Aalto University
School of Science
Degree Programme in Mathematics and Operations Research

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Pricing regulatory capital in over-the-counter derivatives

Master's Thesis
Espoo, August 31, 2017

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ABSTRACT OF MASTER'S THESIS

Author:	Joonas Lanne		
Title:	Pricing regulatory capital in over-the-counter derivatives		
Date:	August 31, 2017	**Pages:**	82
Major:	Systems and Operations Research	**Code:**	SCI3055
Supervisor:	Professor Ahti Salo		
Advisor:	D.Sc. (Tech.) Ruth Kaila		

Tightened regulation requires banks to hold a certain amount of capital to cover for possible losses resulting from their derivatives activities. Capital is provided by the bank's shareholders and thus has a cost. Therefore, incorporating the cost of capital into over-the-counter (OTC) derivatives pricing is vital to conduct sustainable and profitable business. The market price for holding regulatory capital is referred to as capital valuation adjustment (KVA).

In this thesis, we show how a self-financing hedge portfolio for an OTC derivative can be constructed by taking positions in market instruments and setting up bank accounts to fund these positions, collateral and capital. This makes it possible to write an expression for KVA. We focus on KVA for regulatory default risk charge, which is a nested Monte Carlo problem. While this problem could in principle be solved with brute force, we use the American Monte Carlo algorithm to improve computational performance.

Illustrative examples are presented to demonstrate the computational efficiency and accuracy of the model. The numerical examples also highlight the importance of pricing regulatory capital in OTC derivatives.

Keywords: derivatives pricing, regulatory capital, XVA, KVA

Language: English

Aalto-yliopisto
Perustieteiden korkeakoulu
Matematiikan ja operaatiotutkimuksen koulutusohjelma

DIPLOMITYÖN TIIVISTELMÄ

Tekijä:	Joonas Lanne		
Työn nimi:	Regulatorisen pääomavaateen huomioiminen johdannaisten hinnoittelussa		
Päiväys:	31. elokuuta 2017	**Sivumäärä:**	82
Pääaine:	Systeemi- ja operaatiotutkimus	**Koodi:**	SCI3055
Valvoja:	Professori Ahti Salo		
Ohjaaja:	TkT Ruth Kaila		

Tiukentuva regulaatio edellyttää pankeilta riittävästi pääomaa, joka suojaa niitä OTC-johdannaisten (over-the-counter) riskeiltä. Pankit saavat pääoman osakkeenomistajilta, jotka vaativat sijoitukselleen tuottoa. Tämän pääomakustannuksen huomioiminen OTC-johdannaissopimusten hinnoittelussa on tärkeää kestävän ja kannattavan liiketoiminnan kannalta. Pääomakustannusten vaikutusta OTC-johdannaisten hintaan kutsutaan KVA:ksi (capital valuation adjustment).

Tässä työssä näytetään, miten erilaisia markkinainstrumentteja ja pankkitilejä käyttämällä voidaan rakentaa itsensä rahoittava replikointiportfolio. Pankkitilit perustetaan, jotta sijoitukset markkinaistrumentteihin, OTC-johdannaisiin liittyvät vakuudet ja sijoittajilta saatu pääoma voidaan rahoittaa. Replikointiportfolion avulla voimme määritellä KVA:n. Keskitymme regulatoriseen pääomavaateeseen, joka kattaa johdannaisvastapuolien maksukyvyttömyyden riskin. Tästä seuraa kahden sisäkkäisen Monte Carlo -simulaation ongelma, joka voitaisiin periaatteessa ratkaista suoraan, mutta laskennallista tehokkuutta parantaaksemme ratkaisemme ongelman American Monte Carlo -algoritmia käyttäen.

KVA:n estimointimallin laskennallista tehokkuutta ja tarkkuutta tarkastellaan numeerisia esimerkkejä käyttäen. Esimerkit osoittavat, kuinka tärkeää pääomavaateen huomioiminen on OTC-johdannaisten hinnoittelussa.

Asiasanat:	johdannaisten hinnoittelu, regulatorinen pääoma, XVA, KVA
Kieli:	Englanti

Acknowledgements

This study was inspired by my work at Nordea Markets. I am grateful for my time with the company and how the environment full of finance and business experts has contributed to my knowledge and abilities.

I would like to express my sincere gratitude to my instructor Ruth Kaila for the continuous feedback and guidance, for always having time for me, and for her positivity and immense knowledge in financial engineering.

Additionally, I would like to thank professor Ahti Salo for the insightful discussions during this thesis, and during my studies in general.

Last but not least, I would like to thank my family and friends for their continuous support in everything I have decided to do.

Espoo, August 31, 2017

Joonas Lanne

Acronyms

ACVA	Advanced CVA Risk Charge
BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CBOE	Chicago Board Options Exchange
CCP	Central Clearing Party
CDS	Credit Default Swap
CE	Current Exposure
CEM	Current Exposure Method
CSA	Credit Support Annex
CVA	Credit Valuation Adjustment
DVA	Debt Valuation Adjustment
EAD	Exposure At Default
EE	Expected Exposure
EEPE	Effective Expected Positive Exposure
EL	Expected Loss
EONIA	Euro Overnight Index Average
EPE	Expected Positive Exposure
EURIBOR	Euro Interbank Offered Rate
FBA	Funding Benefit Adjustment
FCA	Funding Cost Adjustment

FRA	Forward Rate Agreement
FVA	Funding Valuation Adjustment
FX	Foreign Exchange
IMM	Internal Model Method
IRS	Interest Rate Swap
ISDA	International Swaps and Derivatives Association
KVA	Capital Valuation Adjustment
LGD	Loss Given Default
LIBOR	London Interbank Offered Rate
MV	Market Value
MVA	Margin Valuation Adjustment
NGR	Net-to-Gross Ratio
NYSE	New York Stock Exchange
OIS	Overnight Index Rate
OTC	Over-The-Counter
PD	Probability of Default
PDE	Partial Differential Equation
PFE	Potential Future Exposure
RR	Recovery Rate
RW	Risk Weight
RWA	Risk Weighted Assets
SCVA	Standardized CVA Risk Charge
XVA	Generic name for valuation adjustments

Notations

a	Mean reversion rate in the Hull-White model
$A(t)$	Value of the money market account
α	Regulatory scaling factor
α_C	Amount of the counterparty's bonds in the hedge portfolio
α_1	Amount of the bank's junior bonds in the hedge portfolio
α_2	Amount of the bank's senior bonds in the hedge portfolio
$Addon(t)$	Risk weighted sum of notionals in a netting agreement
$\beta_S(t)$	Bank account that funds the position in S
$\beta_C(t)$	Bank account that funds the position in P_C
$C(\omega; s, t, T)$	Path of cash flows generated by the derivative
$EAD_{CEM}(t)$	Exposure-At-Default calculated under the Current Exposure Method
ϵ_h	Size of the hedge error
ϵ_{h_K}	Capital-dependent part of the hedge error
$\mathcal{F}$	Sigma-algebra
$\tilde{\mathcal{F}}$	Augmented filtration of $\mathcal{F}$
$F(\omega; t)$	Conditional expectation of a derivative's value

$\hat{F}(\omega;t)$	Regression-based estimator of $F(\omega;t)$
ϕ	Usage if capital
G	Inverse standard normal distribution
g_B	The post-default value for the derivative after the bank's default
g_C	The post-default value for the derivative after the counterparty's default
γ_S	Dividend rate of S
γ_K	Total cost of capital for the bank
$H(S)$	Derivative's payoff at maturity
$J(t)$	Compound Poisson process
J_B	Default indicator for the bank
J_C	Default indicator for the counterparty
$K(t)$	Regulatory capital requirement
M	Number of simulated scenarios
M^{pos}	Number of positive realizations in the simulations
μ	Deterministic drift term
$\mathcal{N}$	Standard normal distribution
ω	Element representing a sample simulation path
Ω	The set of all possible realizations of the stochastic environment during a given time interval
$P(t,T)$	Value of a zero-coupon bond maturing at T
$P_C(t)$	Value of the counterparty zero-coupon zero-recovery bond
$P_1(t)$	Value of the bank's own junior bond
$P_2(t)$	Value of the bank's own senior bond
$P_1^-(t)$	Pre-default price for the bank's junior bond
$P_2^-(t)$	Pre-default price for the bank's senior bond

$P_C^-(t)$	Pre-default price for the counterparty's bond
$\mathcal{P}$	Probaility measure on the elements of $\mathcal{F}$
q_S	Financing rate to fund the position in S
q_C	Repo rate to fund the position in P_C
$\mathcal{Q}$	Risk-neutral probaility measure
$R_i, i \in \{1, 2\}$	Recovery rate for P_i
R_B	Proportion of the close-out of the total exposure amount for the bank
R_C	Proportion of the close-out of the total exposure amount for the counterparty
$r(t)$	Risk-free rate
$r_i, i \in \{1, 2\}$	Coupon rate for bond i
r_X	Cost of collateral for the bank
$S(t)$	Price of the market instrument
$S^c(t)$	Continuous part of a jump diffusion process
$\delta(t)$	Amount of market instruments in the hedge portfolio
σ	Deterministic volatility term
$\Sigma(t)$	Value of the hedge portfolio
$\theta(t)$	Drift function in the Hull-White model
U^+	$\max(U, 0)$
U^-	$\min(U, 0)$
$V_i(t)$	Derivative's market value
$V(t)$	Portfolio's market value
$\hat{V}_i(t)$	Derivative's economic value
$W(t)$	Brownian motion
$X(t)$	Collateral balance

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Chapter 1

Introduction

1.1 Over-the-counter derivatives pricing and risks

Financial derivative contracts are financial instruments that derive their price from that of the underlying financial entity. The underlyings can be interest rates, exchange rates or stock prices, for example. When parties enter into a derivative contract, they agree on specific conditions under which payments and deliveries are made between the parties. Counterparty credit risk associated with derivatives is the risk of a loss due to a counterparty's inability to hold its part of the agreement. For example, consider a European call option on a stock, which gives the option buyer the right to buy the underlying stock at a specified price on a specified date. If the underlying stock is quoted higher than the option's strike price, the buyer chooses to exercise the option. However, if the option writer cannot sell the stock, the option expires worthless and the buyer experiences a loss. Counterparty credit risk can be mitigated by using netting and collateral agreements or credit derivatives, for example.

From the option writer's perspective, there is no counterparty credit risk as long as the premium has been paid upfront. Nonetheless, the amount of loss the writer can undergo depends on the price of the underlying stock. If the stock price is significantly above the strike price of the option, the writer has to sell the stock well below its market price. This kind of risk is called market risk. A derivative contract often carries both counterparty credit risk and market risk. Market risk can be hedged, for example, by entering into a reverse contract with another counterparty.

Even though the painful repercussions of defaults have been well established in history, market risk is built into the price of the derivative but counterparty credit risk is not. While counterparty credit risk has been conceptually understood for quite some time, only after the financial crisis in 2007 did market participants realize that large derivative counterparties were actually not too big to fail and begin to incorporate the value of counterparty credit risk in derivatives pricing. This began by traders adjusting the values quoted on derivatives from counterparty to counterparty, which evolved into modeling and calculation of valuation adjustments.

Credit valuation adjustment, commonly referred to as CVA, is the difference in value between a portfolio that has counterparty credit risk and one which consists of the same instruments but is risk-free. CVA is thus the market value of counterparty credit risk. Because CVA is an accounting item, it directly affects the income statement and the balance sheet. During the financial crisis, banks actually suffered severe losses not from counterparty defaults but from their credit valuation adjustments on derivatives as their earnings were negatively affected by credit market volatility.

If the option writer would not consider credit risk in the derivative's price, the price of the option would always be lower from the buyer's point of view. To the option writer, the option is a liability and is adjusted with debt valuation adjustment (DVA). As the flip side to CVA, DVA reflects the credit risk of the entity that writes the contract and provides an equal view to the derivative for both parties. While one can argue that the concept of DVA is counter-intuitive, it is important to remember that DVA has always been included when pricing bonds and can be viewed to extend to derivatives. The combination of CVA and DVA is often referred to as bilateral CVA.

Recently, banks have started to incorporate ever more valuation adjustments to their derivative activities. The funding impact arising from derivatives is taken into account by calculating funding valuation adjustment (FVA), which is normally a cost (FCA) or a benefit (FBA). Collateral posted to or received from a counterparty is considered in margin valuation adjustment (MVA), if the trade is cleared through a central clearing house. Recently, the incremental cost of holding regulatory capital is encapsulated in capital valuation adjustment (KVA). The family of all valuation adjustments is often referred to as XVA.

