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Simulation support for medium-term production planning at an energy utility

School of Science

Thesis submitted for examination for the degree of Master of Science in Technology.

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Tämä diplomityö on tehty energiayhtiö Vantaan Energialle (VE). VE käyttää tuotannon optimointiohjelmaa ennustaakseen kuinka paljon sähköä se tulee tuottamaan seuraavan 1-48 kuukauden aikana. Ohjelmaan syötetään muun muassa tunnittainen sähkön spot hinta, tunnittainen lämpötila ja polttoaineiden kustannukset megawattituntia kohden. Ohjelma tuottaa tunnittaisen tuotantosuunnitelman, josta lasketaan esimerkiksi tuotantomäärät, kustannukset ja voitot. Hypoteesina on, että sähköntuotantomäärät eivät määräydy pelkästään kuukauden keskihintojen perusteella vaan ne ovat herkkiä myös spot hinnan profiilille. Tällöin ainoastaan yhden satunnaisesti luodun hintakäyrän käyttäminen voi johtaa poikkeuksellisen suuriin tai pieniin tuotantoennusteisiin vaikka simuloidun sähkönhintakäyrän kuukausien keskiarvot olisivatkin ennusteen mukaisia. Tämän profiiliriskin arvioimiseksi ja pienentämiseksi, luotiin malli, joka luo päivittäisiä spot-hintoja, joiden kuukausien keskihinnat ovat ennusteiden mukaisia. Stokastinen komponenentti mallinnettiin Markovin regiiminvaihtomallilla, jossa on kolme itsenäistä regiimiä. Simuloidut päivähinnat muutettiin tuntihinnoiksi etsimällä toteutuneista hinnoista sopivia profiileja. Näillä metodeilla saadut kuusisataa spot hinnan simulaatiota syötettiin yksitellen optimointimalliin siten, että muut syötteet pidettiin vakioina. Sähköntuotantomäärät, jotka oli laskettu erilaisilla hintaprofiileilla ja joiden kuukausien keskiarvot olivat samoja, saattoivat erota toisistaan merkittävästi. Hypoteesi piti siis paikkansa. Tulosten pohjalta ehdotetaankin, että tarkkojen tuotantomääräennusteiden saamiseksi lasketaan optimaalinen tuotantomäärä lukuisilla hintaprofiileilla ja käytetään näiden tuotantomäärien keskiarvo.

Avainsanat: Sähkön spot-hinta, Markovin regiiminvaihtomalli, itsenäiset regiimit, ennustaminen, sähköntuotantomäärät, simulaatiot, henkilöstösuunnittelu

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This thesis was made for the energy company Vantaan Energia (VE). VE uses a production schedule optimization program to estimate how much power they will produce in the next 1-48 months. The program's inputs among other things are the hourly spot price curve, hourly temperature curve and fuel costs per megawatt hour. The most important outputs are power production quantities, profits and costs. The hypothesis is that VE's production quantities of power are not only sensitive to the mean spot price, but also to the profile of the spot price curve. Therefore, using a single simulation of the spot price curve might cause the forecast of the power production quantity to be exceptionally large or small. In order to estimate and reduce the profile risk, a spot price simulation model was created. It produces simulations of the daily spot prices, whose monthly means are close to forecasted values. The stochastic component is modeled with a Markov regime-switching model with three independent regimes. These daily spot prices are transformed into hourly spot prices using historical profile sampling (HPS). Six hundred simulations of the hourly spot price were given as inputs to the schedule optimization program one after another while keeping other inputs constant. The results showed that the power production quantities varied significantly, even when they were calculated using price curves which monthly means were close to each others. This means that the hypothesis was correct. Therefore we suggest that the mean production quantity of several simulations should be used as the expected power production quantity instead than the production quantity calculated from a single simulation.

Keywords: Power spot price, Markov regime-switching model, independent regimes, forecasting, power production quantities, simulations, personnel planning

Acknowledgements

This thesis was made for Vantaan Energia's market risk management division. I have learned a lot about the power business during the past year, and writing this thesis has been quite satisfying. For that, I must thank my instructor Tommi and my colleagues Jari-Pekka and Kai. I also want to thank the whole staff of Vantaan Energia for making me feel welcome. You all have made the days fly by. How ever, I want to especially thank Pertti Sahi, who is now enjoying his well earned retirement days. Exchanging ideas with him was very productive, and without his expertize in VBA and optimization, I could not have been able to accomplish everything I wanted.

Keeping the scope of the thesis manageable was a struggle for me. I want to thank my Supervisor Ahti Salo for giving many good tips. I remember him saying that I seemed to be writing more of "an introduction to power business" than a thesis, and he urged me to narrow it down. It was an amusing and a true statement, and I hope I had took the advice more seriously. I wanted to put everything I had learned into the thesis, which made the writing process more demanding than necessary. Nevertheless, writing a thesis is a learning experience and it has made me a better writer than I was when I started.

Lastly, I want to thank my family for encouragement and my friends in Rissittely, who helped me to overcome some of the issues that I faced while writing the thesis. You kept me entertained, and when i had trouble with Latex files, you gave me advice on how to approach the problem. I still cannot believe that rolling my head on the keyboard would make the Latex file compile correctly. Thank you particularly for that peace of wisdom. Although, I was tempted to try it out on my own, even if no one would have suggested it!

Espoo, 23.5.2016

Vili-Matti Ojala

Contents

Abbreviations

ABS	Agent-based simulation models
ANN	Artificial neural network
ARCH	Auto regressive conditional heteroskedasticity
ARMA	Auto regressive moving average
ARX	Auto regressive model with an exogenous variable
CET	Central European time
CHP	Combined heat and power
CI	Computational intelligence
EM	Expectation-Maximization
EPAD	Electricity price area differentials
GARCH	Generalized auto regressive conditional heteroskedasticity
HPS	Historical profile sampling
i.i.d	Independent and identically distributed variable
LTSC	Long term seasonal component
MRS	Markov regime-switching
SARMA	Seasonal auto regressive moving average
SARMAX	Seasonal auto regressive moving average model with an exogenous variable
STSC	Short term seasonal component
SVM	Support vector machine
TSO	Transmission system operator
VE	Vantaan Energia

1 Introduction

In this thesis, a method of simulating hourly power spot prices is presented. We will show that using only one simulation of the spot price curve to estimate the future production quantities of power has a larger risk than when multiple simulations are used. Heat and power producer Vantaan Energia (VE) is used as a case example. The goal is to improve VE's ability to forecast what its power production quantities are on a time scale of 1-48 months into the future.

Forecasting the future power production quantities is an important but a somewhat difficult task for combined heat and power (CHP) producer such as VE. Production quantity forecasts are needed among other things for fuel acquisition and hedging purposes. To forecast the production quantities of power, VE uses a program, which calculates the optimal hourly production schedule based on the inputs it is given while considering the limitations of the production machinery. Production quantities of power and heat, fuel consumption, costs, profits and many other things can be calculated from the schedule. The inputs of the program are among other things the hourly spot prices, hourly temperatures, fuel costs per MWh and maintenance schedules. Many of the inputs, such as the fuel costs, can be forecasted with sufficient accuracies because their volatility is quite low. However, the hourly spot price and temperature curves are impossible to forecast correctly for months into the future. Nevertheless, a price curve needs to be produced. In this thesis, we will show that generating a single price curve with a randomly generated but realistic profile is not the optimal solution, and neither is using price curve with a simple average profile. The risk involved in using a single simulation is that the optimal production schedule is sensitive to the profile of the spot price curve. It is relevant how volatile the prices are or when and where the highs and lows occur in the spot price curve. This is because VE has multiple different power production units, which have their own variable costs. In general, a production unit is used, if its variable costs are lower than the spot price. Therefore, one price profiles can lead to a larger production quantity than the other, even if the monthly mean spot prices are the same. Furthermore, power and heat are produced together and the variable costs are shared between the two products. The colder it is, the more power can be generated so that the waste heat can be utilized in district heating. Some price profiles are aligned more favorably with the temperature curve than others.

Even though the spot price correlates with the temperature [1], for the sake of simplicity, a single simulation of the temperature curve will be used in this thesis, while hundreds of simulations of the spot price are generated independently of the temperature. It will be shown that the production quantities calculated with these simulations can differ substantially, even when the monthly means of the spot prices are close to each other. This confirms the hypothesis that the production schedule is sensitive to the price profile and not only to the monthly mean of the spot price. Therefore, we suggest that when production quantities are forecasted, multiple simulations should be used to calculate the optimal production quantities and their mean should be used as the expected quantity.

1.1 Objective and scope

This thesis has two objectives. First is to provide a method of producing different realizations of hourly spot price, where the monthly means of the simulations are exactly or near predetermined monthly means. These monthly means are forecasts made by experts. The second goal of the thesis is to determine how sensitive the optimal production quantity is to the price profile of the spot price. Different realizations of the spot price are inputted to the production schedule optimization program. The program also requires the hourly temperature, fuel costs per MWh and several other things. However, we will focus on the spot prices. The other inputs are given some realistic set of values, which remain the same every time an optimal production schedule is calculated. We acknowledge that these inputs have a substantial effect on the production schedule, but we do not analyze their effect in this thesis. This is justified by the fact that many of the inputs are known in advance, or they can be forecasted with sufficient accuracy. Furthermore, the sensitivity of the production schedule to the spot price curve is easier to observe when all other inputs are kept constant, while the spot price varies. The optimization program has many detailed outputs, but we will focus on the total power production quantity of each month.

1.2 Structure of the thesis

In section 2, the different physical and financial markets Vantaan Energia (VE) and other Finnish power producers can operate on are introduced. Also, description of the characteristics of the spot prices is given. In section 3, a short introduction of VE and its production capacity is given, and the concepts of price risk and profile risk are explained. How VE estimates its future production quantities with the production schedule optimization program will be explained in section 4. Section 5 is where common approaches for spot price modeling and simulations are reviewed. In section 6, the data is presented, and the components of the spot price model are described. The components are the long-term seasonal component (LTSC), the short-term seasonal component (STSC) and the stochastic component. How simulated daily spot prices are turned into hourly spot prices is presented at the end of section 6. The results are shown in section 7. There we present the model parameters and evaluate how well the model can produce simulations with the same characteristics as the historical spot prices. Then, the simulations are used to examine how much the profile of the spot price affects the production schedules calculated with the optimization program. Conclusion and discussion are presented in section 8.

2 Nordic electricity market

Finland is part of Nord Pool, which is the largest electricity market in Europe and the world's first international power market [2]. It has a day-ahead market called Elspot and an intraday market called Elbas. In both of these markets, power is traded for each hour separately. These markets are governed by Nord Pool Spot AS, which is owned by the transmission system operators (TSO) of Norway, Sweden, Finland, Denmark, Estonia, Lithuania and Latvia [2]. In this thesis, Nord Pool will be used to reference these Nordic and Baltic countries and the power market they share.

In addition to Elspot and Elbas, there is a balancing market in each of the member countries, which is governed by each countries TSO [3] and a financial market Nasdaq OMX Commodities [2]. These markets are shown in Table 1. Market players first operate on the financial market up to 10 years before the delivery hour. Physical market opens the day before delivery. In Elspot, players can place binding bids for buying or selling of actual power for each hour of the next day. If there is any need for adjusting the trades after Elspot has closed, players can participate in the intraday market Elbas, up to one hour before delivery. During the delivery hour, Fingrid monitors the production and consumption of power in Finland and makes sure that they are balanced. If any players consumption or production deviates from what they have settled in Elbas or Elspot, Fingrid makes sure the balance is restored and invoices the party responsible for the disruption [4]. More detailed explanations of these markets are presented in the next sections.

Market	**Provided by**	**Trading starts roughly**	**Trading ends**
Elspot	Nord pool	a day before the delivery hour	12 hours before delivery
Elbas	Nord Pool	a day before the delivery hour	one hour before delivery
Balancing market	TSO	a day before the delivery hour	45 minutes before delivery
Financial market	Nasdaq OMX	10 years before the delivery	one day before delivery

Table 1: The different markets where Finnish power producers can operate in [3]

2.1 Day-ahead market

Elspot is a day-ahead market where most of the trading in Nord Pool takes place. In 2014, 361 TWh of power was traded in Elspot between the Nordic and Baltic countries, whereas only 4.9 TWh was traded in the intraday market Elbas [5]. The day ahead hourly prices of Elspot will be now on referred as the spot prices. There were around 360 participants in this market in 2014. They consist mostly of producers, TSOs, brokers and some large end users [5]. All players trading in Elspot need to be physically connected to the grid [3], and they must have a balancing power agreement with the local TSO [2]. The bidding process is as follows [2]. The buyers need to assess how much electricity they and their possible customers are going to consume in each hour of the next day, and decide how much they are willing to pay for each cumulative MWh. The sellers estimate similarly how much they are prepared to produce in each hour and how much they want for each cumulative MWh. Sellers

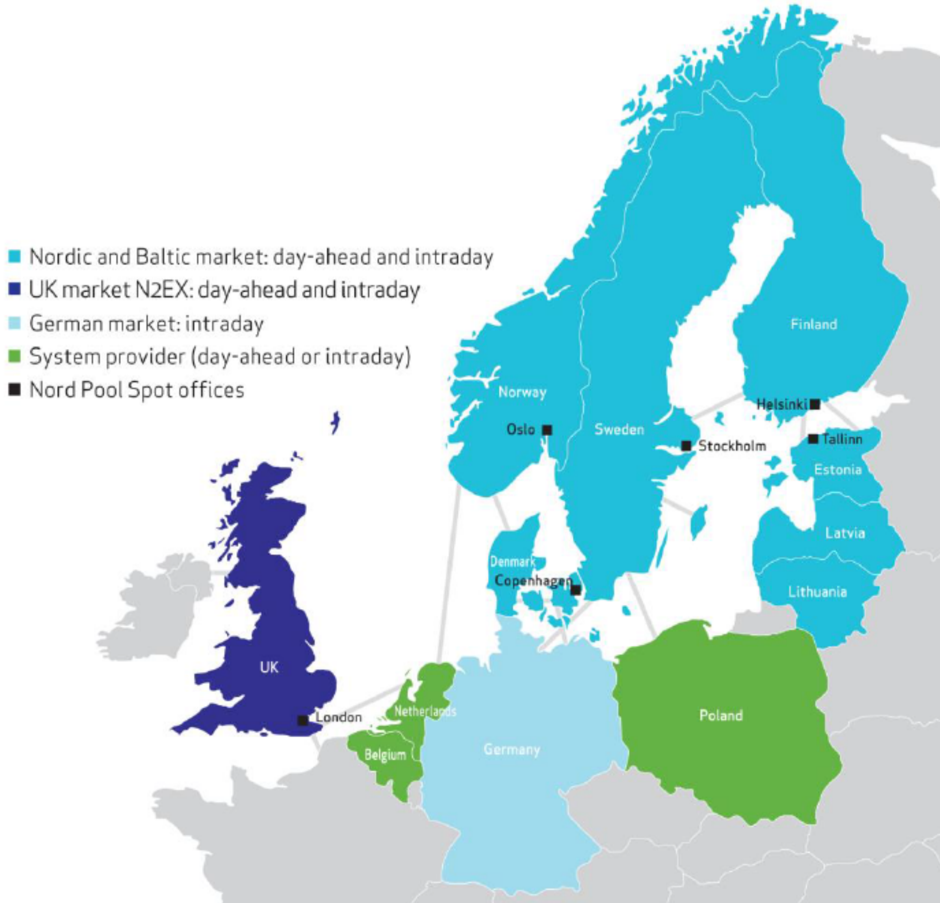

Figure 1: Power markets managed by Nord Pool Spot AS [2]. In this thesis, Nord Pool refers to the Nordic- and Baltic area.

and buyers place their bids for the next day by 12.00 CET. Then, for each hour of the next day, Nord Pool's algorithm calculates one price called the system price and an area prices for each individual bidding area and publishes them at 12.42 CET or later. How the system price and area prices are calculated is explained in [2]. The prices are formed by accepting the production bids starting from the cheapest ones until all of the demand is satisfied, or until no producers can be found, who are willing to produce the demanded amount of power for the offered price. The most expensive bid per MWh that was accepted determines the price per MWh for that hour. System price is a theoretical price, where the markets total supply and demand of the Nordic countries meets. Bids from the Baltic countries are excluded from this calculation. The system prices would realize if there were no transmission capacity limitations between areas. Area prices are calculated the same way, but the limitations in the transfer capacity between the bidding areas are taken into consideration. All sellers whose bid got accepted will receive the area price of that hour for each MWh they sold despite how low their own bid was. Similarly, buyers whose bids got accepted, need to pay the area price despite how high their bid was. Physical deliveries of electricity are done with area prices, whereas the system price provides a reference price for trading and clearing of financial contracts [2]. Possible transfer limitations within the bidding areas are not taken into consideration when the prices are calculated. If there is a bottleneck, the local TSO needs to pay compensation for all producers and suppliers, who are affected by it [4]. However, the local TSO can decide how many bidding areas is within their country. The more bidding areas there are, the less likely the TSO needs to pay congestion compensations. In 2015, Norway was divided into five areas, Sweden into four, Denmark into two, but Finland and the Baltic countries each had only one bidding area per country [2]. Nord Pool's algorithm tries to maximize the social welfare, by guiding the supplied electricity to where the demand is the highest [2]. Therefore, electricity will flow, under the transfer restrictions, from areas with low selling bids to areas with high buying bids. Because the area prices in Finland are usually higher than in other Nordic countries, power is often imported to Finland at the maximum capacity of the transfer lines. The consumers in Finland will pay a high price for the power, but the producers abroad, will not receive all of that money. They are entitled only to the area price of their physical location. The Nordic TSOs will share the difference between the two area prices, according to the agreement they have made [6].

2.2 Intraday market

Market players need to place their bids on Elspot the day before the actual delivery of electricity. Much can happen during the time between placing bids and the delivery. Power plants may break down, weather forecast can change, or large end users factories might shut down unexpectedly. If these unbalances in consumption and production are not mended, the frequency of the power system will deviate from its normal value, which within Nord Pool is 50 Hz [4]. If consumption exceeds production, the frequency will drop and if production exceeds consumption the frequency will rise. Elbas is an intraday market, where demand and supply can be balanced on an hourly basis after Elspot has closed. Elbas opens at 14:00 ECT for the next day and it is a continuous market, where bids can be placed up to one hour before delivery. The highest buying bids and the lowest selling bids will be served first. The volume of trade in Elbas is a fraction of Elspot and the price of power is higher than in Elspot for the price taker [2]. However, it is always less expensive to trade in Elbas, rather than taking the balancing power from the TSO [7].

2.3 Balancing power market

The TSOs of each country in Nord Pool are responsible for maintaining the balance of production and consumption. Fingrid does not have own sufficient production to balance the grid. Therefore, it maintains a balancing market, where market players can sell balancing power, by offering to rapidly increase or decrease their production or consumption [8]. In the Finnish balancing market, bids can be sent up to 45 minutes before the delivery. Fingrid then accepts the bids it needs to secure the balance of the grid, starting from the cheapest one. Parties who fail to produce or consume the amount of power they have sold or bought from Elspot and Elbas will be forced to purchase the balancing power from Fingrid. This can become quite costly for the buyer, because the price is always at least as expensive than the prices in Elspot [7]. For example, for one hour in the morning of 22.1.2016, up-regulating power costed 3000€ per MWh in Finland [2]. This system encourages producers to invest into reliable machinery so that they do not have to rely on the balancing market too often. Similarly, energy companies need to estimate the consumption of their customers as accurately as possible so that they can buy exactly the amount of power they require from Elspot and Elbas.

2.4 Financial market

Elspot, Elbas, and the balancing power market are physical markets, where the actual power is bought and sold. In Nord Pool, these contracts can not be made further into the future than the next day [3]. The short time span between the contract and delivery makes risk management though. Consumers and producers can only know for sure on what price power is traded during the next day. This challenge is aggravated by the fact that, Nord Pool's prices are known to be volatile and can have significant spikes [9]. Therefore, running a business that is heavily dependent on the price of electricity would be extremely risky if one would only rely on the physical market. To reduce the risk, market players can hedge themselves from the volatility of spot price for up to ten years into the future in the financial market. On the other hand, using the financial market to protect oneself from the price risk also prevents one from achieving large profits when the mean spot price moves unexpectedly to a more favorable direction. Therefore, before trading in the financial market, players need to assess carefully what level of price risk they want to expose themselves to.

The financial market for Nord Pool is a part of Nasdaq Commodities [10]. The importance of the financial market becomes apparent when its volume of trade is compared to the physical market. In 2014, the volume of trade in the financial market was 1497 TWh [11]. In Elspot, 361 TWh was exchanged in the same period [5]. The reason for why the financial market has so much more trade in TWh than the physical market is that market players can trade continuously for months and years before delivery. Therefore, they can react to changes in the market by making financial agreements at different times, which partly cancel each other out. They receive money from one agreement, which they use to pay another. The volume of traded is calculated cumulatively and adds up to an amount far larger than in the physical market. Details about the different types of financial agreements are given in [12]. Depending on the contract, market players will be compensated for certain price movements of the spot price. To reduce risk, they can in a way gamble against spot price movements that are favorable for them, so that, if the realization of the spot price is undesirable, they will lose money in the physical market, but they will be compensated in the financial market. If the spot price realizes favorably, they will get larger profits from the physical market, but they need to pay compensation to the other party of the financial agreement. Thus, after making a financial agreement with another party, the combined profits and losses on the physical and financial market will be roughly known, no matter how the spot price realizes.

In Nasdaq Commodities, forward contracts are called Deferred Settlement Futures and they can be made with a settlement period of individual months, quarters or years [12]. Individual months can be traded 6 months ahead. Quarter forwards are available for the next 8-11 quarters. Forwards covering an entire year are available for up to 10 years. Futures can also be traded in Nasdaq OMX and its details can be found in [12]. They are similar to forwards in that parties agree on a time period and a price per MWh to which the realized system price is compared to. The amount and direction of the cash settlement depends on whether or not the spot price realizes higher or lower than the agreed price. One major difference is that, during the agreed time period of a future, the financial settlements are paid daily, instead of at the end of the period, as is done with forwards. In a way, future agreements are a series of daily forward agreements. Futures can be traded similarly to forwards for the next 6 months, 8-11 quarters and 10 years [12]. In addition to these, daily futures can also be traded for the next 3-9 days and weekly futures are available for 6 weeks into the future. Forwards and futures use system price as reference price [12], but physical markets use area prices, in which the grid's limited transfer capacity is taken into account [3]. When the grid becomes congested, some area prices can skyrocket to multiple times larger than the system price. Therefore, only using forwards and futures to lower the prise risk, might not be sufficient. To hedge one self against the area price difference from the system price, one can use Electricity Price Area Differentials (EPAD) [12]. They are future contracts where the reference price is the difference between the area price and the system price. In Finland and Sweden, EPADs can be traded for the next 4 months, 4 quarters and 4 years. All available financial agreements available in Nasdaq Commodities are listed in [12]. EPADs and forwards allow companies such as VE, to hedge their power production quite sufficiently. However, some risk remains concerning the profile of the spot price. This profile risk will be described in section 3.

2.5 Characteristics of the spot price

Elspot is a more significant electricity market than Elbas or the balancing market. Therefore in the literature, and also in this thesis, spot prices refer to the hourly day-ahead power prices or their daily arithmetic means in Elspot. Spot prices exhibit behaviour such as mean-reversion, spikes and seasonality on daily, weekly and annual scale [13]. The main reason for why power spot prices are special is that electricity cannot be stored efficiently in large quantities, and therefore, production has to meet the inelastic demand every hour and second [1]. Demand fluctuates, with the people's somewhat deterministic daily rhythm, and the spot price follows. How much the spot price will change, when the demand shifts, depends on the composition of the available production and how the producers have priced their services.

2.5.1 Merit order

Merit order is a ranking, where all of the available producers are placed in order based on their variable costs from lowest to highest [14], as shown in Figure 2. On the horizontal axis, is the cumulative production capacity of all of the production methods, and on the vertical axis is their variable costs per MWh. The concept of merit order is introduced before listing the characteristics of the spot price because it explains some of them. As explained in [3], in Nord Pool, production methods are used starting from the method with the lowest price per MWh until all of the demand is satisfied, or until no producer is found who is willing to produce power for the offered price. The most expensive production method used to satisfy the demand determines the spot price of that hour [3]. All producers whose bids got accepted will get paid that amount for every MWh they produce. From the merit order, one can estimate how much the spot price would be with a given demand. However, merit order diagrams of this type do not represent the total production costs accurately, because short-term running costs depend on many factors such as fixed costs from turning on a unit and the outside temperature. Furthermore, the efficiency of some production units depends on the power output. Producers take these things into account when placing bids on to Elspot.

The shape of the merit order curve explains the inverse leverage effect. Inverse leverage effect means that the volatility of the spot price increases as the average spot price increases [15]. Let us suppose that the demand is same as in Figure 2, which means that the variable costs of the condensing coal plants determine the spot price. If the demand increases so that gas turbines need to be activated, the spot price can jump significantly. However, if the demand drops or additional production capacity from any cheaper production method is increased, the spot prices will quickly return to more normal levels as the gas turbines are no longer needed. Let us say that in another day, the most expensive production method in use is nuclear power. Even a significant increase in the demand or decrease in the supply will not cause as large jump because CHP is not that much more expensive than nuclear power.

2.5.2 Daily and weekly seasonality

The spot price has a strong positive correlation and causation with consumption, as can be seen from Figure 3. The use of electricity follows the daily rhythm of the people and production must change accordingly because power cannot be stored efficiently. As the demand increases, more expensive production plants are used. Normal consumers use the amount of power they want because usually they pay a fixed price per kWh, and the price risk is taken by the energy company who is selling the power to the customer. Therefore, there is not much price elasticity in the demand. People in general cook and take showers in the mornings, work during the days and sleep during the nights. This kind of behavior causes the spot price to often reach its highest value during the morning between 8 and 12 and then dip a little during 14 - 17. The second peak of the day often occurs around 18 - 21, when people are at home making dinner and using electric devices [4]. Naturally, consumption is low during the nights, which keeps the spot price low. Weekdays

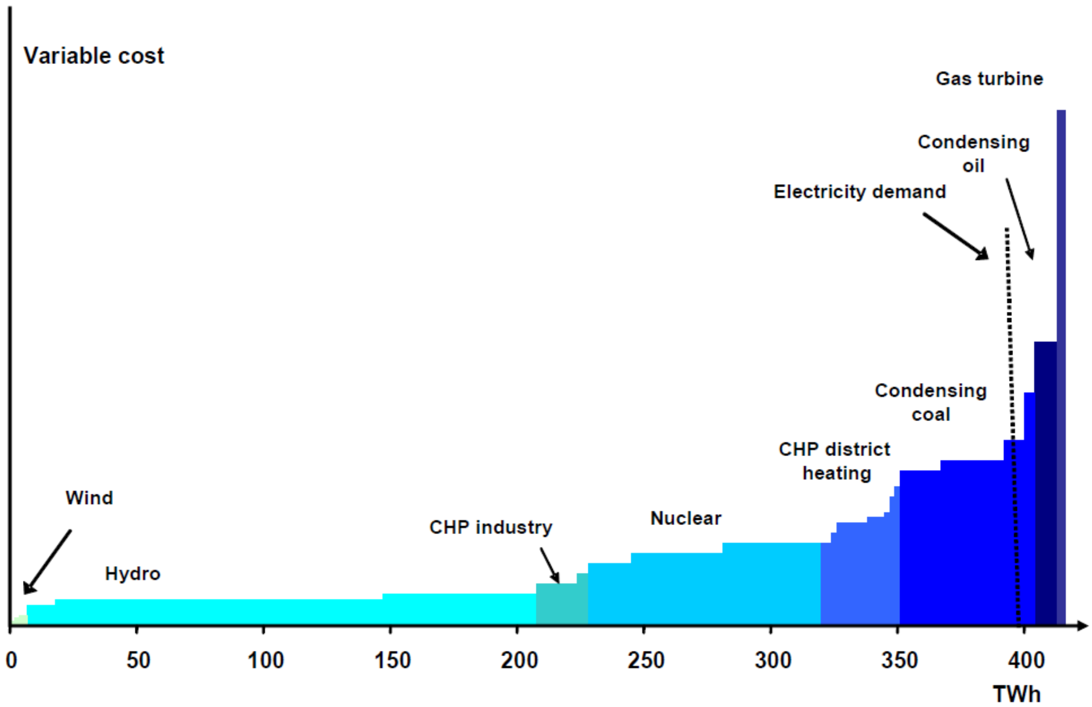

Figure 2: Merit order of the Nordic countries in 2008 [14].

are typically more expensive than weekends or public holidays. This kind of daily and weekly cycle is seen throughout the year in Finland, although it is not always as clear as in Figure 3.

