area they would cover when laid flat on the ground

$$A_f = d_b l \tag{2.28}$$

$$l = \frac{V}{\pi(d_b/2)^2} \tag{2.29}$$

$$A_f = \frac{V_b}{100}\frac{4}{\pi d_b} = \frac{m_b}{25\pi\rho_t d_b}. \tag{2.30}$$

Naturally the branches in the forest do not form a solid wall, but grow into very many different directions. If we assume that branches can grow in all possible directions with same probability the area perpendicular to the velocity vector of the projectile is

$$A_p = \frac{2}{\pi}\frac{m_b}{25\pi\rho_t d_b} = \frac{2m_b}{25\pi^2\rho_t d_b}. \tag{2.31}$$

In reality some branches would be partially behind other branches. However, we will just ignore this here because we are only seeking to form a rough estimate.

If we assume that all the branches are located within the distance $h_b[m]$ from the tree tops as seen in figure 2.8 and their placement is random, we can calculate the probability of the projectile's trajectory intersecting the area covered by a branch in the one meter slice of the forest hectare

$$E(\text{“Number of branches hit”}) = \frac{A_p}{100h_b} = \frac{2m_b}{25\pi^2\rho_t d_b}\frac{1}{100h_b} = \frac{m_b}{1250\pi^2\rho_t d_b h_b}, \tag{2.32}$$

which, when combined with (2.23) and (2.24) gives us

$$c_t = \frac{1}{1250\pi^2\rho_t d_b h_b}, \tag{2.33}$$

Just to give an idea of the scale, for a pine tree farm ready to be harvested[43] $c_{PTF} m_b \approx 0.0085[\frac{1}{m}]$.

Because the probability of hitting branches is represented with exponential distribution, the functions can be readily modified to handle more complicated situations. An example of such situation could be a forest that has several species of trees in it, and different species have different crown heights. In case of two species (2.25) becomes

$$f_b(r) = \begin{cases} 0, & \text{if } 0 < r < \frac{h-h_b}{\sin(\phi)} \\ c_{t1}m_{b1}e^{-c_{t1}m_{b1}(\frac{h}{\sin(\phi)}-r)-c_{t2}m_{b2}\frac{h_{b2}}{\sin(\phi)}}/P_{12}, & \text{if } \frac{h-h_{b1}}{\sin(\phi)} \leq r < \frac{h-h_{b2}}{\sin(\phi)} \\ (c_{t1}m_{b1}+c_{t2}m_{b2})e^{-(c_{t1}m_{b1}+c_{t2}m_{b2})(\frac{h}{\sin(\phi)}-r)}/P_{12}, & \text{if } \frac{h-h_{b2}}{\sin(\phi)} \leq r \leq \frac{h}{\sin(\phi)} \\ 0, & \text{if } r > \frac{h}{sin(\phi)}, \end{cases} \tag{2.34}$$

where $h_{b1} > h_{b2}, c_{t1}, m_{b1}, c_{t2}, m_{b2}$ are parameters for the two species of trees respectively and

$$P = e^{-c_{t1}m_{b1}\frac{h_{b1}}{\sin(\phi)}-c_{t2}m_{b2}\frac{h_{b2}}{\sin(\phi)}}. \tag{2.35}$$

2.3.3 Combining the trunk and branch hit probabilities

Combining Rayleigh and exponential distributions is not difficult, because of all the independence assumptions concerning the probabilities of hitting tree trunks and branches. That means that

$$\begin{aligned} &P(\text{"Projectile hits branch"} \vee \text{"Projectile hits trunk"}) \\ &\quad = 1 - (1 - P(\text{"Projectile hits branch"})(1 - P(\text{"Projectile hits trunk"}). \end{aligned} \tag{2.36}$$

Thus the cumulative distribution function is

$$F_{R\&E}(x) = 1 - (1 - 1 - e^{-\lambda x})(1 - 1 - e^{\frac{-x^2}{2\sigma^2}}) = 1 - e^{\frac{-x^2}{2\sigma^2}-\lambda x}, \tag{2.37}$$

which means that the probability density function is

$$f_{R\&E}(x) = (\frac{x}{\sigma^2} + \lambda)e^{\frac{-x^2}{2\sigma^2} - \lambda x}. \tag{2.38}$$

We can now combine (2.21) and (2.25) to derive the probability density function that takes into account both branches and trunks

$$f(r) = \begin{cases} \frac{\frac{h}{\sin(\phi)} - r}{\sigma^2} \exp\left(\frac{-(\frac{h}{\sin(\phi)} - r)^2}{2\sigma^2} - \lambda_b \frac{h_b}{\sin(\phi)}\right), & \text{if } 0 < r < \frac{h - h_b}{\sin(\phi)} \\ (\frac{\frac{h}{\sin(\phi)} - r}{\sigma^2} + \lambda_b) \exp\left(\frac{-(\frac{h}{\sin(\phi)} - r)^2}{2\sigma^2} - \lambda_b(\frac{h}{\sin(\phi)} - r)\right), & \text{if } \frac{h - h_b}{\sin(\phi)} \le r \le \frac{h}{\sin(\phi)} \\ 0, & \text{if } r > \frac{h}{\sin(\phi)}, \end{cases} \tag{2.39}$$

where $\sigma = \sqrt{\frac{\tan(\phi)h}{nw_0}}$ and $\lambda_b = c_t m_b \approx \frac{m_b}{1250\pi^2 \rho_t T h_b}$. Thus, the cumulative distribution function is

$$F(r) = \begin{cases} 0, & \text{if } r < 0 \\ \exp\left(\frac{-(\frac{h}{\sin(\phi)} - r)^2}{2\sigma^2} - \lambda_b \frac{h_b}{\sin(\phi)}\right), & \text{if } 0 \le r < \frac{h - h_b}{\sin(\phi)} \\ \exp\left(\frac{-(\frac{h}{\sin(\phi)} - r)^2}{2\sigma^2} - \lambda_b(\frac{h}{\sin(\phi)} - r)\right), & \text{if } \frac{h - h_b}{\sin(\phi)} \le r \le \frac{h}{\sin(\phi)} \\ 1, & \text{if } r > \frac{h}{\sin(\phi)}. \end{cases} \tag{2.40}$$

Because there still remains a chance for the projectile to reach the ground and explode there, the cumulative distribution function $F(r)$ has step at $r = 0$.

$$F(0) = \exp\left(\frac{-(\frac{h}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b \frac{h_b}{\sin(\phi)}\right). \tag{2.41}$$

Both functions can also be divided to even more components, if the parameters of the forest drastically change along the projectile's trajectory; this could be the case for example near the edge of the forest. If there is no need to do such further divisions, the probability density function and the cumulative distribution function, respectively, can also be represented as a function of the distance from the ground

$$f_z(r) = \begin{cases} \frac{\frac{h-z}{\sin(\phi)}}{\sigma^2} \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b \frac{h_b}{\sin(\phi)}\right), & \text{if } 0 < z < h - h_b \\ (\frac{\frac{h-z}{\sin(\phi)}}{\sigma^2} + \lambda_b) \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b (\frac{h-z}{\sin(\phi)})\right), & \text{if } h - h_b \le z \le h \\ 0, & \text{if } r > h, \end{cases} \tag{2.42}$$

and

$$F_h(z) = \begin{cases} 0, & \text{if } z < 0 \\ \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b \frac{h_b}{\sin(\phi)}\right), & \text{if } 0 \le z < h - h_b \\ \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b \frac{h-z}{\sin(\phi)}\right), & \text{if } h - h_b \le z \le h \\ 1, & \text{if } z > h. \end{cases} \tag{2.43}$$

Figure 2.9 gives an example of a cumulative distribution function.