While banks were required to hold capital to cover their risks already before the financial crisis, a large set of financial reforms was enacted afterwards. The so-called Basel III, known as the Third Basel Accord, is a global regulatory framework on bank capital adequacy, stress testing and market liquidity. It was agreed upon by the members of Basel Committee on Banking Supervision (BCBS) in 2010-2011. Capital adequacy related to derivatives is fulfilled by having a capital buffer of 8% of the risk weighted assets (RWA). The three major components of RWA are counterparty credit risk, market risk and operational risk. The counterparty credit risk RWA consists of two blocks: the default risk charge and the CVA risk charge to cover losses both due to actual defaults and uncertainty in credit market. Increased regulatory capital requirements have become costly, resulting in many banks seeing low single digits return on capital for their OTC activities and led to them incorporate KVA in derivatives pricing.

While few papers address KVA, Green et al. [2014] demonstrate a semi-replication method for calculating KVA under a standardized method for both counterparty credit risk and market risk capital. Because many banks are approved to use the internal model method (IMM) to calculate counterparty credit risk and market risk capital through simulation-based approaches, an efficient way of calculating KVA under IMM is also required.

1.2 Research objectives

In this thesis, we extend the method presented by Green et al. [2014] for modeling the costs related to holding regulatory capital in over-the-counter (OTC) derivatives pricing, i.e., calculating the capital valuation adjustment for a derivative portfolio that has a regulatory capital requirement calculated under IMM. We address the problem from a financial institution's point of view. The main steps are to (i) construct a simulation model for calculating counterparty default risk capital, (ii) derive KVA for a portfolio of derivatives, (iii) formulate an American Monte Carlo approach for solving the KVA expression and (iv) apply the KVA framework for the capital model. We will assume a back-to-back setup, i.e., market risk is perfectly hedged for all the derivative trades in the financial institution's portfolio, thus no market risk capital is required.

We start by introducing the subject in Chapter 1. Chapter 2 provides a view on the theoretical background of OTC derivatives and the risks associated with them. In Chapter 3, we present the regulatory capital framework and give an example of a simulation-based approach for calculating capital requirements for counterparty default risk. Chapter 4 extends the semi-replication approach to consider simulation-based counterparty default risk capital in KVA and solves for KVA efficiently using American Monte Carlo techniques. After a detailed formulation of the model, we provide numerical examples in Chapter 5. Chapter 6 concludes the thesis.

Chapter 2

Theoretical background

2.1 Over-the-counter derivatives

A financial derivative contract is an instrument that derives its price from one or several underlyings. For example, an equity option depends on the underlying stock's price and volatility. In addition, contractual details such as strike price and maturity date usually affect the derivative's price. The most common underlying assets include stocks, bonds, commodities, currencies, interest rates and market indices. In theory, a derivative can be designed to derive its price from any public piece of information such as inflation or rainfall.

Derivatives have useful applications. Some derivatives are used for hedging and insuring against a risk on an asset. Most interest rate derivatives such as swaps, caps and floors are commonly used to lock down a specific interest rate level or interval paid on a loan. An interest rate swap is an agreement to exchange cash flows calculated on the same notional but different interest rates. Usually, one party agrees to pay a fixed rate on a notional amount in exchange for a floating rate on the same notional amount. By agreeing on paying a floating rate on a loan and then entering into an interest rate swap, a corporation, for example, can effectively have a loan on a fixed interest rate and thus manage its cash flows with less uncertainty. As the other party receives a fixed rate and pays a floating rate, it is exposed to interest rate risk. This other useful application of derivatives is called speculation, a bet on the future development of the underlying.

Derivatives are traded both on formal exchanges, such as the New York Stock Exchange (NYSE) or the Chicago Board Options Exchange (CBOE), and also in the over-the-counter (OTC) market in which dealers act as market makers and execute transactions directly amongst themselves. While derivatives listed on an exchange have better liquidity and can possibly be traded on the secondary market, they are usually very standardized. Most listed options, for example, are traded in blocks of 100 contracts. According to the Bank for International Settlements [2016a], the outstanding notional of exchange-traded derivatives was 4,572 billion US dollars in 2015.

The OTC derivative market is made up mostly of investment banks and hedge funds. As OTC derivatives are usually being traded directly between the parties in the agreement, they can be tailored to fit the exact hedging or speculation purposes. On the downside, OTC derivatives expose the parties to greater counterparty credit risk. OTC derivatives are unfunded bilateral contracts whose both parties can default, causing the non-defaulted party to suffer credit losses. According to the Bank for International Settlements [2016b] the outstanding notional of OTC derivatives was 492,911 billion US dollars in 2015. Because this market is significantly larger than the exchange-traded one, we focus only on the OTC derivatives in this thesis.

2.2 Netting agreements, collateral and credit support annex

Following the purchase of Bear Stearns by JP Morgan Chase for 2 dollars a share, the failure of Lehman Brothers, and the rescue of Merill Lynch by Bank of America in the 2007 financial crisis, it was clear that not even the Triple-A rated entities, global investment banks or sovereigns could ever be regarded as completely risk-free [Sorkin, 2009]. In consequence, counterparty credit risk has become a term which is inextricably associated with financial markets and pricing. Besides taking this into account in pricing, the agonizing impacts of realized credit risk and tightened regulation have forced banks to focus on mitigating counterparty credit risk through netting and collateral agreements [Gregory, 2012].

If two parties have entered into multiple derivative contracts and one of them defaults, the credit losses are determined by the individual market values of the derivatives, because the non-defaulted party suffers losses from all contracts with positive market values, still having the obligation to respect the contracts where it owes the defaulted party. The most common way to mitigate counterparty credit risk is through a netting agreement. While most banks offer their own netting agreements, the most common standardized agreement is the International Swaps and Derivatives Association (ISDA) master agreement. The ISDA master agreement has established international contractual standards governing privately negotiated derivative transactions that reduce legal uncertainty and allow for the reduction of credit risk through netting contractual obligations [International Swaps and Derivatives Association, 2016]. The agreement reduces credit exposure from gross to net exposure.

While netting agreements can significantly reduce counterparty credit risk, the remaining net market value can still be very large especially when the contracts have large notionals. A way to mitigate the credit risk arising from the net market value is to agree on exchanging collateral equal to the amount of net market value. In case of a default, the collateral holder is then able to cover the resulting credit losses with the obtained collateral. A credit support annex (CSA) can be added to a netting agreement to cover the terms of the collateral arrangement between the counterparties. The CSA typically specifies whether shares, bonds or only cash, for instance, are accepted as collateral. Also, any haircuts that are applied to instruments that have uncertain prices, or to cash in foreign currency, are specified in the CSA. While the collateral posting frequency must be specified and honored, the daily posting of, for example, several units of cash would be operationally inefficient. The thresholds for exchanging collateral are thus stated in the CSA to avoid impracticalities while still ensuring proper risk mitigation [International Swaps and Derivatives Association, 2011].

2.3 Central clearing parties

A mandatory central clearing was proposed already during the 2007 financial crisis when many policymakers identified counterparty credit risk in OTC derivatives as a major source of risk to the financial system. When a bilateral OTC derivative is cleared through a central clearing party (CCP), the original counterparties' contracts with one another are replaced with a pair of contracts with the CCP. The CCP then becomes the buyer to the original seller and the seller to the original buyer. If either one of the original counterparties defaults, the CCP is contractually committed to pay all obligations that are owed to the non-defaulting party. To meet its obligation, the CCP has recourse to a variety of financial resources, including collateral posted by those who clear through it and financial commitments, such as default fund contributions, made by its members [Basel Committee on Banking Supervision, 2012].

Because cleared derivatives are daily margined, i.e., the participants post collateral daily to cover possible default losses, the CCP runs regularly with matched books. In contrast, the default fund contributions made by the CCP's members are defined less frequently, typically based on stress testing. The default fund is used as a resource only if the margins are insufficient for covering default costs [Basel Committee on Banking Supervision, 2017].

A drawback of mitigating counterparty credit risk using CCPs is that banks must sacrifice their resources into financial commitments, thus reducing the amount of resources available for profitable investments. This has, however, become mandatory practice since the so-called Dodd-Frank Act was signed into law (2010). The Dodd-Frank Act requires all sufficiently standard derivatives traded by major market participants to be cleared in regulated CCPs. While this act was only signed into U.S. federal law, the European Commission (2010) has also taken similar steps [Duffie and Zhu, 2011].

2.4 Counterparty credit risk measures

Even though netting agreements and CSAs usually reduce risk, there is often a need to measure and quantify the remaining risk. The most important counterparty credit risk measures - current exposure, probability of default, loss given default, exposure at default, and risk exposure amount - are introduced in the next subsections. For a comprehensive presentation of counterparty credit risk measures and models, see [Duffie and Singleton, 2003] and [Engelmann and Rauhmeier, 2006].

2.4.1 Current exposure

Let a portfolio of OTC derivatives between a bank and its client consist of derivative contracts $D = 1, \ldots, n$. Each contract $i \subset D$ has a time-dependent market value $V_i(t)$. The aggregated portfolio-level market value $V(t)$ is

$$V(t) = \sum_{i=1}^{n} V_i(t). \tag{2.1}$$

However, assuming all derivatives are netted under a single netting agreement, a more interesting measure is the net positive exposure. Current exposure (CE) is the net positive market value of the derivative contracts under the netting agreement. It measures the amount of credit loss that would occur immediately if the counterparty were to default, i.e.,

$$V(t)^+ = \max\,(\sum_{i=1}^{n} V_i(t), 0). \tag{2.2}$$

Current exposure is reduced by received collateral and can be increased by paid excess collateral. Current exposure under a CSA is calculated as

$$V_{col}(t)^+ = \max\,(\sum_{i=1}^{n} V_i(t) - X(t), 0), \tag{2.3}$$

where $X(t)$ is the agreement's collateral balance at t.

2.4.2 Probability of default

In addition to quantifying the amount that would be lost in default, it is important to quantify the likelihood of such an event. The probability of default (PD) is the likelihood of a default over a particular time horizon. It is widely used for assessing credit risk associated with derivatives and loans. Usually it is expressed as the probability of default during the next full year.

The credit history of the borrower or counterparty and the nature of the investment are the most important aspects in assessing a PD. External agencies, such as Standard & Poor's or Moody's, can be consulted to get an estimate of the PD of interest. However, banks can also use an internal ratings based method, a sophisticated approach for assessing probabilities of default in each rating grade. One example is the dynamic mechanism developed by Iqbal and Ali [2012] in which implied probability of default is generated through convoluting probability distributions for the total number of defaults and the number of defaults in a specific rating grade.

The PD directly affects the price a bank charges for a mortgage loan by increasing the interest rate paid on the loan amount, for example. In a derivative contract, this can be reflected, for example, in the fixed rate paid in exchange for a floating rate in an interest rate swap.

2.4.3 Loss given default

When a default occurs, a portion of the debt or exposure is usually paid by the defaulted party to its creditors. Recovery rate (RR) is defined as the proportion of the amount of debt than can be recovered. Loss given default (LGD) is the fractional loss the creditors will suffer due to the default

$$LGD = 1 - RR. \tag{2.4}$$

In order to estimate LGD, banks need to consider common characteristics of losses and recoveries identified by numerous academic and industry studies. According to Schuermann [2004], most of the time recovery rate is either very high or very low. Thus, the average recovery rate or LGD can be misleading. However, some industry specific characteristics, such as the amount of tangible assets, seem to matter and allow for the sophisticated estimation of LGD.

2.4.4 Exposure at default

Exposure-at-default (EAD) measures the expected amount of loss that would occur if the counterparty defaults. It is rather easy to quantify the amount of loss that would occur immediately if a counterparty were to default. The main shortcoming of CE is that it does not account for possible losses that may occur in the future. No bank can effectively manage its risks by only preparing on what could happen in a one day's time horizon. While it is never possible to know in advance the losses a bank will suffer in a given year, it is possible to forecast the average level of credit losses that it can expect to suffer. This expected loss is generally viewed as one of the costs of being in the derivatives business. It is, however, possible to experience peak losses that exceed expected levels. One of the main functions of a bank's capital is to provide a buffer which protects the bank's debt holders against these peak losses that can potentially reach eminent levels. Losses above expected levels are usually referred to as unexpected losses, and they can potentially be as high as losing an entire credit portfolio. Because this is a very unlikely scenario, it would be economically inefficient to hold capital against it. Banks have an incentive to minimize the capital they hold, because it can be directed to profitable investments. Therefore, banks and their supervisors must carefully assess the amount of capital that is sufficient to keep the business solvent and profitable [Basel Committee on Banking Supervision, 2005a].