2.5.3 Annual seasonality

In 2014, the average demand of power in Finland was around 10 GWh in the winter months and around 8 GWh in the summer months [4]. One might think that the spot price would be a lot higher during the winter than in the summer. By taking the average Finnish spot price of each month from 2001 to 2014, one can see that spot prices indeed are on average lower during the summer than during the winter. Therefore, annual seasonality must be incorporated in the spot price model. However, when examining individual years, this sinusoidal pattern is not always clear. A possible explanation to this could be the Nordic countries' large capacity of storable hydropower [16]. The ability to store water, partly evens out the highs and lows of spot prices, because it is in the best interest of the producers to save water during low prices so that they have extra supply available when the prices rise. This could explain partly why the price difference is not so dramatic between summer and winter. Furthermore, hydropower is not the only power production method, where the production capacity changes from season to season. Most power plants need to be regularly maintained, and naturally the longest maintenance breaks should be done when the losses of revenue are the smallest. Therefore, maintenance breaks are usually scheduled for the summer, when the spot prices are generally at their

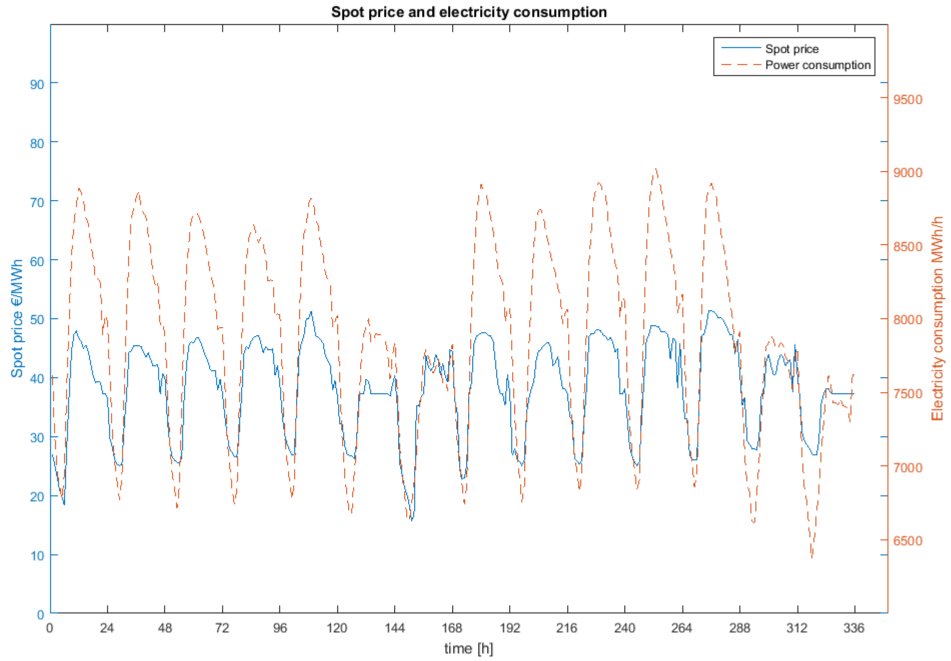

Figure 3: Finnish spot prices [2] and electricity consumption [4] during Monday 14.7.2014 - Sunday 27.7.2014.

lowest. Therefore, even though the demand is lower during the summer, the spot prices does not drop greatly, because also the supply drops.

2.5.4 Spikes and mean reversion

A price spike is a short period when the spot price rises to abnormally high prices. However, there is no generally agreed definition of what is a spike and what is not [17]. This might be because researchers have different needs. Some need more sophisticated methods to make sure they can identify the right kind of spikes and for others, simpler methods suffice. There are many methods, but we will list only a few. In some papers, prices exceeding a predetermined level are spikes [18]. In others, a certain percentage of the highest values are spikes [19]. Other use recursive methods where prices exceeding a certain amount of standard deviations are considered to be spikes and removed or modified [20].

There are also down-spikes, where the price drops significantly below normal values. Prices can drop mainly because of low consumption combined with windy conditions and high hydro levels. Price drops are short and rare events especially in Finland because it is a net importer of power. The transmission capacity into Finland is often congested, so excess supply is uncommon [4]. Therefore, even if the price plummets in Denmark or Sweden, who have plenty of wind power, Finnish area prices might not drop as sharply.

Spikes are a consequence of unexpected or unusual shifts in the demand and/or production. Exceptionally cold weather can increase demand, or a large producer might unexpectedly shut down. When more and more expensive production methods are used to meet the demand, the spot price rises. Usage of production method at the end of the merit order is costly, because they are used irregularly, and they need to pay themselves back to the investors. Furthermore, they need to be able to be activated quickly, which means that only certain type of production methods such as gas turbines can be used. They happen to have high variable costs. Spikes tend to be short lived because the high spot price is a great incentive for producers to increase their production in any way they can. Also, some large end users might shut down their energy intensive operations to save money. Together, these actions will bring the spot price eventually to normal levels.

3 The case company

This thesis is made for Vantaan Energia (VE), which is a producer, retailer and a distributor of power and heat. Some basic statistics can be found in their financial statement available in [21]. In 2014, VE's total income was 274m€, from which 158m€ came from power and 104m€ came from heat. VE produced 1033 GWh of electricity, from which 51% came from own production and 49% from co-owned production facilities. However, VE sold 3237 GWh of electricity to its customers, which meant that they had to purchase about 2204 GWh from the physical market. This makes VE also a significant retailer of power. In addition to power, VE produces and distributes district heat. In 2013 VE sold 1647 GWh of heat and steam energy. Heat is sold to customers in and near Vantaa. Power is sold to a wider area, and actually 64% of all sales are from outside of Vantaa [21].

The city of Vantaa owns 60% of the company, and the city of Helsinki owns the remaining 40%. Vantaan Energia OY is the parent company of Vantaan Energia Sähköverkot OY, which maintains the electricity networks in Vantaa. VE is a co-owner in several other companies, which produce power in the Nordic countries. For example, VE owns 49.6% of Svartisen Holding A/S, which owns a large hydropower plant in Norway. The hydropower plant produced 206 GWh of power for VE in 2014, which is a lot, compared to 527 GWh, which was produced by VE's two CHP plants in Finland. These two CHP plants are fully owned by VE and they form the core of their energy production capacity. The power production of these two CHP plants is what we are attempting to forecast in this thesis.

3.1 Own and co-owned production

VE does not, in general, control its co-owned production facilities. Instead, some other party optimizes the production, and all the owners receive a certain share of the power. This kind of arrangement ensures that the plants' capacity is used as efficiently as possible as a whole. All of VE's nuclear power, wind power and small shares of hydropower are produced like this.

VE's own heat and power production comes mainly from two CHP plants in Vantaa. The first one is a coal powered plant in Martinlaakso. In 2014 it produced 409 GWh of power and 973 GWh of heat while consuming 1198 GWh of coal, 369 GWh of gas and 16 GWh of oil [22]. VE opened a new waste-to-energy (WtE) plant in 2014, which can produce 600 GWh of power and 920 GWh of heat annually [22]. However, in 2014 it produced 114 GWh of power and 545 GWh of heat while using 665 GWh worth of municipal waste, 113 GWh of gas and 1 GWh of oil [22]. The output was not near full capacity, mainly because of two reasons. Firstly, the plant did not operate a full year. The first experimental batches were burned on May. Secondly, the low spot prices kept the power production down. Power can be produced at the WtE plant together with heat by burning waste, but the main capacity of power production in this facility is in the gas turbines. However, low spot prices made power production with gas unprofitable for most of the year.

In addition to these plants, VE has several smaller heat and steam plants, which mostly run on gas. Their combined maximum thermal power was 548 MW at the end of 2013 and in 2014, their individual output varied from 0 to 121 GWh [22]. They do not produce any power.

3.2 Spot price risk

In this thesis, one of the goals is to forecast the total power production quantities of the two CHP plants, introduced in section 3.1, for months and years into the future. Their production quantities depend heavily on the future spot prices, which are unknown. Obviously, other factors, especially the temperature, affects the production quantities, because both plants produce heat and power. However, in this thesis, we will be focusing on simulation of the spot prices. Therefore, we will be using a single simulation of temperature and leave the simulation of temperature for future research.

Let us define price risk as the undesired realization of the monthly mean spot prices, compared to current expectations. We are examining it from the perspective of a power producer. If the mean spot price of the following months and years realizes lower than expected, the risk realizes. Fortunately, a producer can partially protect oneself from this risk, by using the financial market. As explained in section 2.4, market players can protect themselves from unfavorable movements in the mean spot prices by forming forward agreements.

However, financial agreements cannot protect a power producer from the another risk involving the spot price, which is the profile risk. Let us define profile risk as the undesirable realization of the hourly profile of the spot price, with a given mean spot price. The profile risk is realized when VE is not able to take full advantage of the spot price mostly due to start-up time constraints or inconvenient outside temperatures. Start up time constraints prevent production units to produce power when necessary. Therefore, if a spot spike is too short, certain production units cannot be turned on at that time, and the full benefit of the increased spot prices is not realized. This is unfortunate because the price spike will increase the mean spot price of that month, which decreases the amount of money VE will get from the financial agreement. Another way how the profile risk can realize has to do with the way spot price and temperature curve interact. In a CHP plant, power is ideally produced together so that the waste heat from power production is utilized for district heating. Because the variable costs are shared by power generation and district heating, VE can offer to produce power with lower spot prices when it is cold outside, and the demand for district heat is high. Therefore, their bids will be more likely to be accepted, which means that they should produce more power. When the demand of heat is low, all of the waste heat of power cannot be utilized, which means that the efficiency of the plant decreases and VE must increase the price that they want for each MWh of power. This decreases the likelihood of the bids going trough. Let us concider a month, where the first half is cold and the second half is warm. In the first scenario, the spot price profile is such that it is high in the first half and low in the second half. In this scenario the amount of power produced is large and so are the profits. In the second scenario the temperature is the same and so is the mean spot price, but now the spot prices are low in the first half and high in the second half. The profile risk realized in the second example. Now the amount of power that is generated is most likely lower than in the first scenario. These were just a few examples of how VE's production machinery has advantages and disadvantages. Each production unit works optimally under certain conditions and less optimally under some other conditions. Whether or not the profile risk realizes, depends on the spot price, temperature, fuel costs and other variables and parameters. The existence of the profile risk is why multiple simulations of spot prices should be used, when production quantities, profits, and other relevant variables are forecasted. Ideally, in addition to the spot price, other inputs of the program would also be simulated, but that is left for future research. In section 3.3, we will be discussing how VE, as a power producer, can protect it self from the risks concerning spot prices.

3.3 Risk management

The key risk management tool against the decrease of the mean spot price is forming of forward agreements, which can effectively lock in the future selling price of a predetermined amount of power. In this section, we explain how the uncertainty of the price profile makes forming of financial agreements difficult.

As explained in section 3.2, the mean spot prices of days, month and longer periods are not sufficient information for accurately estimating the future power production quantities and profits. VE's production capacity produces more power, with a certain price profile, than another, even if the arithmetic mean of the spot prices were the same in both cases. This is unfortunate because forward agreements are cleared using the mean spot prices [12]. Furthermore, a quantity of power needs to be specified by the parties making the forward agreement. The uncertainty about the profile of the future spot price makes it difficult to estimate the expected power production quantity. Let us assume that the current forward price of one MWh of power in month m is currently F_m€/MWh, and VE believes that it is the expected mean spot price of month m. They might want to reduce the risk of declining spot prices by hedging a certain percentage of their expected production quantity in month m. Therefore, they need to estimate how much power they will produce in that month if the spot price is on average F_m€/MWh. This can be done, for example, by generating a single realization of the hourly spot price $\mathbf{H} = (H_1, H_2, ..., H_H)$ which arithmetic mean $\overline{H}$ is F_m€/MWh. The spot price curve is then inputted to the production schedule optimization program, with several other inputs, such as the hourly temperature, fuel costs per MWh and maintenance schedules. However, we will ignore them for now. The program gives several outputs, from which one is the optimal production quantity Q_m of month m, assuming that the spot price realizes as $\mathbf{H}$. The problem here is that, if only one realization of the spot price is used, how confident can we be about Q_m? The realization $\mathbf{H}$ might be exceptional in some way and cause an unusually small or large production quantity. To get a more robust estimate of Q_m, we propose of simulating S realizations of the spot price whose mean is close to F_m€/MWh. Then, they are inserted individually to the production schedule optimization program, which calculates an optimal production quantity Q_m^s for each simulation of the spot price $\mathbf{H}^s, s = (1, 2, ..., S)$. The more robust, expected production quantity $\overline{Q}_m$ could be for example

$$\overline{Q}_m = \frac{1}{S} \sum_{s=1}^{s=S} Q_m^s, \tag{1}$$

where $\overline{Q}_m$ is the arithmetic mean of all of the production quantities given by the optimization program. If the set of simulated spot prices $\mathbf{H}^s, s = (1, 2, ..., S)$ contains the right number of different kinds of simulations, then $\overline{Q}_m^s$ can be considered as sort of a expected production quantity of month m. "Kind" refers to the characteristics of the spot price such as volatile, flat, decreasing trend or increasing trend. Once we acquire a robust estimate Q_m of the production quantity of month m, the forward agreements can be made with more confidence. This increases the chances that the risk policy of the company is followed more accurately, which means that the company can expose itself to the level of prise risk it wants. Obviously, the probability of production quantity $\overline{Q}_m$ realizing is small, because there are many uncertainties about the inputs.

4 Production planning and forecasting

In this section, the production schedule optimization program will be introduced. As any energy company, VE needs to make plans and decisions years, months, days and hours before delivery. Some decisions need to be made years in advance of delivery so that there is enough time to execute the plan. Other decisions, need to be postponed minutes, hours or days before delivery because relevant information is not available any sooner. Planning is often divided into three different classes, which are long-, medium- and short-term planning [23]. In energy production business, long-term planning covers the period from a few years to decades forward [23]. Long-term planning composes of strategic decisions, such as making large investments, like building new production facilities. Medium-term planning covers the period from a few weeks to few years into the future [23]. These are decisions such as fuel acquisitions, maintenance scheduling and market risk management. One of the most important goals of market risk management for VE is to ensure stable profits so that budgeting for the next year can be done with confidence. Stable and predictable profits can be achieved with successful hedging. Short-term planning, however, covers the time from present to few weeks into the future [23]. Short-term planning includes various operational decisions such as sending bids to the physical market and creation of accurate production schedules for each production unit.

We will focus on medium-term planning and especially on hedging, which requires estimations of the future production quantities by using the production schedule optimization program, which is introduced in sections 4.1, 4.2 and 4.3.

4.1 Production schedule optimization program

The production schedules are made using an in-house optimization program, which cannot be covered here in detail. Instead, only a short introduction is provided. It calculates the optimal hourly production schedule, which minimizes costs while satisfying the demand for district heat. Inputs include time series and parameters such as the hourly spot price, hourly temperature, fuel costs per MWh and maintenance schedules. The heat consumption is calculated from the inputted hourly temperature forecast, by utilizing the known habits of each customer. Profits gained from power production are treated as negative costs, which means that power is produced only when it is beneficial, where as heat demand must be satisfied despite the costs. From the schedule, different statistics can be calculated. These are, for example, production quantities of power and heat, fuel consumption, costs and profits. This information is crucial for fuel acquisition, market risk management and budgeting. The limitations of the production capacity are taken into consideration in the program. They are mostly physical restrictions of the production units, such as minimum/maximum loads and ramp up/down times. A typical period that is optimized is one year, but several years can be optimized in a row. Short-term planning is made using a different method.

4.2 Inputs

When creating an optimal hourly production schedule, the program needs to know various things such as the hourly spot price, temperature, fuel prices and maintenance outages. Some of the inputs such as the spot price, temperature and maintenance outages are given at an hourly resolution. Others such as fuel prices and carbon taxes are given at a monthly resolution. There are also dozens of parameters, which describe the limitations of the production units and tell how efficiently fuel is converted into heat and power. These parameters require updating, only when there are changes in the machinery or fuels. The previously mentioned inputs have a direct effect on the schedule. Any changes in them might change for example, how much power is produced. In addition to these, there are parameters such as the selling price for district heat. They do not have an effect on the optimal schedule, but they do affect outputs such as income and profits. How much money VE gets from each kWh of heat it sells to a customer, is not relevant information when the production schedule is optimized. The demand of heat must be satisfied in the most cost efficient way, and the selling price of heat is not relevant information. However, these parameters are needed so that expenses can be allocated and profits calculated.

Next, the sensitivity of the optimal production schedule to different inputs is discussed. Fuel costs affect the variable costs directly, which means that even a small change in them, can significantly affect the outputs of the program. However, fuel costs are quite stable, and their costs can be stabilized by hedging them with forwards. Therefore, the most important and influential inputs are the spot price and the temperature. They are also harder to produce, than the other inputs, because they are required to be forecasted at an hourly resolution. There is also much uncertainty and volatility involved in them, compared to many of the other inputs. To keep matters relatively simple in this thesis, all the other inputs, except the hourly spot price, are maintained at their original values when the different optimal production schedules are created. This means that for example the temperature curve has realistic down-spikes and other characteristics, but it remains the same during each run of the optimization program while the spot price curve is changed. This is done so that the effect of the profile risk of the spot price can be studied with a reasonable number of simulations. However, we must acknowledge that the profile of the hourly temperature curve has a large effect on how the plants are run. Therefore, we suggest that if further study is done on the matter, the temperature curve should also be simulated. The exact values given to the inputs are not presented in this thesis for confidentiality reasons. Only the monthly means of the spot prices are given, and a few examples of the price curves are plotted.

In sections 4.2.1 and 4.2.2, the impact of temperature and spot price to the optimal production schedule is discussed. Section 4.3 explains how using multiple simulations of the spot price can result in more robust forecasts of production quantities and profits.

4.2.1 Temperature

Temperature affects the production schedule in two ways. First, the outside temperature affects how efficiently the combustion power production units convert fuel to power. The colder it gets, the more power can be generated with the same amount of fuel. Second, and more importantly, outside temperature affects the heat consumption of the district heat customers. The hourly demand for heat is calculated for each customer separately, based on their consumption profiles and the outside temperatures. VE has data of the historical consumption rates of each customer, form which they can make a profile, which can be used to calculate the approximate heat demand of the customer for a given temperature during each day of the week and hour of the day. The demand for district heat has a large effect on how much power can be produced efficiently because power and heat are ideally produced together. The variable cost of producing power and heat is low when all of the waste heat of power production can be used in district heating. If the temperature is high, so that all of the waste heat cannot be utilized, the optimal production quantity of power will be low because the spot price will be less often above the variable costs of power production. In this thesis, the temperature curve will be kept the same when optimal production schedules are calculated while the spot price curve is changed. However, to get even more robust forecasts, the temperature curve should also be simulated.

4.2.2 Spot price

There are several reasons why the spot price is the most interesting and most significant input of the optimization program. First, it has a large effect on the production quantities and profits of the company. Secondly, it is volatile, which produces a significant risk. Thirdly, some of the risk concerning spot price can be managed with financial agreements. Therefore, something can be done to the risk, with market risk management tools. In contrast, the risk concerning temperature cannot be managed similarly, because there is no financial market for that purpose. These reasons justify why this thesis revolves around forecasting and simulating future spot prices rather than simulating temperatures or other variables.

In this thesis, several realizations of hourly spot prices are made, once the expected mean spot price F_m of each month is given. These monthly means are given by experts. However, even if the monthly mean of the spot price would be forecasted correctly, the exact shape of the price profile is unknown. Therefore, we suggest using several different simulations of the spot price, which monthly means are close to F_m. Each simulated time series of the spot price has a different number of price spikes and down-spikes and the highs and lows of the spot price occur at different times. This simulates the random shorter-term volatility of the spot price, which is mainly caused by exceptional weather or outages of large power producers. In some simulations, the spot price curve realizes favorably with the temperature curve, which means that the CHP-plants can be used efficiently, and the production quantities of power and the profits of VE should be relatively high. In other simulations, the spot price curve will be less favorable, which leads to smaller profits and different production quantities. If only a single simulation of the hourly spot price would be used to forecast the future, it could have a profile, which leads to exceptionally large or small production quantities. Therefore, multiple simulations are used to increase the robustness of the forecasted production quantities, profits, fuel consumption and other important variables.

4.3 Outputs

The main goal of this thesis is to provide a simulation method which can be used to produce different realizations of the spot price so that the future production quantities can be more robustly estimated, for hedging purposes. However, in addition to production quantities, the outputs of the program include values such as fuel consumption and profit. By taking arithmetic means of these outputs calculated using multiple different price curves, we assume that more robust forecasts of them can be made, compared to taking only the outputs of a single simulation. This should mean that the risk involved in fuel consumption and profits should reduce. Profit is an important variable for several reasons. Firstly, if profit can be estimated more accurately, budgeting becomes easier. Secondly, it acts as a reference point. For example, the monetary value of a possible modification of the production machinery can be measured. For example, if VE is considering changing a burner A to a more expensive burner of type B. Burner B could, for example, decrease the cold start time of a production unit, or it could increase its maximum power output. Hundreds of different realizations of the spot price can be simulated and inserted into the optimization program, which production unit parameters are set for burner A, and relevant statistics can be gathered from the outputs. Arguably, the most important one of which is the average profits. Then, the process is repeated, but the parameters are changed to match the characteristics of burner B. The performance of the two burners, will most likely be pretty similar in most simulations, because the ramp up time, or maximum capacity are not necessarily limiting factors during each day. However, when enough different simulations are used, differences between the two components should eventually become visible. Then one can calculate, is burner B a worthwhile investment or not. If only one realization of the spot price is used, the benefits of burner B can be under- or overestimated.

5 Electricity price forecasting

In this section, the pretreatment and decomposition of the spot price into different components are introduced and popular models for modeling the spot price are presented.

Before model parameters can be estimated from the data, the data should be pretreated. The need for pretreatment depends on the model, but it includes among other things deseasonalizing, detrending, removal of outliers and taking the logarithm of the prices [1]. As stated in [24], often the first thing that is done, is to take the logarithm of the spot prices. However, we will be using the actual spot prices instead. The reason behind this decision is that we are using a Markov regime-switching model (MRS) and [13], [24] and [25] suggest that MRS models might perform better with actual prices than their logarithms. Outliers are errors in the data or rare uncharacteristic behaviour of the spot price [17]. These could be extraordinary spikes and down-spikes of spot price or times when the volatility was extremely high or low. If those data points are not removed or modified, they can interfere with the calibration of the models. However, spikiness is a characteristic of the spot price and if one wants to capture it in the model, one should not remove all of the spikes, but limiting the spikes in some way might be beneficial [17]. Some models such as jump diffusion and time series models require the data to be deseasonalized and detrended before the model is fitted to the stochastic part of the spot price [1]. This requires the identifying and removal of the deterministic trend and the seasonal components. Therefore, spot price P_t is often divided into the deterministic component f_t and the stochastic component X_t [26], so that

$$P_t = f_t + X_t, \qquad t = (1, 2, ..., T). \tag{2}$$

The term f_t can be divided further into a long-term seasonal component (LTSC) L_t and the short-term seasonal component (STSC) S_t

$$f_t = L_t + S_t, \qquad t = (1, 2, ..., T). \tag{3}$$

LTSC is supposed to capture the annual seasonality and STSC is expected to capture the seasonality within a week. If the data is in hourly resolution, a daily seasonality can be added or incorporated to the STSC. However, in this thesis, we will be simulating daily spot prices, which are turned into hourly data with a method called Historical Profile Sampling (HPS), which is presented in section 6.6. Therefore, we do not need to model the daily seasonality into the STSC. The annual component is often estimated by fitting a sinusoidal function, with a linear component, to the daily spot prices [17] [27]. This function should capture the expected oscillation between winter and summer prices and the long-term linear trend of the spot price. Other popular options for the LTSC are wavelets, moving average or piecewise constant functions [1]. We will be using a piecewise linear function for the LTSC and a dummy variable function for STSC. These models will be introduced in sections 6.2 and 6.3 respectively. Whereas the models for the seasonal components are often quite simple, the models for the stochastic component are often much more advanced

[1]. Although, in some models the deterministic component and the stochastic component are not clearly separated. In the next section 5.1, we will be introducing some popular methods of modeling the spot price.

5.1 Different electricity price models

Typical models that are used to simulate and forecast electricity prices are presented in Figure 4. Some of the models are used to simulate the spot price and some are used to model only the stochastic component. Each of the models has their advantages and disadvantages and for this reason, they are sometimes combined into hybrid models, so that the best possible outcome can be achieved. The information in sections 5.1.1 to 5.1.5 is mostly acquired from [1], which is a comprehensive article about different methods used for electricity price forecasting. We give a much more detailed description of the reduced-form models in section 5.1.5, than any other type of model. The reason is that we chose to use a reduced-form model, and more precisely, a Markov regime-switching model to model the stochastic component of the spot price.

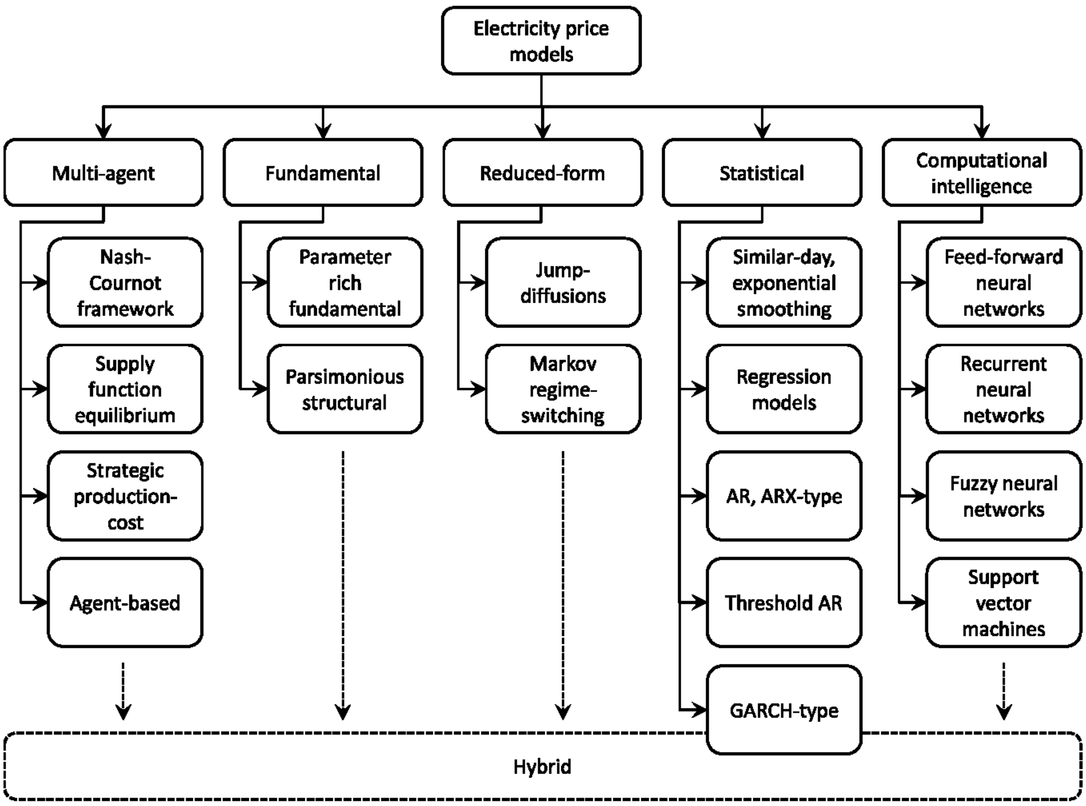

Figure 4: Different types of spot price models [1].

5.1.1 Multi-agent

According to [1], multi-agent models use the capacity and strategy of the market players to find the market equilibrium, where supply meets demand. Nash-Cournot framework models use algebraic equations, whereas supply function equilibrium models use differential equations. This makes the latter cumbersome to calculate, and simplifications such as linearization of the demand and supply curves are sometimes done to ease the calculations [1].

Strategic production cost models are even more simplified because they do not necessarily aim to find the Nash equilibrium, where no producer can profit from deviating from it alone [28]. Instead, the bid of each producer is calculated based on their individual production costs and a strategic parameter. The parameter is the slope of the residual demand function for each amount of production for that company [1]. Therefore, the model does not consider the chain of actions and reactions to each players strategies, when determining the equilibrium, which makes the method fast enough to use for real time analysis [1].

All three multi-agent models have limitations on how competition between players can be modeled [1]. Furthermore, the static equilibrium is often hard to solve, and heuristics are required in finding it. Agent-based simulation models (ABS) are used to work around these limitations. The interaction of possibly heterogeneous agents, with different strategies and ways of influencing and affecting each other's decisions and actions can be modeled trough simulations. Each party needs to forecast the impact of their decisions to the system and determine the best course of action. [1]

The strength of these four models lies in their flexibility, which is especially found in ABS [1]. These models can provide excellent means to simulate the outcome of the market. However, defining the strategies for all parties can become a burden because the role and characteristics of each player are not necessarily clear. Producers are often also buyers and retailers of power. They can have several different production methods, and their capacity and restrictions can be unknown. Creating an accurate model of the market and its players can be therefore difficult.