The combined probability distribution can also be modified to handle more complicated situations in the same way in which the branch hit probability was modified in (2.34). If the different tree species are relatively similar in their heights, it is enough to modify the probability of hitting branches. Using the example with two different species of trees again, (2.43) becomes

$$F_h(z) = \begin{cases} 0, & \text{if } z < 0 \\ \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_{b1} \frac{h_{b1}}{\sin(\phi)} - \lambda_{b2} \frac{h_{b2}}{\sin(\phi)}\right), & \text{if } 0 \le z < h - h_b \\ \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_{b1} \frac{h-z}{\sin(\phi)} - \lambda_{b2} \frac{h_{b2}}{\sin(\phi)}\right), & \text{if } h - h_{b1} \le z < h - h_{b2} \\ \exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - (\lambda_{b1} + \lambda_{b2}) \frac{h-z}{\sin(\phi)}\right), & \text{if } h - h_{b2} \le z \le h \\ 1, & \text{if } z > h, \end{cases} \tag{2.44}$$

where $h_{b1} > h_{b2}, \lambda_{b1}, \lambda_{b2}$ are parameters for the two different species' branches

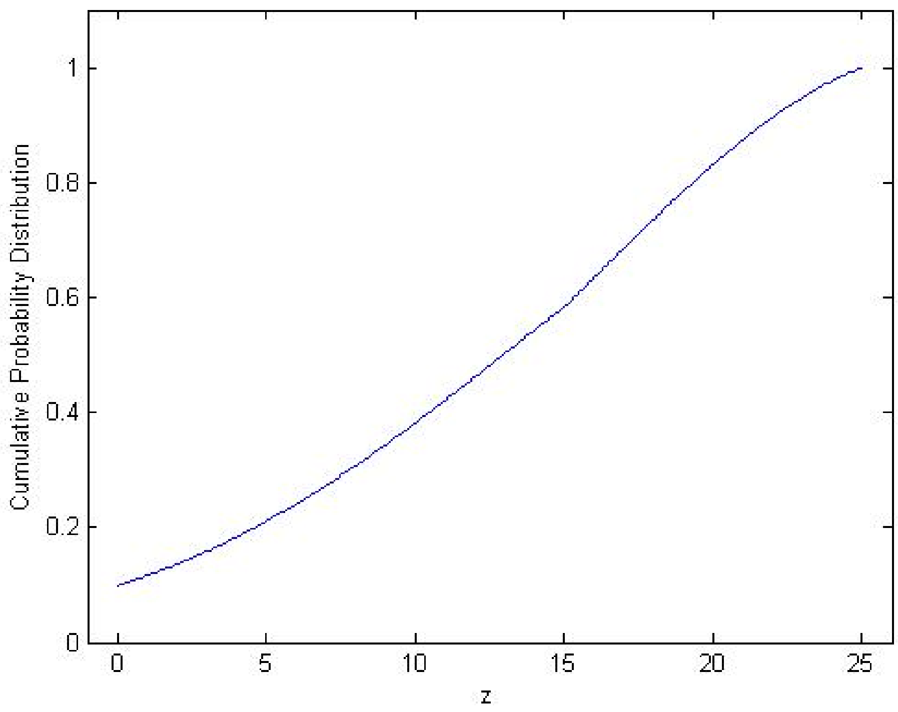

Figure 2.9: The cumulative distribution function $F_h(z)$ with $\phi = 30°$, $h = 25$ and $h_b = 10$. The parameters $\sigma \approx 42$ and $\lambda = 0.01$ represent a thick pine forest and a superquick fuze.

respectively.

2.3.4 Combining the tree hit probability with projectile trajectory dispersion

If the forest is homogeneous along the projectiles' trajectory and the projectiles' dispersion pattern is known, we can derive the three-dimensional probability density from (2.9)

$$f_{xyz}(x, y, z) = f_{xy}(x, y + \text{cotan}(\phi)z)f_z(z), \tag{2.45}$$

where f_{xy} is the probability distribution function of projectile's impact point on the ground level. The ground level is xy plane and the projectile's trajectory's projection to xy plane is perpendicular to y-axis similar to what is shown in figure 2.4, but with the exception that the origin point can be fixed anywhere. If we assume that impact point's probability distribution is bivariate normal distribution with zero correlation, as is commonly done [2], we get

$$f_{xyz}(x, y, z) = f_x(x)f_y(y + \text{cotan}(\phi)z)f_z(z), \tag{2.46}$$

where

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x}\right), \tag{2.47}$$

which is the probability density function of normal distribution with mean value μ_x and standard deviation σ_x. The function $f_y(y)$ is similar. In this case the point $(\mu_x, \mu_y, 0)$ is the aim point of the artillery fire.

Cumulative distribution functions are not applicable in more than one dimension. However, if we seek to calculate the probability that the explosion happens within the rectangular cuboid limited by x_1, x_2, y_1, y_2, z_1, and z_2, we get

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} f_x(x)f_y(y+\text{cotan}(\phi)z)f_z(z)\ \mathrm{d}z\ \mathrm{d}y\ \mathrm{d}x$$
$$= \int_{x_1}^{x_2} f_x(x)\ \mathrm{d}x \int_{z_1}^{z_2}\left(\int_{y_1}^{y_2} f_y(y+\text{cotan}(\phi)z)\ \mathrm{d}y\right) f_z(z)\ \mathrm{d}z$$
$$= \Big/_{x_1}^{x_2} [F_x(x)] \Big/_{z_1}^{z_2} \Big/_{y_1}^{y_2} \Bigg[F_y(y+\text{cotan}(\phi)z)\exp\left(\frac{-(\frac{h-z}{\sin(\phi)})^2}{2\sigma^2} - \lambda_b\frac{h_b}{\sin(\phi)}\right)$$
$$- \frac{2}{\sqrt{\frac{\sigma_y^2}{\sin(\phi)^2\sigma^2}+\text{cotan}(\phi)}}\exp\left(-\frac{(\mu_y-y)^2}{2\sigma_y^2} - \frac{h^2}{2\sin(\phi)^2\sigma^2} - \lambda_b\right)$$
$$\cdot \exp\left(-\frac{1}{2}\frac{(\frac{\text{cotan}(\phi)(\mu_y-y)}{\sigma_y^2}+\frac{h}{\sin(\phi)\sigma^2}+\lambda_b)^2}{\frac{\text{cotan}(\phi)}{2\sigma_y^2}+\frac{1}{2\sin(\phi)\sigma^2}}\right)$$
$$\cdot\frac{1}{2}(1+\text{erf}\left(\left(\frac{1}{\sin(\phi)^2\sigma^2}+\frac{\text{cotan}(\phi)}{2\sigma_y^2}\right)z - \frac{1}{2}\left(\frac{\mu_y-y}{\sigma^2}+\frac{h}{\sin(\phi)^2\sigma^2}+\lambda_b\right)\right)\Bigg] \tag{2.48}$$

for $z_1, z_2 \in [h-h_b, h]$.