While the approach to determine the amount of capital a bank must hold has evolved along with regulations, the current approach focuses on the frequency of bank insolvencies, defined as the failure to meet its senior obligations. Stochastic credit portfolio models make it possible to estimate the amount of loss which would lead to an insolvency. Thus, the size of the capital buffer can be set so that the probability of reaching this amount of unexpected losses is very low [Basel Committee on Banking Supervision, 2005a].

Exposure-at-default is the estimated amount of loss that a bank can be exposed to in case of a possible default. Together with PD and LGD, EAD is used to calculate the regulatory capital requirement for banks to cover their counterparty credit risk. Exposure at default can be calculated through methods such as the standard current exposure method (CEM) and the more sophisticated internal model method (IMM) which we discuss in Chapter 3.

2.4.5 Risk weighted assets

Because the credit quality of a counterparty varies, EAD is insufficient in measuring counterparty credit risk. Thus, what determines the size of the buffer that a bank needs to hold to fulfill its capital adequacy is the Risk Weighted Assets (RWA). RWA is calculated by multiplying EAD with a counterparty-specific risk weight (RW). The underlying assumption is that the capital buffer needed against a certain exposure does not depend only on the exposure amount but also on the probability of default and the portion of the exposure that is lost if a default occurs. RWA is thus dependent on the modeling of EAD, PD and LGD, i.e.,

$$RWA = EAD \cdot RW(PD, LGD). \tag{2.5}$$

The Basel Committee on Banking Supervision (BCBS) has set out a proposal for an internal ratings based approach (IRB) to assess capital requirements for counterparty credit risk. Building on the capability of assessing credit risk by categorizing exposures into broad, qualitatively different layers of risk, for each exposure class (e.g. corporate, retail, sovereign), the IRB approach provides a single framework for translating a given set of risk components into minimum capital requirements [Basel Committee on Banking

Supervision, 2005a].

2.5 Market risk

When investors consider investment risks, they usually think about market risk first. Market risk is the risk of loss due to fluctuations in market factors, such as interest rates or equity prices. In fact, market risk can be divided into specific primary sources: interest rate risk, equity price risk, foreign exchange risk, and commodity risk as described in the following subsections. See [Carol, 2009] for a comprehensive framework for modeling market risk.

2.5.1 Interest rate risk

Interest rate risk is the risk of experiencing losses due to fluctuating interest rates. Interest rate risk covers basis risk, options risk, term structure risk and repricing risk. In addition to derivatives, this risk is often recognized in loan and bond portfolios.

2.5.2 Equity price risk

Equity price risk arises from the volatility of equity prices. Besides the primary sources, market risk is usually either unsystematic or systematic. A good example of systematic risks in the equities market would be a general decline in indices due to a global crisis. Unsystematic risk relates to a specific factor. An example would be the bankruptcy of a particular company that makes its stock worthless.

2.5.3 Foreign exchange risk

Foreign exchange risk is a form of market risk that reflects the uncertainty in foreign exchange rates. For example, a perfectly hedged interest rate swap in a foreign currency is still exposed to market risk via possible losses due to changes in the foreign currency's exchange rate.

2.5.4 Commodity risk

Commodity risk refers to the market risk which arises from volatile commodity prices. Consider for example a utilities sector company that provides oil to its customers. A major source of that company's market risk is related to oil prices. If the company chooses to hedge its risks by entering into oil swaps, futures or forwards, it can substantially mitigate its commodity risk in exchange for paying a premium.

2.6 Valuation adjustments

Since the seminal paper by Black and Scholes [1973], derivatives pricing has been based on the Black-Scholes framework. However, the simplifying assumptions of credit risk-free counterparties and non-capital consuming derivatives are clearly a problem. Thus, incorporating the risks related to own and counterparty's credit, funding and capital are of increasing importance in the environment of tightening regulation. These are taken into consideration by introducing valuation adjustments to the classic Black-Scholes price of a derivative [Green, 2015].

2.6.1 Credit valuation adjustment

As noted previously, derivatives often carry counterparty credit risk. While netting and collateral agreements mitigate this risk, it is vital not only to quantify this risk but also to account for it in pricing derivatives. A derivative contract with a well-rated and stable counterparty is worth more than a derivative with a risky counterparty, because the likelihood of getting the agreed payments is greater. Credit valuation adjustment (CVA) is the market price of the default risk for a derivative or portfolio of derivatives. In other words, CVA is the price one would be willing to pay to hedge the derivative's or a portfolio of derivatives' counterparty credit risk.

Many market participants have developed sufficient systems and models to manage CVA, but they are still not standardized and may vary among market participants. Typically, methodologies range from relatively simple to highly complex techniques used by the largest investments banks with extensive resources. The simplest approach is to adjust the discount rates that are used in the discounted cash flow framework to account for the counterparty's credit spread and to compare that with the risk-free price. The difference between the two represents the CVA amount. More advanced methodologies involve simulation modeling of market risk factors in hundreds or thousands of scenarios [Gregory, 2014].

2.6.2 Debt valuation adjustment

When two parties enter into a derivative contract, both parties will calculate CVA based on the counterparty's credit quality. To get an equal view on the derivative's price, both parties' credit qualities must be taken into account. Debt valuation adjustment (DVA) is the impact of company's own credit risk on the value of a derivative. While pricing a company's own credit risk into a derivative is arguably counter-intuitive, it has always been included in bond prices. The issuer's own credit risk is taken into account when pricing bonds, because their fair value is less than that of a risk-free bond. DVA is an analogous adjustment for derivatives.

It can be confusing that when a company's own credit quality deteriorates, this actually has a positive impact on the income statement, because of increasing DVA. On the other hand, if the institution is looking to fund its derivatives or other investments through a loan, the deteriorating credit quality obliges it to pay a higher interest rate for the loan and thus offsets the benefits gained from DVA [Gregory, 2014].

2.6.3 Funding valuation adjustment

When a financial institution operates in the derivatives business, it decides to use its capital to cover the risks arising from dealing with derivatives. To compensate for that, it requires profit in the form of margins or commissions from its clients. Typically, a corporate or sovereign-type client seeks to buy protection against fluctuating market factors and enters into a derivative agreement with a bank. Because the bank oftentimes does not want to leave any market risk open, it enters into a reverse agreement with another counterparty, usually from the interbank market. The bank then structures a situation in which it fully serves its client and hedges the market risk while charging a margin from the corporate.

While most corporate customers have insufficient resources or infrastructure to arrange daily margining of derivatives, interbanks usually operate under CSAs [International Swaps and Derivatives Association, 2015]. The resulting imbalance from hedging uncollateralized derivatives with collateralized ones creates a funding gap for banks. If the original derivative with the corporate has positive market value, i.e., the bank is in the money, the hedge derivative is out of the money. In this case, the bank is required to post collateral to the hedge counterparty while not receiving any funding from the corporate. The CSA stipulates the rate of interest that is received from posted collateral; this is typically the overnight index rate (OIS) such as the Euro Overnight Index Average (EONIA). Because OIS is currently considered to be the best proxy for a risk-free rate, it is unreasonable to assume that any bank could borrow funding for the collateral at this low rate. In this case, the spread between (i) the interest rate the bank has to pay to fund any posted collateral and (ii) the OIS rate received from it creates a funding cost. In the financial sector, the spread between the three month London Interbank Offered Rate (Libor) and the three month OIS spread is usually considered the indicator of short-term funding costs [Hull and White, 2012].

Taking the funding costs into consideration in derivatives pricing is essential for reflecting the differences between funded and unfunded positions. The adjustment made to a derivative's price is called funding valuation adjustment (FVA), which can refer to funding cost or funding benefit, depending on the setup and market movements. To revert back to the previous example, an out of the money derivative with the corporate would yield a funding benefit, because the collateral received from the hedge counterparty could have been invested at Libor which exceeds the OIS rate paid for the collateral. In addition to funding collateral positions, traders may need to seek funding to arrange proper hedging or to pay cash for acquiring an in the money derivative. The approaches for modeling FVA vary from deal specific to homogeneous cost-views and funding strategies using either in-house treasuries or money market directly. A comprehensive framework for FVA modeling is given by Pallavicini et al. [2011].

2.6.4 Capital valuation adjustment

Given that holding capital is a legal requirement for financial institutions in the derivatives business, it is striking that so few papers address the topic of including cost of capital in derivatives pricing. In fact, the topic of quantifying capital costs in derivatives pricing was presented as late as in 2014 by Green et al. [2014]. Because the tightening regulation requires banks to hold ever more capital to cover their risks, it is essential to take into account the cost associated with holding capital when pricing derivatives. The adjustment is called capital valuation adjustment (KVA).

As mentioned in the previous sections, most trades tend to be hedged with either a reverse position with another counterparty or by having offsetting positions in other market instruments. While this hedge is appropriate for eliminating market risk, it usually creates two counterparty credit risky positions which increase the bank's capital requirement. Because investors and shareholders require adequate compensation for involvement in derivatives business, the capital required to be set aside has a cost.

In a sense, KVA can be considered to be the oldest of all valuation adjustments. This is because banks have historically used capital hurdles and risk-adjusted return measures to price derivatives. However, not until recently has this been quantified as an absolute amount.

KVA can be seen as an upfront amount that would generate returns equal to the cost of capital over the full lifetime of the derivative. Thus, calculating KVA makes it necessary to model expected regulatory capital until a trade's maturity. This unfortunately creates an inconsistency across the banking sector, because banks tend to use different models for calculating capital requirements. In addition, changing regulation can cause uncertainties in future capital requirements, which makes modeling even trickier.

Chapter 3

Regulatory capital charges

3.1 The Basel committee on banking supervision

The Basel Committee on Banking Supervision (BCBS) provides a forum for regular cooperation on banking supervisory matters. Its objective is to contribute to the understanding of key supervisory issues and to improve the quality of banking supervision worldwide. Basel III, released in 2011, is a comprehensive set of reform measures, developed by the BCBS, to strengthen the regulation, supervision and risk management of the banking sector. The objective of the document is to improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the effect of the financial sector's realized risks on the real economy [Bank for International Settlements, 2016c].

The reform package addresses the lessons of the 2007 financial crisis. One of the main reasons for the crisis was that the banking sector had excessive on- and off-balance sheet leverage accompanied with an insufficient quality of capital. Therefore, the banking sector was not able to absorb losses resulting from systemic worsening of credit qualities, for instance.

According to the package, the framework aims to capture all material risks. More specifically, the amount of capital required is determined by quantifying counterparty credit risk, market risk and operational risk [Basel Committee on Banking Supervision, 2006].

The capital consists of Tier 1, Tier 2 and common equity Tier 1. In addition to common shares issued by the bank, Tier 1 capital primarily consists of stock surplus, retained earnings and cash reserves. According to Basel Committee on Banking Supervision [2010], the minimum limits for capital are for common equity, Tier 1 capital and total capital at 4.5%, 6.0% and 8.0% of risk weighted assets, respectively.

3.2 Counterparty credit risk charge

The counterparty credit risk capital charge depends on the amount of default risk and CVA risk. Next, we introduce the two sources of counterparty credit risk in detail and present a framework for the modeling of default risk capital charge.

3.2.1 Default risk charge

The default risk charge is the most fundamental form of counterparty credit risk capital charges. The capital reserved for default risk aims to cover any risks arising from the possibility of counterparty defaults. Banks can (i) calculate the amount of required capital using either a standardized current exposure method (CEM) or (ii) apply for a more sophisticated internal model method (IMM) approval. IMM based calculations tend to rely on more realistic methods, such as volatility calibration and collateral algorithms, whereas the standard CEM approach uses pre-determined ratios and is limited in considering risk-mitigating collateral. Most IMM models for default risk rely on Monte Carlo-based approaches, because they tend to outperform other methodologies in terms of mean-squared-error [Ghamami and Zhang, 2014].