5.1.2 Fundamental models

In [1] the following description of fundamental models is given. Fundamental models are often complex hybrids of several models. What fundamental models have in common, is that they try to identify and model the behaviour of the fundamental determinants of spot price and then use them to form the spot price. The fundamental determinants should be appropriately interdependent, for example, with feedback loops. Popular basic determinants are temperature, consumption, fuel prices, hydro levels and past spot prices. The determinants can be further divided into components until an appropriate level of precision is achieved. For example, the temperature of different areas could be modeled separately instead of using one average temperature. Also, the hydro levels of each region could be dependent of the temperatures. Parameter rich fundamental models take this idea quite far and can have dozens of variables. The variables are functions of each others and stochastic components so that the causality between the variables is captured. Building and fine tuning such a model can take months or years, and because the world is ever changing, the model is newer finished. Production, consumption, fuel prices, transfer capacity and all the other determinants keep changing. Therefore, the model needs to be regularly updated to keep it relevant. Due to the complexity of the model, advanced optimization algorithms may be needed to calculate the spot prices within a reasonable time.

Another subgroup of fundamentals models discussed in [1] are the parsimonious structural fundamental models. They are often simpler models than parameter-rich models, and their defining character is that they use the supply and demand curve to calculate the spot price. The complexity of these curves varies. Some just pick them at random from a predetermined set, while others construct them according to determinants such as available capacity or fuel costs and the cost of carbon emissions [1]. Structural models can be quite good at capturing the behaviour of the spot price while keeping the model simple enough that the complexity of the model is not overwhelming. Fundamental models are more often used to predict daily rather than hourly prices, which is understandable due to the amount of work one would need to do to build an hourly fundamental model [1].

5.1.3 Statistical modeling

In [1], Weron stated the following of the statistical models. Statistical models are fitted to the historical data of the spot price and possible other exogenous variables such as fuel costs, weather and hydro levels. The spot price is predicted with its own past values and possibly with exogenous variables and statistical components which follow certain distributions, which parameters are estimated during the fitting process. Weron divided statistical models into similar-day- and exponential smoothing models, regression models and several different time series models. The simplest models are similar day- and exponential smoothing models. They use the average of prices from the history to predict the future prices. Naturally different components can be added to to make the models more advanced, but their simplicity brings a distinct advantage. They can be used as benchmarks. More advanced models should outperform the simple ones before their usage can be justified.

Regression methods are one of the most common statistical methods. In the basic regression model, spot price is calculated as a weighted sum of current or previous values of other variables and a stochastic component. The parameters are then calibrated by fitting the model to historical data and using the least squares method. Complexity of regression models can be increased by, for example, making them nonlinear, autoregressive, regime-switching or seasonal [1]. Their advantage and disadvantage is their simplicity. They are easy to understand and use, but cannot provide accurate forecasts of the spot price is all cases.

Auto Regressive Moving Average (ARMA) is a time series model, where the spot price is the weighted sum of its previous values and the previous and current noise values. The weights and the noise parameters are chosen by fitting the model to historical data and using, for example, the most likelihood method or the prediction error method. Spot price should depict at least weekly seasonality, and that can be acchieved by adding a seasonal component to the ARMA model making it a SARMA model. For example the spot price could be dependent of its last six previous values and the spot price of the same hour during the previous week. However, forecasting spot price with its past values has its limitations. For example sudden and large price spikes, will not occur unless the volatility of the model is increased to unrealistic levels. Therefore adding exogenous variables such as production capacity or fuel costs can help to achieve realistic variability of the spot price. These are called ARX and SARMAX models. Lags can also be added to the model so that the first few previous values of variables are omitted, if they are not relevant. For example, a drop in the temperature might not necessarily lead to an instant increase in demand and price of power.

Time series models are versatile, and they can be combined with other types of models to create even more advanced models. Their advantage is that no fundamental understanding of the power market is necessarily required to use them. However, they do have disadvantages. Calibrating the parameters of the model requires a long history of spot prices. Furthermore, the times series might not be applicable as it is, because most time series models require the data to be somewhat stationary. Therefore, trends and outages need to be removed or replaced with more normal values, and there is no strong consensus on how that should be done [1].

5.1.4 Computational intelligence

The most common computational intelligence (CI) models that are used to model the power market are artificial neural networks (ANN), fuzzy neural networks and support vector machines (SVM) [1]. CI models in general handle complexity and non linearity better than other models mentioned in section 5. They are able to this by utilizing learning, fuzziness or evolution [1].

Artificial neural networks are divided in [1] into feed-forward and recurrent neural networks. In [1], the differences between these two are described to be as follows. They both have one or more inputs and output nodes and possibly hidden layers. Nodes take information and modify it before sending it onward. The programmer chooses a learning algorithm, which determines how the weights of the network adapt to inputs. The difference with feed-forward and recurrent neural networks is that in the first type there are no loops, but in the second one there are. This difference makes feed-forward networks better at forecasting and recurrent neural networks better at pattern classification. ANNs can be used with fuzzy logic. For example, the inputs or the outputs can be pre-categorized with fuzzy logic or the learning algorithm could utilize fuzzy logic. The last CI model mentioned in [1], is support vector machines (SVM). SVMs are good at categorizing and at non-linear regression. SVMs bypass the problem caused by non-linear regression, by mapping the input data to a higher dimensional space. By utilizing the extra dimension, it is possible to find a linear function which approximates the nonlinear dependency of the input data variables [29]. SVMs have been reported to be less prone to over fitting than ANNs [30]. As a conclusion, CI models can model non-linearity well, but they are a diverse group so finding the optimal model to use is difficult [1].

5.1.5 Reduced-form

A reduced-form model is used to simulate the stochastic component of the spot price in this thesis. Therefore, a much more detailed description of the reduced form models is given here, compared to previous models. The exact model that is used, is a Markov regime-switching (MRS) model with three independent regimes.

Structures of reduced-form models are relatively simple. Unlike fundamental models, they do not explicitly formulate how different relevant variables affect the spot price. Instead, simpler 'reduced form' components are used, which attempt to replicate the main characteristics of the spot price [1]. Jump-diffusion models and Markov regime-switching (MRS) models are the two main categories of reduced form models introduced in [1]. Jump diffusion models, which are used for modeling of the power spot price, are based on this stochastic differential equation

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t + dq(X_t, t), \tag{4}$$

where X_t is the stochastic component of spot price, $\mu(X_t, t)$ is the drift term, dW_t are the increments of standard Wiener process and $dq(X_t, t)$ are increments of the pure jump process [1]. As can be seen from equation 4, components can be made dependent of the value of X and time t. This is useful because it is a known fact that volatility of the spot price increases as the spot price rises [15]. However, volatility and the frequency of spikes are often set to a constant level to keep the model simple [1]. One way of capturing the mean reversion of the spot price, is to set the drift term $\mu(X_t, t)$ into $(\alpha - \beta X_t)$ [1]. The model presented in equation 4 is a continuous model, which fits in the world of stock exchanges, where the stock price changes continuously. However, in Nord Pool, the price of power is fixed for each hour. Therefore, discrete models are a natural way of modeling the spot prices. An example of a discrete mean reverting jump-diffusion model is given in equation 5.

$$dX_t = -\alpha(\mu - X_{t-1}) + \sigma\epsilon_t + \sum_{i=0}^{n_t} Z_t, \tag{5}$$

where $\epsilon \sim N(0, 1)$, $Z_t \sim N(\mu_z, \sigma_z)$ and $n_t \sim POI(\lambda)$ [31]. The larger the α is, the faster the process will revert to its mean after deviating from it. The problem with this simple model is that the same α is responsible for mean reversion after a spike, and during normal random walk close to the mean [31]. If α is large enough to bring down the spot price after a spike, it might prevent the normal variation around the mean. If α is small enough to allow the normal variation around the mean, it might not be able to bring down the spot price fast enough after a spike. In reality, spot price returns quickly close to its long-term mean after a spike [32]. Therefore, a simple jump-diffusion process is not ideal for simulating spot prices. However, Markov regime-switching models (MRS) can change their dynamics momentarily, which enables simulating spot prices, with spikes that dissipate quickly [31]. MRS-models have r regimes, where the characteristics of the simulated spot prices are different. We will be using an MRS model to simulate daily spot prices. For this reason, we will start using notation d instead of t to display time. The spot price can be, for example, modeled with a following parameter changing MRS, where the spot price is modeled with the same random processes, but the parameters are different in each regime

$$X_d = \alpha_i + (1-\beta_i)X_{d-1} + \epsilon_{d,i}, \qquad i = (1, 2, ..., r) \tag{6}$$

where $\epsilon_{d,i} \sim N(\mu_i, \sigma_i^2)$ and i is the regime the system is in day d [1]. Let us define a variable $R_d = (1, 2, ..., r)$, which expresses on which regime the system is in day d. Popular choices for regimes are base ($R_d = 1$), spike ($R_d = 2$) and down-spike ($R_d = 3$) [1]. The same regimes will be used in the model that is used in this thesis. During the base regime, volatility and mean reversion are moderate, but when the system enters the spike regime, the mean of the spot price rises, and so does the volatility. Down-spike regime can be used to bring the spot price down after a spike, with high mean reversion. Down-spikes can also be used to model exceptionally low prices. A latent Markov chain R_d determines the regime the system is in at any time. Its transition probabilities are stored in a Markov transition matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1r} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{r1} & \pi_{r2} & \dots & \pi_{rr} \end{bmatrix}, \tag{7}$$

where π_{ij} is the transition probability from regime i to j during the next time unit $\pi_{ij} = p(R_{d+1} = j | R_d = i)$ [33]. The system needs to stay in the same regime or move to another regime during each time step, which means that $\sum_{j=1}^{r} \pi_{ij} = 1, \forall i$. If the current regime $R_d = i$ is known, and there are three regimes, the regime of the next time unit R_{d+1} can be produced according to equation 8.

$$R_{d+1} = \begin{cases} 1, & \text{if } r_d \leq \pi_{i1} \\ 2, & \text{if } \pi_{i1} < r_d < \pi_{i1} + \pi_{i2} \\ 3, & \text{if } \pi_{i3} \leq r_d \end{cases}, \tag{8}$$

where r_d is a uniformly distributed variable between 0 and 1. The initial state R_1 needs to be defined by the modeler.

Although parameter switching MRS models can change their mean reversion level, they are still not able to reproduce quick changes between regimes. The transition between regimes is not instant, because of the autocorrelation in equation 6. If the previous value X_{d-1} was from the spike regime, and X_d is in the base regime, the high X_{d-1} keeps the value of X_d somewhat higher than is characteristic for the base regime. It might take several time steps until the effect of the spike regime becomes insignificant. However, if the regimes are modeled independently of each other, the transition between regimes, will cause an instant change in the dynamics of the spot price. In the power market, spikes are caused, for example, by the sudden outage of a major power producer. The spot prices should return to normal level starting as soon as they are able to fix the problem. The spot price model should be able to replicate this kind of behaviour. [1] defined independent regime process X_d as follows

$$X_d = \begin{cases} X_{d,1} & \text{if } R_d = 1 \\ \vdots & \\ X_{d,r} & \text{if } R_d = r, \end{cases} \tag{9}$$

where at least one regime is modeled by

$$X_{d,i} = \alpha_i + (1 - \beta_i) X_{d-1} + \sigma_i |X_{d-1,i}|^{\gamma_i} \epsilon_{d,i}, \qquad i = 1, \tag{10}$$

where $\epsilon_{d,i} \sim N(0,1)$. Regimes which do not follow equation 10 are modeled by independently and identically distributed (i.i.d.) random variables [1]. For example, equation 10 can be used for base regime, and equation 11 can be used to model the spike and down regime.

$$X_{d,i} \sim N(\mu_i, \sigma_i^2), \qquad i = (2,3). \tag{11}$$

The spot price of the base regime $X_{d,1}$ becomes latent for each day d where $R_d \neq 1$. Therefore, $X_{d,1}$ is calculated for each day, regardless on what regime the system is in each day [1]. The difference between a normal MRS model and an independent MRS model, becomes apparent when the system returns to the base regime after deviating from it. The previous value $X_{d-1,1}$ is now drawn from the latent time series instead of the previous value of the spot price X_{d-1}. The regimes, which are modeled with i.i.d.s, do not have autocorrelating components. Therefore, their values do not need to become latent when the regime changes. The benefit of the independent regimes is that they guarantee that the dynamics of the simulated spot price will change immediately when the system changes from one regime to another.

One of the disadvantages of MRS models is that the calibration of the model parameters is somewhat difficult, especially when independent regimes are used [25]. A popular way to estimate the parameters of regime-switching models is a two step iterative procedure called the Expectation-Maximization (EM) algorithm. Description of the exact MRS-model used in this thesis is given in section 6.4.2 with an explanation of the used EM algorithm.

In conclusion, reduced form models can capture the spiky and mean reverting nature of spot price quite well while maintaining a relatively simple structure [1]. Another upside is that they do not require any information about the future, such as consumption forecasts, which some of the other models may require. However, because they only replicate the basic characteristics of the spot price, the spot price time series they produce, are more like simulations than forecasts. Fortunately, the lack of accurate forecasts is not a large disadvantage, because we simulate spot prices for market risk management purposes up to several years into the future. No model can be expected to forecast spot prices accurately over such extended period. The goal is to generate realistic spot prices, which have a sensible number of spikes and down-spikes. They are then used to calculate the optimal production quantities of power on a monthly scale. We do not need to predict the precise locations of time when a spike might occur. We just need to produce simulations where the distribution of spikes is sufficiently realistic.

6 Spot price simulations

In this section, we will describe how we produce spot prices for n months into the future on an hourly resolution. We begin with $\mathbf{F} = (F_0, F_1, ..., F_{n+1})$, which contains the forecast for the arithmetic means of $n + 2$ months. Why we require the two additional monthly values F_0 and F_{n+1}, will come apparent in section 6.2.1. Vector $\mathbf{F}$ is turned into hundreds of simulations of the daily spot prices, which are eventually turned into hourly spot prices.

The daily spot price P_d composes of three components

$$P_d = L_d + S_d + X_d, \qquad d = (1, 2, ..., D), \tag{12}$$

where L_d is the long-term seasonal component (LTSC), S_d is the short-term seasonal component (STSC) and X_d is the stochastic component. Vector $\mathbf{P}$, contains a single simulation of D daily spot prices. We will produce hundreds of different and independent simulations of the daily spot price $\mathbf{P}$. Therefore, a index $s = (1, 2, ..., S)$ is used when there is a need to distinguish one realization from another, which gives us $\mathbf{P}^s = (P_1^s, P_2^s, ..., P_D^s)$. Using vector notation, $\mathbf{P}^s$ can be written as

$$\mathbf{P}^s = \mathbf{L} + \mathbf{S} + \mathbf{X}^s, \qquad s = (1, 2, ..., S). \tag{13}$$

As can be seen from equation 13, the STSC $\mathbf{S}$ and the LTSC $\mathbf{L}$ remains the same in each simulation s, whereas $\mathbf{X}^s$ changes. Even though the LTSC is is the same in each simulation, it is still one of the most important components because it determines the expected mean $\overline{P}_m^s$ of each simulated month. The LTSC is often modeled with sinusoidal-, wavelet- or piecewise models, which are estimated form the history and extrapolated into the future [27]. However, these options do not provide the flexibility we need, because our intention is to simulate spot prices which monthly mean spot prices we can control and change. We will create a method that can provide and answer to the question "How much power VE will produce in each month if the means of the spot price are approximately these?" Therefore, we have chosen to generate the LTSC with a flexible method, so that we can directly control its monthly arithmetic means. This allows us to create simulations in the price levels we want, instead of extrapolating the historical trend and seasonality.

In our price model, the LTSC $\mathbf{L}$ has, in general, the greatest influence to the spot price $\mathbf{P}^s$, of all the three components. It is the backbone, to which $\mathbf{S}$ and $\mathbf{X}^s$ are added. The terms $\mathbf{S}$ and $\mathbf{X}^s$ are modeled to have an approximate mean of zero, and therefore, they do not have a strong effect to the monthly means of the simulated spot prices. Instead, their purpose is to bring variance on a day to day level around $\mathbf{L}$. For the most days, the values of X_d^s and S_d are few euros below or above zero. However, $\mathbf{X}^s$ can increase the daily price buy a hundred euros or so, but these spikes tend to last only for a short period. How each of the components are modeled and simulated, will be explained in detail in sections 6.2, 6.3 and 6.4. The data that is used is described in section 6.1. In section 6.6, the process of transforming simulated daily prices into hourly data with historical profile sampling (HPS) is explained.

6.1 Data

The most essential data used in this thesis, consists of hourly spot prices of power in Finland from 1.1.2007 to 31.12.2015 presented in Figure 5. These 78888 data points were gathered from VE's own data archives. The time series containing these original hourly spot prices is from now on referred as $\tilde{\mathbf{H}}^H = (\tilde{H}_1^H, \tilde{H}_2^H, ..., \tilde{H}_{78888}^H)$. Spot prices until 31.8.2015 are used for estimation and the last 4 months of 2015 are used for out-of-data validation. The original data contains exceptionally high spot prices of up to 1400.11€/MWh. Occasional high spot prices are a normal characteristic of the spot price, but leaving extremely high spot prices can be counterproductive [17]. Including these rare events in the estimation data could distort the parameters of the regime-switching model used to model the stochastic component. The model parameters responsible for generating price spikes might be drawn to values, which enable rare and extremely large jumps, but this could deteriorate the modeling capability of more normal jump characteristics. Therefore, a limit of 200€/MWh was arbitrarily chosen, and all data points over that limit were given a value of 200€/MWh. In total 72 data points were modified out of 78888, which is less than 0.1% of the data. The lowest spot prices in the realized spot prices were 0€/MWh. Because they are quite close to the arithmetic mean of the estimation data of 40.99€/MWh, they are not considered to be outliers. Furthermore, they are not likely errors in the data, because they were located during the summer nights, when the spot prices are usually very low.

As a result, we get a vector $\mathbf{H}^H = (H_1^H, H_2^H, ..., H_{78888}^H)$, where H_1^H is the spot price of the first hour of 1.1.2007 and H_{78888}^H is the spot price of the last hour of 31.12.2015. This time series is plotted in Figure 5. From vector $\mathbf{H}^H$, average prices for each day from the period of 1.1.2007 to 31.8.2015 were calculated, which gave us the estimation time series $\mathbf{E} = (E_1, E_2, ..., E_{3165})$, where E_1 is the average spot price of the day 1.1.2007 and E_{3165} is the average spot price of the day 31.8.2015. Plot of $\mathbf{E}$ is presented in Figure 6.

6.2 Long term seasonal component

In our model, the long-term seasonal component (LTSC) $\mathbf{L}^s$ can be used to control the monthly means of the simulated spot prices. The LTSC is modeled with a piecewise linear function, which is generated so that we are able to simulate daily spot prices which monthly means are close to values given in the forecast vector $\mathbf{F}$.

One popular method of estimating the long-term seasonal component $\mathbf{L}$ is fitting a sinusoidal or other type of function to the historical prices [17]. Forecast of the LTSC is done by simply extrapolating the function onwards. However, this rigid approach does not serve our needs. What we need to know is, how much power will VE produce in each month, when the monthly mean spot prices $\mathbf{F}$ are given. The vector $\mathbf{F} = (F_0, F_1, ..., F_{n+1})$ contains the mean spot prices the modeler wishes the monthly mean spot prices of the simulations to cluster around. It can contain the best forecasts available of the future mean prices, or perhaps experimental mean prices, which are used to test how what happens to different outputs of the opti-

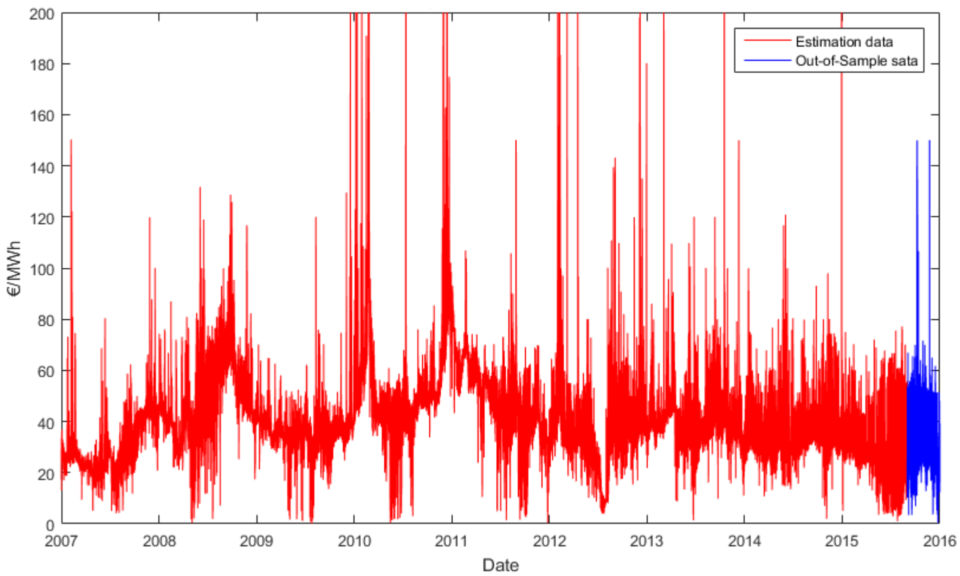

Figure 5: Realized Finnish spot prices that are cut off at 200€/MWh. From 2007 to 2015 the spot price exceeded 200 euros during 72 hours, and the maximum hourly spot price was 1400€/MWh.

mization program when the mean spot prices are at certain price levels. We want to create simulations $\mathbf{P}^s, s = (1, 2, ..., S)$ so that for $m = (1, 2, ..., n)$

$$\overline{P}_m = \frac{1}{S}\sum_{s=1}^{s=S}\overline{P}_m^s = \frac{1}{S}\sum_{s=1}^{s=S}\frac{\sum_{d=d_1^m}^{d=d_b^m} P_d^s}{d_b^m - d_1^m + 1} = \frac{1}{S}\sum_{s=1}^{s=S}\frac{\sum_{d=d_1^m}^{d=d_b^m}(L_d + S_d + X_d^s)}{d_b^m - d_1^m + 1} = F_m, \quad (14)$$

where d_1^m is the index of the first day in month m and d_b^m is the index of the last day in month m. Therefore, we allow the monthly means $\overline{P}_m^s$ of individual simulations to deviate from F_m, but we want the overall mean of all simulations in each month $\overline{P}_m$ to equal to F_m. Similarly to $\overline{P}_m^s$ in equation 14, let us define notations $\overline{L}_m$, $\overline{S}_m$ and $\overline{X}_m^s$ to signify the arithmetic mean of the daily values of each component in month m of realization s. Whereas, $\overline{X}_m$ is the arithmetic means of the stochastic components daily values in month m calculated across all simulations S similar to $\overline{P}_m$ in equation 14. However, because the LTSC and the STCS remains the same in each simulation $s = (1, 2, ..., S)$, $\overline{L}_m$ and $\overline{S}_m$ are used in both instances. We can freely choose the monthly mean of the LTSC $\overline{L}_m$ which is very useful, because the STCS is modeled in a way that its monthly mean $\overline{S}_m$ is not in general zero. We can calculate the magnitude of the error induced by $\mathbf{S}$, and adjust $\mathbf{L}$ so that the monthly mean of the deterministic component $\overline{f}_m$ is at the desired level. We will adjust the LTSC so that equation 15 holds

$$\overline{f}_m = \overline{L}_m + \overline{S}_m = \frac{\sum_{d=d_1^m}^{d_b^m}(L_d + S_d)}{d_b^m - d_1^m + 1} = F_m, \qquad \forall\ m, \quad (15)$$

where F_m is a target monthly mean for the simulations. The stochastic component will have have a mean of zero, but the monthly means $\overline{X}_m^s$ will deviate from zero more or less. Therefore, equation 14 does not hold exactly, but the more simulations are made, the closer it becomes of holding. The remaining error in that equation is fixed by scaling all of the spot prices slightly with a method shown in section 6.5.

6.2.1 Daily spot prices

In this section, we will show how $\mathbf{F} = (F_0, F_1, ..., F_{n+1})$ is used to generate the long-term seasonal component (LTSC) $\mathbf{L} = (L_1, L_2, ..., L_D)$, where L_d is the LTSC of the day d. Let us begin, by defining vectors $\mathbf{d}^m, m = (1, 2, ..., n)$, where m is the index of the forecasted month. These n vectors are of form $\mathbf{d}^m = (d_1^m, d_2^m, ..., d_b^m)$, where d_1^m is the index of the first day of month m, and d_b^m is the index of the last day of month m. For example, if the first forecasted month is January 2016, then $\mathbf{d}^2$ refers to February and $\mathbf{d}^2 = (32, 33, ..., 60)$.

A simple way to change monthly prices $\mathbf{F}$ into daily resolution $\mathbf{L}$ is to assign the day in the middle of each month m to a value $\tilde{F}_m$, and then make all the other days linear combinations of the two nearest values of $\tilde{F}_m$. The values of $\tilde{F}_m$ are stored in a vector $\tilde{\mathbf{F}} = (\tilde{F}_0, \tilde{F}_1, ..., \tilde{F}_{n+1})$. The goal is to find the optimal vector $\tilde{\mathbf{F}}$, so that equation 16 holds.

$$\overline{f}_m = \overline{L}_m + \overline{S}_m = \frac{\sum_{d=d_1^m}^{d_b^m} (L_d + S_d)}{d_b^m - d_1^m + 1} = F_m, \qquad m = (1, 2, ..., n), \tag{16}$$

where f stands for deterministic component, L_d is calculated with equation 17. The STSC $\mathbf{S}$ needs to be produced, with a method described in section 6.3 before the LTSC can be formed. An iterative process of finding the optimal values for $\tilde{\mathbf{F}}$, so that equation 16 holds, will be introduced after we describe how $\mathbf{L}$ is generated with a given $\tilde{\mathbf{F}}$. First, we need to define a new vector and some functions. In vector $\mathbf{D} = (D_0, D_1, ..., D_{n+1})$ the cell D_m is the index of the day that is in the middle of the month m. If the forecast is made starting from 1.1.2016 and $n = 12$, then $\mathbf{D} = (-15, 16, 46, ..., 382)$. The cell $D_0 = -15$ represents the index of the middle day in December 2015, and it is required so that equation 17 would be defined for $0 < d \leq 15$. The cell $D_{n+1} = 382$ represents similarly the index of the day in the middle of the month after the last forecasted month n. It is added so that the equation 17 would be defined for $351 < d \leq 366$. Function $A(d) \in \{0, 1, ..., 30\}$ gives the distance from d to the previous middle of the month day, and function $B(d) \in \{1, 2, ..., 31\}$ gives the distance from d to the next day that is in the middle of the month. Similarly, function $M_{Lm}(d, \tilde{\mathbf{F}})$ returns the price $\tilde{F}_m$ of the previous middle of the month day relative to d, and function $M_{Nm}(d, \tilde{\mathbf{F}})$ returns the price $\tilde{F}_m$ of the next middle of the month day. Finally, the long-term seasonal component can be formed with $\tilde{\mathbf{F}}$, which contains the middle of the month day prices for each month

$$L_d = \frac{B(d)}{A(d) + B(d)} M_{Lm}(d, \tilde{\mathbf{F}}) + \frac{A(d)}{A(d) + B(d)} M_{Nm}(d, \tilde{\mathbf{F}}), \qquad \forall d \tag{17}$$

In equation 17, the daily price L_d is the linear combination of the two nearest middle of the month prices, which are weighted by d's distance to them in the calender. Next, we will explain the iterative process, which adjusts the values of $\tilde{\mathbf{F}}$ so that equation 16 holds for all m. We will iteratively increase or decrease the values of $\tilde{\mathbf{F}}$, which are the output of functions $M_{Lm}(d)$ and $M_{Nm}(d)$ until the arithmetic mean $\overline{f}_m$ of each month is close to F_m. During each iteration i, new values $\tilde{\mathbf{F}}_i = (\tilde{F}_{0,i}, \tilde{F}_{1,i}, ..., \tilde{F}_{n+1,i})$ are calculated

$$\tilde{F}_{m,i} = \tilde{F}_{m,i-1} + \epsilon_m(\tilde{\mathbf{F}}_{i-1}), \qquad i \in (1, 2, ..., k), \forall\ m, \tag{18}$$

where $\tilde{F}_{m,0} = F_m$, $\epsilon_m(\tilde{\mathbf{F}}_0) = 0$ and $\epsilon_m(\tilde{\mathbf{F}}_{i-1})$ is the error between the arithmetic mean $\overline{f}_{m,i-1}$ and F_m

$$\epsilon_m(\tilde{\mathbf{F}}_i) = F_m - \overline{f}_{m,i} = F_m - \frac{\sum\limits_{d=d_1^m}^{d_b^m} [L_{d,i} + S_d]}{d_b^m - d_1^m + 1}, \qquad \forall\ m, \tag{19}$$

where d_1^m is the index of the first day and d_b^m is the index of the last day of month m. The value of $L_{d,i}$ is calculated using equation 17 with $\tilde{\mathbf{F}}_i$. During each iteration, the months which mean was too low will have an increase to their $\tilde{F}_{m,i+1}$, and the months with too high mean prices will experience a decrease in $\tilde{F}_{m,i+1}$. The iterations continue until the mean absolute error MAE_i

$$\text{MAE}_i = \frac{1}{n} \sum_{m=1}^{n} |\epsilon_m(\tilde{\mathbf{F}}_i)| \tag{20}$$

drops below 0,0001 € or the maximum number of iterations $i = 50$ is reached. The end results is an optimal vector $\tilde{\mathbf{F}}$ and the LTSC $\mathbf{L}$, which in practice fulfills the equation 16. The small error of 0,0001€ that remains has no effect on the simulations.