Because most practical applications will be dealing with several different species of trees and coordinate systems other than the Cartesian coordinates, deriving analytical solutions for the probability of an explosion happening within certain area becomes very difficult and impractical. Thus, calculating a numerical estimate is usually a better solution.

Chapter 3

Model Validation

3.1 Forest parameters

To test the method presented in this thesis, we first define the forest parameters. Table 3.1 shows a list of the different parameters needed.

Most common species of trees in Finland grow to a height of 15–30m depending on the soil and other factors. [39] Thus, if one wants to model a typical mature Finnish forest h can be chosen to be 20 or 25 metres. If one wants to model specific forest area, average height of trees can be found from the Metla file service. [34]

Table 3.1: Forest parameters

Parameter	Unit	Explanation
h	m	height of trees
h_b	m	height of trees' crowns
w_0	m	diameter of trees at stump
N	$1/\mathrm{m}^2$	number of trees per hectare
m_b	kg/ha	biomass of branches per hectare
c_t	m	branch hit coefficient

The height of trees' crown is the distance from the top to lowest living branches. The proportional height of the crown varies by tree species and many other factors. [46] Figure 3.1 shows the heights of the crown bases as a function of a tree's height in the most common species of trees in Finland. The figures are based on averages from several different environments, but are good enough for our purposes. [46] The height of the crowns can be approximated as

$$h_{b,pine} = 0.2h + 3 \tag{3.1}$$

$$h_{b,birch} = 0.5h + 1 \tag{3.2}$$

$$h_{b,spruce} = \begin{cases} 0.75h, & \text{if } h < 20 \\ h - 5, & \text{if } h \geq 20. \end{cases} \tag{3.3}$$

The calculation of more accurate estimates is possible, but it would call for employing many parameters, that are not easily available. It would be easier to just measure the heights of the tree crowns than measure all the parameters required for the most accurate estimate functions.[46]

According to Repola et al. [41], the diameter of a tree's at stump can be calculated from the tree's diameter at chest height using the formula

$$w_0 = 0.02 + 1.25 w_{1.3}. \tag{3.4}$$

In the context of this thesis the formula should only be used for smaller trees, because tree trunks were assumed to be cone shaped. The cone assumption works better for larger trees when the base diameter of the cone is calculated directly from the diameter at 1.3 meters, i.e.,

$$w_0 = w_{1.3} \frac{h}{h - 1.3}. \tag{3.5}$$

For an average mature Finnish forest we have $w_{1.3} \approx 0.3$m, which represents the average width of a tree farm that should be restocked. [17] Another alternative is to model a specific forest area and use the average width from

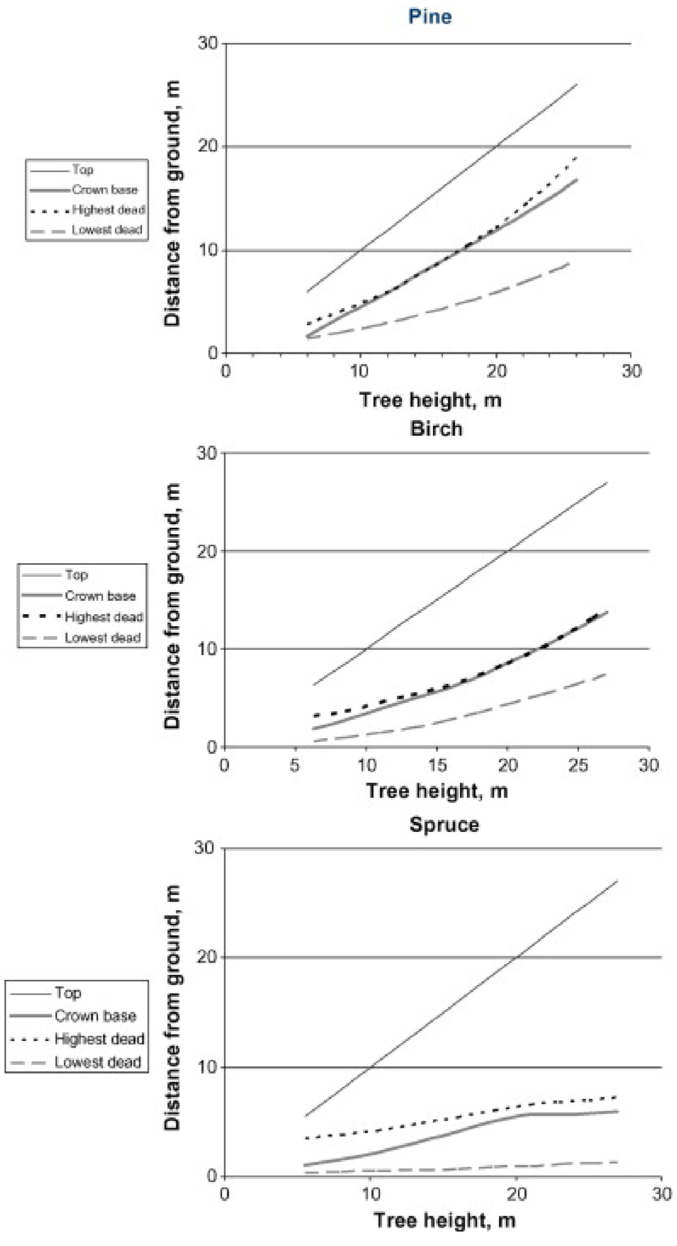

Figure 3.1: The height of branches as a function of tree height in the most common species of trees in Finland. [46]

Table 3.2: Dry densities [47]

Species	ρ_t
Pine	510kg/m^3
Spruce	430kg/m^3
Birch	670kg/m^3

Metla file service. [34]

The number of trees per hectare varies greatly depending on the species of the trees and whether the forest in question is a tree farm or a natural forest. Most forests in Finland are used for forestry. That means that the number of trees per hectare in a mature forest is 900–1100 for spruce, 900–1300 for pine, and for silver birch 700–800. [17] In natural forests the number of trees is typically higher. If a specific forest area is modeled the number of trees per hectare can be easily calculated from the stand basal area and the stand mean width found in Metla file service. [34]

Biomass of branches per hectare varies depending on the species of the trees and other factors. For a mature pine forest $m_{b,pine} \approx 15000$kg/ha and a spruce forest $m_{b,spruce} \approx 20000$kg/ha. [43] Data for specific forest areas can be found in Metla file service. [34] In most cases biomass of living branches can be used as an estimate for the total branch mass, because dead branches would be less likely to cause the fuze to act.

Because field test data is not publicly available, it is best to estimate c_t using (2.33). The densities of trees are well known, and thus finding values for ρ_t for different species of trees is not difficult. [47] It should be noted that tree biomass is most often expressed as dry weight [41] and thus appropriate densities should also also be used. Dry densities for pine, spruce and birch can be found in table 3.2. For branch diameter d_b no fixed value exists. It can be thought as a diameter of a branch that is strong enough to cause an artillery fuze to act when hit and thus depends on the type of fuze used. In this thesis a value of $d_b = 0.05$m is used.