3.2.1.1 Current exposure method

According to Basel Committee on Banking Supervision [2013a], the regulatory exposure at default (EAD) under the current exposure method (CEM) is calculated by adding potential future exposure (PFE) to the uncollateralized current exposure (CE) of a netting set

$$EAD_{CEM}(t) = CE(t) + PFE(t), \tag{3.1}$$

where PFE is based on an agreement-specific addon, which is calculated as the risk-weighted sum of the trade notionals under the netting set. The weight each notional gets is mainly dependent on the trade type and its residual maturity. On top of that, banks are allowed to use regulatory matching in these trades so that same products with same underlyings and maturities are allowed to offset each other in terms of their addon. Finally, partial offsetting of trades is considered by incorporating the net-to-gross ratio (NGR) into the calculation, so that netting sets that have trades with both negative and positive market values are considered less risky. Thus,

$$EAD_{CEM}(t) = V(t)^+ + (0.4 + 0.6 \cdot NGR(t)) \cdot addon(t), \tag{3.2}$$

where $V(t)^+$ is the net-positive market value of the portfolio, $addon(t)$ is the risk-weighted sum of notionals and NGR is formally defined as

$$NGR(t) = \frac{[\sum_{i=1}^{n} V_i(t)]^+}{\sum_{i=1}^{n} [V_i(t)]^+}. \tag{3.3}$$

3.2.1.2 Internal model method

Because the CEM methodology is rather simple and conservative, many banks choose to allocate their resources to modeling counterparty credit risk through simulation-based approaches. According to Basel Committee on Banking Supervision [2010], banks applying the internal model method (IMM) must have a collateral management unit that is responsible for calculating and making margin calls, managing margin call disputes and reporting levels of independent amounts, initial margins and variation margins accurately on a daily basis. This unit must control the integrity of the data used to make margin calls, and ensure that it is consistent and reconciled regularly with all relevant sources of data within the bank. This unit must also track the extent of reuse of collateral and the rights that the bank gives away to its respective counterparties for the collateral that it posts. Furthermore, these internal reports must indicate the categories of collateral assets that are reused, and the terms of such reuse including instrument, credit quality and maturity. The unit must also track concentration to individual collateral asset classes accepted by the banks. Senior management must allocate sufficient resources to this unit for its systems to have an appropriate level of operational performance, as measured by the timeliness and accuracy of outgoing calls and response time to incoming calls. Senior management must ensure that this unit is adequately staffed to process calls and disputes in a timely manner even under severe market crisis, and to enable the bank to limit its number of large disputes caused by trade volumes. These requirements for the (IMM) approval can be demanding for smaller banks, thus making the CEM approach their best alternative.

Pykhtin and Zhu [2006] present a universal framework for modeling counterparty credit risk through scenario-based simulation. This approach can be used to obtain the entire exposure distribution and it makes it possible to calculate standard deviations and percentile statistics in addition to the expected exposure profile. The main components of the model are:

- **Scenario generation**. Future development of the market factors are simulated for a fixed set of simulation dates using stochastic evolution models. A sample path is represented by element ω.

- **Instrument valuation**. The derivative portfolio is valued for each simulation date and scenario.

- **Portfolio aggregation** For each simulation date and scenario, the aggregated exposure is obtained by applying the portfolio level exposure formula in (2.1).

In this thesis, we construct a simulation-based model for EAD calculation. We assume a finite, discrete and evenly spaced time interval $[0, T]$ with K time steps and a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the set of all possible realizations of the stochastic environment between 0 and T, $\mathcal{F}$ is the sigma-algebra, i.e., a closed collection of subsets of distinguishable events at time T and $\mathcal{P}$ is a probability measure on the elements of $\mathcal{F}$. An augmented filtration of $\mathcal{F}$ is an increasing and indexed family of sub-sigma-algebras of $\mathcal{F}$. It is generated by instrument price processes ω_i. Let such an augmented filtration be $\tilde{\mathcal{F}} = \{\mathcal{F}_t; t \in [0, T], \mathcal{F}_T = \mathcal{F}\}$. We assume no arbitrage opportunities, i.e., a risk-neutral measure $\mathcal{Q}$ exists for the stochastic environment. The risk-neutral measure is defined as a measure under which each instrument has a price equal to the discounted expectation of its future price. In contrast, the real probability measure is based on actual realizations of instrument price processes and can provide arbitrage opportunities [Bingham, 2004].

The simulation process results in M realizations of agreement-level exposure for each simulation date. While the aggregated market value of each netting set can be positive or negative, only positive realizations, i.e., the ones where the bank is in the money, are interesting from a counterparty credit risk perspective. M^{pos} is the number or positive realizations at t_k. At each time point t_k, the positive realizations of $V(\omega, t_k)$ compose a distribution that can be used for obtaining the expected exposure (EE)

$$EE(t_k) = \mathbb{E}_{\mathcal{Q}}[V(t_k)^+ | \mathcal{F}_{t_k}] = \frac{1}{M^{pos}} \sum_{i=1}^{M^{pos}} V^i(\omega, t_k)^+, \tag{3.4}$$

where the expectation is taken under $\mathcal{Q}$ and conditional on the information set $\mathcal{F}_{t_k}$. Because the process is computationally heavy, daily intervals of the simulation points are rarely considered. Instead, a common approach is to calculate the exposure distributions daily or weekly up to a month, then monthly up to a year and yearly up to five years, and so on. The regulatory default risk capital charge requires that the risk calculations are done over the first year, or - if all the contracts within a certain agreement mature before the end of the first year - up to the longest maturity of the agreement. The expected positive exposure (EPE) is the time-weighted average of the positive realizations, i.e.,

$$EPE(t_k) = \int_{t_k}^{T} EE(t)dt, \tag{3.5}$$

where T is either $t_k + 1$ or the longest maturity of the trades in the portfolio if all of them mature within one year. The integral is usually evaluated using numerical integration techniques [Pykhtin and Zhu, 2006].

In addition to being limited to positive exposures, the regulatory approach for exposure modeling requires that the process is non-decreasing. The effective expected positive exposure (EEPE) is defined as the time-weighted average of non-decreasing average positive exposures [Basel Committee on Banking Supervision, 2005b]. By starting from the beginning and taking the maximum of a given simulation date's EE and the previous date's EE, the

EEPE is obtained recursively as

$$EEPE(t_k) = \int_{t_k}^{T} \max\left(EE(t_k), \ldots, EE(t)\right) dt. \tag{3.6}$$

Exposure-at-default (EAD) is obtained by multiplying EEPE with a supervisory multiplier α, which is by default equal to 1.4, unless modeled internally [Basel Committee on Banking Supervision, 2005b]. EAD can then be expressed as

$$EAD(t_k) = \alpha \cdot EEPE(t_k). \tag{3.7}$$

To obtain the EE profile, we start by modeling the underlying market factors to generate the desired scenarios for our exposure calculations. Not all banks necessarily have the full IMM approval for all product types. In this thesis, we limit the model to consider only interest rate and foreign exchange (FX) related instruments and formulate the IMM EAD model to generate appropriate market scenarios for the valuation of these instruments. The scenarios are usually generated via stochastic differential equations. For foreign exchange rates, we use geometric Brownian motion given by

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{3.8}$$

where $S(t)$ is the FX rate, μ is a deterministic drift term, σ is a deterministic volatility term and $W(t)$ is Brownian motion. Similarly to Wilmott [2001], we solve this by applying Itô's lemma to $d(\ln S(t))$, i.e.,

$$d(\ln S(t)) = \frac{dS(t)}{S(t)} - \frac{1}{2S(t)^2}(dS(t))^2. \tag{3.9}$$

From (3.8), we solve

$$d(S(t))^2 = \sigma^2 S(t)^2 dt, \tag{3.10}$$

where $dt^2 = 0$ and $dW(t)^2 = dt$ are used. Inserting this into (3.9) gives

$$d(\ln S(t)) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW(t). \tag{3.11}$$

Assuming an arbitrary intial solution S_0 gives

$$\ln \frac{S(t)}{S_0} = (\mu - \frac{\sigma^2}{2})t + \sigma W(t). \tag{3.12}$$

By taking the exponential of both sides, we can write a general realization at t as

$$S(t) = S(0)e^{(\mu - \frac{\sigma^2}{2})t + \sigma dW(t)}. \tag{3.13}$$

Depending on whether the FX rates are simulated using the risk-neutral or the real probability measure, the drifts are calibrated either arbitrage-free or to the historical movements, respectively. A risk-neutral drift for the FX rates is simply given by the difference between domestic and foreign interest rates. Volatilities are calibrated to option implied volatilities [Hull, 1988].

Interest rate models vary from one-factor short rate models, e.g., the Vasicek model presented by Vasicek [1977], the Hull-White model presented by Hull and White [1990] and the Cox-Ingersoll-Ross model presented by Cox et al. [1985] to more comprehensive modeling of the entire yield curve, e.g., the Heath-Jarrow-Morton model presented by Heath et al. [1990] and principal component analysis. Short rate models describe interest rate movements as a result of changes in one source of market risk, whereas the Heath-Jarrow-Morton model, for example, is a framework with multiple sources of randomness.

In this thesis, we model the future development of short interest rates r with the Hull-White model. The model is defined with a stochastic differential equation

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t), \tag{3.14}$$

where a is mean reversion rate, σ is volatility of the short rate, $W(t)$ is Brownian motion and $\theta(t)$ is a drift function. To solve the expressions for the yield curve, we follow closely the approach presented by Hull and White [1993]. In the case of a one-factor term structure, the risk-neutral price for a zero-coupon bond maturing at T has the dynamics

$$dP(t,T) = r(t)P(t,T)dt + v(t,T)P(t,T)dW(t), \tag{3.15}$$

where $v(t,T)$ is volatility, $W(t)$ is Brownian motion and $r(t)$ is the short rate. Because the bond has zero volatility at maturity, $v(t,t) = 0$. By using Itô's lemma, we can write the dynamics for any times T_1 and T_2, $T_1 < T_2$ as

$$d\ln P(t,T_1) = \left[r - \frac{v(t,T_1)^2}{2}\right]dt + v(t,T_1)dW(t), \tag{3.16}$$

$$d\ln P(t,T_2) = \left[r - \frac{v(t,T_2)^2}{2}\right]dt + v(t,T_2)dW(t). \tag{3.17}$$

We denote the forward rate from T_1 to T_2 as $f(t,T_1,T_2)$

$$f(t,T_1,T_2) = -\frac{\ln P(t,T_2) - \ln P(t,T_1)}{T_2 - T_1}. \tag{3.18}$$

By using the results from (3.16) and (3.17), we get

$$df(t,T_1,T_2) = \left[\frac{v(t,T_2)^2 - v(t,T_1)^2}{2(T_2 - T_1)}\right]dt - \left[\frac{v(t,T_2) - v(t,T_1)}{T_2 - T_1}\right]dW(t). \tag{3.19}$$

We denote the instantaneous zero rate at t until T as $R(t,T)$

$$R(t,T) = f(0,t,T) + \int_0^T df(u,t,T). \tag{3.20}$$

Inserting the results from (3.14) gives

$$\begin{aligned} R(t,T) =& f(0,t,T) + \int_0^t \left[\frac{v(u,T)^2 - v(u,t)^2}{2(T-t)}\right]du - \\ & \int_0^t \left[\frac{v(u,T) - v(u,t)}{T-t}\right]dW(u). \end{aligned} \tag{3.21}$$

When the time interval decreases and T approaches t, $R(t,T)$ approaches $r(t)$ and the future forward rate $f(0,t,T)$ becomes equal to the the instantaneous forward rate $F(0,t)$

$$r(t) = F(0,t) + \int_0^t \frac{\partial}{\partial t}\frac{v(u,t)^2}{2}du - \int_0^t \frac{\partial}{\partial t}v(u,t)dW(u). \tag{3.22}$$

We differentiate with respect to t so that we can write the dynamics of $r(t)$ as

$$\begin{aligned} dr(t) =& \left[\frac{\partial F(0,t)}{\partial t} + \int_0^t \left(v(u,t)\frac{\partial^2 v(u,t)}{\partial t^2} + \left(\frac{\partial v(u,t)}{\partial t}\right)^2\right) du - \right. \\ & \left. \int_0^t \frac{\partial^2 v(u,t)}{\partial t^2} dW(u)\right] dt - \frac{\partial v(u,t)}{\partial t}|_{u=t} dW(t). \end{aligned} \tag{3.23}$$

In the Hull-White model, $v(t,T) = \sigma(1-e^{-a(T-t)})/a$. Inserting this into (3.22) gives

$$r(t) = F(0,t) + \frac{\sigma^2}{a^2}(1-e^{-at}) - \frac{\sigma^2}{2a^2}(1-e^{-2at}) - \int_0^t \sigma e^{-a(t-u)} dW(u), \tag{3.24}$$

or

$$\int_0^t \sigma a e^{-a(t-u)} dW(u) = aF(0,t) + \frac{\sigma^2}{a}(1-e^{-at}) - \frac{\sigma^2}{2a}(1-e^{-2at}) - ar(t). \tag{3.25}$$

Equation (3.23) gives

$$dr(t) = \left[\frac{\partial F(0,t)}{\partial t} + \frac{\sigma^2}{a}(e^{-at} - e^{-2at}) + \int_0^t \sigma a e^{-a(t-u)} dW(u)\right] dt - \sigma dW(t). \tag{3.26}$$