With the method described above, we create the LTSC $\mathbf{L} = (L_1, L_2, ..., L_D)$, where L_d is the value of the LTSC in day d. This LTSC will be used in all simulations $s = (1, 2, ..., S)$. Therefore, the mean spot price in month m of simulations s will be

$$\overline{P}_m^s = \overline{f}_m + \overline{X}_m^s = \frac{\sum_{d=d_1^m}^{d=d_b^m} [L_d + S_d + X_d^s]}{d_b^m - d_1^m + 1} = F_m + \overline{X}_m^s, \qquad \forall m, s. \tag{21}$$

The stochastic component causes the $\overline{P}_m^s$ to deviate from F_m. How ever, we will set the mean of the stochastic component to zero

$$\overline{X}^s = \frac{1}{D} \sum_{d=1}^{d=D} X_d^s = 0, \qquad \forall s. \tag{22}$$

Therefore, the expected value for $\overline{X}_m^s$ is zero for all m and s. This means that

$$E[\overline{P}_m^s] = F_m, \qquad \forall m, s. \tag{23}$$

However, the stochastic component can be volatile, and $\overline{X}_m^s$ can be dozens of euros above or below zero. This means that equation 14 will not hold unless an infinite number of simulations are made. Therefore, we will be performing small adjustment of the spot prices in section 6.5 so that equation 14 would hold.

6.3 Short term seasonal component

The short-term seasonal component (STSC) $\mathbf{S} = (S_1, S_2, ..., S_D)$ is added to the simulation model so that the price profile within a week would be realistic. Furthermore, STSC is needed to deseasonalize the historical data so that the stochastic component can be estimated from it. As described in section 2.5.2, the relative price between each day of the week has certain deterministic characteristics. Days from Monday to Friday, that are workdays, have in general higher spot prices compared to non-working days, which are Saturdays, Sundays and public holidays. STSC is supposed to capture this weekly profile. Therefore, when d is a workday, S_d is in general slightly positive, and when d is not a workday, it is slightly negative. If a public holiday is positioned on a weekday, or on a Saturday, it is treated as a Sunday. Because the calender is the same in each simulation $s = (1, 2, ..., S)$, there is only one vector $\mathbf{S} = (S_1, S_2, ..., S_D)$, which is used in all S simulations.

Researchers often model the STSC as a mean week, by calculating the arithmetic mean or median for each type of day from Monday to Sunday from the detrended historical prices, from which the long-term seasonal component is removed [33], [17]. The term S_d would then have seven different values, one for each type of day. However, we have decided to make the STSC also dependent of the month, because assuming that the STCS would remain the same for the entire year is questionable. In [34], Lucia and Schwartz concluded after thorough analysis, that the volatility of spot prices in Nord Pool is consistently different between warm and cold periods. This makes sense, because the demand for power is lower in the summer than in the winter, mostly due to changes in the weather [16]. Therefore, it is not unreasonable to assume that the deterministic component of the weekly price profile, could be different in each month. The value S_d is set to depend on the type of the day d and the month d is in. This requires estimation of a separate weekly profile for each twelve months. This type of STSC can be modeled with the following function

$$S_d = V(t(d), m(d)), \qquad \forall\ d, \tag{24}$$

where $t(d)$ is the type of the day d and $m(d)$ is the month the day d is in, and $V(t, m)$ is a piecewise function, where $t \in \{1, 2, ..., 7\}$ and $m \in \{1, 2, ..., 12\}$. Naturally, $m = 1$ corresponds to January and $m = 12$ corresponds to December. The term $t(d)$ is defined in equation 25.

$$t(d) = \begin{cases} 1, & \text{if } d = \text{Monday} \\ 2, & \text{if } d = \text{Tuesday} \\ 3, & \text{if } d = \text{Wednesday} \\ 4, & \text{if } d = \text{Thursday} \\ 5, & \text{if } d = \text{Friday} \\ 6, & \text{if } d = \text{Saturday} \\ 7, & \text{if } d = \text{Sunday or a public holiday.} \end{cases} \tag{25}$$

The STSC is estimated from $\mathbf{E}^S = (E_1^S, E_2^S, ..., E_{3165}^S)$, which contains Finnish daily mean spot prices from 1.1.2007 to 31.8.2015, from which the long-term seasonal component is removed.

$$E_d^S = E_d - L_d^H, \qquad d = (1, 2, ..., 3165), \tag{26}$$

where E_d are historical daily spot prices, introduced in section 6.1, and L_d^H is the LTSC that is constructed to have the same monthly means than $\mathbf{E}$. How this historical LTSC is made, will be explained next. The arithmetic mean spot prices of each month of $\mathbf{E}$ from December 2006 to January 2016 can be seen in Table 2.

Table 2: Mean spot prices calculated from the realized hourly spot prices that are limited to 200€/MWh.

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Jan	-	27.4	46.1	41.1	60.1	69.0	38.8	41.6	40.2	33.8	37.83
Feb	-	30.1	39.8	38.3	81.6	64.6	52.6	39.4	34.2	33.2	-
Mar	-	23.7	31.9	34.9	55.2	60.9	36.5	45.0	31.2	29.4	-
Apr	-	22.2	43.6	34.5	43.7	52.9	36.5	43.9	31.5	30.1	-
May	-	22	38.3	33.1	39.5	54.4	33.3	37.3	36.6	25.9	-
Jun	-	26.9	57.6	35.4	41.9	48.6	27.4	38.6	35.4	21.5	-
Jul	-	22.4	59.1	33.8	48.1	42.2	13.7	37.0	36.8	27.6	-
Aug	-	26.9	65.2	37.3	43.2	49.0	38.1	43.5	38.4	31.1	-
Sep	-	32.2	73.4	35.6	51.2	38.9	41.0	47.8	38.3	31.8	-
Oct	-	37.2	60.4	35.1	51.2	36.9	38.6	46.0	36.7	33.5	-
Nov	-	45.6	52.5	36.7	56.6	42.0	36.9	38.0	35.4	31.7	-
Dec	32.0	43.6	44.4	43.9	91.2	33.3	46.5	35.7	37.1	26.6	-

We create a vector $\mathbf{F}^H = (F_0^H, F_1^H, ..., F_{105}^H)$, which contains values from Table 25 so that F_1^H is the mean spot price of January 2007 and F_{104}^H is the mean spot price of August 2015. The cell $F_0^H = 32.02$€ is the monthly mean of December 2006, and $F_{105} = 31.75$€ is the monthly mean of September 2015. These two extra values are needed so that the first and the last 15 days of the LTSC can be calculated as explained in section 6.2.1. However, the historical LTSC will only be constructed for the time period of January 2007 and August 2015. So that the historical LTSC can be made, optimal middle of the month values $\tilde{\mathbf{F}}^H = (\tilde{F}_0^H, \tilde{F}_1^H, ..., \tilde{F}_{105}^H)$ need to be found. They are searched with the iterative process described in section 6.2.1, with one modification. The error term in equation 27 is used instead of equations 19.

$$\epsilon_m(\tilde{\mathbf{F}}_i^H) = F_m^H - \overline{L}_{m,i}^H = F_m^H - \frac{\sum_{d=d_1^m}^{d_b^m} L_{d,i}^H}{d_b^m - d_1^m + 1}, \qquad m = (1, 2, ..., 104), \tag{27}$$

where $L_{d,i}^H$ is calculated using equation 17 with the values $\tilde{\mathbf{F}}_i^H$. Once the historical LTSC $\mathbf{L}^H$ is acquired, it is removed from the historical spot prices $\mathbf{E}$ as in equation 26. The historical spot prices $\mathbf{E}$ and the historical LTSC $\mathbf{L}^H$ are presented in Figure 6, as is the time series $\mathbf{E}^S = \mathbf{E} - \mathbf{L}^H$.

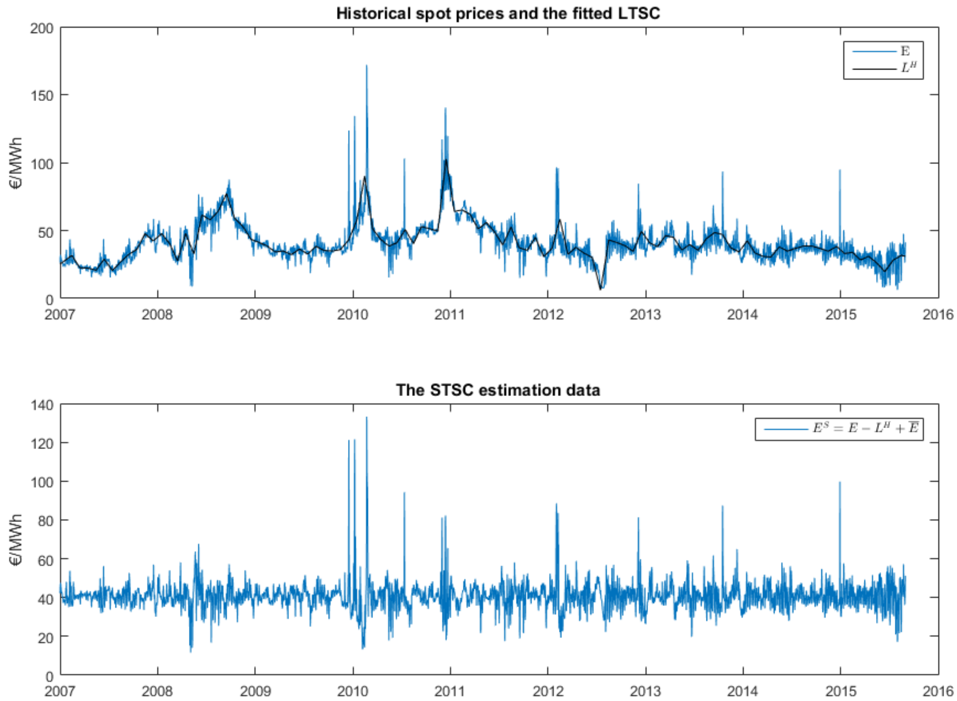

Figure 6: The daily realized spot prices with the LTSC fitted to them, and the estimation data for the STSC.

Each day of $\mathbf{E}^S$ has a certain type $t(d) = (1, 2, ..., 7)$ and a month $m(d) = (1, 2, ..., 12)$. Corresponding dummy variables are attached to each day. The value $\tilde{V}(t, m)$ is the median price of all the days, which day of the week is of type t and the month is m. The median is chosen over the mean, because it is less affected by extreme prices. Therefore, it should capture the deterministic profile more robustly. Finally, $V(t, m)$ is formed from $\tilde{V}(t, m)$ by removing the arithmetic mean of the week from each daily value of $V(t, m)$

$$V(t, m) = \tilde{V}(t, m) - \frac{1}{7}\sum_{k=1}^{7} \tilde{V}(k, m), \qquad \forall\, m, t. \tag{28}$$

Therefore, it applies that $\sum_{t=1}^{7} V(t, m) = 0$, $\forall\, m$. This feature is important, because $\mathbf{S}$ should only bring variations to $\mathbf{P}^s$ on a day to day level, but not produce any systematic long-term deviations in the mean monthly prices. The monthly arithmetic means of S_d would ideally be zero. However, most months do not only consist of full weeks. Some months have more Sundays, public holidays and Saturdays compared to weekdays than other months. Therefore, the mean of S_d^s during each month will not be zero. This would cause the monthly means of the deterministic component to deviate from desired values unless nothing is done. The solution executed in this thesis, is to keep $\mathbf{S}$ as it is described here, but the long-term seasonal component $\mathbf{L}$ is adjusted, so that the systematic error caused by **S** is countered by increasing or decreasing the values of the LTSC. The LTSC is adjusted so that $\overline{f}_m = \overline{L}_m + \overline{S}_m = F_m$ for all m. Details how **L** is produced are given in section 6.2.1.

The weekly short-term seasonal component for each month and day is given in Table 3. It seems that the mean profile of the week does change from season to season. The absolute difference between working days and the non-working days increases in the summer and decreases in the winter. The term S_d can be easily extrapolated into the future, because the day of the week and month of each day d are known in advance. It is simply assigned with the value of $V(t, m)$ corresponding to the day of the week and month of each future day d.

Short term seasonal component S_d												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Now	Dec
Mon	2.55	1.03	0.92	1.90	1.58	3.16	1.50	3.59	1.98	1.93	1.00	2.12
Tue	2.40	0.68	2.11	1.86	2.25	3.92	1.68	2.38	1.05	1.09	1.78	2.06
Wed	1.58	1.07	1.72	1.63	2.08	3.08	1.79	1.92	2.92	1.33	1.23	3.77
Thu	-0.15	2.40	1.22	0.99	1.92	1.49	1.60	2.50	1.80	1.75	1.11	0.75
Fri	1.02	1.46	1.14	2.18	2.21	0.77	1.72	0.61	1.23	1.18	0.96	0.69
Sat	-3.47	-2.34	-2.76	-3.59	-4.83	-4.22	-2.30	-4.78	-3.68	-3.00	-2.54	-3.14
Sun	-3.93	-4.31	-4.36	-4.96	-5.216	-8.19	-6.01	-6.22	-5.32	-4.29	-3.54	-6.25

Table 3: The values which are given to the short-term seasonal component. Public holidays are considered to be Sundays.

6.4 Stochastic component

The stochastic component **X** is the part of the spot price that is not deterministic. However, it has some characteristics which can be attempted to capture and reproduce. These characteristics are, for example, the volatility and mean reversion. As stated in section 4, the model needs to be able to reproduce these characteristics, because the profile of the spot price affects the optimal production schedule.

Mean reverting regime-switching jump diffusion (MRS) model with three independent regimes is used in this thesis to model the stochastic component. MRS models were introduced briefly in section 5.1.5, but a more detailed description is given in section 6.4.1. The estimation process of the model parameters with Expectation-Maximization algorithm will be explained in section 6.4.2. Section 6.4.3 explains how the different realizations of the stochastic components $\mathbf{X}^s, s = (1, 2, ..., S)$ are generated with the obtained MRS-model.

6.4.1 Model

A discrete mean reverting regime-switching jump diffusion model with independent regimes will be used to model the stochastic component $\mathbf{X} = (X_1, X_2, ..., X_D)$ of the spot price. The basic theory of MRS model was introduced in section 5.1.5 and the exact model will be defined next. The model has 3 regimes, which are base ($R_d = 1$), spike ($R_d = 2$) and down-spike ($R_d = 3$). Therefore,

$$X_d = \begin{cases} X_{d,1}, & \text{if } R_d = 1 \\ X_{d,2}, & \text{if } R_d = 2 \\ X_{d,3}, & \text{if } R_d = 3, \end{cases} \quad ,\forall\ d. \tag{29}$$

The base regime is modeled as in [25], so that

$$X_{d+1,1} = \alpha_1 + (1-\beta_1)X_d + \sigma_1 |X_{d,1}|^{\gamma_1} \epsilon_{d,1}, \tag{30}$$

where $\epsilon_{d,1} \sim N(0,1)$. The absolute value of $X_{d,1}$ is taken as a precaution in case of negative values. What is noteworthy in this model is that the model is heteroscedastic. The inverse leverage effect of spot price is captured by raising $|X_{d,1}|$ to a power of γ_1. Inverse leverage effect is an important characteristic of the spot price. It means that the volatility of spot price increases when the spot price rises [15]. The spike regime is modeled with an i.i.d. with shifted lognormal distribution as in [24]

$$\log(X_{d,2} - L_2) \sim N(\mu_2, \sigma_2), \qquad X_2 > L_2, \tag{31}$$

where L_2 is a arbitrarily chosen lower limit for spikes. Only spot prices that are higher than L_2 can be classified as being in the spike regime. However, all values over L_2 are not automatically part of the spike regime, because they can also belong to the base regime. The down-spike regime is modeled similarly with an i.i.d.

$$\log(L_3 - X_{d,3}) \sim N(\mu_3, \sigma_3), \qquad X_3 < L_3, \tag{32}$$

where L_3 is the upper limit for a down-spike. The limits are meant to help the expectation maximization algorithm (EM) to estimate the model parameters from appropriate data points. Without them, it is possible that the spike regime will end up modeling all extreme values, including the exceptionally small values. Furthermore, EM algorithm is known to converge often to the smallest and most frequent spikes, when it estimates the parameters of the spike regime [1]. By assigning thresholds for the regimes, these features of the EM algorithm can be controlled.

Transition from regime to regime is governed by a Markov transition matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}, \tag{33}$$

where $\sum_{j=1}^{3} \pi_{ij} = 1, \forall\ i$. Cell π_{ij} is the transition probability from regime i to j during the next time unit $\pi_{ij} = P(R_{d+1} = j | R_d = i)$. The transition probabilities and the model parameters in equations 30, 31 and 32 are estimated with the EM algorithm, which will be described in section 6.4.2.

6.4.2 Estimation

Parameters of the MRS-model needs to be estimated from a detrended and deseasonalized time series $\mathbf{E}^X = (E_1^X, E_2^X, ..., E_D^X)$ [1] of period 1.1.2007 to 31.8.2015.

$$E_d^X = E_d - L_d^H - S_d^H + \overline{\mathbf{E}}, \qquad d = (1, 2, ..., D), \tag{34}$$

where E_d are the historical daily mean spot prices introduced in section 6.1, L_d^H is the historical LTSC from section 6.3, S_d^h is the historical STCS and $\overline{\mathbf{E}}$ is the arithmetic mean of $\mathbf{E}$

$$\overline{E} = \frac{1}{D} \sum_{d=1}^{k=D} E_d = 40.99. \tag{35}$$

The historical STCS is generated like any STCS that would be used for forecasting. The value of S_d^H depends on what day of the week d is and in what month it is. For more details see section 6.3.

As a result, we get a deseasonalized and detrended time series $\mathbf{E}^X$, which arithmetic mean is 41.13. There is a small difference between the means of $\mathbf{E}^X$ and $\mathbf{E}$. The monthly means of the STCS are not exactly zero, which causes a small deviation. However, this is not a significant problem. The reason the mean of $\mathbf{E}^X$ is set to be close to $\overline{\mathbf{E}}$ and not any other arbitrarily chosen positive value, is because the estimation data needs to be in the correct price level so that the parameter γ_1 from equation 30, would be calibrated correctly. The term γ_1 is responsible for modeling the inverse leverage effect, which causes the volatility of the price to increase as the spot price increases.

The ability to switch regimes is a great benefit. It enables natural simulations of the spot price. However, because the state of the process is governed by an unobservable latent variable R_d, the estimation of the model parameters is not straightforward. The reason is that there is no certainty on what regime the system was during each historical data point [1]. Therefore, it is not trivial which data points should be used to calibrate the model parameters of each regime. The Expectation-Maximization (EM) algorithm developed by Dempster et al. [35] provides the basis for the solution of this problem. Hamilton [36] and Kim [37] improved the method, but even Kim's approach is not optimal for us because it is designed for MRS models which regimes are not independent. Using it to solve the parameters of a model with independent parameters would lead to extreme computational burden [25]. Proposals how to modify Kim's EM algorithm, so that it can be used with independent regimes, have been made at least by Jong [38] and by Janczura & Weron [25]. In this thesis, we will use the latter method [25], which deviates from Kim's EM algorithm in the definition of the probability density function (pdf) $f(X_t|R_d = i; \mathbf{X}_{t-1}; \theta(n))$. In this section we will first present Kim's version of the EM algorithm as described in [25], and then we will explain what happens when the pdf is changed.

It the beginning of the EM algorithm, we create an initial parameter vector $\theta^{(0)} = (\alpha_i^{(0)}, \beta_i^{(0)}, \sigma_i^{(0)}, \gamma_i^{(0)}, \mu_i^{(0)}, \mathbf{\Pi}^{(0)}, \rho_i^{(0)})$, where $i = 1, 2, 3$. The term $\mathbf{\Pi}^{(0)}$ is the transition probability matrix for base regime, $\rho_i^{(0)}$ is the probability that the system is in regime i during the first time step $\rho_i^{(0)} = p(R_1 = i)$. The terms $\alpha_i^{(0)}, \beta_i^{(0)}, \sigma_i^{(0)}, \gamma_i^{(0)}$ and $\mu_i^{(0)}$ are model parameters from equations 30, 31 and 32. However, $\mu_i^{(0)}$ is not defined for the first regime, and only parameters $\mu_i^{(0)}$ and $\sigma_i^{(0)}$ are defined for the other two regimes. The EM algorithm has two steps. In the E-step, inferences of the process being in each regime are estimated based on the historical prices and the parameter vector $\theta^{(n)}$ [25] [37]. In the M-step, new parameter vector $\theta^{(n+1)}$ is calculated utilizing the inferences obtained in the E-step [25] [37]. These steps are iterated, until a local maximum of the likelihood function is found.

Next, we will describe Kim's EM algorithm as it is presented in [25]. However, we are using three regimes instead of two. Let us define that $\mathbf{x}_d = (x_1, x_2, ..., x_D)$ is the vector containing deseasonalized and detrended historical daily mean spot prices. In this case $x_d = E_d^X$ for all d. Let us assume that $\theta^{(n)}$ is the initial parameter vector, or it is obtained during the previous M-step. The E-step of the EM algorithm is divided into two parts, which are the filtering and the smoothing. During the filtering, the conditional probabilities for the system being in each regime at day d are calculated using only the information available up to day d. Filtering is done iteratively for $d = (1, 2, ..., D)$ and all i

$$P(R_d = i|\mathbf{x}_d; \theta^{(n)}) = \frac{P(R_d = i|\mathbf{x}_{d-1}; \theta^{(n)}) f(x_d|R_d = i; \mathbf{x}_{d-1}; \theta^{(n)})}{\sum_{j=1}^{3} P(R_d = i|\mathbf{x}_{d-1}; \theta^{(n)}) f(x_d|R_d = i; \mathbf{x}_{d-1}; \theta^{(n)})}, \tag{36}$$

where $f(x_d|R_d = i; \mathbf{x}_{d-1}; \theta^{(n)})$ is the probability density function (pdf) of the observed spot price x_d. The conditional probabilities of the regime being in each state $i = (1, 2, 3)$ during the next time step are calculated for $d = 1, 2, ..., D-1$

$$P(R_{d+1} = i|\mathbf{x}_d; \theta^{(n)}) = \sum_{j=1}^{3} \pi_{ji}^{(n)} P(R_d = j|\mathbf{x}_d; \theta^{(n)}), \tag{37}$$

where $P(R_1 = i; \mathbf{x}_0; \theta^{(n)}) = \rho_i^{(n)}$ and $\pi_{ij}^{(n)}$ is a cell from the transition matrix $\mathbf{\Pi}$.

In the smoothing part, the conditional probabilities calculated during filtering are used to calculate iteratively the smoothed inferences for the system being in state $i = (1, 2, 3)$ for $d = (D-1, D-2, ..., 1)$ so that

$$P(R_d = i|\mathbf{x}_D; \theta^{(n)}) = \sum_{j=1}^{3} \frac{P(R_d = i|\mathbf{x}_d; \theta^{(n)}) P(R_{d+1} = j|\mathbf{x}_D; \theta^{(n)}) \pi_{ij}^{(n)}}{P(R_{d+1} = j|\mathbf{x}_d; \theta^{(n)})}. \tag{38}$$

The density function $f(x_t|R_d = i; \mathbf{x}_{t-1}; \theta^{(n)})$, required in equation 36, can be derived from the equations 30, 31 and 32. The base regime should follow Gaussian distribution with the mean $\alpha_1 + (1-\beta_1)x_{d-1}$ and a standard deviation $\sigma_i|x_{d-1}|^{\gamma_i}$. Therefore the pdf for base regime is

$$f(x_d|R_d = 1; \mathbf{x}_{d-1}; \theta^{(n)}) = \frac{1}{\sqrt{2\pi}\sigma_1^{(n)}|x_{d-1}|^{\gamma_1^{(n)}}} \cdot \exp\left(-\frac{(x_d - (1-\beta_1^{(n)})x_{d-1} - \alpha_1^{(n)})^2}{2(\sigma_1^{(n)})^2|x_{d-1}|^{\gamma_1^{(n)}}}\right). \tag{39}$$

In the spike and down-spike regimes, the spot price is modeled with i.i.d.'s with shifted lognormal distributions. Therefore, the pdf for spike regime is

$$f(x_d|R_d = 2; \mathbf{x}_{d-1}; \theta^{(n)}) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_2^{(n)}(x_d - L2)} \cdot \exp\left(\frac{-(\ln(x_d - L2) - \mu_2)^2}{2(\sigma_2^{(n)})^2}\right), & \text{if } x_d > L_2, \\ 0, \ \text{if } x_d \leq L_2, \end{cases} \tag{40}$$

where L_2 is the arbitrarily chosen lower limit for spikes, μ_2 and σ_2 are the mean and standard deviation, which were estimated during the previous M step. Details about the M-step will be discussed later in this section. The down-spike regime's pdf is

$$f(x_d|R_d = 3; \mathbf{x}_{t-1}; \theta^{(n)}) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_3^{(n)}(L3-x_d)} \cdot \exp\left(\frac{-(\ln(L_3-x_d)-\mu_3)^2}{2(\sigma_3^{(n)})^2}\right), \text{ if } x_d < L_3, \\ 0, \text{ if } x_d \geq L3, \end{cases} \tag{41}$$

where L_3 is the arbitrarily chosen upper limit for down-spikes, μ_3 and σ_3 are the mean and standard deviation, which were estimated during the previous M step. As the result of the E-step of the EM algorithm, we have the smoothed inferences, which are used to calculate the new model parameters in the M-step.

In the M-step the a parameter vector $\theta^{(n+1)} = (\alpha_i^{(n+1)}, \beta_i^{(n+1)}, \sigma_i^{(n+1)}, \gamma_i^{(n+1)}, \mu_i^{(n+1)}, \mathbf{\Pi}^{(n+1)}, \rho_i^{(n+1)})$ is constructed. The easiest parameter to estimate is $\rho^{(n+1)}$, which contains the probabilities of the Markov process being in each regime at time $d = 1$. As in [36] we simply assign

$$\rho_i^{(n+1)} = P(R_1 = i|\mathbf{x}_D; \theta^{(n)}), \qquad i = (1, 2, 3). \tag{42}$$

As in [37], the new transition parameters are calculated for all i and j

$$\begin{aligned} \pi_{ij}^{(n+1)} &= \frac{\sum\limits_{d=2}^{D} P(R_d = j, R_{d-1} = i|\mathbf{x}_D; \theta^{(n)})}{\sum\limits_{d=2}^{D} P(R_{d-1} = i|\mathbf{x}_D; \theta^{(n)})} \\ &= \frac{\sum\limits_{d=2}^{D} P(R_d = j|\mathbf{x}_D; \theta^{(n)}) \frac{\pi_{ij}^{(n)} P(R_{d-1}=i|\mathbf{x}_{d-1};\theta^{(n)})}{P(R_d=j|\mathbf{x}_{d-1};\theta^{(n)})}}{\sum\limits_{d=2}^{D} P(R_{d-1} = i|\mathbf{x}_D; \theta^{(n)})}. \end{aligned} \tag{43}$$

The rest of the model parameters are chosen so that they maximize the following weighted log-likelihood function [33]

$$\max_{\theta^{(n+1)}} \log[L(\theta^{(n+1)})], \tag{44}$$

where

$$\log[L(\theta^{(n+1)})] = \sum_{i=1}^{3} \sum_{d=1}^{D} P(R_d = i|\mathbf{x}_D; \theta^{(n+1)}) \log\left[f(x_t|R_d = i; \mathbf{x}_{d-1}; \theta^{(n+1)})\right]. \tag{45}$$

Exact maximum likelihood estimates for $\alpha_1^{(n+1)}, \beta_1^{(n+1)}, \sigma_1^{(n+1)}$ and $\gamma_1^{(n+1)}$ are $\hat{\alpha_1}, \hat{\beta_1}, \hat{\sigma_1}$ and $\hat{\gamma_1}$ respectively. They can be calculated by setting the partial derivatives of the equation 45 to zero [25]

$$\hat{\alpha_1} = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}(x_d - (1-\hat{\beta}_1)x_{d-1})]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}]}, \tag{46}$$

$$\beta_1 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})x_{d-1}|x_{d-1}|^{-2\hat{\gamma}_1}B_1]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})x_{d-1}|x_{d-1}|^{-2\hat{\gamma}_1}B_2]}, \tag{47}$$

$$B_1 = x_d - x_{d-1} - \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}(x_d - x_{d-1})]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}]}, \tag{48}$$

$$B_2 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})x_{d-1}|x_{d-1}|^{-2\hat{\gamma}_1}]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}]} - x_{d-1}, \tag{49}$$

$$\hat{\sigma_1}^2 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|x_{d-1}|^{-2\hat{\gamma}_1}(x_d - \hat{\alpha_1} - (1-\hat{\beta}_1)x_{d-1})^2]}{\sum_{d=2}^{D} P(R_d = 1|\mathbf{x}_D; \theta^{(n)})}. \tag{50}$$

The last parameter $\hat{\gamma}_1^{(n+1)}$ can be estimated by numerically maximizing the log-likelihood function in equation 45. How this is done, will be explicitly written once we have introduced the modifications that will be done to EM algorithm. Similarly, the simple process of estimating $\mu_i^{(n+1)}$ and $\sigma_i^{(n+1)}$ for the spike and down-spike regimes will be explained later in this section. The new parameters for the base regime are $\hat{\alpha_1}, \hat{\beta}_1, \hat{\sigma_1}, \hat{\gamma_1}$. Therefore, we can create the new parameter vector $\theta^{(n+1)} = (\alpha_i^{(n+1)}, \beta_i^{(n+1)}, \sigma_i^{(n+1)}, \gamma_i^{(n+1)}, \mu_i^{(n+1)}, \mathbf{\Pi}^{(n+1)}, \rho_i^{(n+1)})$ and the E-step can be started again to gather new smoothed inferences. The iteration of these two steps should be continued until a local maximum is located.