Table 3.3: Forest locations used for test calculations. The forest is named after the dominant species of trees in the area. The coordinates given are in the ETRS-TM35FIN coordinate system. The forest parameters are from Metla file service. [34]

Forest	E	N	h	$w_{1.3}$	N	m_{spruce}	m_{birch}	m_{pine}
Spruce1	620073	6849808	17.5	0.2	859	19090	0	590
Birch	620224	6849838	12.8	0.11	1789	0	9110	0
Pine	621923	6852671	17.6	0.2	700	2850	2130	7600
Spruce2	265950	6814925	24.8	0.29	469	23330	830	760

For the test calculations three forest locations in Eastern Finland were chosen. Forest parameters for those locations were then taken from Metla file service [34]. Each location has a different dominant species of trees. They are all in forestry use, so they are neither very old or very dense. The location with birch as dominant species has somewhat younger forest than the other two. To complement these three sites, a fourth one was chosen from Western Finland with old Spruce forest. Coordinates and forest parameters for different locations can be found in table 3.3.

3.2 The prototype software

To test the method presented in this thesis a prototype software was created using Python 2.7.8 [1]. The Software calculates probabilities for air burst locations and expected losses for target units in forest terrain. To calculate losses caused to the targets, the program uses physical model for fragmenting ammunition software, EETU, that is also used by military operation analysis tool Sandis 2 of Finnish Defence Forces Research Agency. [23] The working principles of the fragmentation model are described in subsection 2.1.

The program calculates the probability for a target element being hit by adding together the probabilities of target being hit by a fragment or a blast wave from numerous discrete calculation points. The probability of the target

Table 3.4: The fragment fan parameters for 81mm HE shell used. The angles are given starting from the nose of the projectile.

	Front Fan	Side fan	Rear fan
Start angle (deg)	0	65	170
End angle (deg)	10	115	180
Initial velocity of fragments (m/s)	1200	1200	1200
Total number of fragments	482	2568	161
Average fragment mass (g)	1.15	1.5	1.15

Table 3.5: The fragment fan parameters for 120mm HE shell used. [21] The angles are given starting from the nose of the projectile.

	Nose	Front Fan	Side fan	Rear fan
Start angle (deg)	0	0	65	170
End angle (deg)	5	10	115	180
Fragment initial velocity (m/s)	1200	1200	1200	1200
Total number of fragments	1	1046	5580	349
Average fragment mass (g)	30	1.63	1.63	1.63

being hit from a single calculation point is calculated by

$$P(\text{The target hit from } (x,y,z)) = \\ P(\text{An air burst at } (x,y,z) \text{ hurts the target})P(\text{Air burst happens at } (x,y,z)) \tag{3.6}$$

P(An air burst at (x, y, z)hurts the target) is calculated by EETU software for given target and air burst locations. The artillery shell parameters used for calculations were found in public sources. The parameters for 120mm [22] and 155mm [21] shells have been used in previous studies and the parameters for 81 mm shell were modified from them. The values for fragment fans of the shells can be found in tables 3.4, 3.5, and 3.6.

The probability for an air burst happening within the vicinity of the calculation point is calculated by using (2.45). When $f_{xy}(x, y + \text{cotan}(\phi)z)$ is

Table 3.6: The fragment fan parameters for 155mm HE shell used. [22] The angles are given starting from the nose of the projectile.

	Front Fan	Side fan	Rear fan
Start angle (deg)	0	70	170
End angle (deg)	10	110	180
Initial velocity of fragments (m/s)	1200	1200	1200
Total number of fragments	381	2030	127
Average fragment mass (g)	14.34	14.34	14.34

Table 3.7: Vulnerable areas and armor thickness of the prone soldier target. The armor value is given in steel millimeters. It is assumed that fragments with enough energy to penetrate 1.5mm of steel can wound the soldier target. [44]

	Top	Front	Rear	Side
Armor thickness (mm)	1.5	1.5	1.5	1.5
Vulnerable area (m^2)	0.61	0.08	0.08	0.38

assumed to be a constant around each calculation point, we get

$$F_{xyz}(x, y, z) = A \cdot f_{xy}(x, y + \text{cotan}(\phi)z)F_h(z), \tag{3.7}$$

where A is the area represented by the calculation in xy-plane. It would have also been possible to use (2.48) to produce more accurate results, but it gets even more complicated than it already is when you take into account different species of trees, and the fact that $f_z(z)$ is piecewise defined and (2.48) only covers one piece. Using (3.7) also has the added benefit that it can easily be converted to a cylindrical coordinate system by replacing term $f_{xy}(x, y + \text{cotan}(\phi)z)$, which could be useful in other applications.

For all the test cases target area is one hectare square that is divided to one hundred 10m x 10m squares with a prone soldier target in the middle of each of them. The parameters for prone soldier targets can be found in table 3.7. The aim point is chosen so that the rounds are distributed around the center of the target area. The forest parameters used in the tests include the parameters presented in table 3.3, and an open field where there is no forest.

3.3 Comparison with field tests

There are very few public sources that contain data from actual artillery firing field tests and most of them did not take place in a forest environment. The only source, with detailed enough information to allow easy comparison, that was found was a study by Keinonen [20] which contains relatively detailed data about the results of a mortar field test in forest environment conducted in 1953. Light and heavy mortar platoons were used to fire in one hectare target areas with 36 prone soldier sized targets. Areas were located both in an "average thickness Finnish forest" and in the open. Both light and heavy mortar platoons fired one strike to each area after which the results were recorded.

The test software described in subsection 3.2 was used to recreate the test firings. Because the exact nature of the forest that was in the target area is not fully known, the simulated test was repeated in all four different forest environments found in table 3.3. There were other parameters that required some assumptions too. The shells were assumed to be similar enough to modern mortar shells that the modern parameters could be used. Superquick fuzes were used. The standard deviations of fire was assumed to be 1/4 of the reported width and length of shot patterns. The angle of fall (AOF) for the artillery shells was assumed to be 45°, because accurate information was not available. The smaller firing angle minimizes the deviation from target, and mortars' minimum firing angle is often 40-50°.[2] A full list of the test parameters can be found in table 3.8. The forest parameters used can be found in table 3.3.

Table 3.9 shows the percentage of air bursts for each test forest type as calculated by the test software. Keinonen [20] lists the percentages of air bursts as about 40% for light mortars and 39% for heavy mortars. Because the birch forest is very young and the trees are thus smaller and the pine forest is not very thick, the two spruce forests best match the description given by

Table 3.8: The parameters used by the test software.

	81mm mortars	120mm mortars
AOF (deg)	45	45
Projectile velocity (m/s)	200	300
$\sigma_{x,forest}$ (m)	29	29
$\sigma_{y,forest}$ (m)	40	50
$\sigma_{x,open}$ (m)	33	21
$\sigma_{y,open}$ (m)	33	42
Rounds fired	54	24

Table 3.9: The expected percentage of air bursts caused by hitting trees in different types of forests. The forest parameters for each forest can be found in table 3.3.

Forest	Tree hits
Spruce1	34.9%
Birch	26.8%
Pine	27.6%
Spruce2	38.5%

Keinonen. The difference between the percentage of air bursts calculated by the test software and the actual test firings is thus relatively small.