Inserting the result from (3.25) to (3.26) gives

$$dr(t) = \left[\frac{\partial F(0,t)}{\partial t} + aF(0,T) + \frac{\sigma^2}{2a}(1-e^{-2at}) - ar(t)\right] dt - \sigma dW(t). \tag{3.27}$$

This expression is the Hull-White model

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t), \tag{3.28}$$

with

$$\theta(t) = \frac{\partial F(0,t)}{\partial t} + aF(0,T) + \frac{\sigma^2}{2a}(1-e^{-2at}). \tag{3.29}$$

By using (3.21), we can express the complete representation of the yield curve as

$$\begin{aligned} R(t,T) =& f(0,t,T) + \frac{\sigma(e^{-aT} - e^{-at})}{a(T-t)} \int_0^t e^{au} dW(u) + \\ & \frac{\sigma^2[e^{-2a(T-t)} - e^{-2aT} - 1 + e^{-2at} - 4e^{-a(T-t)} + 4e^{-aT} + 4 - 4e^{-at}]}{4a^3(T-t)}. \end{aligned} \tag{3.30}$$

By using the result from (3.24)

$$\sigma \int_0^t e^{au} dW(u) = -r(t)e^{at} + F(0,t)e^{at} + \frac{\sigma^2}{a^2}(e^{at} - 1) - \frac{\sigma^2}{2a^2}(e^{at} - e^{-at}), \quad (3.31)$$

$$\ln P(t,T) = -R(t,T)(T-t), \quad (3.32)$$

$$\frac{P(0,T)}{P(0,t)} = e^{-f(0,t,T)(T-t)}, \quad (3.33)$$

and denoting

$$\beta(t,T) = \frac{1}{a}(e^{-a(T-t)} - 1), \quad (3.34)$$

$$\alpha(t,T) = -F(0,t)\beta(t,T) + \ln \frac{P(0,T)}{P(0,t)} + \frac{\sigma^2}{4a}\beta^2(t,T)(e^{-2at} - 1), \quad (3.35)$$

the yield curve can be expressed as

$$R(t,T) = \frac{1}{T-t}\alpha(t,T) - \frac{1}{T-t}\beta(t,T)r(t). \quad (3.36)$$

After generating market scenarios for all simulation dates, the next step is to value the instruments in the credit portfolio. Because the valuation for a certain instrument needs to be done multiple times for multiple dates, computationally demanding models are out of question and analytical approximations are preferred to ensure the efficient calculation of credit exposure.

When the instrument market values under each scenario have been obtained, the aggregated exposure for a portfolio is calculated under each scenario. The distribution composed by these valuations is then used to calculate the EE for each simulation date. By constructing the non-decreasing EE curve, one can use numerical integration techniques to get the EAD value for the portfolio.

Under Basel III, banks must use the greater of the default risk charge calculated with current market data and the default risk charge calculated with stressed calibration. The comparison between these methods is done at the portfolio-level, i.e., the market data used must be the same for all counterparties. The period used must coincide with increased credit spreads and thus is usually either the 2008-09 or the 2011-12 period.

3.2.1.3 Regulatory risk weights

Because exposures towards different counterparties are not equally risky due to different credit qualities, the capital a bank must hold is calculated as a risk-weighted sum of EAD values towards different counterparties [Basel Committee on Banking Supervision, 2005a]. Risk weight (RW) for each EAD is calculated as

$$\begin{aligned} RW =& 12.5 \cdot LGD \cdot [\mathcal{N}(\frac{\mathcal{G}(PD_0) + \sqrt{\rho_{os}} \cdot \mathcal{G}(0.999)}{\sqrt{1-\rho_{os}}}) - PD_o] \\ &\cdot \frac{1 + (M - 2.5) \cdot b}{1 - 1.5 \cdot b}, \end{aligned} \tag{3.37}$$

where LGD is the loss given default, $\mathcal{N}$ is the standard normal distribution, $\mathcal{G}$ is the inverse standard normal distribution,

$$R = 0.12 \cdot \frac{1 - e^{-50PD}}{1 - e^{-50}} + 0.24 \cdot (1 - \frac{1 - e^{-50PD}}{1 - e^{-50}}), \tag{3.38}$$

$\rho_{os} = 1 - R$, M is the maturity and

$$b = (0.11852 - 0.05478 \cdot \ln(PD))^2 \tag{3.39}$$

is the smoothed maturity adjustment. RWA is thus obtained by

$$RWA = EAD \cdot RW. \tag{3.40}$$

We assume a general capital requirement for default risk charge as

$$K = RWA \cdot 8\% = EAD \cdot RW \cdot 8\%. \tag{3.41}$$

3.2.2 Credit valuation adjustment risk charge

CVA risk charge is a capital reserve which accounts for the risk of losses due to credit market volatility, i.e., increased CVA [Basel Committee on

Banking Supervision, 2013b]. Similarly to the default risk charge, banks are allowed to choose between a standard model and an advanced model. The capital requirement under the standard method is called standardized CVA risk charge (SCVA) and it is calculated by using EAD, effective maturities and possibly, purchased CDS hedges.

The advanced approach relies on a similar approach compared to the IMM EAD model described in Section 3.2.1.2. Advanced CVA risk charge (ACVA) is calculated using the EE profile obtained from simulating market scenarios. ACVA is calculated by combining a counterparty's EE curve with a full credit spread curve of the counterparty. Similarly to SCVA, purchased CDS hedges may be taken into account to lower the capital requirement for CVA risk.

3.3 Other capital charges

In addition to counterparty credit risk, banks are required to hold capital to cover possible losses resulting from market risk, operational risk and various additional requirements, generally referred to as Pillar II. In this thesis, we focus on counterparty credit risk related capital charges and thus, will not present the formulae for the calculation of other capital items [Basel Committee on Banking Supervision, 2006].

Chapter 4

Quantifying valuation adjustments

In this chapter, we show how the valuation adjustments for uncertainty and costs related to own and counterparty's credit quality, funding and capital can be derived by constructing a replicating portfolio to hedge a derivative contract. After obtaining the expressions for each valuation adjustment, we show how KVA for regulatory default risk capital can be solved using American Monte Carlo techniques.

4.1 Semi-replication and pricing partial differential equation with capital extension

Burgard and Kjaer [2013] present a semi-replication model which addresses valuation adjustments for credit and funding and is simple enough to be extended to include simulation-based capital costs. This method was extended to consider regulatory capital calculated under a simplified model by Green et al. [2014]. We closely follow these methods by considering a derivative contract between a bank B and a counterparty C with an economic value of $\hat{V}$ that incorporates the risk of default of both counterparty and bank and any net funding and capital costs the bank may encounter. By introducing four tradable instruments, it is possible to derive a general semi-replication strategy which allows the bank to perfectly hedge out market risk and counterparty default risk, but which may not be appropriate for providing a perfect hedge in the event of the bank's own default. The instruments in this strategy are (i) a counterparty zero-coupon zero-recovery bond P_C, (ii) the bank's own junior bond P_1 with recovery rate R_1 and yield r_1, (iii) the bank's own senior bond P_2 with recovery rate R_2 and yield r_2 and (iv) a market instrument S that can be used to hedge the market factor underlying the derivative contract. The following standard dynamics are assumed for the instruments:

$$\begin{aligned} dS(t) &= \mu S(t)dt + \sigma S(t)dW(t) \\ dP_C &= r_C P_C^-(t)dt - P_C^-(t)dJ_C \\ dP_i &= r_i P_i^-(t)dt - (1 - R_i)P_i^-(t)dJ_B, \end{aligned} \tag{4.1}$$

where $W(t)$ is Brownian motion, J_B and J_C are default indicators for the bank and the counterparty and P_i^- and P_C^- are the pre-default bond prices. Let P_1 be the price of the junior bond and P_2 the price of the senior bond, i.e., $R_1 < R_2$ and $r_1 > r_2$. Let us also assume that the basis between bonds of different seniorities is zero such that

$$r_i - r = (1 - R_i)\lambda_B, \tag{4.2}$$

where r is the risk-free rate and λ_B is the spread of a zero-coupon zero-recovery bond of the bank.

We assume that if either the bank or the counterparty defaults, the surviving party pays the defaulting party in full if owing money. Also, the defaulting party pays the surviving party as much as it can if owing money. The received amount is assumed to be a fixed recovery rate times the market value.

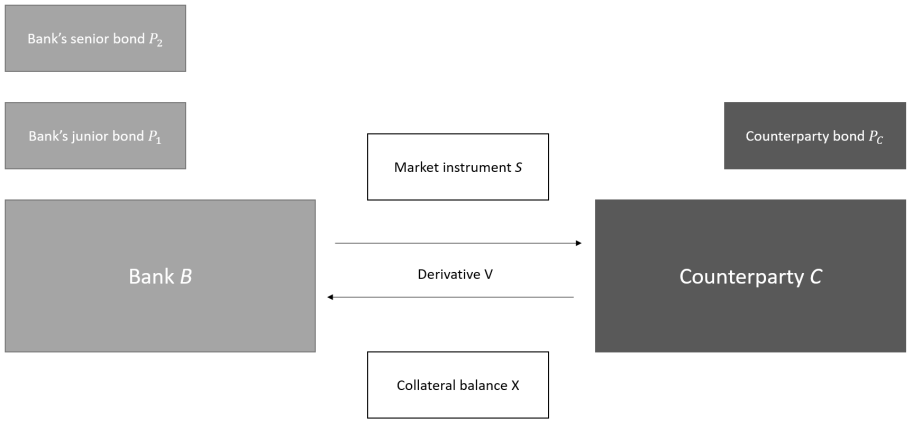

Figure 4.1: The setup consists of the bank, the counterparty, four tradable instruments and the collateral account.

Let $V(t, S(t), J_B, J_C)$ be the classic risk-free value of a derivative to the bank. $\hat{V}(t, S(t), J_B, J_C)$ is the economic value of the derivative that takes into account possibilities of defaults and both funding and capital costs. If the bank defaults ($J_B = 1$) and the derivative's market value is higher than the collateral balance $X(t)$, the economic value is $V(t)$. However, if the collateralized market value is negative, the economic value is also negative but takes into account the recovery rate of the bank. The proportion of exposure lost due to a default is called a close-out amount, which is equal to $1 - RR$ times the exposure. The dynamics are similar in the case of the counterparty's default. Using the notations $U^+ = \max(U, 0)$ and $U^- = \min(U, 0)$, the value of the derivative has boundary conditions

$$\begin{aligned}\hat{V}(t, S(t), 1, 0) &= (V(t) - X(t))^+ + R_B(V(t) - X(t))^- + X(t)\\ \hat{V}(t, S(t), 0, 1) &= (V(t) - X(t))^- + R_C(V(t) - X(t))^+ + X(t),\end{aligned} \tag{4.3}$$

where $V(t)$ is the classic risk-free value of the derivative that does not consider funding and capital costs either, and R_B and R_C are the proportions of close-out amounts of the total exposure for B and C, respectively.

The semi-replication strategy is set up by constructing a hedge portfolio Π

from the instruments P_C, P_1, P_2 and S. Let the amounts of each instrument in the portfolio be α_C, α_1, α_2 and σ, respectively. In addition to these instruments, one collateral and two cash accounts for funding positions in S and P_C are set up. The hedge portfolio is

$$\begin{aligned}\Pi(t) =&\delta(t)S(t) + \alpha_1(t)P_1(t) + \alpha_2(t)P_2(t) + \alpha_C(t)P_C(t)+\\ &\beta_S(t) + \beta_C(t) - X(t),\end{aligned} \tag{4.4}$$

where β_S funds position in S and β_C funds position in P_C, i.e., $\sigma(t)S(t) + \beta_S(t) = 0$, $\alpha_C P_C(t) + \beta_C(t) = 0$ and $X(t)$ is the collateral balance. In order to present the expressions in a more readable format, we leave out the time dynamics from now on. The dynamics of the hedge portfolio are then

$$d\Pi = \delta dS + \alpha_1 dP_1 + \alpha_2 dP_2 + \alpha_C dP_C + d\beta_S + d\beta_C - dX, \tag{4.5}$$

where changes in the collateral balance, dX do not include re-balancing.