However, as previously mentioned, the approach described above is not suitable for a model with independent regimes. The reason is that the pdf for the autoregressive base regime, displayed in equation 39, becomes computationally intensive to calculate if the regimes are independent. As stated in [25], once the system leaves the mean-reverting base regime, the process in the base regime becomes latent, and the distribution of x_d becomes dependent of the whole history $(x_1, x_2, ..., x_{d-1})$. The memory capacity needed especially in the E-step of the EM algorithm increases rapidly with the size of the data sample. To overcome this challenge, [25] suggests replacing the pdf for base regime in equation 39, with

$$f(x_d|R_d = 1; \mathbf{x}_{d-1}; \theta^{(n)}) = \frac{1}{\sqrt{2\pi}\sigma_1^{(n)}|\tilde{x}_{d-1}|^{\gamma_1^{(n)}}} \cdot \exp\left(-\frac{(x_d - (1-\beta_1^{(n)})\tilde{x}_{d-1} - \alpha_1^{(n)})^2}{2(\sigma_1^{(n)})^2|\tilde{x}_{d-1}|^{\gamma_1^{(n)}}}\right), \tag{51}$$

where $\tilde{x}_{d-1,1} = E(X_{d-1,1}|\mathbf{x}_{d-1}; \theta^{(n)})$, which can be calculated with equation 52.

$$\begin{aligned} E(X_{d,1}|\mathbf{x}_d; \theta^{(n)}) = & P(R_d = 1|\mathbf{x}_d; \theta^{(n)})x_t \\ & + P(R_d \neq 1|\mathbf{x}_d; \theta^{(n)}) \cdot (\alpha_1^{(n)} + (1-\beta_1^{(n)})E(X_{d-1}|\mathbf{x}_{d-1}; \theta^{(n)})). \end{aligned} \tag{52}$$

Therefore, changes needs to be done in the formulation of equations 46 - 50, because they are derived from the log-likelihood function, which uses the smoother inferences that are calculated using the new pdf. New exact maximum likelihood model parameters for the base regime are [25]

$$\hat{\alpha_1} = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}(x_d - (1-\hat{\beta}_1)\tilde{x}_{d-1})]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}]}, \tag{53}$$

$$\hat{\beta}_1 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})\tilde{x}_{d-1}|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}B_1]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})\tilde{x}_{d-1}|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}B_2]}, \tag{54}$$

$$B_1 = x_d - \tilde{x}_{d-1} - \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}(x_d - \tilde{x}_{d-1})]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}]}, \tag{55}$$

$$B_2 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})\tilde{x}_{d-1}|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}]}{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}]} - \tilde{x}_{d-1}, \tag{56}$$

$$\hat{\sigma_1}^2 = \frac{\sum_{d=2}^{D}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})|\tilde{x}_{d-1}|^{-2\hat{\gamma}_1}(x_d - \hat{\alpha_1}^{(n)} - (1-\hat{\beta}_1^{(n)})\tilde{x}_{d-1})^2]}{\sum_{d=2}^{D} P(R_d = 1|\mathbf{x}_D; \theta^{(n)})}. \tag{57}$$

The term $\hat{\gamma}_1$ will be assigned the value which minimizes the following equation

$$\min_{\hat{\gamma}_1}\Bigg| - \sum_{d=1}^{D-1}[P(R_d = 1|\mathbf{x}_D; \theta^{(n)})\log(|\tilde{x_d}|)] + \sum_{d=1}^{D-1}\left[\frac{\log(|\tilde{x_d}|)(x_{d+1} - (1-\hat{\beta})(\tilde{x_d}) - \hat{\alpha})^2 P(R_d = 1|\mathbf{x}_D; \theta^{(n)})}{\hat{\sigma}_1^2|\tilde{x_d}|^{2\gamma_1}}\right]\Bigg|, \tag{58}$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}^2$ are calculated with equations 53 - 57 and the initial value for $\hat{\gamma}_1 = \gamma_1^{(n)}$ [39].

The estimation processes for the spike and down-spike regimes are much simpler. The parameters which are estimated for the spike regime in the M-step are $\mu_2^{(n+1)}$ and $\sigma_2^{(n+1)}$.

$$\mu_2^{(n+1)} = \frac{\sum\limits_{d \in \mathbf{d}^{L_2}} [\log(x_d - L_2)P(R_d = 2|\mathbf{x}_D; \theta^{(n)})]}{\sum\limits_{d \in \mathbf{d}^{L_2}} P(R_d = 2|\mathbf{x}_D; \theta^{(n)})}, \tag{59}$$

where $\mathbf{d}^{L_2}$ is a vector containing all the indexes of days which spot price is higher than L_2, and $P(R_d = 2|\mathbf{x}_D; \theta^{(n)})$ is the smoothed inference obtained during the E step [39]. The days which price does not exceed the arbitrarily chosen limit L_2, cannot be classified as spikes and are excluded from the estimation process. The variance of the spike regime is estimated as follows [39]

$$(\sigma_2^{(n+1)})^2 = \frac{\sum\limits_{d \in \mathbf{d}^{L_2}} [(\log(x_d - L_2) - \mu_2^{(n+1)})^2 P(R_d = 2|\mathbf{x}_D; \theta^{(n)})]}{\sum\limits_{d \in \mathbf{d}^{L_2}} P(R_d = 2|\mathbf{x}_D; \theta^{(n)})}. \tag{60}$$

Parameters for the down-spike regime are estimated similarly

$$\mu_3^{(n+1)} = \frac{\sum\limits_{d \in \mathbf{d}^{L_3}} [\log(L_3 - x_d) P(R_d = 3|\mathbf{x}_D; \theta^{(n)})]}{\sum\limits_{d \in \mathbf{d}^{L_3}} P(R_d = 3|\mathbf{x}_D; \theta^{(n)})}, \tag{61}$$

where $\mathbf{d}^{L_3}$ is the vector containing all the indexes of days which value is lower than L_3 [39]. The days which price was higher than the arbitrarily chosen limit L_3, cannot be classified as down-spikes and are excluded from the estimation of the model parameters for the down-spike regime. The variance for the third regime is calculated as follows [39]

$$(\sigma_3^{(n+1)})^2 = \frac{\sum\limits_{d \in \mathbf{d}^{L_3}} [(\log(L_3 - x_d) - \mu_3^{(n+1)})^2 P(R_d = 3|\mathbf{x}_D; \theta^{(n)})]}{\sum\limits_{d \in \mathbf{d}^{L_3}} P(R_d = 3|\mathbf{x}_D; \theta^{(n)})}. \tag{62}$$

By using the above methods, we form a new parameter vector $\theta^{(n+1)}$ and continue the iteration, until a local maximum of the log-likelihood function is found. The algorithm stops when the differences in the parameters between two iterations are less than 10^{-8} or when the maximum number of 100 iterations is reached. We use notation θ to refer to the parameter vector that is calculated during the last iteration. For more information about the estimation of a MRS model with independent regimes, see [25].

6.4.3 Simulations

This section explaines how we formed S simulations of the stochastic component $\mathbf{X}^s = (X_1^s, X_2^s, ..., X_D^s), s = (1, 2, ..., S)$, which are one of the components of the simulated daily spot prices

$$\mathbf{P}^s = \mathbf{L} + \mathbf{S} + \mathbf{X}^s, \qquad s = (1, 2, ..., S). \tag{63}$$

The simulations of the stochastic component are made with a Markov regime-switching mean reverting jump-diffusion (MRS) model specified in section 6.4.1, and defined by equations 29-33. The optimal model parameters θ for the MRS model are visible in the Results section in table 4. The simulations produced with this model are marked with a tilde $\tilde{\mathbf{X}}^s = (\tilde{X}_1^s, \tilde{X}_2^s, ..., \tilde{X}_D^s), s = (1, 2, ..., S)$ because they are not used in equation 63 as they are. We want the long-term mean of the stochastic component $\mathbf{X}^s$ to be zero. This allows the LTSC $\mathbf{L}^s$ to determine the expected price level of each month, and the STSC $\mathbf{S}$ and the stochastic component $\mathbf{X}^s$ would only bring variation on day to day level. A suitable vector $\mathbf{X}^s$ can be created by subtracting the mean of $\tilde{\mathbf{X}}^s$ from each of its value $\tilde{X}_d^s$

$$X_d^s = \tilde{X}_d^s - \frac{1}{D} \sum_{k=1}^{k=D} \tilde{X}_k^s, \qquad \forall\ s, d. \tag{64}$$

As a result we get $\mathbf{X}^s = (X_1^s, X_2^s, ..., X_D^s)$, which mean $\overline{X}^s$ is zero. This is the stochastic component that is used in creation of the daily spot price of simulation s.

Next, we will discuss, how the simulations of $\tilde{\mathbf{X}}^s$, $s = (1, 2, ..., S)$ are made. Each realization of $\tilde{\mathbf{X}}^s$ is produced using a modified version of the Matlab script that is available in [40]. The original script does not differentiate weekdays from weekends. The probability of a spike was not set to be conditional to the type of day, which causes spikes to occur on weekends and public holidays. We see this as a problem. By using the EM algorithm to estimate the MRS model parameters, we also get the smoothed inferences, which are the conditional probability for the realized daily spot prices being in each regime. There were significant differences in the density and size of spikes which occurred during workdays and non-working days. During non-working days, the probability of a spike was about half of a workday. Furthermore, the spikes were smaller than spikes on workdays. More details about the statistics are given in section 2.5.4. We decided to simulate the spot prices, so that spikes are allowed to happen only during workdays. The reason behind this decision is that for a CHP producer, it is not irrelevant on what weekdays the spot price spikes occur, because of the codependency of power and heat production. In general, if heat demand is low, as it is during non-working days, the quantity of power that can be produced efficiently is not maximal for CHP production plants, because the waste heat of power production cannot be necessarily fully utilized in district heating. Therefore, the variable cost of producing power is relatively high. If a spike, occurs during a workday, demand for heat is in general higher, and therefore larger quantities of power can be produced more efficiently compared to non-working days. Therefore, we decided to prevent spikes of occurring during the weekends and public holidays and move the spikes to the next available working day.

This was accomplished by manipulating the transition probabilities of the variable R_d in certain situations. When the regime for the next day R_{d+1} would be assigned to be a spike by the Markov process when $d + 1$ is not a workday, the system is forced to go to base regime instead. Every time a spike is changed forcefully to the base regime, a counter c is increased by one. If $c > 0$ and d is a workday that is assigned to be in the base regime by the Markov process, it will be forced to become spike regime instead, and the value of c is subtracted by one. The spikes that would occur during non-working days, are effectively moved forward until a suitable day is found. This means that, the expected number of spikes and down-spikes in the simulations should match to what what was estimated from the historical data. This is confirmed in section 2.5.4.

This is not an optimal solution to the problem, but we feel that allowing spikes to occur during weekends would deteriorate the reliability of our results even more. Furthermore, this restriction can be easily removed if necessary. The differences in the characteristics of the spot price during working days and non-working days suggest that perhaps they should be modeled with different models. However, we leave this for further research.

Each simulation is started from the base regime, and the first value X_1 is set to the median spot price of the time series from which the MRS model was estimated from $\overline{E}^X = 41.13$. The regimes of the next days are randomly generated from a distribution defined by the transition probability matrix, but spikes are not allowed to occur during non-working days. The spot prices for each day are generated with the function defined for each regime. This gives us $\tilde{\mathbf{X}}^s, s = (1, 2, .., S)$. After using equation 64, we get S different simulations of the stochastic component $\mathbf{X}^s$, which arithmetic mean is zero and which has no spikes on the non-working days. We also gather the information on what regime $R_d^s \in \{1, 2, 3\}$ the system was in each day. This information is needed when daily prices are turned into hourly prices with a method described in section 6.6.1. Therefore, during each simulation, we also save a vector $\mathbf{R}^s = (R_1^s, R_2^s, ...R_D^s)$, where R_d^s is the regime the system was in day d in simulation s.

6.5 Daily spot prices

In this section, it will be explained how the final time series of daily spot prices are created. The spot price of day d of simulation s is a combination of three components

$$P_d^s = L_d + S_d + X_d^s, \qquad d = (1, 2, ..., D), s = (1, 2, ..., S), \tag{65}$$

where L_d is the LTSC described in section 6.2, S_d is the STSC described in section 6.3 and X_d^s is the stochastic component described in section 6.4. Once the components are produced, they are added up into $\mathbf{P} = (P_1^s, P_2^s, ..., P_D^s), s = (1, 2, ..., S)$. However, they still need to be adjusted in two ways. Firstly, the daily spot prices are given a lower limit of one euro. All days, whose price is lower than that is changed into one euro. This is done because, the stochastic component produces down-spikes, which can cause the spot price of some days to be negative. However, Finnish spot prices from 2007 to 2015 have all been positive. Secondly, the spot prices in each month will be increased or decreased slightly. As explained in section 6.2, we would like the arithmetic mean of all daily spot prices in month m to be F_m

$$\overline{P}_m = \frac{1}{S} \sum_{s=1}^{s=S} \overline{P}_m^s = \frac{1}{S} \sum_{s=1}^{s=S} \frac{\sum_{d=d_1^m}^{d=d_b^m} P_d^s}{d_b^m - d_1^m + 1} = F_m, \tag{66}$$

where d_1^m is the index of the first day in month m and d_b^m is the index of the last day in month m. The monthly means of the deterministic component is F_m for all months, but only the expected value of the monthly mean of the stochastic component is zero. Therefore, $\overline{P}_m$ approaches F_m as more and more simulations are made, but performing dozens or thousands of simulations does not guarantee that outcome. Furthermore, when the daily spot prices were limited to a minimum of one euro, the mean spot price of some simulations increased, which caused further disturbance. Therefore, the spot prices are scaled slightly with the following method so that equation 66 would hold.

$\tilde{\mathbf{P}}^s = (\tilde{P}_1^s, \tilde{P}_2^s, ..., \tilde{P}_D^s), s = (1, 2, ..., S)$ are the simulated daily spot prices, which minimum value is set to one euro. The error $e_m = \overline{\tilde{P}}_m - F_m$ can be calculated and removed from the daily spot prices of each month. Each simulated daily price $\tilde{P}_d^s$ in month m will be increased or decreased by the same amount, so that the average of daily prices in that month equals to F_m. The final version of the daily spot prices $\mathbf{P}^s = (P_1^s, P_2^s, ..., P_D^s)$ of simulation s is generated according to equations 67 and 68.

$$P_d^s = \tilde{P}_d^s - e(d), \qquad \forall d, s, \tag{67}$$

where $e(d)$ is a piecewise constant function, which has n different values. One for each month $m = (1, 2, ..., n)$. The term $e(d)$ is defined for all d

$$e(d) = \begin{cases} e_1 = \overline{\tilde{P}}_1 - F_1, & \text{if } d \text{ is in month } 1 \\ e_2 = \overline{\tilde{P}}_2 - F_2, & \text{if } d \text{ is in month } 2 \\ \vdots & \\ e_n = \overline{\tilde{P}}_n - F_n, & \text{if } d \text{ is in month } n, \end{cases} \tag{68}$$

where $\overline{\tilde{P}}_m$ is the arithmetic mean of all simulated daily spot prices $\tilde{\mathbf{P}}^s$, $s = (1, 2, ..., S)$ in month m. It can be calculated similar to $\overline{P}_m$ in equation 66. As a result, we get the final simulated daily spot prices $\mathbf{P} = (P_1^s, P_2,^s, ..., P_D^s), s = (1, 2, ..., S)$. These prices can then be transformed to hourly spot prices, with a method described in section 6.6.

6.6 Historical profile sampling

In this section, we will go over how simulations of daily spot prices

$$P_d^s = L_d + S_d + X_d^s, \qquad d = (1, 2, ..., D), s = (1, 2, ..., S) \tag{69}$$

are turned into hourly spot prices $\mathbf{H}^s = (H_1^s, H_2^s, ..., H_H^s), s = (1, 2, ..., S)$, where H_h^s is the spot price of hour h in simulation s. We will be using a variation of the historical profile sampling (HPS) method presented in [41]. In HPS, a historical counterpart d^H for each simulated day d is retrieved. Then, we take the 24 hour price profile of that historical day $\mathbf{H}_{d^H}^H = (H_{d^H,1}^H, H_{d^H,2}^H, ..., H_{d^H,24}^H)$ and use it to create an hourly price profile $\mathbf{P}_d^s = (P_{d,1}^s, P_{d,2}^s, ..., P_{d,24}^s)$, so that the arithmetic mean $\overline{P}_d^s$ of the 24 hours is equal to the value of the simulated daily spot price P_d^s. The daily prices are turned into hourly prices independently of each others.

6.6.1 Finding a suitable historical day

The historical day d^H is chosen by minimising the following equation

$$\min_{d^H} |P_d^s - \overline{\mathbf{H}}_{d^H}^H|, \tag{70}$$

where P_d^s is the simulated daily mean spot price of day d and $\overline{\mathbf{H}}_{d^H}^H$ is the mean of the 24 historical hourly spot prices of day d^H. In [41], the historical day d^H is chosen at random from a pool of days that satisfy similar constraint that we use. The differences between our constraints and the constraints in [41] are that we have three regimes instead of two, and the ranges from within the days are searched in equation 76 are larger. There are two constraints that d^H needs to fulfill. They are meant to ensure that the retrieved historical day most likely has the similar profile as the simulated day d should have. Firstly, the type of the historical day $T(d^H)$ needs to be same as the type of the simulated day $T(d)$

$$T(d^H) = T(d), \tag{71}$$

where

$$T(d) = \begin{cases} 1, & \text{if } d \text{ is a workday, for which } R_d = 1 \\ 2, & \text{if } d \text{ is a workday, for which } R_d = 2 \\ 3, & \text{if } d \text{ is a workday, for which } R_d = 3 \\ 4, & \text{if } d \text{ is a Saturday} \\ 5, & \text{if } d \text{ is a Sunday or a public holiday} \end{cases} \tag{72}$$

Workdays are weekdays from Monday to Friday, which are not public holidays. We divide workdays into groups based on their regime $R_d \in \{1, 2, 3\}$. The regimes are base, spike, and down-spike respectively. During the estimation process of the MRS-model, we retrieved conditional probabilities for each historical day d^H being in each regime $i = 1, 2, 3$. Therefore, we can estimate the regime the system was in each historical day. As in [25], if $P(R_{d^H} = i) > 0.5$, then we assume that $R_{d^H} = i$. If $P(R_{d^H} = i)$ is not larger than 0.5 for any i, then we define that day to belong into the base regime. As described in section 6.4.3, the regime R_d of each simulated day is explicitly known. Therefore, we can match each simulated workday with a historical day of the correct type. As stated in section 6.4.3, most historical Saturdays, Sundays and public holidays belonged either to the base regime or down-spike regime. The probability of these non-working days being in spike regime was about half of the working days. Furthermore, because down-jumps are quite rare, we decided not to divide non-working days further based on their regimes to guarantee that the pools from which the days are drawn from are of sufficient size. Therefore, we have in total five different types of days in equation 72.

The second condition is that the date of d^H must be approximately from the same season as d. Each day d and historical day d^H is assigned a number $N(d) = (1, 2, ..., 366)$, which specifies the position of the day within a year. Naturally, 1 is given to 1.th of January and 366 is given to 31.th of December on a leap year. The year from which each day is originated from does not matter. As in [41], the position of the historical day d^H must be within an arbitrarily chosen limit of the position of the simulated day d. This can be achieved if d^H is chosen so that, one of the equations 73 - 75 holds

$$N(d) - N(d^H) + 366 \leq K(T(d)), \tag{73}$$

$$|N(d) - N(d^H)| \leq K(T(d)), \tag{74}$$

$$-N(d) + N(d^H) + 366 \leq K(T(d)), \tag{75}$$

where $K(T(d))$ is the arbitrarily chosen maximum allowed difference in days between the positions of d and d^H in the calender

$$K(T(d)) = \begin{cases} 30, & \text{if } T(d) = 1 \\ 60, & \text{if } T(d) \geq 2. \end{cases} \tag{76}$$

Equation 74 holds in most cases when d and d^H are within $K(T(d))$ days of each others in the calender. However, for example, it does not allow choosing a d^H from December, if d is in January. To allow this possibility, equation 73 was added. Similarly, equation 75 allows choosing d^H which is in January when d is in December. If one of the equations 73 - 75 holds, then d^H fulfills the second constraint.

Because of the constraints, it could be possible that, no acceptable historical day could be found for some simulated days. However, we checked that for all d and $T(d)$, it is possible to find a day d^H with the same type within $K(T(d))$ days. This might not be the case for all data sets.

6.6.2 Transforming daily prices to hourly spot prices

Once a historical day d^H is selected, its hourly spot prices $\mathbf{H}_d^H = (H_{d^H,1}^H, H_{d^H,2}^H, ..., H_{d^H,24}^H)$ are used to to create the hourly price profile of the simulated day d. We use the same process as in [41]. First, each historical hourly spot price from day d^H is divided by the arithmetic mean on that day

$$r_{d,h}^s = \frac{H_{d^H,h}^H}{\overline{H}_{d^H}^H}, \qquad h = (1, 2, ..., 24). \tag{77}$$

As a result we get a vector $\mathbf{r}_d^s = (r_{d,1}^s, r_{d,2}^s, ..., r_{d,24}^s)$, which is used to create the hourly profile for the simulated day. The hourly simulated spot prices are generated by multiplying each cell of $\mathbf{r}_d^s$ with P_d^s:

$$P_{d,h}^s = P_d^s r_{d,h}^s, \qquad h = (1, 2, ..., 24) \tag{78}$$

This gives us a vector $\mathbf{P}_d^s = (P_{d,1}^s, P_{d,2}^s, ..., P_{d,24}^s)$, which arithmetic mean $\overline{\mathbf{P}}_d^s$ is P_d^s, and the hourly price profile is similar to the historical day $\mathbf{H}^{d^H}$. With the method described above, each simulated daily spot price P_d^s can be transformed into 24 hourly spot prices. Once an hourly profile $\mathbf{P}_d^s$ is generated for all d, an hourly spot price vector $\mathbf{H}^s$ can be constructed from them by arranging them one after another.

$$\mathbf{H}^s = (P_{1,1}^s, P_{1,2}^s, ..., P_{1,24}^s, P_{2,1}^s, ..., P_{d,h}^s, ..., P_{D,24}^s) \tag{79}$$

$$\mathbf{H}^s = (H_1^s, H_2^s, ..., H_H^s) \tag{80}$$

$\mathbf{H}^s$ is the hourly spot price curve of simulation s, which will be inputted to the production schedule optimization program as it is.

7 Results

In section 7.1, we present the model which was estimated from the historical prices using the EM algorithm presented in section 6.4.2. In section 7.2 the simulation period and how the spot prices were simulated is explained. In section 7.3, validation of the model is made by comparing statistics calculated form the realized prices to the statistics calculated from the simulated prices. In section 7.4, optimal production quantities calculated using the simulations are analyzed.

7.1 Model estimation

We use a Markov regime-switching model with independent regimes to model the stochastic component of the spot price. Details about the model are given in sections 5.1.5 and 6.4. It is estimated from the deseasonalized and detrended Finnish daily spot prices from January 2007 to August 2015. Details about the estimation data $\mathbf{E}^X$ are given in section 6.4.2. The parameters of the model were estimated using the EM algorithm presented in section 6.4.2. The initial parameters $\theta^{(0)}$ and the estimated parameters θ are presented in Table 4. The EM algorithm was run with

Table 4: Parameters of the Markov regime-switching model

			The initial parameters $\theta^{(0)}$				
Regime	$\alpha_i^{(0)}$	$\beta_i^{(0)}$	$\sigma_i^{2(0)}$	$\gamma_i^{(0)}$	$\mu_i^{(0)}$	$L_2^{(0)}$	$L_3^{(0)}$
Base i=1	15	0.4	1	0	-	-	-
Spike i=2	-	-	1	-	2	43.3267	-
Down spike i=3	-	-	0.5	-	2	-	37.9052
		$\mathbf{\Pi}$					
Base i=1	0.9	0.05	0.05				
Spike i=2	0.1	0.79	0.11				
Down spike i=3	0.1	0.11	0.79				
			Estimated parameters θ				
Regime	α_i	β_i	σ_i^2	γ_i	μ_i	L_2	L_3
Base i=1	13.9067	0.3404	0.0041	1.0028	-	-	-
Spike i=2	-	-	1.0337	-	1.629	43.3267	-
Down-spike i=3	-	-	0.4808	-	1.7491	-	37.9052
		$\mathbf{\Pi}$			$P(R_d = i)$		
Base i=1	0.9321	0.0448	0.0231		0.8218		
Spike i=2	0.3899	0.5784	0.0317		0.0974		
Down-spike i=3	0.2199	0.0532	0.7269		0.0808		
$\log[L(\theta)]$=-7728.226							

different variations of the initial parameters $\theta^{(0)}$, but the results remained the same. Therefore, it was deemed that a systematic search for the perfect initial parameters was unnecessary. This is fortunate because there are many parameters and going trough different combinations would become computationally intensive quite fast. However, changing the values of the parameters L_2 and L_3 did have a small effect on the value of the log likelihood function and to the values of the estimated parameters θ. Therefore, a systematic empirical search for the optimal values for L_2 and L_3 was performed. The EM algorithm was run with different values of L_2 and L_3, which were changed with a resolution of one percentile. The log likelihood function reached its maximum of -7722.8 when $L_2 = 43.32$€/MWh and $L_3 = 37.91$€/MWh. These values made sure that only 27% of the largest values of the estimation data could be classified as spikes, and 23% of the lowest values could be classified as down-spikes. The final set of parameters θ was obtained using these bounds for the spikes and down-spikes.

The estimation data with the estimated regimes of each day are presented in Figure 7. The probabilities of each day being in each regime are the smoothed inferences, which are calculated during the EM algorithm. As in [25] and [39], if the probability of historical day d^H being in regime i is larger than 0.5, that day is selected to belong to regime i. However, if the probability of belonging to any regime does not rise above 0.5, then that day is chosen to belong to the base regime. From the Figure 7, few issues become apparent. Firstly, spikes and down-spikes seem to often cluster together, especially during the year 2010. Spikes are followed by down-spikes and down-spikes by spikes. However, when the historical daily spot prices from the same periods in the year 2010 are examined from Figure 6, no clear down-spikes are visible. The movement seems to have been more between spike and base regime. Nevertheless, many days from that period are estimated to belong to the down-spike regime. The cause of this phenomenon lies in the deseasonalization process, which was described in sections 6.3 and 6.4.2. Effectively, the linear combination of the means of the two nearest months from each realized daily spot price was removed from each day. Therefore, when the spot prices were exceptionally high and volatile in some months, the mean spot price became substantially higher than the median spot price. When the mean spot price is removed from all spot prices, the normal days, which are close to the median, were lowered quite a lot. The EM algorithm then estimated that these normal days between spikes belong to the down-regime because they were now significantly lower than the mean of the whole estimation data. It seems that the method we used for estimating the LTSC was too sensitive to local volatility. A more robust method might have been better.