The expected losses for the soldier targets calculated by the test software can be found in table 3.10. The expected losses in the forest terrain are very close to those reported by Keinonen [20]. He lists the losses in the forest for the light mortars as 72% and for the heavy mortars 49.1% for a target area that was centered around the strike pattern. For the open target area the

Table 3.10: The expected loss percentages for the soldier targets in different types of forests. The forest parameters for each forest can be found in table 3.3. Open is an area without any forest.

Forest	54 81mm HE shells	24 120mm HE shells
Open	75.58%	63.74%
Spruce1	74.65%	59.78%
Birch	74.85%	59.37%
Pine	74.53%	59.44%
Spruce2	73.57%	59.11%

numbers were 59.3% and 58.3% respectively.

The differences between the results from the test software and actual field tests can be explained by that (I) the test program only calculates expected losses, (II) reality rarely matches expectations exactly, and (III) the shells used for calculations were modern, and thus more effective than those that were used in 1953.

The only value that differs considerably is the loss percentage for 81mm mortars in the open target area. The size of the shot pattern is so much larger in the open than in the forest that it might indicate that the aiming was off. In addition, the firing distance for the 81mm mortars is much shorter in the case of the open target area so the angle of fall might have also been different than in the other cases.

Another intriguing result is the fact that forest type seems to have very little if any effect in the expected losses predicted by the test software in this case. In fact most of the difference between the open target area and forest one seems to come from the slightly different standard deviations used. Figures 3.2 and 3.3 show the the expected losses for each soldier target in a hectare area for 81mm and 120mm mortars respectively.

The figures show side by side the expected losses in open terrain and in the thickest forest used. The differences between the two are barely distinguishable. That means that forest has very little effect on the losses sustained by the target when using parameters like this. The reasons for this are better explained in subsection 3.4

3.4 The mass test run results

Because there was relatively little hard field test data with which to compare the results given by the prototype software, and because software always

0.47	0.59	0.69	0.75	0.78	0.78	0.75	0.69	0.59	0.47
0.55	0.68	0.77	0.82	0.85	0.85	0.82	0.77	0.68	0.55
0.61	0.73	0.82	0.87	0.89	0.89	0.87	0.82	0.73	0.61
0.64	0.77	0.85	0.9	0.92	0.92	0.9	0.85	0.77	0.64
0.66	0.78	0.86	0.91	0.92	0.92	0.91	0.86	0.78	0.66
0.65	0.77	0.86	0.9	0.92	0.92	0.9	0.86	0.77	0.65
0.62	0.75	0.83	0.88	0.9	0.9	0.88	0.83	0.75	0.62
0.56	0.69	0.79	0.84	0.87	0.87	0.84	0.79	0.69	0.56
0.49	0.62	0.72	0.78	0.81	0.81	0.78	0.72	0.62	0.49
0.41	0.52	0.62	0.68	0.72	0.72	0.68	0.62	0.52	0.41

0.46	0.57	0.67	0.73	0.76	0.76	0.73	0.67	0.57	0.46
0.53	0.65	0.75	0.8	0.83	0.83	0.8	0.75	0.65	0.53
0.59	0.71	0.8	0.85	0.88	0.88	0.85	0.8	0.71	0.59
0.63	0.75	0.83	0.88	0.9	0.9	0.88	0.83	0.75	0.63
0.64	0.76	0.85	0.89	0.91	0.91	0.89	0.85	0.76	0.64
0.64	0.76	0.84	0.89	0.91	0.91	0.89	0.84	0.76	0.64
0.61	0.74	0.82	0.87	0.89	0.89	0.87	0.82	0.74	0.61
0.56	0.69	0.78	0.84	0.86	0.86	0.84	0.78	0.69	0.56
0.5	0.62	0.72	0.78	0.81	0.81	0.78	0.72	0.62	0.5
0.42	0.54	0.63	0.69	0.73	0.73	0.69	0.63	0.54	0.42

Figure 3.2: The calculated loss probabilities to soldier targets when 54 81mm mortar shells are fired in the target area. The firing unit is located to the south. The figure on the left is from an open field and the one on the right is from an old spruce forest, Spruce2. The same standard deviations were used for both.

0.39	0.49	0.57	0.62	0.65	0.65	0.62	0.57	0.49	0.39
0.43	0.53	0.62	0.67	0.7	0.7	0.67	0.62	0.53	0.43
0.45	0.56	0.65	0.71	0.73	0.73	0.71	0.65	0.56	0.45
0.47	0.58	0.67	0.73	0.75	0.75	0.73	0.67	0.58	0.47
0.47	0.59	0.67	0.73	0.76	0.76	0.73	0.67	0.59	0.47
0.47	0.58	0.66	0.72	0.75	0.75	0.72	0.66	0.58	0.47
0.44	0.55	0.64	0.7	0.73	0.73	0.7	0.64	0.55	0.44
0.41	0.52	0.6	0.66	0.69	0.69	0.66	0.6	0.52	0.41
0.37	0.47	0.55	0.61	0.64	0.64	0.61	0.55	0.47	0.37
0.31	0.41	0.48	0.54	0.57	0.57	0.54	0.48	0.41	0.31

0.38	0.48	0.56	0.61	0.64	0.64	0.61	0.56	0.48	0.38
0.42	0.52	0.61	0.66	0.69	0.69	0.66	0.61	0.52	0.42
0.45	0.56	0.64	0.7	0.73	0.73	0.7	0.64	0.56	0.45
0.47	0.58	0.67	0.72	0.75	0.75	0.72	0.67	0.58	0.47
0.48	0.59	0.68	0.73	0.76	0.76	0.73	0.68	0.59	0.48
0.47	0.58	0.67	0.73	0.75	0.75	0.73	0.67	0.58	0.47
0.46	0.56	0.65	0.71	0.74	0.74	0.71	0.65	0.56	0.46
0.43	0.53	0.62	0.68	0.7	0.7	0.68	0.62	0.53	0.43
0.39	0.49	0.57	0.63	0.66	0.66	0.63	0.57	0.49	0.39
0.34	0.43	0.51	0.57	0.6	0.6	0.57	0.51	0.43	0.34

Figure 3.3: The calculated loss probabilities to soldier targets when 24 120mm mortar shells are fired in the target area. The firing unit is located to the south. The figure on the left is from an open field and the one on the right is from an old spruce forest, Spruce2. The same standard deviations were used for both.

Table 3.11: The parameters used by the test software for the mass test runs.

	81mm shells	120mm shells	155mm shells
Projectile velocity (m/s)	200	300	300
σ_x (m)	29	29	29
σ_y (m)	40	50	50

needs to be tested for programming errors, a large number of test runs were done in addition to the cases described in subsection 3.3. These test runs were all ran using the same parameters for all the test runs done with the same ammunition type. Only angle of fall and forest terrain varied. The fixed parameters for each ammunition type can be seen in table 3.11. Unlike in the test calculations that were compared to field test data, same standard deviations were used for open and forest terrain to make the results easier to compare.

The forest parameters for the mass test runs were the same and they can are shown in table 3.3. In addition to tests in forest terrain calculations were also made in open terrain for comparison purposes. The angle of fall was varied between 20° and 80° to better see how that affects the probability of an air burst happening and the expected losses sustained by target unit.