To hedge all the cash flows, except for possibly the ones resulting from the bank's own default, this strategy must satisfy

$$\hat{V} + \Pi = 0. \tag{4.6}$$

We assume that the hedge position $\beta_S S$ is collateralized with financing rate q_S and hedge position $\beta_C P_C$ is repo-ed costing a repo rate q_C. Repo refers to repurchase agreement, which is an agreement to borrow an instrument for a period of time in exchange for a repo rate. If the dividend rate associated with S is γ_S, the cash accounts β_S and β_C are paying net rates of $q_S - \gamma_S$ and q_C, respectively. Here, a positive collateral balance, i.e., $X > 0$ indicates received collateral from the bank's point of view and costs rate r_X. The growth in the cash accounts (prior to rebalancing, see Burgard and Kjaer

[2010]) then follow the dynamics

$$\begin{aligned} d\beta_S &= \sigma(\gamma_S - q_S)Sdt \\ d\beta_C &= -\alpha_C q_C P_C dt \\ dX &= r_X X dt. \end{aligned} \tag{4.7}$$

In addition, we have to account for the use of the capital offsetting funding requirements. The amount of capital associated with the derivative depends on counterparty rating C, derivative value V, its sensitivities, i.e., market risk and balance on the collateral account X. The capital associated with the replicating portfolio depends on the positions in the stock δ and counterparty bond α_C. Thus, the amount of capital is a function

$$K = K(t, V, \text{“}market\ risk\text{”}, X, C, \delta, \alpha_C). \tag{4.8}$$

The change in the cash account β_K which reflects changes in the capital requirement can be treated as a borrowing action such that capital is borrowed from stakeholders who then receive rate equal to the cost of capital as a compensation for supporting derivatives activities. If the cost of capital including possible dividend yield is λ_k, we can formulate the dynamics of the account as

$$d\beta_K = -\gamma_K K dt. \tag{4.9}$$

Inserting the dynamics in (4.7) and (4.9), and using the instrument standard dynamics in (4.5) gives

$$\begin{aligned} d\Pi =& (r_1\alpha_1 P_1 + r_2\alpha_2 P_2 + (r_C - q_C)\alpha_C P_C + (\gamma_S - q_S)\delta S - r_X X - \gamma_K K)dt + \\ & (\alpha_1 R_1 P_1 - \alpha_1 P_1 + \alpha_2 R_2 P_2 - \alpha_2 P_2)dJ_B - \alpha_C P_C dJ_C + \delta dS. \end{aligned} \tag{4.10}$$

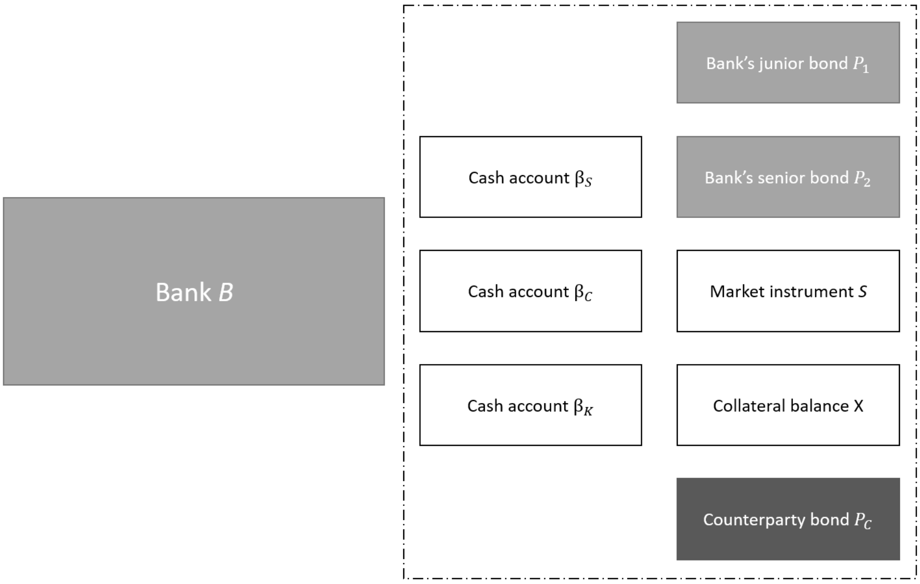

Figure 4.2: The bank funds the hedge portfolio and its capital requirement with cash accounts β_S, β_C and β_K.

If the use of capital is ϕK and the positions in the bank's own bonds are used to fund any additional cash that is not funded with the collateral, we can state the following funding constraint

$$\hat{V} - X + \alpha_1 P_1 + \alpha_2 P_2 - \phi K = 0. \tag{4.11}$$

For the evolution $d\hat{V}$ of the derivative's value, we consider Itô formula for jump diffusions [Cont and Tankov, 2004] and apply it to geometric Brownian processes with known jump sizes. A jump diffusion process

$$S = S^c + J \tag{4.12}$$

consists of a continuous part S^c and a compound Poisson process J

$$J = \sum_{i=1}^{N} \Delta V_i. \tag{4.13}$$

Let S be a geometric Brownian motion and $\hat{V} = f(S)$ where f is a twice continuously differentiable function. $T_i, i = 1 \dots N_T$ are the jump times of S. Between T_i and T_{i+1}, S evolves according to

$$dS = dS^c = \mu S dt + \sigma S dW. \tag{4.14}$$

By applying the Itô's formula to the dynamics of S we obtain

$$\begin{aligned} \hat{V}_{T_{i+1}} - \hat{V}_{T_i} &= \int_{T_i}^{T_{i+1}} \frac{\sigma^2}{2} S^2 f''(S) dt + \int_{T_i}^{T_{i+1}} f'(S) dS \\ &= \int_{T_i}^{T_{i+1}} (\frac{\sigma^2}{2} S^2 f''(S) dt + f'(S) dS^c). \end{aligned} \tag{4.15}$$

Because we are between two jump times, S has the same properties as its continuous part, i.e., $dS = dS^c$. If a jump of size ΔS occurs, it results in a change of $f(S + \Delta S) - f(S)$ in $\hat{V}$. Combining this with (4.15) gives the total change in $\hat{V}$ as

$$\begin{aligned} f(S) - f(S_0) = \int_0^t \frac{\sigma^2}{2} S^2 f''(S) ds + \int_0^t f'(S) dX^c + \\ \sum_{0 \le s \le t, \Delta S \ne 0} [f(S + \Delta S) - f(S) - \Delta X f'(S)]. \end{aligned} \tag{4.16}$$

Expressing this in differential notation and replacing changes in f due to a jump with known changes resulting from defaults gives

$$d\hat{V} = \frac{\partial \hat{V}}{\partial t} dt + \frac{\partial \hat{V}}{\partial S} dS + \frac{\sigma^2}{2} \frac{\partial^2 f}{\partial S^2} S^2 + \Delta \hat{V}_B dJ_B + \Delta \hat{V}_C dJ_C, \tag{4.17}$$

where $\Delta \hat{V}_B = \hat{V}(t, S, 1, 0) - \hat{V}(t, S, 0, 0)$ and $\Delta \hat{V}_C = \hat{V}(t, S, 0, 1) - \hat{V}(t, S, 0, 0)$ are the changes in $\hat{V}$ due to bank's and counterparty's default, respectively.

Combining the derivative's evolution in (4.17) with the evolution of the replicating portfolio in (4.10) gives the total evolution of the hedged derivative

$$\begin{aligned}
d\hat{V} + d\Pi =&(\frac{\partial \hat{V}}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{V}}{\partial S^2}S^2 + r_1\alpha_1 P_1 + r_2\alpha_2 P_2 + \\
&(r_c - q_c)\alpha_C P_C + (\gamma - q)\delta S - r_x X - \gamma_K K)dt + \\
&(\alpha_1 R_1 P_1 - \alpha_1 P_1 + \alpha_2 R_2 P_2 - \alpha_2 P_2 + \Delta\hat{V}_B)dJ_B + \\
&(-\alpha_C P_C + \Delta\hat{V}_C)dJ_C + \\
&(\delta + \frac{\partial \hat{V}}{\partial S})dS.
\end{aligned} \tag{4.18}$$

The risks associated with underlying instrument price and counterparty default can be eliminated by choosing

$$\alpha_C P_C = \Delta\hat{V}_C, \tag{4.19}$$

$$\delta = -\frac{\partial \hat{V}}{\partial S}. \tag{4.20}$$

By (i) using the results from (4.19) and (4.20), (ii) denoting the pre-default bank bond position $\alpha_1 P_1 + \alpha_2 P_2$ with P and the post-default position $\alpha_1 R_1 P_1 + \alpha_2 R_2 P_2$ with P_D and (iii) using the funding constraint in (4.11), we can formulate (4.18) as

$$\begin{aligned}
d\hat{V} + d\Pi =&(\frac{\partial \hat{V}}{\partial t} + \mathcal{A}\hat{V} - r_X X + r_1\alpha_1 P_1 + r_2\alpha_2 P_2 + \lambda_C\Delta\hat{V}_C - \gamma_K K)dt + \\
&(\Delta\hat{V}_B + V + P_D - X)dJ_B,
\end{aligned} \tag{4.21}$$

where $\mathcal{A} = \frac{\sigma^2}{2}\frac{\partial^2}{\partial S^2}S^2 + (q_S - \gamma_S)S\frac{\partial}{\partial S}$ and $\lambda_C = r_C - q_C$.

The zero basis relation in (4.2) and the funding constraint in (4.11) allows us to write (4.21) as

$$\begin{aligned}
d\hat{V} + d\Pi =&(\frac{\partial \hat{V}}{\partial t} + \mathcal{A}\hat{V} - s_X X - (r + \lambda_B + \lambda_C)\hat{V} + \\
&\lambda_C g_C + \lambda_B g_B - \epsilon_h\lambda_B)dt + \epsilon_h dJ_B,
\end{aligned} \tag{4.22}$$

where $g_B = \hat{V} + \Delta\hat{V}_B$ and $g_C = \hat{V} + \Delta\hat{V}_C$ are the post-default values for the derivative after bank's and counterparty's default, respectively, and $s_X = r_X - r$. From the jump term, we can see that in case of the bank's own default there is a hedge error of

$$\begin{aligned} \epsilon_h =& g_B + P_D - X - \phi K \\ =& \epsilon_{h_0} + \epsilon_{h_K}, \end{aligned} \tag{4.23}$$

where ϵ_{h_K} is the capital-dependent part of the error. On the other hand, unless the bank defaults, it will gain a corresponding incremental cost/gain of $\epsilon_h \lambda_b dt$. As long as the bank is alive the combination of the derivative and hedge portfolio is risk-free.

Let us now assume that, while alive, the bank wants the strategy to be self-financing and perfectly replicate the derivative value. The direct implication of this feature is that the drift term in (4.22) must be zero, i.e.,

$$\frac{\partial \hat{V}}{\partial t} + \mathcal{A}\hat{V} - (r + \lambda_B + \lambda_C)\hat{V} = s_X X - \lambda_C g_C - \lambda_B g_B + \epsilon_h \lambda_B + (\gamma_K - r\phi)K, \tag{4.24}$$

with a final condition $\hat{V}(T, S) = H(S)$, where $H(S)$ is the payoff at maturity. Recall that V was used for the classic Black-Scholes price of the derivative whereas $\hat{V}$ is the value taking into account valuation adjustments. If the combined valuation adjustment is U, we can write

$$\hat{V} = V + U. \tag{4.25}$$

Because V satisfies the Black-Scholes partial differential equation (PDE)

$$\frac{\partial V}{\partial t} + \mathcal{A}V - rV = 0, \tag{4.26}$$

with the final condition $V(T, S) = H(S)$, we can write the following PDE for the valuation adjustment

$$\begin{aligned} &\frac{\partial U}{\partial t} + \mathcal{A}U - (r + \lambda_B + \lambda_C)U = \\ &V\lambda_C - g_C\lambda_C + V\lambda_B - g_B\lambda_B + \epsilon_H\lambda_B + s_X X + \gamma_K K - r\phi K, \end{aligned} \tag{4.27}$$

with the final condition $U(T, S) = 0$.