The second issue that can be seen from Figure 7 is that some of the days that are estimated to belong into the down-spike regime are not individual down-spikes, but instead they are the lowest values of what seems to be heavily autocorrelated random walk of the spot price. For example in fall 2009 and 2010, the spot price decreases several days in a row and then starts to increase again. The few lowest values in the middle of this movement are classified as down-spikes. They should perhaps be categorized as base regime, and the base regime should be modeled with sufficient volatility and auto correlation so that it can produce spot prices that behave in this way. We could force the EM algorithm to categorize these values to the base regime by decreasing the value of L_3, but it decreased the value of the log

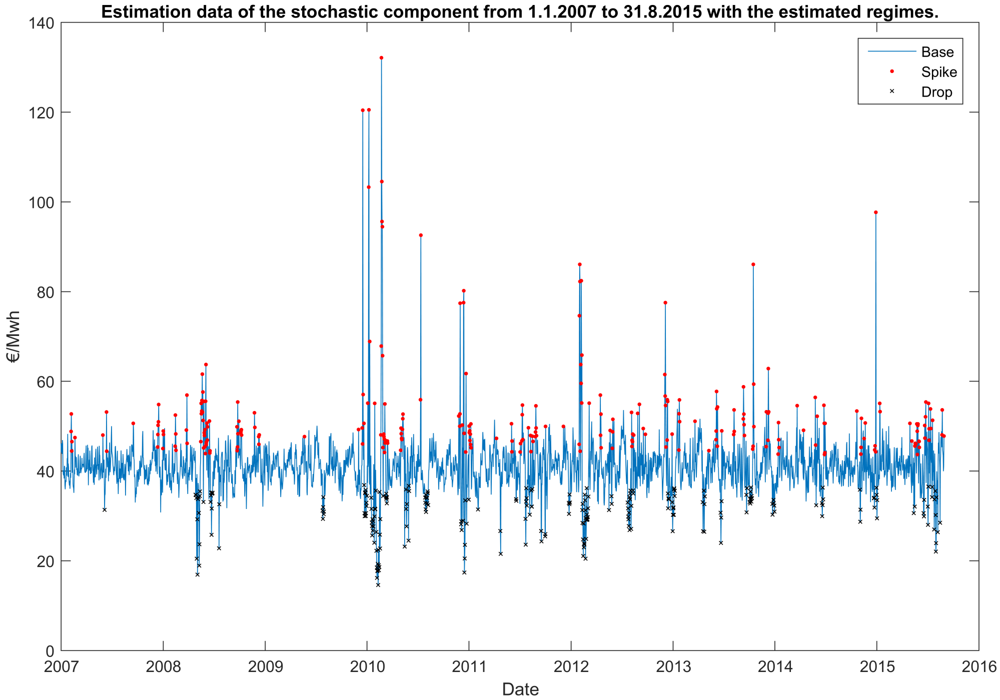

Figure 7: The data used for estimating the parameters of the MRS model and the regimes

likelihood function. Therefore, we abandoned this idea for now. The MRS model will most likely not be able to produce simulations where this kind of behaviour is visible because the likelihood of a down-spike is not conditional to the values of the base-regime. Therefore, it is not able to produce decreasing base regime spot prices, which are followed by a few down-spikes and then increasing base regime days. Instead, the down-spikes will appear more evenly throughout the simulation period in small clusters.

The third issue involves the apparent homoscedasticity of the realized spot prices. In Figure 7 there are long periods where the spot price stays in base regime which are followed by periods where there are a lot of spikes and down-spikes. Because, in the MRS model the probability of the regime of the next day is dependent of the current regime, the simulations can produce simulations where this homoscedasticity is partially observed. The transition probabilities that are presented in table 4 make it so, that when the system enters a regime, it is likely to stay in it for more than one day. Furthermore, spike/down-spike is more likely to appear after a down-spike/spike than after a base regime day. Therefore, volatile spot prices will most likely cluster together in the simulations. However, in Figure 7, it seems that the volatile periods last for weeks or months. The MRS model which regime depends only on the previous day is not likely to produce such simulations. Instead, the spikes and down-spikes will be more evenly distributed over the whole simulation period.

7.2 Simulations

We will simulate daily spot prices, for the period of 1.1.2007 - 31.12.2015. These simulations are compared to the historical hourly spot prices $\mathbf{H}^H = (H_1^H, H_2^H, ...H_{78888}^H)$, which are from that period and have been limited to a maximum of 200€ per MWh. The monthly means that are calculated from $\mathbf{H}^H$ are visible in Table 2. We want to pick the deterministic component so that its mean in each month is equal to the monthly means of the historical prices. This can be achieved by creating the LTSC with the method described in section 6.2 with a vector $\mathbf{F} = (F_0, F_1, ..., F_{109})$, where F_0 is assigned the mean spot price of December 2006, F_1 is assigned the monthly mean of January 2007 and so forth. The last cell F_{109} has the mean spot price of January 2016. After the deterministic component was done, the stochastic component was added and the spot prices were modified with methods described in sections 6. We made 600 simulations of the daily spot prices $\mathbf{P}^s$ of the time period of 2007-2015, where the deterministic component $\mathbf{f} = \mathbf{L} + \mathbf{S}$ remained the same in each one, but the stochastic component $\mathbf{X}^s$ changed. Equation 81 holds for these simulations.

$$\overline{P}_m = \frac{1}{S}\sum_{s=1}^{s=S}\overline{P}_m^s = \frac{1}{S}\sum_{s=1}^{s=S}\frac{\sum_{d=d_1^m}^{d=d_b^m} P_d^s}{d_b^m - d_1^m + 1} = \frac{1}{S}\sum_{s=1}^{s=S}\frac{\sum_{d=d_1^m}^{d=d_b^m}(L_d + S_d + X_d^s)}{d_b^m - d_1^m + 1} = F_m, \quad (81)$$

where $S = 600$ and $m = (1, 2, ..., 108)$.Then, these simulations $\mathbf{P}^s, s = (1, 2, ..., S)$, were transformed into hourly data $\mathbf{H}^s, s = (1, 2, ..., S)$ with historical profile sampling (HPS), which is described in section 6.6.

7.3 Validation

We are not trying to make a model which can robustly forecast a single realizations of the future spot prices once the expected monthly means are given. Instead, we are attempting to build a model, which can create several different simulations of the spot prices where the deterministic component remains the same and where the stochastic component varies. The simulations are then used to estimate the future production quantities of power. Because, the price profile of each simulation is different, the production schedule optimization program will calculate different optimal production quantities for each simulation. By analysing these production quantities, it can be determined how sensitive they are to the profile of the spot price. What matters, here is that the distribution of the hourly spot prices, is realistic and different in each simulation. To measure this, we create statistics of the realized spot prices and compare them to the simulations. If the statistics calculated from the realized prices are in general close to the mean of the statistics calculated from the simulations, the model is deemed good enough.

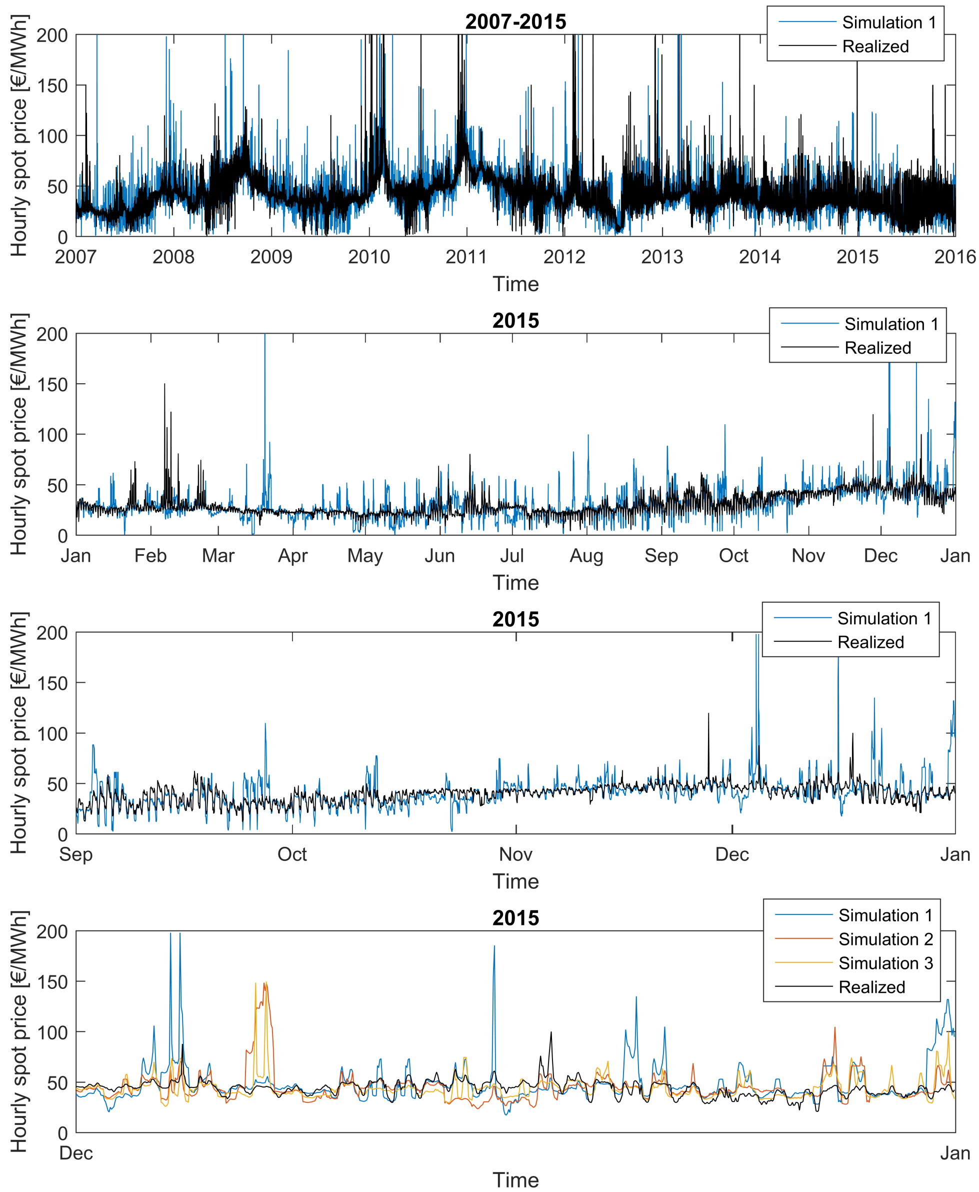

Figure 8: Realized and simulated hourly spot prices which have been cut off at 200€/MWh

The performance of the simulation model is tested by examining the percentiles of the realized and simulated spot prices and by examining the frequency, distribution and relative size of the spikes and down-spikes. The tests will be performed on different time spans. First, the whole period from 2007 to 2015 will be examined. Then, each year will be tested separately. Finally, some months will be examined in detail. We will begin with a visual inspection of the simulations. In Figure 8, the realized spot price is presented with a simulation in different time periods. The simulated spot prices seem to be quite homogeneous throughout the years, whereas the realization is more heterogeneous. The simulations resemble the realized spot price better in some periods than the others, which is expected. Based on the visual inspection of the simulations, the model seems to be good enough for the intended. Same conclusions can be made from Figures 14 to 17, which are in the Appendix. They show the realized spot prices of each year and a simulation side by side. From them, it is clear that the characteristics of the realized spot prices have changed quite drastically from season to season. The volatility and overall shape of the realized spot price curve changes but the simulated price curves are much more homogeneous. This is caused by the fact that the model parameters are fixed, which means that the simulations cannot depict such variability of characteristics as observed in the realized spot prices.

7.3.1 Percentiles

Percentiles of the realized spot prices were calculated for each year and then compared to the statistics of the percentiles calculated from the simulations. The results are gathered in table 8 in the Appendix and visualized in Figure 9. Percentiles $p = [5\%, 10\%, 20\%, ..., 80\%, 90\%, 95\%]$ were calculated for 600 simulations individually and then statistics of the values for each percentile were gathered. These were the mean, standard deviation and minimum and maximum. Figure 9 shows that the simulations have in general similar distributions as the realized spot prices, but the percentiles of the realized spot prices are not always within the standard deviation of the mean of simulations. Sometimes, they are not even within the minimum and maximum values of the percentiles calculated from the simulations. Therefore, the same conclusion that was done with the visual inspection of the simulations can also be done here. The realized spot prices are more versatile than the simulations, and the model cannot in every case produce simulations that are very close to the realized spot prices. However, when the parsimonious structure of the model is taken into consideration, the overall performance seems to be good.

The percentiles were also calculated on a monthly resolution, but this was done only for the year 2015. The Year 2015 was chosen because it is the most recent year and because its last four months are out-of-sample data. The results are presented in Table 9 in the Appendix and visualized in Figure 10. The year 2015 had exceptionally low spot prices, which could make it difficult to produce simulations with similar percentiles as the realized year. Furthermore, the percentiles calculated from the entire year 2015 in Figure 9, show that 2015 was one of the most exceptional years. However, the results seem to be relatively good, but there are some issues. The realized percentiles are within the minimum and maximum percentiles gathered from the 600 simulations in all months and percentiles, with the exception of two cases in October. In many cases, they are even within the standard deviation of the mean. The realized percentiles in the first 5 months seem to increase gradually, and the simulations can follow that trend quite well. In the remaining 7 months, the

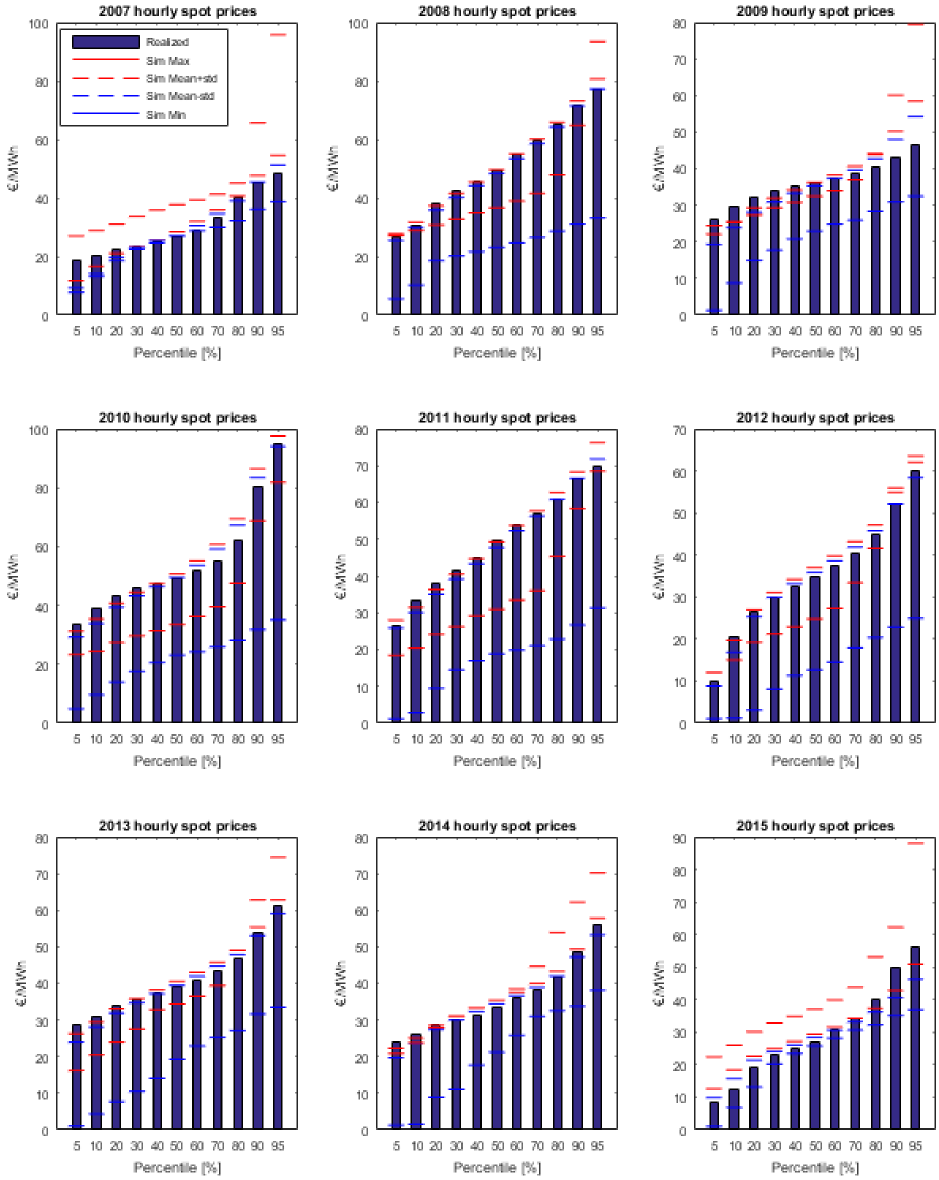

Figure 9: Percentiles of the realized hourly spot prices from the year 2007 to the year 2015 and statistics about the percentiles of 600 simulations.

percentiles of the simulations seem to overshoot in the low percentiles and undershoot in the high percentiles. Therefore, the performance of the simulations in the out-of-sample period from September to December is not great. Reasons for this can be found by examining the spot prices in Figure 8. The visual characteristics of the spot price vary between different season and the spot prices from 5/2015 to

12/2015 seem a bit different than the spot prices earlier in that year. Furthermore, the percentiles of each year visualized in Figure 9 show that each year is different, and 2015 is one of the most exceptional years. The lowest percentile is low, and the highest percentile is relatively high. The S-shape is not as clear in the percentiles of realized spot prices than in the percentiles of the simulated prices.

The model parameters are fixed, and they are estimated from several years of spot prices, which means that the model is only able to produce simulations which characteristics are a hybrid of some of the different characteristics of the realized spot prices. Therefore, the characteristics of the spot prices are not as versatile as the characteristics of the realized spot price. The parsimonious structure of the model cannot accurately match the realized percentiles of all arbitrarily chosen periods. Therefore, it could be beneficial to analyze and divide the realized spot prices to different categories and estimate a model for each category. Estimation data could be divided, for example, by season, year, volatility or mean spot price. However, we leave this out of the scope of this thesis. The percentiles of the simulations seem to be close enough of the realized percentiles taking into consideration the simplicity of the model. The results seem to be good enough so that the model can be used to produce simulations of the spot price for medium term planning.

7.3.2 Spikes

In this section, the statistics of the distribution and relative size of the realized and simulated spikes and down-spikes are compared. So that the data would be comparable, deseasonalized and detrended data of the daily spot prices are used. The time series $\mathbf{E}^X$ represents the realized prices, and $\tilde{\mathbf{X}}^s, s = (1, 2, ..., 600)$ represent the simulated prices. The time series $\mathbf{E}^X$ is used for estimation of the parameters of the MRS model, and more information about is given in section 6.4.2. The vector $\tilde{\mathbf{X}}^s$ is the s.th simulation made using the MRS model and it is described in more detail in section 6.4.3. The vector $\mathbf{E}^X$ covers only the period of 1.1.2007 to 31.8.2015 and therefore only values from that time period are taken from $\tilde{\mathbf{X}}^s, s = (1, 2, ..., S)$. First we test that the simulations resemble the model estimated from the estimation data to make sure there are no obvious problems in the simulation proses. The transition matrix $\mathbf{\Pi}$ and the probabilities $P(R_d = i)$ for a simulated day being in regime $i = (1, 2, 3)$ are visible in Table 4. They suggest that around 82.2% of the simulated days should be in the base regime, 9.7% should be in the spike regime and 8.1% should be in the down-spike regime. The percentages of days that belong to these regimes in $\tilde{\mathbf{X}}^s, s = (1, 2, ..., 600)$ were the same to one decimal. Therefore, the frequency of each regime in the simulations matches the transition parameters in $\mathbf{\Pi}$. However, when the percentages of days estimated to belong in these regimes in $\mathbf{E}^X$ were calculated, small differences were found. The percentages were 83.6%, 8.5% and 7.9% respectively. Therefore, the estimated model does not exactly reproduce the distribution of spikes and down-spikes that is observed in the realized spot prices. Instead, it increases the frequency of spikes and down-spikes. A possible explanation for this phenomenon might be lie in the way spikes and down-spikes are distributed in the estimation data $\mathbf{E}^X$. As previously stated, they seem to be clustered in certain

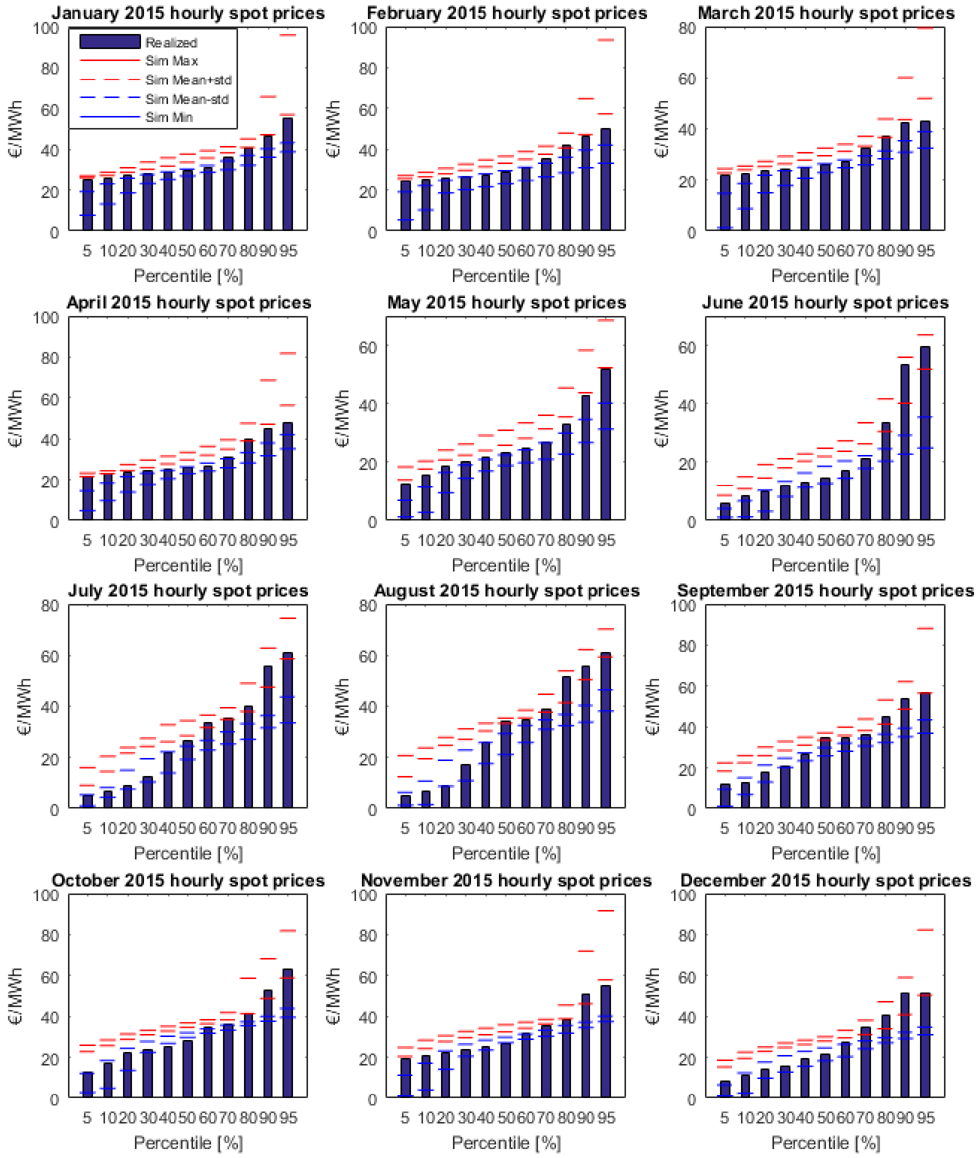

Figure 10: Percentiles of the realized hourly spot prices and statistics about the percentiles of 600 simulations in the year 2015.

periods of weeks and months. However, the MRS model cannot reproduce this kind of clusters without increasing the probability of spikes and down-spikes. This could have been the reason the EM algorithm was driven to these transition parameters.

Because the realized spikes and down-spikes seem to cluster together, we want to try to estimate how well the simulations repeat this behaviour. In Table 5, statistics about the frequency of spikes and down-spikes are calculated from all of the 108 realized months and they are compared to the statistics calculated from $600 \times 108 = 64800$ simulated months. Each month has 28 to 31 days which makes the measurement nonstandard, but many comparisons in this thesis are made with time spans of months, which makes it logical to be used here too. The results seem

Table 5: Statistics of the frequency and size of spikes and down-spikes during a period of a month. The statistics are gathered from realized spot prices $\mathbf{E}^X$ and simulations $\tilde{\mathbf{X}}^s, s = (1, 2, ..., 600)$ of the period 1.1.2007 - 31.8.2015. Therefore, the size of data samples for realized spot prices was 108 months, and the data sample for the simulations was $108 \times 600 = 64800$ months. The statistics are calculated from these pools respectively.

		Frequency		**Size**	
		Realized	Simulations	Realized	Simulations
Spikes	Average	2.57	2.95	52.91	51.85
	STD	3.25	2.84	12.58	11.49
	min	0	0	43.68	43.38
	max	15	22	132.11	473.11
Down-spikes	Average	2.41	2.46	30.59	30.57
	STD	3.87	3.45	4.76	5.78
	min	0	0	14.60	-94.60
	max	19	29	36.86	37.65

to be reasonable and unsurprising. The average frequency of spikes in a month is significantly higher in the simulations than in realized prices. The standard deviation calculated from the number of spikes and down-spikes seems to be higher in the realized spot prices than in the simulations. This again suggests that the spikes and down-spikes have realized in clusters, which have been followed by more stable periods. The MRS-model cannot replicate this kind of localization of spikes and down-spikes in the simulations. Instead, the spikes are distributed more evenly among different periods of time. The minimum and maximum numbers of spikes and down-spikes are sensible. The minimum number in all cases is 0, which is caused by the facts that there have been stable periods in the realized prices and when this many simulations are made it is likely that there are months where there are no spikes or down-spikes. Similarly, the large number of simulations explains why the maximum numbers of spikes and down-spikes calculated from all of the simulations are so much larger than the ones calculated from a single realization of the spot price. In conclusion, the frequency of spikes and down-spikes in the simulations seems to be acceptable according to these statistics.

Next, we will analyze the size of the spikes and down-spikes in $\mathbf{E}^X$ and $\tilde{\mathbf{X}}^s$, $s = (1, 2, ..., 600)$ presented in Table 5. The fact that spikes and down-spikes are more frequent in the simulations than in the realized data is balanced by the fact that they seem to be less extreme in size. The simulated spikes seem to be on average about one euro smaller, and the down-spikes are about 35 cents larger than the realized spikes and down-spikes. Because we have placed limits L_2 and L_3 for the size of spikes and down-spikes, the minimum spike size and the maximum down-spike sizes are very close to the ones calculated from the realized prices. However, because the spikes and down-spikes are drawn from a shifted lognormal distribution, there is chance of drawing extremely large values for spikes and extremely low values for down-spikes. Therefore, there is nothing alarming about the minimum and maximum sizes of the spikes or down-spikes. The standard deviation of the size of the spikes and down-spikes in the simulations seem to be quite close to the ones calculated from the realized spot prices, but there are some differences. The smaller standard deviation of the realized down-spikes can be explained by the fact that realized spot prices have always been positive. There is no such limitation in the simulations. Therefore, the down-spikes can be larger and their size varies more, which increases the standard deviation. However, for some reason the standard deviation of the size of the spikes of the realized spikes is larger in the standard deviation of the simulated spikes. This suggest that the simple shifted lognormal distribution is not perhaps accurately capturing the characteristics of the spikes. Based on this analysis, the sizes of the simulated spikes and down-spikes seem to be sufficiently realistic.