Figure 3.4 shows how the expected losses 120mm shell change based on angle of fall in different terrains. Similar figures were also produced for 81mm and 155mm shells, but the results were so similar that those figures would not bring any additional information. The figure helps explain why there was so little difference between forest and open terrain in the test runs that were compared to field test data. Forest terrain seems to have very little impact on losses sustained compared to open terrain when the angle of fall is over 40°. These results are very similar to those shown in figure 3.5. The figure can be found in several educational materials produced by Finnish Defence Forces. [2][3]

On the other hand the forest terrain becomes a very important factor in

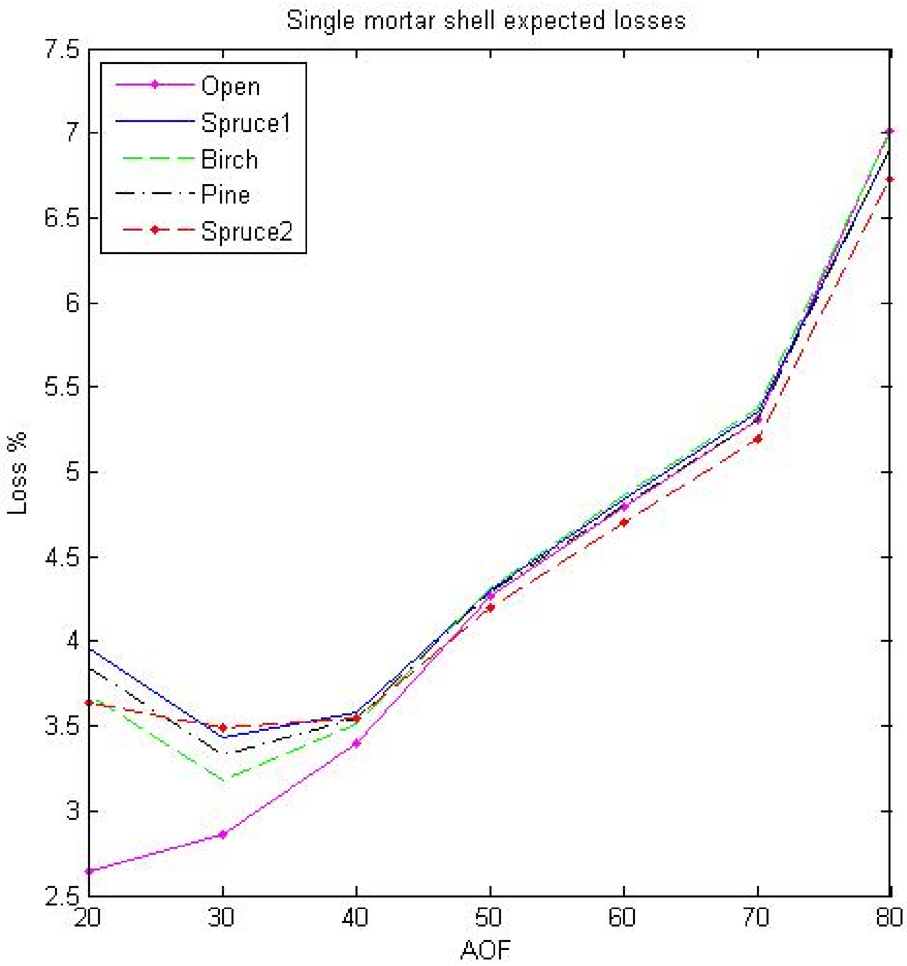

Figure 3.4: The expected loss percentages for soldier targets from a single 120mm shell falling in different angles. Compare to figure 3.5.

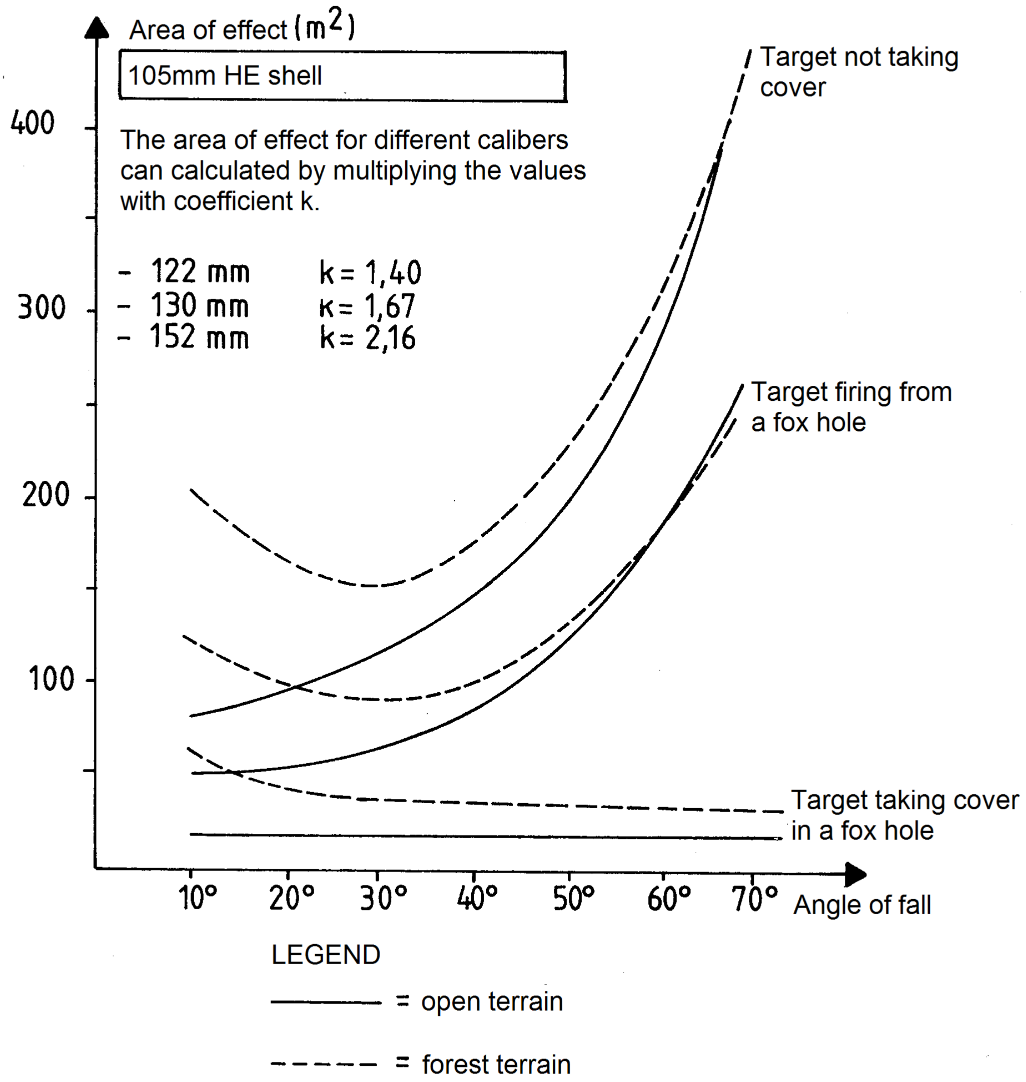

Figure 3.5: 105mm HE shell's area of effect as a function of the angle of fall. The original figure is from educational material used by Finnish Defence Forces. [2] The text on the figure was translated to English to make it easier to read. Compare to simulated results in figure 3.4