To solve the PDE, we consider Feynman-Kac theorem [Kac, 1951] applied to the Black-Scholes PDE by Janson and Tysk [2006]. The Feynman-Kac theorem provides a link between parabolic second order PDEs and stochastic processes and can be used to solve PDEs by simulating random paths of a stochastic process. While the approach is used for solving the price of European call options, it can be extended to solve (4.27) which, for simplicity, can be written in a more general form

$$\frac{\partial U}{\partial t} + \mathcal{A}U - \kappa U = \omega, \tag{4.28}$$

with the final condition $U(T, S) = 0$. According to the Feynman-Kac theorem, the solution for (4.28) can be expressed as the expectation

$$U = -\mathbb{E}_t[\int_t^T e^{-\int_t^u \kappa(S_s,s)ds}\omega(S_u, u)du], \tag{4.29}$$

which allows us to write the solution for (4.27) as

$$\begin{aligned} U = -\int_t^T e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds}\mathbb{E}_t[\lambda_C(u)V(u)- \\ \lambda_C(u)g_C(V(u), u) + \lambda_B(u)V(u) - \lambda_B(u)g_B(V(u), u)+ \\ \lambda_B(u)\epsilon_{h_0}(u) + s_X X(u) + (\gamma_K(u) - r(u)\phi)K(u) + \lambda_B\epsilon_{h_K}]du. \end{aligned} \tag{4.30}$$

By grouping the equation so that we have terms dependent on counterparty's credit quality, bank's credit quality, funding and capital, we can write

$$U = CVA + DVA + FVA + KVA, \tag{4.31}$$

where

$$\begin{aligned}
CVA &= -\int_t^T e^{-\int_t^u \tilde{r}(s)ds} \lambda_C(u)\mathbb{E}_t[V(u) - g_C(V(u), X(u)]du \\
DVA &= -\int_t^T e^{-\int_t^u \tilde{r}(s)ds} \lambda_B(u)\mathbb{E}_t[V(u) - g_B(V(u), X(u)]du \\
FVA &= -\int_t^T e^{-\int_t^u \tilde{r}(s)ds} (\lambda_B(u)\mathbb{E}_t[\epsilon_h(u)] - r(u)\phi\mathbb{E}_t[K(u)]+ \\
&\qquad s_X(u)\mathbb{E}_t[X(u)])du \\
KVA &= -\int_t^T e^{-\int_t^u \tilde{r}(s)ds} \gamma_K(u)\mathbb{E}_t[K(u)]du,
\end{aligned} \tag{4.32}$$

and $\tilde{r}(s) = r(s) + \lambda_B(s) + \lambda_C(s)$.

4.2 American Monte Carlo algorithm

While most XVAs can be easily approximated via simple numerical integration techniques, KVA is more complicated due to the capital term $K(t)$ in the integrand which is usually computed via simulation based techniques. A simple brute force solution could be applied to solve the nested Monte Carlo problem, but to enhance computational performance, we consider American Monte Carlo techniques presented by Longstaff and Schwartz [2001]. The method was originally developed for the valuation of American options, hence the name. American options differ from European options in that the options can be exercised before maturity. The decision whether or not to exercise the option must be determined at all times when the option is exercisable to come up with the optimal exercise strategy and thus a price for the instrument. The exercise decision at t is done by comparison of the value from immediate exercise and continuation. The key concept of American Monte Carlo techniques is to estimate the conditional expectation of payoff from continuation by cross-sectional information in the simulation using least-squares regression, thus removing the need for nested simulations. This means that, instead of starting a new simulation from each time point in each path (ω, t_k), we estimate the conditional expectation of the option's value at (ω, t_k) with the information that is provided by all paths ω and available at t_{k-1}. Valuing American options and calculating KVA are similar in that both require calculations of conditional expectations at each time step. To allow for efficient use, we have chosen American Monte Carlo approach for the calculation of KVA. To formulate the American Monte Carlo algorithm, we follow the Longstaff and Schwartz [2001] approach closely but apply a few changes to make it more applicable to our problem.

As in Chapter 3, we assume a finite, discrete and evenly spaced time interval [0, T] with K time steps and a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the set of all possible realizations of the stochastic environment between 0 and T, $\mathcal{F}$ is the sigma-algebra, i.e., a closed collection of subsets of distinguishable events at time T and $\mathcal{P}$ is a probability measure on the elements of $\mathcal{F}$. An augmented filtration of $\mathcal{F}$ is an increasing and indexed family of sub-sigma-algebras of $\mathcal{F}$. It is generated by the instrument price processes ω_i. Let such an augmented filtration be F: $F = \{\mathcal{F}_t; t \in [0, T], \mathcal{F}_T = \mathcal{F}\}$. We assume no arbitrage opportunities, i.e., a risk-neutral measure $\mathcal{Q}$ exists for the stochastic environment. In the original approach, $C(\omega, s; t, T)$ is defined as the path of cash flows generated by the option given that the option is alive at t and the optionholder uses optimal exercise strategy for all $s, t < s \leq T$. In other words, $C(\omega, s; t, T)$ represents the payoff received from exercising the option at some t. The assumption of N discrete time points when the option is exercisable allows us to write the value of continuation at t_k as the expectation of the discounted cash flows $C(\omega, s; t_k, T)$

$$F(\omega; t_k) = \mathbb{E}_{\mathcal{Q}}[\sum_{j=k+1}^{N} e^{-\int_{t_k}^{t_j} r(\omega,s)ds} C(\omega, t_j; t_k, T)|\mathcal{F}_{t_k}], \tag{4.33}$$

where $r(\omega, s)$ is the riskless discount rate and the risk-neutral expectation is taken under $\mathcal{Q}$ and conditional on the information set $\mathcal{F}_{t_k}$. Prior to maturity, the optionholder chooses to exercise the option if the value given by (4.33) is less than the value from immediate exercise. We will use the notation $C(\omega, s; t, T)$ for a path of cash flows generated by a generic derivative product, given that the derivative is alive and if it carries possibilities of exercise, optimal exercise strategy is used. The value of a derivative without exercise possibilities is then simply the value of continuation given by (4.33). We also assume that the generic derivative can be decomposed into sub-components that generate only one cash flow. For example, interest rate swaps can be decomposed into a series of consecutive forward rate agreements (FRA), caps into series of caplets, etc.

The conditional expectation $F(\omega; t_k)$ is an element of a space of square-integrable functions, i.e., the integral of the square of the absolute value is finite. Because the space has a countable orthonormal basis, $F(\omega; t_k)$ can be presented as a linear combination of a finite set of $\mathcal{F}_{t_k}$-measurable basis functions. The implication of this assumption is that in the case of valuation of American options, we can write the value of the option as a function of the underlying market factors and then approximate the value by regression. For example, the value of the asset S underlying the option can be used as the explanatory variable.

After decomposing the derivative contracts into single FRAs, caplets, etc., we assume that each future cash flow is dependent on one underlying market factor. Let the explanatory variable be $X(t)$. We construct the set of basis functions as simple polynomials of increasing powers of $X(t)$ and write the linear combination in the general form as

$$F(\omega; t_k) = \sum_{j=0}^{\infty} a_j X(t_{k-1})^j, \tag{4.34}$$

where the a_j coefficients are constants. The basis functions can be easily expanded to take into account multiple underlying market factors by replacing $a_j X(t_{k-1})^j$ with $a_j X(t_{k-1})^j + b_j Y(t_{k-1})^j$, for example.

Illustrative figures of 1-year interest rates and market values for a receive floating, pay fixed interest rate swap are shown in Figures 4.3 and 4.4. The Hull-White model is calibrated with $a = 0.2$ and $\sigma = 0.015$. 1-year interest rate starts from 3.50%. In the case of a linear derivative product, such as an interest rate swap, the paths for the underlying interest rate and the derivative are similar. Figure 4.5 illustrates a simple linear regression $F(\omega; t_k) = a_0 + a_1 X(t_{k-1})$ for the four scenarios when simulated swap values at t_k are regressed onto the basis function at t_{k-1}. First order regression is likely insufficient for options and other products which have more complex payoff functions.

Let us denote the approximation with the first $m < \infty$ terms as $F_m(\omega, t_k)$. Once the basis functions where $a_j \neq 0$ are specified, we start backwards from maturity t_K by estimating $F_m(\omega, t_{K-1})$ with $\hat{F}_m(\omega, t_{K-1})$ by regressing the values of cash flows $C(\omega; s, t_K, T)$ onto the basis functions for all simulation paths. By doing this, we construct pairs of derivative values and underlying market factors $(X_{t_{K-1}}, V(X_{t_K}))$ and estimate the regression parameters a_j from these pairs by minimizing least squares

$$\min_{a_j} \ (F(\omega; t_k) - \hat{F}_m(\omega, t_k))^2. \tag{4.35}$$

Based on the regression coefficients, we can estimate the conditional expectation of the derivative's value when we know the underlying market factor in each scenario. It can be shown that the fitted regression value $\hat{F}_m(\omega, t_k)$ converges in probability and in mean square as the number of simulation paths approaches infinity [White, 1984]. Also, Theorem 1.2.1 of Amemiya [1985] implies that the estimate is the best linear mean-squared metric based unbiased estimator of $F_m(\omega, t_k)$. The fitted regression value $\hat{F}_m(\omega, t_{K-1})$ represents the expected value of the derivative at t_{K-1} for path ω. We then continue stepwise by estimating $F_m(\omega, t_{K-i}), i = 2, \ldots$ by regressing the values of $C(\omega; s, t_{K-1}, T)$ onto the basis functions at t_{K-2}, values of $C(\omega; s, t_{K-2}, T)$ onto the basis functions at t_{K-3} and so on. By averaging over all paths, we obtain an expected exposure (EE) profile for the derivative. Providing a general convergence rule for the algorithm may be difficult as the number of time points, basis functions and paths go to infinity unless certain limits are set.

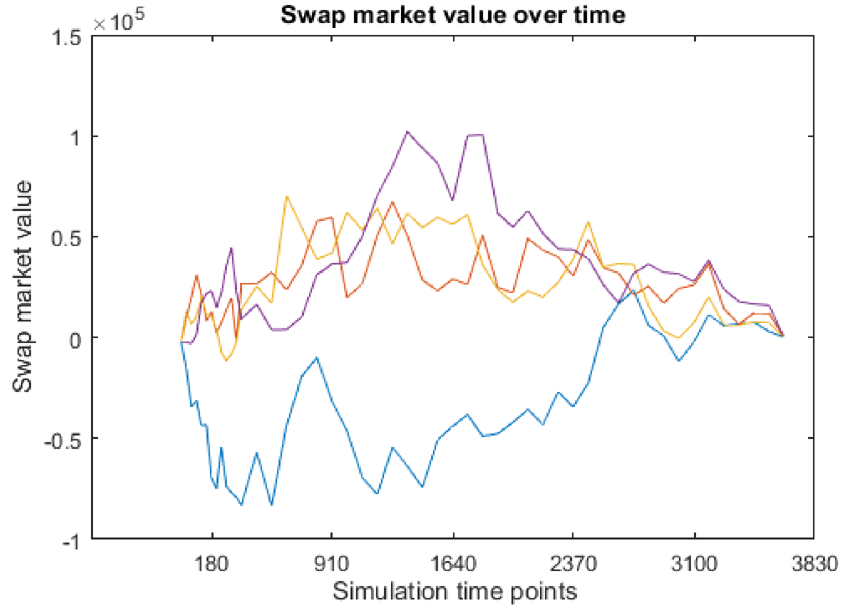

Figure 4.3: Interest rate swap market value development in four randomly chosen scenarios. The Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$. Time points in the x-axis represent days.

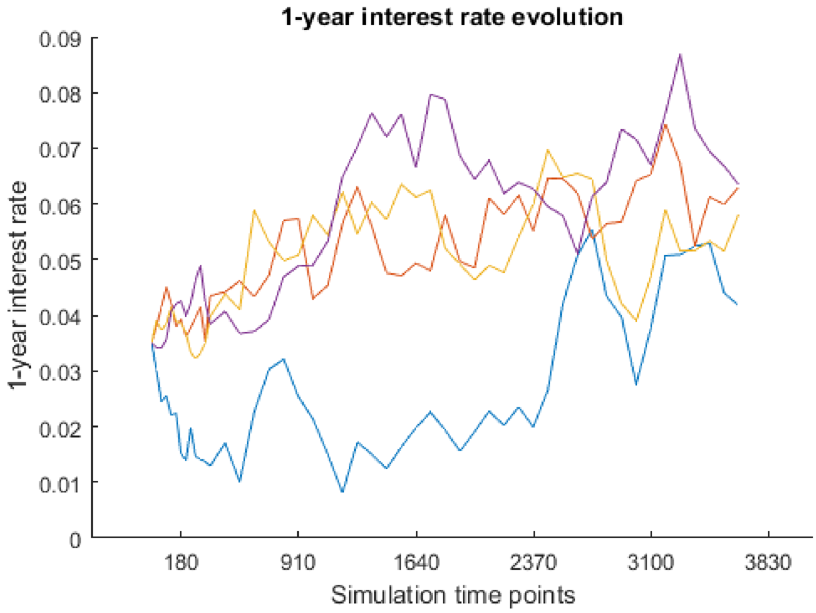

Figure 4.4: 1-year interest rate development in four illustrative scenarios. The 1-year interest rate is 3.50% at $t = 0$ and the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$. Time points in the x-axis represent days.