Next, we will analyze the distribution and size of spikes and down spikes between different type of days. The days are divided into workdays from Mondays to Friday and to non-working days that are Saturdays or Sundays. Public holidays are all categorized as Sundays. The results are visible in Table 6. This data is calculated from the realized and simulated daily spot prices from the period of 1.1.2007 to 31.8.2015. This analysis is important because, as explained in section 6.4.3, we decided to prevent spikes from happening during the weekends, and move them to the next available workday that is originally assigned to be in the base regime by the Markov process. It is clear that spikes do not occur often during Saturdays, but the number of spikes in Sundays and public holidays is about the same as in Fridays. However, it is important to notice that days of type 7 are the most frequent day type, which means that the probability of a spike on a Sunday or a public holiday is in reality quite low. Furthermore, the spikes during non-working days were on average smaller than during workdays. Therefore, as explained in section 6.4.3, forbidding spikes from occurring during the weekends seems to be justified. However, a better solution could be to model non working days separately or add a fourth regime for spikes that occur during the non-working days. These spikes could be less frequent and smaller in size. In the simulations spikes occurring in non-working days are pushed to the next available workday that is not originally a spike, which tilts the distribution of spikes heavily towards to the beginning of the week. However, the same kind of trend, all though not as strong, is visible in the realized spot prices. Spikes tend to have realized more frequently at the beginning of the week than at the end of the week. The distribution of realized down-spikes seems to be more even than the distribution of spikes, with the exception of Sundays and public holidays. The frequency of spikes during Sundays and public holidays was about double compared to other days. Days of type 7 are the most frequent day, which

Table 6: Statistics about the distribution and size of spikes and down-spikes in different type of days in the realized spot prices and in the simulated spot prises in the time period of 1.1.2007 - 31.8.2015. The statistics for the simulations are mean values calculated from 600 simulations.

	Working days					Non-working days	
	Mon	Tue	Wed	Thu	Fri	Sat	Sun or a holiday
Type $t(d)$	1	2	3	4	5	6	7
Number of days	445	443	446	441	427	435	528
	Spikes in realized spot prices						
Number	57	48	44	39	30	19	31
Mean size [€]	54.5	52.0	53.9	55.1	54.4	49.0	48.2
	Spikes in simulated spot prices						
Mean number	81.8	66.9	58.7	52.5	47.9	0.0	0.0
Mean size [€]	51.9	51.9	51.8	51.9	51.9	-	-
	Down-spikes in realized spot prices						
Number	37	26	26	33	29	36	64
Mean size [€]	31.8	31.9	30.5	30.7	29.8	29.4	30.4
	Down-spikes in simulated spot prices						
Mean number	35.7	35.9	36.2	36.0	34.9	35.3	42.6
Mean size [€]	30.5	30.5	30.6	30.6	30.5	30.6	30.6

means that a small elevation is expected, but it is obvious that down-spikes seem to occur more often during Sundays and public holidays than any other day type. The average size of down-spikes was consistent pretty much the same between all type of days. The down-spikes in the simulations are evenly distributed among the different types of days, but the frequency of down-spikes during Sundays and public holidays is elevated because it is the most frequent day type. However, the simulations do not reproduce the same level of concentration of down-spikes on Sundays and public holidays.

The size and distribution of spikes of the realized spot prices seem to be adequately reproduced in the simulations for the purpose these simulations are intended to be used in. The validation made so far is not conclusive, but it is sufficient to make sure that there were no large technical problems during the estimation and simulation processes. Further validation, for example, for the robustness of the parameters gathered with the EM algorithm is left for future research.

7.4 Power production quantities

To determine whether or not multiple simulations of the hourly spot prices are needed to robustly estimate the expected production quantities is necessary, the following test was executed. The 600 simulations of the spot price for the year 2015 were inputted one by one to the production schedule optimization program, introduced in section 4.1, and the production quantities of power in GWh during each month were collected. The other inputs such as hourly temperature curve, fuel prices per MWh and maintenance breaks, were the same in each run. They were given arbitrarily chosen values, which are more or less close to the values they could have in the future. We will not go into detail about them for confidentiality reasons. Let us refer to these simulations of the hourly spot prices as $\mathbf{H}^s = (H_1^s, H_2^s, ..., H_H^s)$, $s = (1, 2, ..., 600)$, where H_1^s is the spot price of the first hour of 1.1.2015 and H_H^s is the spot price of the last hour of 31.12.2015 in simulation s. The optimal power production quantity of power in month m that is calculated using the spot price simulation $\mathbf{H}^s$ is Q_m^s, and it is measured in gigawatt hours. The mean spot prices of the 600 simulations of month m is exactly the mean spot price of the realized spot prices in that month

$$\overline{H}_m = \frac{\sum_{s=1}^{s=600} \overline{H}_m^s}{600} = \frac{\sum_{s=1}^{s=600}}{600} \frac{\sum_{h=h_1^m}^{h=h_b^m} H_h^s}{h_b^m - h_1^m + 1} = F_m, \qquad m = (1, 2, ..., 12), \tag{82}$$

where h_1^m is the index of the first hour and h_b^m is the index of the last hour in month m, and F_m is the mean price of the realized spot prices in month m, which have been limited to 200€ /MWh. Therefore, the mean spot price of month m in simulation s is likely not exactly F_m, but the average spot price of all 600 simulations $\overline{H}_m$ is. The idea here is that once the production quantities Q_m^s are gathered, the mean production quantity $\overline{Q}_m$ in each month is intended to be used as the expected production quantity for that month

$$\overline{Q}_m = \frac{1}{600} \sum_{s=1}^{s=600} Q_m^s, \qquad m = (1, 2, ..., 12). \tag{83}$$

In order to determine whether multiple simulations should be used, a few comparisons are made. First, the distribution of the production quantities Q_m^s are are examined by using a scatter plot, where each Q_m^s of month m is placed in the y-axis against the corresponding mean spot price $\overline{H}_m^s$. If there are substantial differences in the optimal production quantities that have been calculated from simulations which have approximately the same mean spot price, it is clear that the monthly mean spot price does not robustly determine the expected production quantity. Therefore, the profile of the spot price has a significant effect on the optimal production quantity. This suggests that using only one simulation is riskier than using several simulations and taking their mean production quantity. However, confirming this claim does not automatically mean that using dozens of hundreds of simulations is necessary. It might be possible to create a robust simulation $\mathbf{H}^R$ of the spot price and use it to calculate a robust estimate of the production quantities Q_m^R for each month. To test this idea, we created a reference spot price curve $H^R = (H_1^R, H_2^R, ..., H_H^R)$ and ran it trough the optimization program and acquired the production quantities Q_m^R for each month $m = (1, 2, ..., 108)$. The daily resolution reference spot price curve

$\mathbf{P}^R$ = is created from the same deterministic component $\mathbf{f} = \mathbf{L} + \mathbf{S}$ than the other simulations, but no stochastic component is added

$$P_d^R = L_d + S_d, \qquad d = (1, 2, ..., D). \tag{84}$$

Therefore, its monthly means $\overline{P}_m^R$ are exactly F_m for all m. A modified version of the historical profile sampling (HPS) method introduced in section 6.6 is used to transform these daily values $\mathbf{P}^R$ to hourly spot prices $\mathbf{H}^R$. In the HPS, the daily means of the hourly spot prices equal to the daily spot prices, which means that the monthly means of the hourly spot prices $\overline{H}_m^R$ are also equal to F_m. The modification that is made to HPS is that instead of finding a suitable day d^h from all of the realized days from the period of 1.1.2007 to 31.8.2015, only seven predetermined days are used. A mean 24-hour price profile is created for each type of day $\tilde{t}(d) = (1, 2, ..., 7)$. Workdays from Monday to Friday are given the type 1 to 5 respectively. Saturdays are of type 6 and Sundays and public holidays are days of type 7. A reference day with a 24-hour price profile was created for each of these seven types, by taking the median spot price from the realized hourly spot prices, which are of the correct type. For example, to create the 24-hour profile for Mondays, all the hourly spot prices of days of type 1 were gathered from the realized hourly spot prices from 1.1.2007 to 31.8.2015 and the median value for each 24-hour of the day was taken. The same was done for the remaining 6 types. As a result, a 168 hour reference week was acquired, which is presented in Figure 11.

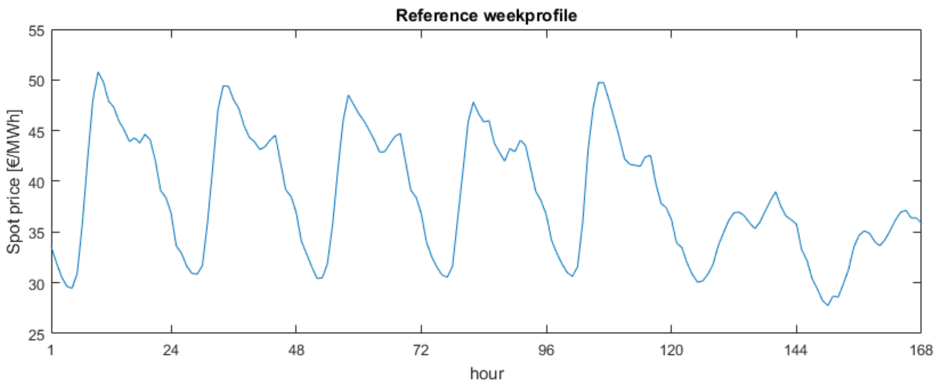

Figure 11: The median price profile of spot prices from 1.1.2007 - 31.8.2015. This profile is used to create the reference curve $\mathbf{H}^R$.

The simulated days in $\mathbf{H}^R$ have similar profiles to the ones in the figure, but they are scaled so that their mean matches the simulated daily spot price P_d^R, with the method described in section 6.6. The end product $\mathbf{H}^R$ is a simple simulation of the future spot prices. It has no spikes or down-spike and it is completely deterministic. The optimal production quantities $Q_m^R, m = (1, 2, ..., 12)$ calculated using these spot prices, serves as a reference production quantities to which the production quantities calculated from the 600 simulations can be compared to. If the mean production quantities $\overline{Q}_m$ are close to the reference production quantities Q_m^R, the usage of multiple simulations in determining the expected production quantities might be unnecessary. Why use a more complicated method, when a simpler one yields the same results.

The production quantities of the simulations are also compared with the production quantities Q_m^{Re} calculated using the realized spot prices $\mathbf{H}^H$, which have been limited to 200 euros per MWh. If the production quantities $Q_m^{Re}, m = (1, 2, ..., 12)$ deviate systematically from Q_m^s, it is a sign that the simulated spot prices are not realistic enough. However, as previously stated, due to the simplicity of the simulation model, the simulations do not depict as wide of a range of characteristics as the realized spot prices. Therefore, only if Q_m^{Re} are more extreme than any of the Q_m^s in several months, there is a cause for concern.

7.4.1 Statistics

The results of the production quantities calculated using the reference curve and the realized spot prices are presented in Table 7 with the mean production quantities $\overline{Q}_m$ calculated using the 600 simulations. Based on these results, it can be said that

Table 7: Calculated optimal production quantities of power [GWh].

	Calculated using	Simulations $\mathbf{H}^s$, $s = (1, 2, ..., 600)$	Reference curve $\mathbf{H}^R$	Realized spot prices $\mathbf{H}^H$		
	Mean spot price €/MWh	$\overline{Q}_m = \frac{\sum_{s=1}^{s=600} Q_m^s}{600}$	Q_m^R	Q_m^{Re}	$\overline{Q}_m - Q_m^R$	$\frac{\overline{Q}_m - Q_m^R}{Q_m^R}$
Jan	33.8	76.9	76.4	70.9	0.53	0.7 %
Feb	33.2	73.7	82.7	79.3	-9.04	-10.9 %
Mar	29.4	58.9	51.6	57.0	7.35	14.2 %
Apr	30.1	51.1	50.6	50.4	0.53	1.1 %
May	25.9	24.5	21.7	25.6	2.72	12.5 %
Jun	21.5	14.9	14.8	13.5	0.05	0.4 %
Jul	27.6	17.9	17.8	17.8	0.09	0.5 %
Aug	31.1	18.4	18.1	18.1	0.34	1.9 %
Sep	31.8	20.4	19.9	20.5	0.49	2.5 %
Oct	33.5	50.2	50.0	50.8	0.23	0.5 %
Nov	31.7	47.9	44.2	46.8	3.72	8.4 %
Dec	26.6	63.0	59.6	63.1	3.44	5.8 %

there are large differences in the production quantities $\overline{Q}_m$ and Q_m^R in 4-5 months and in the remaining months the difference is insignificant. The value of $\overline{Q}_m$ was larger than Q_m^R in all other months, but in February, it was 9 GWh smaller, which was the largest absolute difference between them. It is clear that in some cases the mean production quantity $\overline{Q}_m$ taken from the production quantities of several spot price simulations can be very different from the production quantity Q_m^R, which is optimized using a single simple simulation $\mathbf{H}^R$. The production quantities Q_m^{Re}

calculated using the realized spot prices are used as references to see if there would be some systematic difference compared to $\overline{Q}_m$ or Q_m. It seems that there are no large or systematic differences which would suggest critical insufficiency of the simulated spot prices compared to the realized spot prices. These results show that the production quantities calculated using several simulations can differ substantially from the production quantities calculated using a simple spot price simulation. This is especially true during the winter months. Therefore, it might be useful to take the extra time to run the simulations instead of using only one price curve.

7.4.2 Scatter plots

Next, we will discuss the results that can be gathered from the scatter plots of the production quantities $Q_m^s, s = (1, 2, ..., 600)$ in month m against the mean spot prices $\overline{H}_m^s, s = (1, 2, .., 600)$ that were used to calculate them. The scatter plots of the 12 months for year 2015 were made, but they are not all presented in this thesis for confidentiality reasons. Only two interesting months are shown here, and even then the scales of the axises are hidden. Also, the origos of the plots are moved from the true zeros. The hypothesis was that using several simulations instead of one, two benefits can be achieved. First of all, when several price profiles are used, the price profile risk decreases. The concept of price risk was introduced in section 3.2. It means that two simulations of the spot price with the same monthly mean spot price, can result in very different production quantities. Figure 12 demonstrates that even though the production quantity clearly correlates with the mean spot price, production quantities calculated from realizations with the same mean spot price can be very different. There is no scale on the Y-axis in Figure 12, but it can be said that the difference between the highest and the lowest production quantity in the middle of the X-axis is over 20 GWh. This is a large spread compared to the mean production quantities in Table 7, which range from 14.9 GWh to 76.9 GWh. Therefore, if only one simulation of the spot price is used, it can be hard to say weather or not the production quantity calculated from that spot price curve is exceptionally large or small. This uncertainty can be diminished by running several simulations of the spot price and calculating optimal production quantities for them and taking their average. The production quantity Q_m^R calculated using the reference spot price curve $\mathbf{H}^R$ was close to the mean production quantity $\overline{Q}_m$, which suggests that using a single simulation to get a robust estimate of the production quantity might be possible under some conditions. However, as can be seen from the results in Table 7, $\overline{Q}_m$ and Q_m^R are quite different in many months. Therefore, even though Q_m^R was close to $\overline{Q}_m$ in this month, it is not the case every time.

The main result that can be obtained from Figure 12 is that the profile risk is real at least in some months. The optimal production quantity depends on the profile of the spot price, which means that if only one price curve is used, it should not be generated randomly. Using many simulations decreases the profile risk. However, because $\overline{Q}_m$ and Q_m^R were quite close to each other in this month, it seems that using a single designed price curve can give sufficiently accurate production quantities under the right conditions.

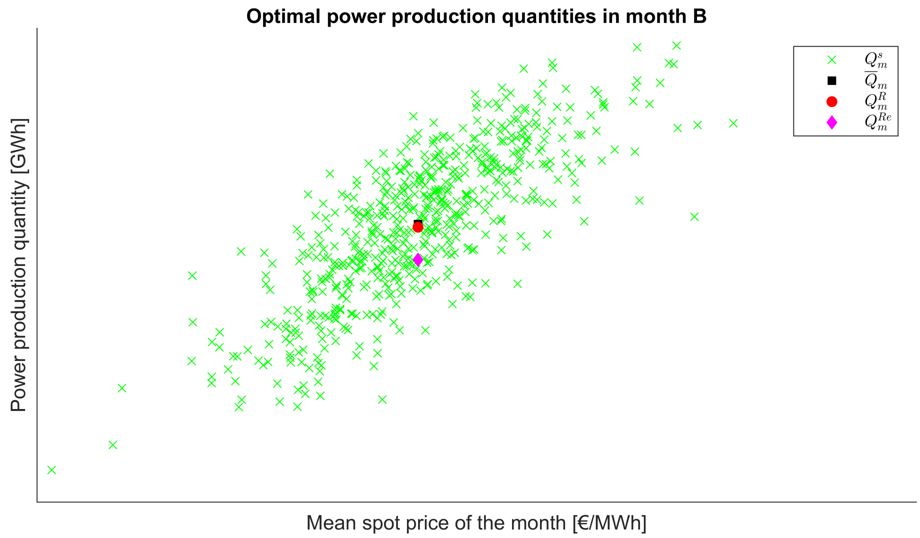

Figure 12: Optimal production quantities that are plotted against the monthly mean spot prices of the 600 different spot price curves that were used to obtain them. The axes are hidden for confidentiality reasons, but it can be said that the difference between the highest and the lowest production quantity in the middle of the x-axis is over 20 GWh. This image shows clearly that the production quantity can be very different between spot price curves with the same mean spot price.

The second scatter plot in Figure 13 demonstrates the other benefit that is gained from using several simulations. In Figure 13, a clear lower bound for the production quantity is visible. Of course, upper bounds on power production can also be observed in certain situations. These bounds are caused by the physical restraints of the production units and fixed starting costs. In month A, it is not profitable to shut down a large CHP production unit, because it is the most affordable way to satisfy the demand of district heat. How ever, because the spot prices are relatively low, it is run most of the time at minimum power. In some simulations, the shape of the spot price curve is such that more than the minimum amount of power is produced. The reason the boundary is so clear is because the same temperature curve was used in each case. The price points where these boundaries appear depend on the characteristics of the production units, temperatures and fuel costs, which are all inputted to the optimization program. Therefore, the location of the boundaries can change as the inputs change. With simulations, these boundaries can be discovered and that information can maybe be used for hedging. It is relevant information to know that the production quantity will not drop below a certain level, assuming that the mean spot price of that month realizes somewhat as forecasted and the other inputs realize as forecasted.

Figure 13 is an excellent example, which shows us why using multiple simulations can be beneficial. It shows that the optimal production quantity can be non-linearly dependent of the mean spot price. The mean production quantity $\overline{Q}_m$ is larger than

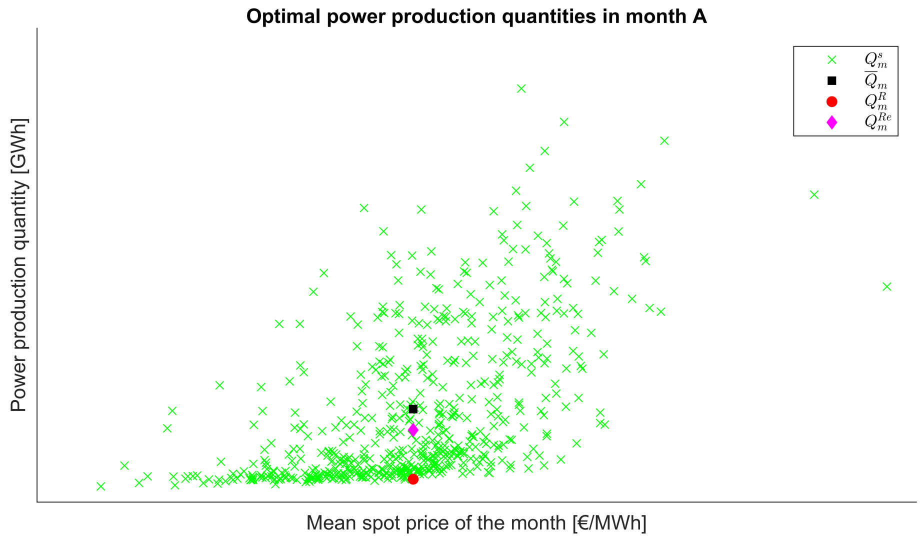

Figure 13: Optimal production quantities in a month calculated using different spot price profiles. The axes are hidden for confidentiality reasons, but it can be said that the difference between the highest and the lowest production quantity is over 10 GWh. This image shows clearly how the production quantity is not a linear function of the mean spot price.

Q_m^R or Q_m^{Re}, which is not a coincidence. The distribution of production quantities have often a lower cap, which causes the mean to be larger than the median. If only one conservative spot price curve such as $\mathbf{H}^R$ is used, the forecasted production quantity will be most likely lower than the mean production quantity of multiple realistic simulations. This effect can be seen in table 7, where $\overline{Q}_m$ is larger than Q_m^R in all months expect February. How ever, these results are only valid for this specific set of mean spot prices, temperatures and other parameters.

7.4.3 Utility of simulations in hedging

Next, we will discuss whether or not, based on these results, using several simulations produces better results compared to one spot price curve, regarding hedging. Based on the results in Figure 12, it is clear that production quantities calculated using only one price curve that is produced randomly should not be used. The optimal production quantities calculated using two simulations of the spot price with the same monthly mean, can differ by more than 20 GWh. However, the differences between $\overline{Q}_m$ and Q_m^R were significantly smaller. Therefore, it might be possible to produce a single conservative simulation of the spot price curve, which would be used for forecasting.

The differences between the production quantities $\overline{Q}_m$ and Q_m^R in different months were between -9.04 GWh and 7.35 GWh. So that the significance of these differences can be understood, we need to establish that what are the consequences of estimating the production quantity of one month incorrectly regarding hedging. Let us assume that the trader is required by the companies risk strategy to hedge $x\%$ of the estimated production quantity Q_m of month m. If the forecasted production quantity Q_m is larger than the real expected value, the company will end up hedging a larger amount of its production that they intended to. Therefore, if the spot prices in month m realize higher than expected, the company's profits will be smaller than if they would have hedged less. However, if the spot prices realizes lower than anticipated, they will benefit from hedging a larger amount of power than they intended to. Similarly, if the forecasted production quantity Q_m is smaller than the true expected power production quantity, they will benefit more of the increasing spot prices than if they had hedged a larger amount of power. However, if the spot prices decrease, their profits will decrease more than if the had hedged a larger portion of their production. Therefore, sometimes it is beneficial to over- or underestimate the future production quantities. This means that mistakes in the forecasts do not necessarily lead to losses. Therefore, an error of a few GWh in the production forecasts caused by poor spot price simulation(s) is not necessarily a large problem. However, using only one simulation exposes the company to a different amount of price risk than it wants to be exposed to. Each company needs to decide themselves is is worth to use resources to run simulations, which can produce more robust estimates of the production quantities. The observed differences in the production quantities of $\overline{Q}_m$ Q_m^R and Q_m^{Re} were less than 10 GWh and often close to zero. Therefore, using several simulations of the spot price curve does not change the results dramatically compared to production quantities calculated using simple methods such as Q_m^R. However, the results obtained in this thesis are valid only with this particular set of temperatures, mean spot prices, fuel costs and other parameters. If the spot prices continue their current trend and fall even further, the significance of the shape of the price curve might change.

The ability to produce realistic simulations of the spot price with different price profiles can be useful in other areas than hedging. For example, when changes to the production machinery are planned, the possible consequences of these actions might be more accurately measured by creating optimal production schedules with different simulations of the spot price, than using only one type of price profile. For example, the profitability of improving the flexibility of a production unit can be tested in different scenarios more thoroughly by using multiple and unique price curves than just one price curve. The upgrade might become useful only under certain conditions, and the results gathered using one price curve might under- or overestimate the benefits.

7.4.4 Suggestions

The optimal production schedules are optimized on an hourly resolution and an hourly spot price curve needs to be provided so that the calculations can be carried out. We have shown that, if only one simulation is made using a stochastic model presented in this thesis, the production quantity calculated using that price curve, should not be trusted. That price curve might have caused exceptionally large or small production quantities. Therefore, we suggest using multiple simulations to decrease this risk. However, we acknowledge that solving the optimal production schedule several hundred times, takes a lot of computing time, which could perhaps be used to solve more urgent matters. However, if only one simulation is used, special attention should be put into its design. The simple reference curve $\mathbf{H}^R$ does not most likely provide the best results. The realized spot prices have spikes and down-spikes, which should be incorporated to the simulated price curve. If there are no spikes, the usage of gas turbines and other production methods with high variable costs will be used too rarely, and the estimated production quantity will be underestimated. We leave the design of this price curve for future research. A compromise between these two approaches could be using a suitable number, for example, 5-20 simulations of the spot price, which have different characteristics. After the production quantities are calculated using these price curves, they could be weighted by the assumed realization probability of each simulation. The mean of these weighted production quantities could be considered as the expected production quantity. Some of the simulated spot price curves should be more volatile than others, and the highs and lows of the price curves should be located in different parts of the month so that the interaction between the temperature curve would be different in each case. Choosing what these classes of simulations are and assessing their probabilities can be difficult. In this thesis, that part was bypassed by assuming that the MRS model created different simulations in the correct distribution so that more common spot price curves were more frequently produced than rarer ones. Therefore, when the mean production quantity $\overline{Q}_m$ was calculated, weighting of the different realizations was no longer necessary and that $\overline{Q}_m$ was already the expected production quantity. Of course, the assumption that the MRS model can produce different type of simulations in the right distribution will not most likely hold under scrutiny. Simulations of the temperature curve should also be done in the future. Furthermore, because the spot price is correlated to the temperature, they should perhaps be modeled together.

8 Conclusion and discussion

The first goal of this thesis was to create a model that can create multiple simulations of the hourly spot price for years into the future, once the expected mean spot prices $\mathbf{F} = (F_1, F_2, ..., F_n)$ for each month are given. The values of $\mathbf{F}$ are conservative forecasts of the future mean spot prices generated by experts. The second goal was to answer whether or not using several simulations of the spot price is necessary to get accurate forecasts of the future power production quantities, or will a single spot price curve suffice. Production quantities are forecasted by using a production schedule optimization program, which takes inputs such as the hourly spot price curve, hourly temperature curve, fuel costs per MWh and maintenance outages. It outputs the hourly production schedule of power and heat production units of Vantaan Energia (VE). From this schedule, among other things, the optimal power production quantity in each month can be calculated. The motivation behind this thesis is to improve the accuracy of the forecast of the future power production quantities so that hedging could be done more accurately according to the risk policy.

The spot price model was introduced in sections 6. The simulations were made in two stages. In the first stage, the daily spot prices were created, so that their monthly means are close to the forecast $\mathbf{F}$. In the second stage, they were modified into hourly spot prices using a simple method called the Historical Profile Sampling (HPS). The daily spot price consist of a deterministic component and the stochastic component. The deterministic component is further divided into two components, which are the short-term seasonal component (STSC) and the long-term seasonal component (LTSC). The STSC models the weekly predictable fluctuation between each day of the week, while the LTSC controls the monthly means of the simulations. The LTSC is adjusted so that the mean of the deterministic component is equal to the forecasted monthly means. The stochastic component is modeled with a Markov regime-switching (MRS) model. It has three independent regimes which are base, spike and down-spike. The model parameters were estimated using a modified expectation–maximization algorithm. The long-term mean of the stochastic component was adjusted to zero, so that when it is added to the LTSC and STSC, the monthly means of the daily spot prices remain, in general, close to the forecasted mean price. Even though the long-term mean of the stochastic component is zero, its monthly means vary. We want this kind of fluctuation to occur, but we also want to control it. Therefore, once the desired number of simulations were done, they were are all adjusted slightly so that the combined monthly means calculated from all of the simulations matched the forecast vector $\mathbf{F}$. After this procedure, the daily spot prices were transformed into hourly spot prices one by one with historical profile sampling (HPS), which was introduced in section 6.6. In HPS, a realized day which is similar to the simulated day is retrieved, and its 24-hour price profile is used to create the price profile for the simulated day.

The thorough validation of the MRS model was left for future research. Instead, general statistics of the simulations and the realized spot prices were examined. The distribution of spikes and down-spikes in the realized daily spot prices were compared to the distributions of the simulated daily prices from year 2007 to 2015. Also, percentiles of the spot prices from different time periods were calculated from the realized hourly spot prices and compared to statistics calculated from the percentiles of the simulations. These results showed that the realized spot prices change characteristics from season to season and year to year. For example, a few volatile months can be followed by several months where the volatility is low. In the simulations, the volatility seemed to be more even throughout the year. The realized spot prices display diverse characteristics, which the parsimonious model could not accurately reproduce. Despite of these shortcomings, the simulations were deemed to exhibit the same characteristics of the realized spot prices well enough, so that they can be used as inputs to the production schedule optimization program.

The results supported the hypothesis, that the production quantities are not only sensitive to the monthly mean of the spot price, but also to its profile. The optimal production quantity was not linearly dependent on the mean spot price, because the production units have fixed starting costs and their variable costs depend on many things such as the outside temperature. Therefore, it is relevant what kind of price profile one uses to forecast the future production quantities. The usage of multiple simulations and taking their mean production quantity decreases the risk that the used price curve results in an exceptionally large or small optimal production quantities.