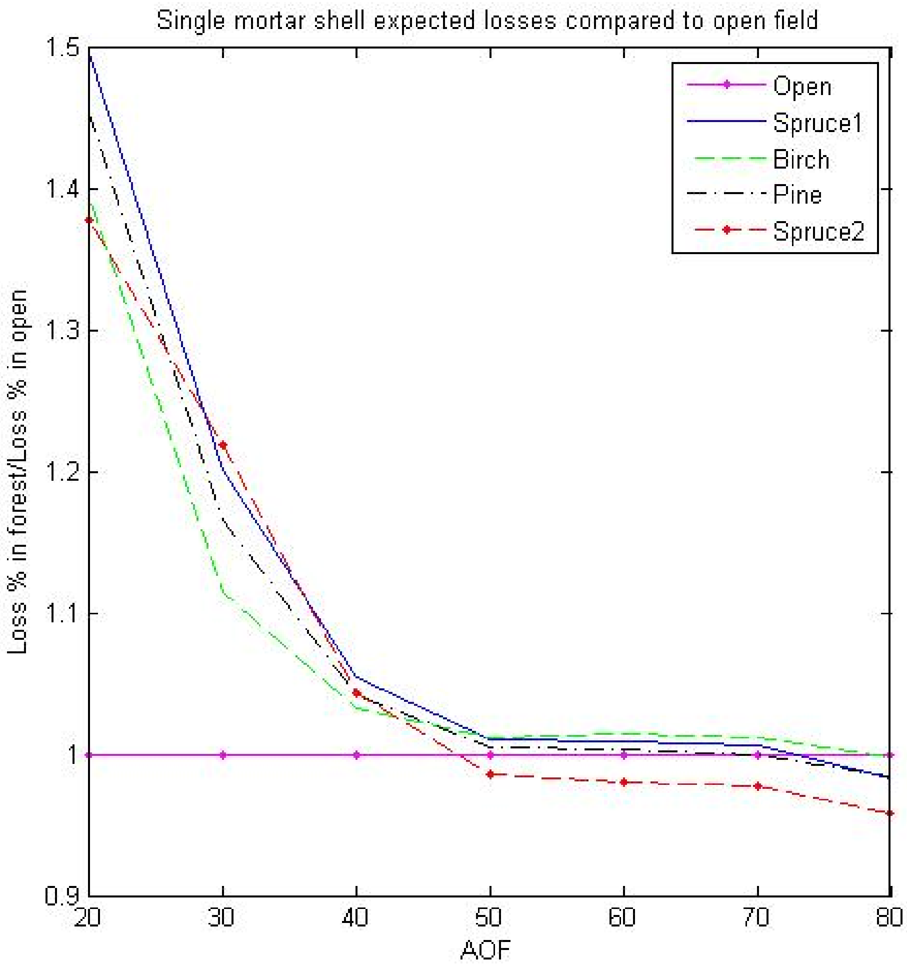

Figure 3.6: A comparison for expected loss percentage for soldier targets between different types of forest and open terrain from a single 120mm shell falling in different angles.

losses sustained by target unit when the angle of fall is 30° or less, as can be more easily seen in figure 3.6. The expected losses for soldier targets within certain forest terrains are 1.5 times higher compared to open terrain when the AOF is 20°. It can also be seen that with a higher AOF certain forest types even offer a slight protection from artillery fire.

Figures 3.7 and 3.8 help explain why the AOF has such a significant impact on losses sustained by the target unit. As can be seen in figure 3.7, when the projectile's AOF is over 40°, the majority of the shells will not hit the trees and will thus explode at the ground level making the results similar to those in the open terrain. Similar results have also been reported by several other sources.[8, 40]

When the AOF is low the situation becomes quite different. A low AOF is not very effective in open terrain because majority of the fragments from the HE shells' explosions will fly towards the sky or hit the ground at the impact location.[2] In the case of an air burst the situation is reversed however. An air burst over a target taking cover will cause the fragments to rain on the target instead, making them highly effective.[8] Figures 3.7 and 3.8 show that majority of the shells achieve an air burst at a low AOF and the average height where the shells explode is between 5 and 15 meters depending on forest terrain.

One additional noteworthy finding from the results of the mass test runs is that the specific characteristics of the forest terrain do not have a high impact on the losses sustained by the target, and that for terrains that are more similar to each other results are more similar too. This makes sense from a physical point of view and makes using the model easier. If small changes in the forest parameters were to cause large changes in the results, it would follow that that model is most likely flawed in some way. In particular, this also means that relatively accurate results can be achieved without accurate information about the forest terrain, which is advantageous from usability point of view.

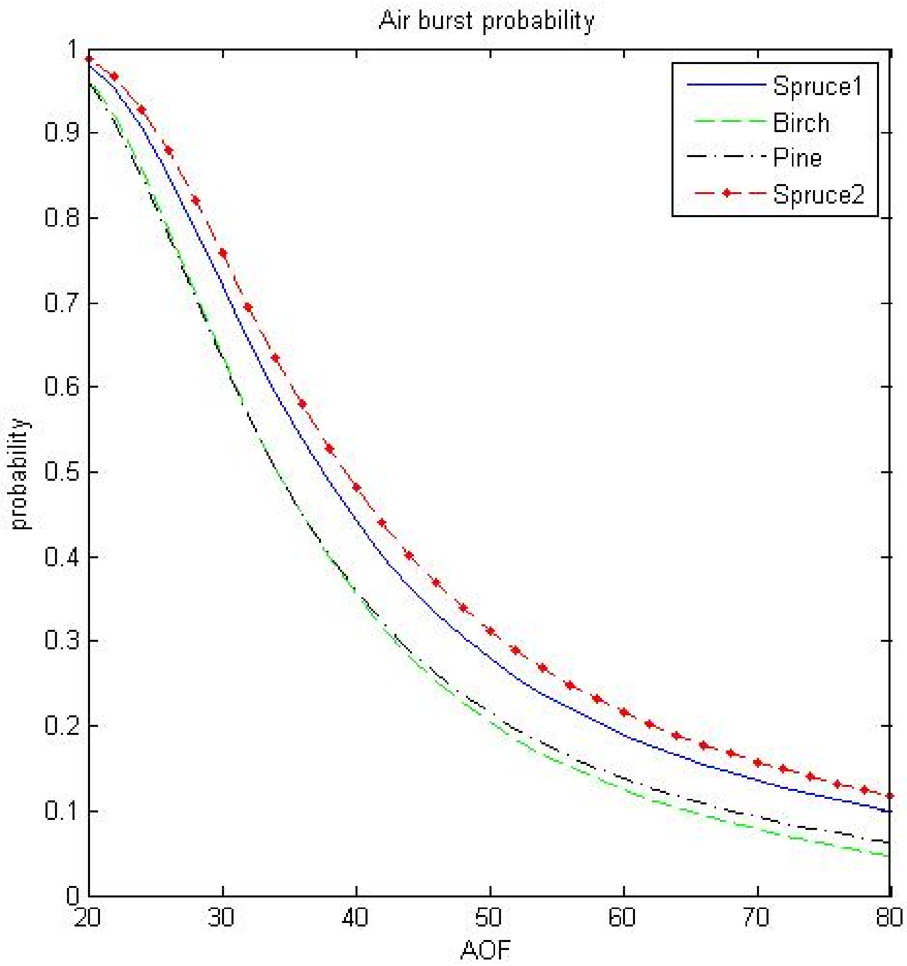

Figure 3.7: Probability of an artillery shell with a SQ fuze hitting a tree and exploding in the different forest terrains.

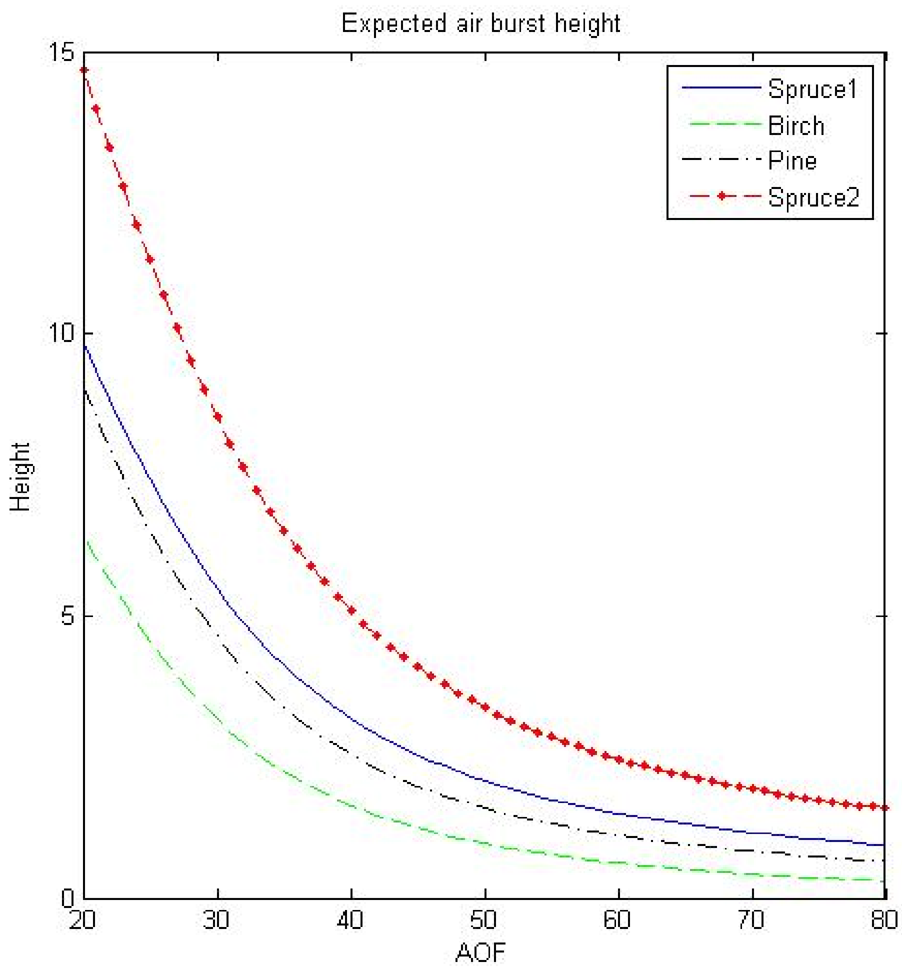

Figure 3.8: Expected value of the height at which artillery shell explodes in the different forest terrains.

Chapter 4

Summary

4.1 Conclusions

This thesis's purpose was to develop a mathematical model that is capable of estimating locations of artillery fire air bursts in forest terrain. This model is based on a physical perspective, which makes it easier to understand, modify, and verify compared to models derived only from statistical data. The mathematical model is robust and tractable enough that it can be easily be modified to solve different simulation cases.

To test the model, methods were developed to produce parameters matching the most common Finnish forest types, but the model itself is easily extended to handle different or more complex forest types. The methods for parameter generation in thesis are very well suited for handling most cases that can arise when trying to model artillery fire in a Finnish forest environment.

The tests that were run on the model show that the model produces results that correspond to field test data and other reference data. The results of the tests also make sense from a physical point of view and do not give reason to believe that there are any obvious mistakes in the model. The testing would have benefited from some additional reference data and field test results, but such results were not found in publicly available sources. Even so, the model seems to produce accurate enough results for the purpose for which it was developed.

The test runs also served another purpose by revealing useful information about in which types of modeling scenarios the forest should be accounted for. When the angle of fall of the projectiles is very high, the forest's effect on the losses suffered by the target are minuscule in a typical Finnish forest. That means that most modeling cases using mortars do not necessarily need a forest model at all to produce accurate results.

The situation changes drastically when the angle of fall of the projectiles is low. When the angle of fall is 20° targets in forest may suffer 50% more losses compared to targets in the open as can be seen in figure 3.6. Low angles of fall often become relevant when modeling the effectiveness of howitzers and field guns. It is in cases like these that some kind of forest model becomes absolutely necessary.

Overall, it can be said that this thesis has met its objectives: the mathematical model can predict the probabilities for air bursts and specific air burst locations, the literary review indicated that there were not any similar models already publicly available, the parameters produced for the test cases can be used in further research, and the test program made it possible to validate the model and can be used as basis for implementing the model as part of larger wholes.

4.2 Future research

There are still ample opportunities for future research and further development. The most immediate way to proceed is implementing the forest model as a part of a more advanced indirect fire or military modeling software such as Sandis 2[23]. This would have the clear advantages of not having to derive or program the damage calculations from scratch and the weapon parameters that already exist in the software could also be used.

The test software produced as a part of this thesis could be extended to handle more complicated situations too, but it is severely hindered by having to rely on another software to calculate the actual effects of the shells and shell fragments. The dependence on an outside program results in calculation times becoming unnecessarily long because over 95% of the calculation time is spent on the fragment model. Currently the test program's calculation takes over 10 minutes per parameter set. That is too slow for it to be applicable to any larger scale simulation. Because the fragmentation model is essentially a black box, it is impossible speed up the calculation without programming the fragmentation model again from the beginning.

One of the most interesting possibilities for future would be to combine the forest model with a terrain model that can take into account the shapes of the ground. Example of such terrain model was presented for example by Lappi et al. [30]. The terrain model is already implemented in Sandis 2 military modeling software[23] which makes Sandis 2 all the more attractive as a platform to implement the forest model. It would make it possible to analyze the combined effects of the forest and the terrain models, in addition to all the other benefits that an actual military modeling software would bring over a test software that was written for a single scenario. It would also make sense to implement some kind of terrain database to the software at the same time. The forest data available at Metla file service[34] could be used as a source for the forest data.

Another useful future research subject would be to test further the forest model with different test cases. The reference data for testing within the constraints of this thesis was not quite as comprehensive as would have been ideal. This was due to the fact that few organizations actually own the kind of weaponry that can be used to perform actual artillery field tests, and those that do, usually do not publish the results. Any further testing would thus probably have to be done in cooperation with such an organization.

One final interesting subject for future research would be to explore how well the model can handle forests that differ greatly from Finnish forests. The mathematical model itself is generic enough that it can be easily used to model jungle instead of pine forest. Multiple-layered rain forest canopy can be represented within the forest model using similar methods that were used to represent different species of trees in this thesis. The biggest obstacle in doing research into artillery fire in jungle terrain is once again lack of publicly available field test data that the simulated results could be compared to. If there was such data publicly available, modeling artillery fire in jungle environment would have probably already been included in this thesis.

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