Based on the results presented in this thesis, the use of several simulations is advisable when future production quantities are forecasted. The optimal production quantities can differ by dozens of percentages especially during the winter months. However, the increase in the reliability of the forecast does not translate directly into substantial benefits in hedging. The reason is that each company has their own risk policy. Some hedge 100% others 0% of their estimated production. Whatever hedging percentage is used, it should not be detrimental to the company, if the actual production quantity that is hedged is off by ten percent or so due to an error in the production forecast. Hedging a smaller quantity increases risk and hedging a larger quantity decreases risk. Which one was the better decision is only revealed when the spot prices realize. Therefore, the benefit of multiple simulations seems to be moderate at best regarding hedging, but the ability to produce realistic simulations of the spot price in any given price range can be beneficial in other areas. For example, when possible modifications to the production units are considered. The benefits of the modifications can be estimated by modeling the changes into the production schedule optimization program and calculating the profits, fuel consumption, production quantities and other relevant information under different conditions. Therefore, the costs of the modification can be compared to the possible increase of profits. Using only one simulation of the spot price curve to get these results might under- or overestimate the impact of the modification. Using multiple spot price profiles, decreases that risk.

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Appendix A

Table 8: Percentiles calculated from the realized prices and statistics of the percentiles calculated from the 600 simulations.

Year		Percentile of the hourly spot price [€/MWh]										
		5 %	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	95 %
2007	Realized	18.90	20.35	22.20	23.50	24.89	26.54	28.89	33.42	40.27	44.96	48.57
	Sim Mean	10.43	15.35	20.33	22.91	25.09	27.77	31.26	35.24	39.79	46.47	52.79
	Sim STD	1.19	1.28	0.66	0.48	0.47	0.61	0.72	0.63	0.84	1.13	1.66
	Sim Min	7.12	10.94	17.98	21.36	23.74	26.00	29.43	33.43	37.56	43.08	48.44
	Sim Max	14.14	18.29	22.02	24.38	27.00	29.74	33.53	37.51	42.44	50.44	58.59
2008	Realized	26.68	30.75	38.29	42.21	45.44	49.45	55.00	59.77	64.93	71.89	77.20
	Sim Mean	26.56	30.79	36.54	40.81	44.80	48.97	54.21	59.31	65.03	72.27	78.88
	Sim STD	1.16	0.90	0.75	0.68	0.61	0.67	0.81	0.80	0.76	0.90	1.75
	Sim Min	21.44	28.10	34.14	38.40	42.84	47.23	51.50	57.00	62.50	69.32	74.63
	Sim Max	30.01	33.46	38.31	42.74	46.82	51.22	56.49	61.84	66.85	75.12	85.60
2009	Realized	26.27	29.58	32.38	33.90	35.06	36.25	37.48	38.80	40.42	42.91	46.54
	Sim Mean	20.67	24.61	28.67	31.41	33.72	35.77	37.77	40.13	43.40	49.12	56.36
	Sim STD	1.36	0.70	0.62	0.53	0.46	0.45	0.44	0.51	0.69	1.11	2.14
	Sim Min	14.85	21.93	26.60	29.91	32.24	34.25	36.20	38.36	41.11	45.38	50.06
	Sim Max	24.04	27.14	30.60	33.18	35.34	37.17	39.05	41.68	45.12	52.88	62.97
2010	Realized	33.77	39.07	43.07	45.74	47.54	49.20	51.65	55.20	62.32	80.27	95.02
	Sim Mean	30.28	34.60	40.03	43.89	46.94	50.08	54.37	59.96	68.38	84.97	95.80
	Sim STD	1.04	0.83	0.69	0.59	0.54	0.64	0.77	0.85	1.03	1.53	1.78
	Sim Min	26.19	32.01	38.04	42.27	45.20	48.31	52.41	57.61	65.49	80.08	90.40
	Sim Max	32.78	37.00	42.38	45.71	48.57	51.73	56.22	62.94	72.18	88.66	101.68
2011	Realized	26.56	33.20	38.06	41.68	45.03	49.84	53.79	56.83	61.08	66.29	69.92
	Sim Mean	26.92	30.77	35.68	39.84	44.01	48.50	53.02	56.99	61.73	67.41	74.04
	Sim STD	1.05	0.70	0.64	0.71	0.66	0.81	0.73	0.69	0.92	0.87	2.22
	Sim Min	22.49	28.57	33.80	37.81	41.83	45.70	50.15	55.03	59.23	65.11	68.22
	Sim Max	31.25	34.35	38.62	42.30	45.94	50.52	54.94	58.77	64.46	70.80	81.02
2012	Realized	10.11	20.54	26.44	30.01	32.45	34.87	37.32	40.41	45.02	52.11	60.04
	Sim Mean	10.45	18.30	26.13	30.48	33.68	36.44	39.21	42.54	46.44	53.49	60.20
	Sim STD	1.62	1.48	0.83	0.62	0.53	0.52	0.58	0.67	0.72	1.36	1.79
	Sim Min	3.91	13.95	23.19	28.29	31.63	34.61	37.52	40.30	44.07	49.61	55.39
	Sim Max	14.96	22.25	28.58	32.27	35.21	37.84	41.09	44.46	48.80	57.89	66.96
2013	Realized	28.68	31.08	33.84	35.81	37.37	39.07	40.91	43.39	46.80	53.72	61.17
	Sim Mean	25.13	28.75	32.49	35.29	37.71	40.01	42.49	45.21	48.44	54.22	60.97
	Sim STD	1.16	0.72	0.60	0.57	0.50	0.54	0.55	0.51	0.62	1.11	1.89
	Sim Min	19.73	25.64	30.42	33.74	36.42	38.57	40.99	43.74	46.83	51.27	56.09
	Sim Max	27.85	30.44	34.07	36.91	39.39	41.93	44.43	46.75	50.73	58.10	69.27
2014	Realized	24.05	26.04	28.61	30.11	31.54	33.37	35.97	38.18	41.93	48.46	56.10
	Sim Mean	20.98	24.38	27.97	30.50	32.80	34.93	37.08	39.42	42.63	48.31	55.53
	Sim STD	1.31	0.70	0.64	0.53	0.51	0.48	0.44	0.51	0.67	1.11	2.29
	Sim Min	16.08	21.65	26.05	28.96	31.23	33.43	35.73	38.03	40.93	45.50	50.60
	Sim Max	23.87	26.29	29.83	32.20	34.34	36.27	38.61	41.24	44.64	52.74	62.86
2015	Realized	8.71	12.30	19.03	23.04	24.83	26.89	30.87	34.48	39.96	49.97	55.97
	Sim Mean	11.22	17.01	21.97	24.49	26.64	28.93	31.10	33.74	36.76	41.75	48.56
	Sim STD	1.35	1.34	0.64	0.46	0.54	0.51	0.55	0.56	0.60	1.08	2.30
	Sim Min	7.09	12.15	20.19	23.06	25.17	27.37	29.72	32.10	34.92	39.22	43.66
	Sim Max	15.48	19.78	23.74	26.00	28.31	30.38	32.86	35.44	38.39	45.05	59.25

Table 9: Percentiles calculated from the realized prices and statistics of the percentiles calculated from the 600 simulations.

		Percentile of the hourly spot price [€/MWh]										
2015		**5 %**	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**95 %**
Jan	Realized	25.11	26.03	27.25	28.10	28.82	29.72	31.17	35.95	40.57	46.71	55.39
	Sim Mean	22.67	25.06	27.32	28.91	30.41	32.05	33.97	36.26	38.97	43.67	50.01
	Sim STD	3.49	2.15	1.41	1.26	1.41	1.65	1.83	1.94	1.91	3.40	6.86
	Sim Min	27.02	28.77	31.00	33.75	35.84	37.69	39.29	41.21	45.03	65.62	95.77
	Sim Max	7.68	13.12	18.48	23.16	25.39	27.04	28.74	29.99	32.18	36.10	38.75
Feb	Realized	24.67	25.14	26.08	26.55	27.40	29.17	31.06	35.57	42.28	46.02	50.07
	Sim Mean	22.35	24.30	26.41	28.12	29.73	31.36	33.16	35.41	38.40	43.39	49.62
	Sim STD	3.28	2.24	1.64	1.63	1.70	1.82	1.99	2.19	2.33	3.77	7.60
	Sim Min	27.18	28.77	30.77	32.80	34.84	36.60	38.90	41.47	47.86	64.71	93.41
	Sim Max	5.38	10.22	18.50	20.17	21.63	23.10	24.69	26.53	28.58	31.00	33.17
Mar	Realized	21.76	22.59	23.75	24.21	24.87	25.88	27.42	32.14	37.08	42.20	43.09
	Sim Mean	18.72	21.28	23.59	25.02	26.39	27.91	29.51	31.32	34.02	39.45	45.39
	Sim STD	4.03	2.73	1.66	1.36	1.45	1.57	1.63	1.91	2.65	4.14	6.46
	Sim Min	24.44	25.46	27.28	29.27	30.68	32.45	33.94	36.97	43.79	60.08	79.45
	Sim Max	1.22	8.71	14.86	17.69	20.74	22.89	24.81	25.89	28.33	30.87	32.45
Apr	Realized	21.58	22.71	23.96	24.49	25.06	25.71	26.68	31.08	39.99	45.04	48.03
	Sim Mean	18.06	20.73	23.11	24.63	26.16	28.01	30.09	32.63	36.16	42.53	49.18
	Sim STD	3.54	2.30	1.45	1.33	1.56	1.74	1.92	2.31	2.84	4.55	7.09
	Sim Min	23.22	24.38	27.42	29.64	31.46	33.44	36.29	39.54	47.56	68.69	81.87
	Sim Max	4.81	9.66	13.89	17.63	20.56	23.06	24.23	25.91	28.25	31.76	35.11
May	Realized	12.28	15.53	18.58	20.40	21.56	23.17	24.59	26.98	32.78	42.90	51.65
	Sim Mean	10.41	14.59	18.64	20.74	22.46	24.13	26.20	28.98	32.68	39.15	46.27
	Sim STD	3.49	3.11	2.16	1.70	1.54	1.66	1.97	2.35	2.74	4.55	6.09
	Sim Min	18.37	20.37	24.14	26.19	29.15	30.90	33.42	36.01	45.33	58.34	68.52
	Sim Max	1.20	2.77	9.53	14.51	17.00	18.83	19.89	21.03	22.82	26.67	31.28
Jun	Realized	5.78	8.20	9.92	11.80	12.97	14.63	17.08	21.41	33.58	53.34	59.66
	Sim Mean	6.34	8.79	12.49	15.74	18.36	20.28	22.09	24.26	27.56	34.66	43.67
	Sim STD	2.27	2.08	2.12	2.32	2.02	1.69	1.67	2.00	2.96	5.46	8.28
	Sim Min	12.00	15.03	19.22	21.23	22.86	24.80	27.30	33.45	41.60	55.97	63.56
	Sim Max	1.09	1.20	3.12	8.15	11.35	12.68	14.51	17.90	20.38	22.83	24.92
Jul	Realized	5.05	6.66	9.21	12.28	21.79	26.48	33.59	35.53	40.06	55.61	60.93
	Sim Mean	7.25	11.49	18.43	21.96	24.24	26.51	29.20	32.44	35.58	42.01	51.12
	Sim STD	1.92	3.14	3.33	2.42	1.96	2.04	2.46	2.41	2.36	5.66	7.48
	Sim Min	16.16	20.49	23.93	27.52	32.76	34.37	36.56	39.40	49.11	62.84	74.42
	Sim Max	1.03	4.33	7.65	10.48	14.03	19.25	22.92	25.27	27.06	31.61	33.50
Aug	Realized	4.96	6.54	9.12	17.37	26.07	34.03	34.46	38.99	51.64	55.82	60.95
	Sim Mean	9.40	15.19	21.86	25.00	28.08	31.35	34.02	36.20	39.07	45.46	52.91
	Sim STD	3.16	4.35	2.90	2.10	2.25	2.03	1.45	1.53	2.29	5.14	6.53
	Sim Min	20.80	23.78	27.91	31.15	33.32	35.40	38.46	44.66	53.91	62.23	70.24
	Sim Max	1.27	1.51	8.84	11.05	17.68	21.17	25.86	31.04	32.49	33.80	38.08
Sep	Realized	11.89	13.04	18.04	20.94	26.37	34.40	34.50	36.27	44.78	53.42	56.92
	Sim Mean	13.97	18.84	23.68	26.60	29.22	31.65	33.90	36.20	38.87	44.01	49.87
	Sim STD	4.47	3.65	2.33	1.84	1.91	1.86	1.85	1.94	2.48	4.65	6.49
	Sim Min	22.40	26.03	30.15	32.83	34.93	37.06	39.90	43.82	53.07	62.27	88.14
	Sim Max	1.03	6.78	13.11	20.22	23.41	25.80	28.13	30.62	32.31	35.14	36.87
Oct	Realized	12.69	17.03	22.19	23.82	24.93	28.11	34.45	35.88	41.23	52.67	63.43
	Sim Mean	17.54	22.06	26.56	29.42	31.55	33.32	35.14	36.96	39.31	44.35	51.37
	Sim STD	5.32	3.67	2.15	1.64	1.29	1.27	1.29	1.29	2.02	4.37	7.60
	Sim Min	25.90	28.43	31.38	33.08	35.12	36.74	38.31	41.76	58.78	68.29	81.84
	Sim Max	2.78	4.91	13.67	22.43	26.83	29.75	31.78	33.26	35.54	37.56	39.51
Nov	Realized	19.58	21.03	22.50	23.56	24.85	26.89	31.84	35.09	37.99	50.96	55.08
	Sim Mean	15.86	20.72	25.45	28.00	29.68	31.13	32.78	34.74	37.16	41.57	48.98
	Sim STD	4.60	3.59	2.27	1.65	1.30	1.30	1.48	1.60	1.61	4.46	8.97
	Sim Min	24.84	28.34	30.58	32.66	34.24	35.83	37.14	38.48	45.46	72.01	91.60
	Sim Max	1.14	4.13	14.13	20.52	23.51	27.07	28.80	30.23	31.82	34.49	37.45
Dec	Realized	8.57	11.47	14.11	15.92	19.17	21.84	27.03	34.71	40.61	51.34	51.45
	Sim Mean	10.91	15.97	20.32	22.79	24.61	26.27	27.92	29.57	31.81	36.49	42.51
	Sim STD	4.38	3.59	2.58	2.02	1.68	1.63	1.52	1.51	2.17	4.26	7.77
	Sim Min	18.63	22.51	25.04	27.02	28.35	30.04	33.11	38.03	47.12	59.16	82.28
	Sim Max	1.09	2.48	9.88	12.91	15.61	18.35	20.39	24.16	27.09	29.20	31.06

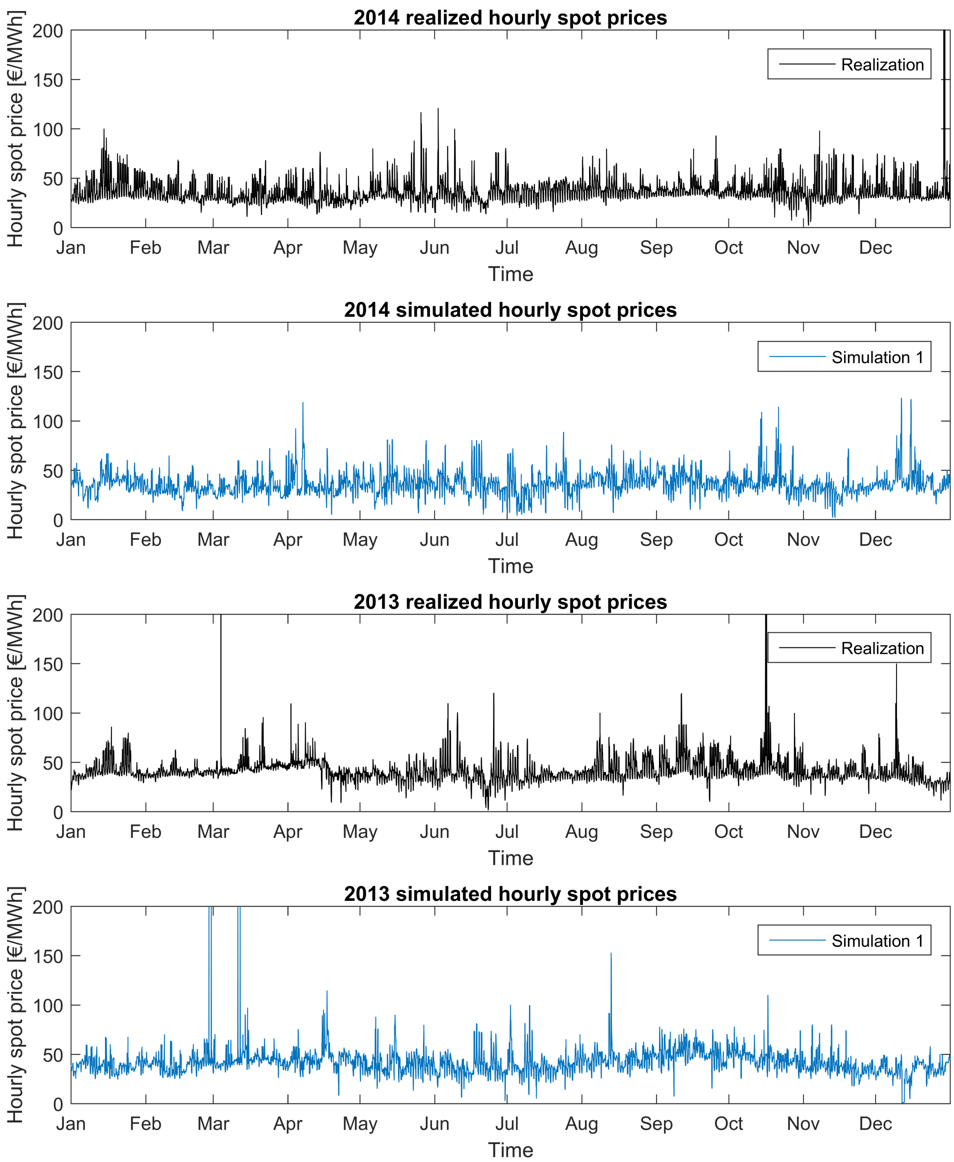

Figure 14: Realized and simulated hourly spot prices which have been limited to 200€/MWh.

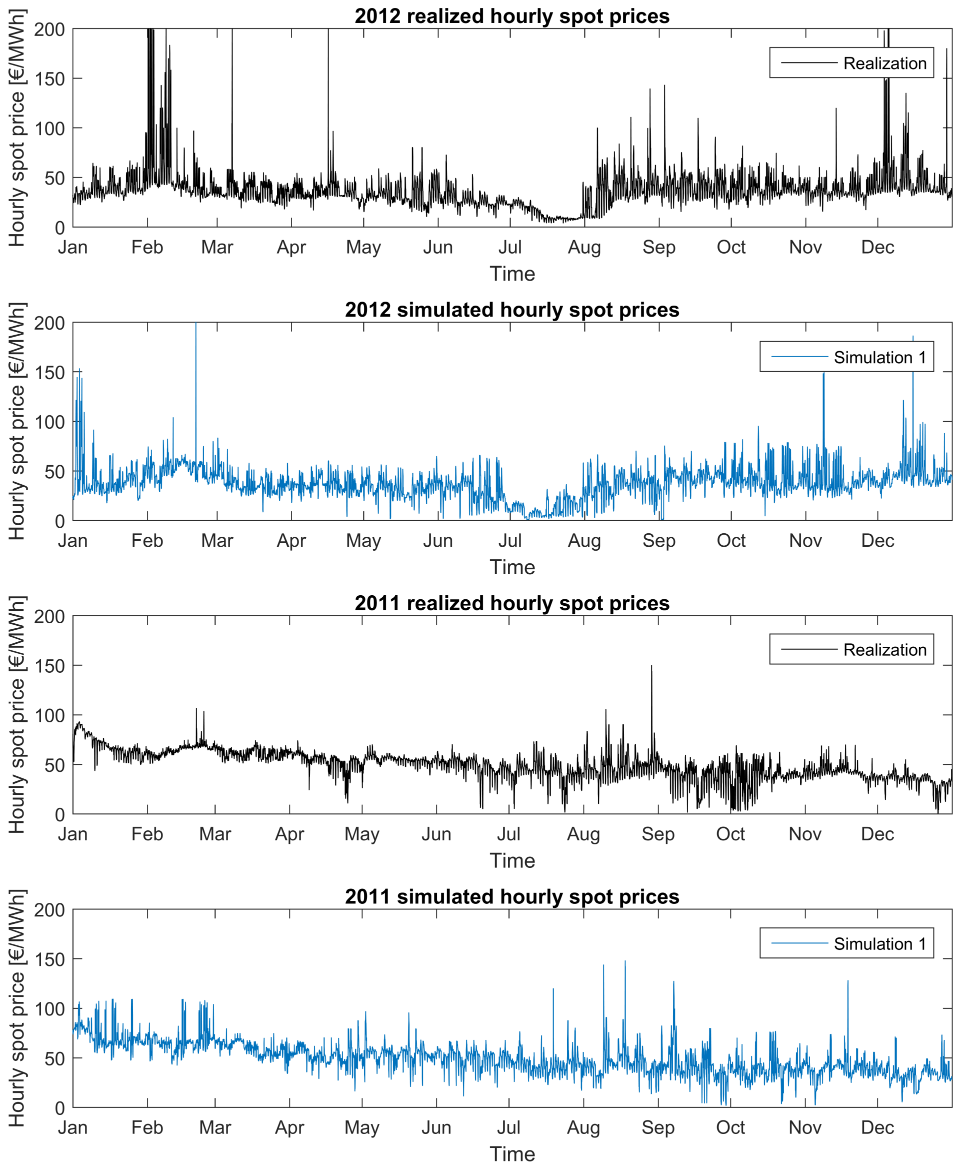

Figure 15: Realized and simulated hourly spot prices which have been limited to 200€/MWh.

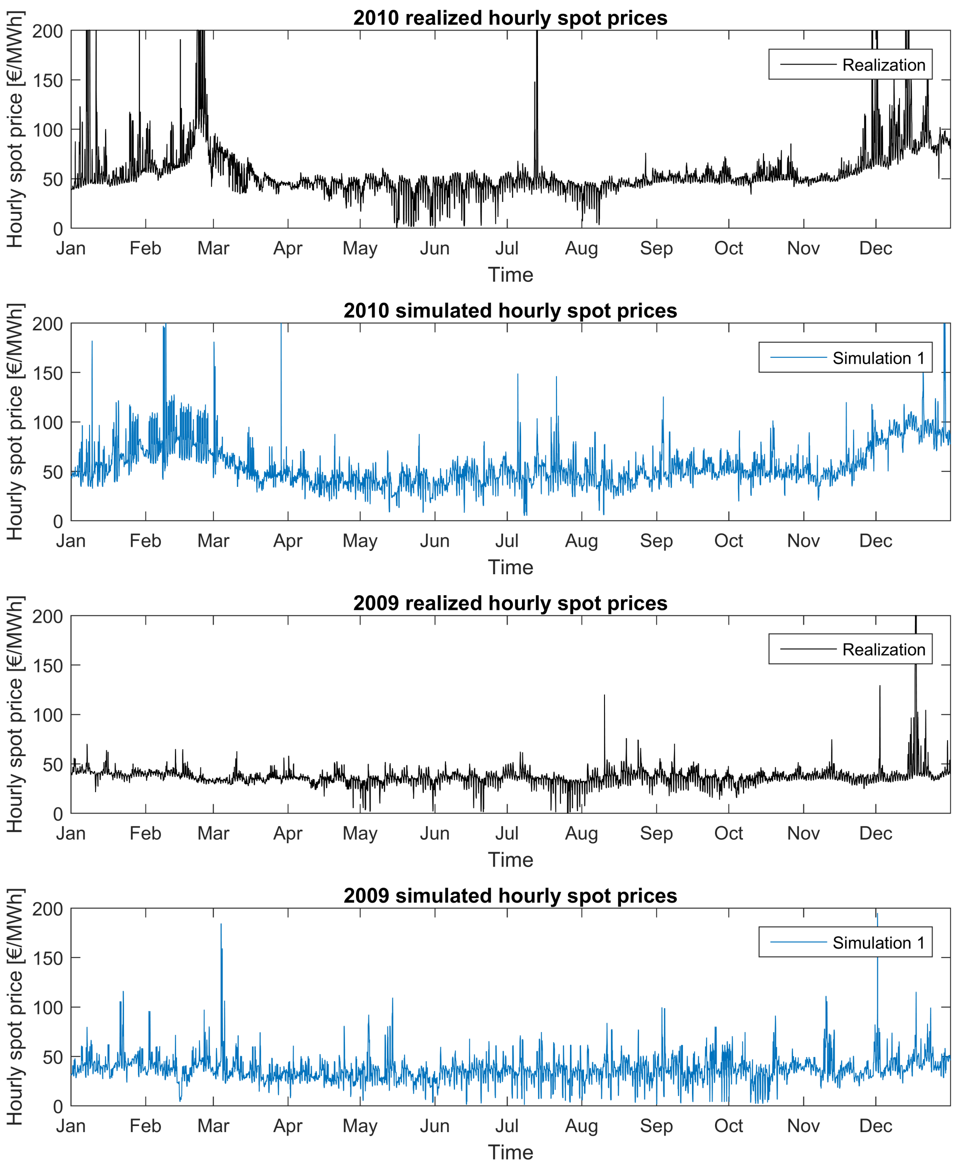

Figure 16: Realized and simulated hourly spot prices which have been limited to 200€/MWh.

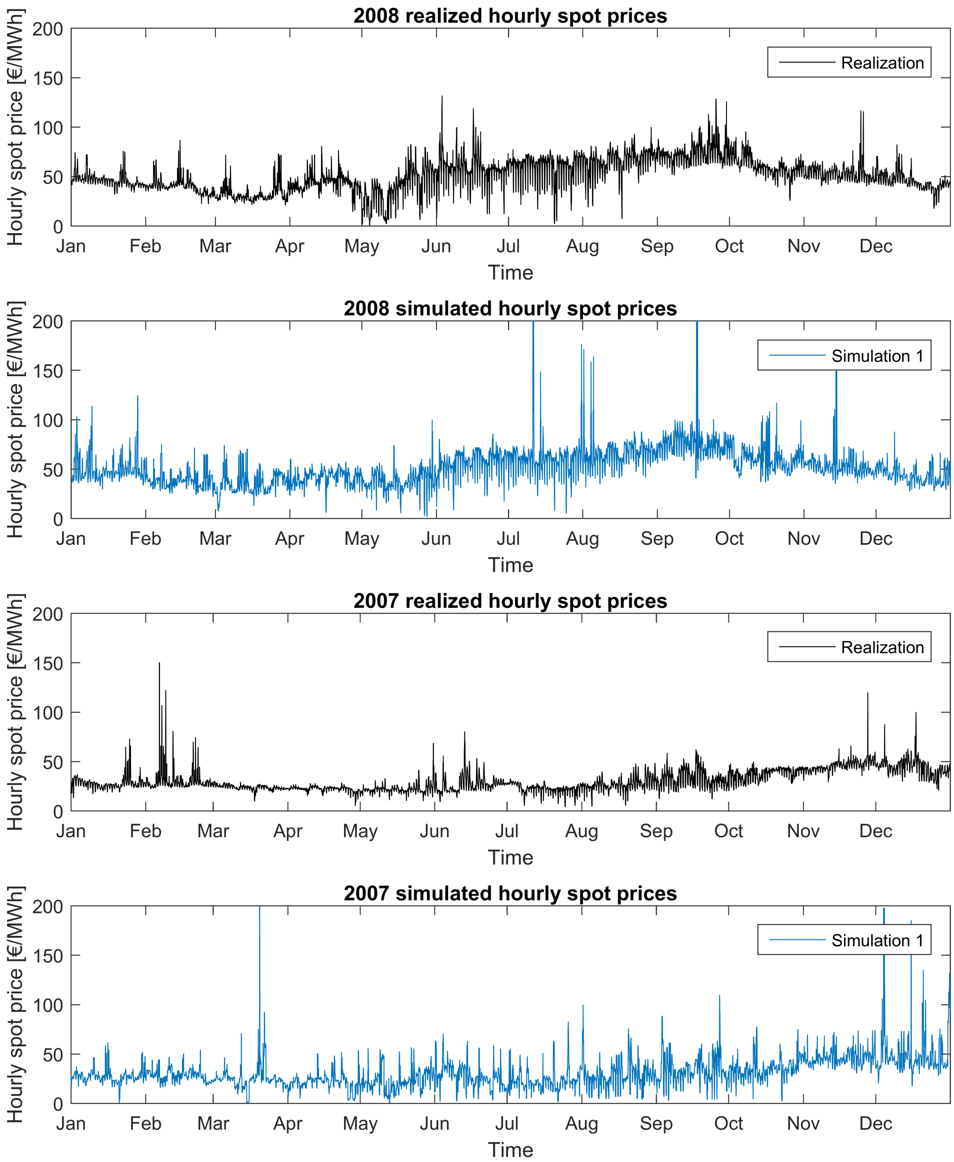

Figure 17: Realized and simulated hourly spot prices which have been limited to 200€/MWh.