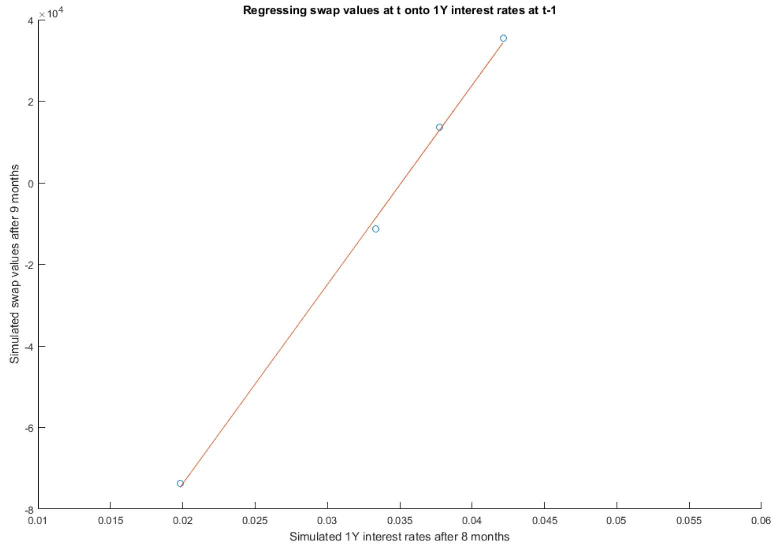

Figure 4.5: Linear regression of swap values at t onto the basis function at $t-1$

4.3 Capital valuation adjustment for regulatory default risk charge

After the general representation of the algorithm, we now consider the default risk capital requirement $K(t_k)$ for a portfolio of derivatives shown in (3.41) and substitute EAD to EE using (3.7) and (3.6)

$$K(t_k) = RW \cdot 8\% \cdot \alpha \cdot \int_{t=t_k}^{T} \max\left(EE(t_k), \ldots, EE(t)\right) dt, \tag{4.36}$$

where the risk weight RW is assumed to be constant, T is either the one-year point or the maturity of the longest dated contract if all the contracts mature within a year.

For the modeling of the EE profile, we re-consider the pseudo-algorithm in Chapter 3.2.1.2. We start by generating M market scenario paths for all underlying market factors X_i. Specifically, interest rates are generated with the Hull-White model presented by Hull and White [1990]

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t), \tag{4.37}$$

and FX rates with geometric Brownian motion

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t). \tag{4.38}$$

We decompose all derivatives into sub-components that generate only one cash flow. After obtaining the cash flows $C(\omega; s, t_K, T)$ generated by each derivative at maturity t_K, we use the basis functions and least-squares regression to obtain conditional expectations $\hat{F}_m(\omega, t_{K-1})$ for each derivative's value at t_{K-1} conditional on the information set at t_{K-1}. As each derivative's value is now dependent only on one discounted cash flow, the relevant explanatory variables are trivial. For example, in the case of an interest rate product, such as a floating for fixed FRA, the underlying floating interest rate is the only explanatory variable. In this case, we use the underlying interest rate at t_{K-1} as an explanatory variable and construct pairs of those and the FRA's values at t_K for each simulation path. By considering all simulation paths, we can then find the optimal regression parameters and estimate the conditional expectation of the FRA's price using only the underlying interest rate at t_{K-1}. By starting from maturity and continuing backwards until the current time point, this makes it possible to approximate an individual trade's expected exposure for each time point.

We use $\hat{F}^i_m(\omega, t_k)$ to denote the estimate for derivative i's market value at t_k in path ω obtained by recursively generating the conditional expectations via fitted regression. An estimate for portfolio level EE profile at t_k is then obtained by averaging over the estimated positive portfolio-level realizations

$$EE(t_k) \approx \frac{1}{M^{pos}} \sum_{\omega} [\sum_{i=1}^{n} \hat{F}^i_m(\omega, t_k)]^+, \tag{4.39}$$

where M^{pos} is the number of positive portfolio-level realizations at t_k. EEPE is calculated by using the trapezoidal rule for numerical integration of the non-decreasing EE curve

$$EEPE(t_k) = \sum_{j=1}^{J} \frac{\mathcal{E}(t_k, j-1) + \mathcal{E}(t_k, j)}{2}(t_{k+j} - t_k), \tag{4.40}$$

where $\mathcal{E}(t_k, j) = \max\left(EE(t_k), \ldots, EE(t_k + j)\right)$ is the non-decreasing EE curve and $t_{k+J} = T$.

The expected capital requirement $\mathbb{E}_t[K(t_k)]$ needed to solve KVA in (4.32) is approximated with $\tilde{K}(t_k)$

$$\tilde{K}(t_k) = RW \cdot 8\% \cdot \alpha \cdot EEPE(t_k). \tag{4.41}$$

By using this, we can solve KVA at t_k from

$$KVA = -\int_{t_k}^{T} e^{-\int_t^u \tilde{r}(s)ds} \gamma_K(u) \mathbb{E}_t[K(t_k)]du. \tag{4.42}$$

by using the trapezoidal rule and the approximation $\mathbb{E}_t[K(t_k)] \approx \tilde{K}(t_k)$

$$KVA(t_k) = \sum_{j=1}^{J} \frac{\mathcal{K}(t_k, j-1) + \mathcal{K}(t_k, j)}{2}(t_{k+j} - t_k)\gamma_K(t_{k+j+1}), \tag{4.43}$$

where $\tilde{r}_{t_{k+j}}$ is the discount rate from t_k to t_{k+j} and $\mathcal{K}(t_k, j) = e^{-\tilde{r}_{t_{k+j}}}\tilde{K}(t_{k+j})$ is the discounted capital charge at t_{k+j}.

Chapter 5

Numerical examples

We demonstrate the model for estimating KVA with an interest rate swap between a bank and a counterparty.

5.1 Setting and assumptions

We assume a situation between a bank and a counterparty which (i) is an investment grade company with a risk weight of 0.3 and (ii) does not have a credit support annex (CSA). The parties have entered into an interest rate swap. The bank has an annual total capital cost of $\gamma_K = 10\%$ and uses the regulatory scaling factor $\alpha = 1.4$ for EAD. We use Actual/365 date convention and linear regression to obtain interest rates between two tenors in the yield curve. The Hull-White interest rate model

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t) \tag{5.1}$$

is calibrated with mean reversion rate $a = 0.24$ and volatility parameter $\sigma = 0.015$. See Gurrieri et al. [2009] for a comprehensive framework of calibrating the Hull-White model parameters. The yield curve at $t = 0$ is shown in Figure 5.1. Each simulation is run with 1000 scenarios and one

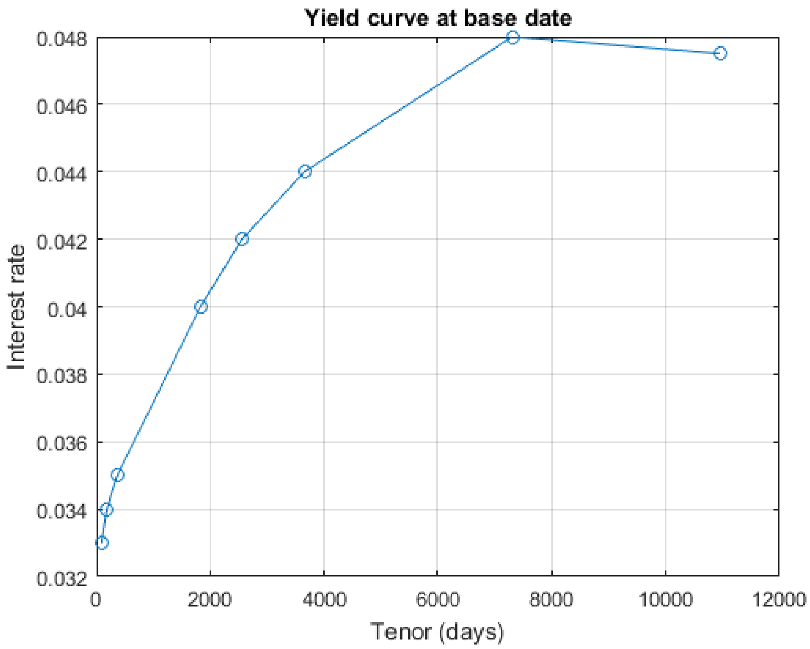

Figure 5.1: Yield curve at base date

time step represents one day. We determine the capital costs the bank needs to incorporate in the valuation of the interest rate swap.

5.2 Capital valuation adjustment for an at-the-money interest rate swap

The counterparty has agreed on paying floating rate to the bank in exchange for fixed 3.95 % on a notional of 10000 units in the domestic currency for 10 years. Details are shown in Table 5.1.

The evolution of 1-year interest rates is shown in Figure 5.2. The evolution of the full yield curve for a randomly chosen scenario is in Figure 5.3.

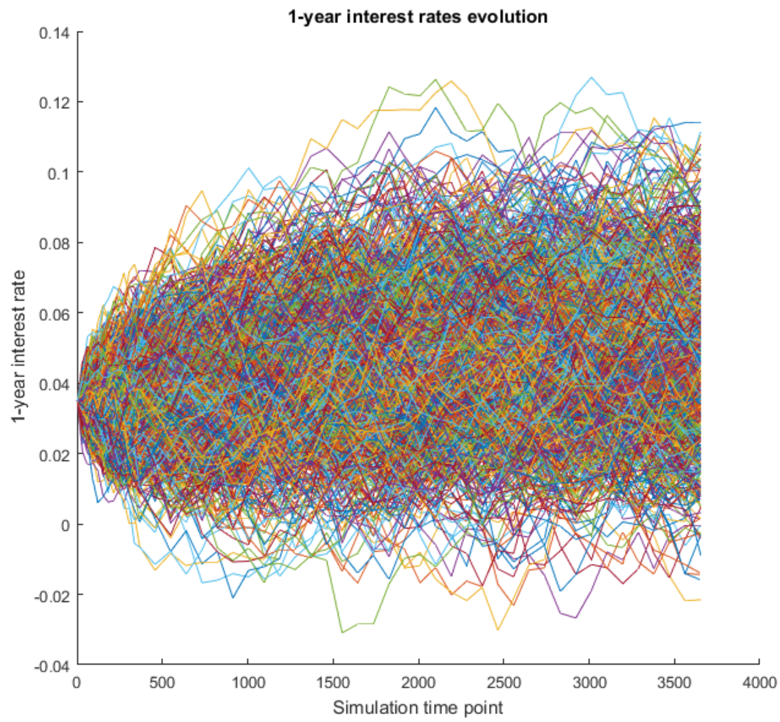

Figure 5.2: 1-year interest rates evolution in 1000 scenarios for each simulation time point during the 10 years lifetime of the trade. 1-year interest rate is 3.50% at $t = 0$ and the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$.

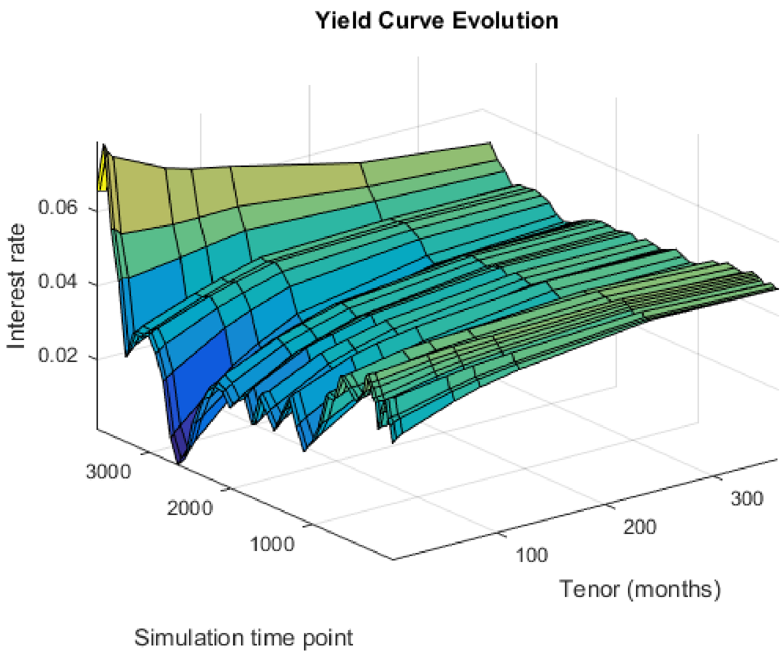

Figure 5.3: Full yield curve evolution in a randomly chosen scenario. 1-year interest rate is 3.50% at $t = 0$ and the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$.

Table 5.1: Details of the at-the-money interest rate swap

Swap	Notional	Tenor (years)	Pay leg rate	Receive leg rate
1	10000	10	Fixed 3.95%	Floating

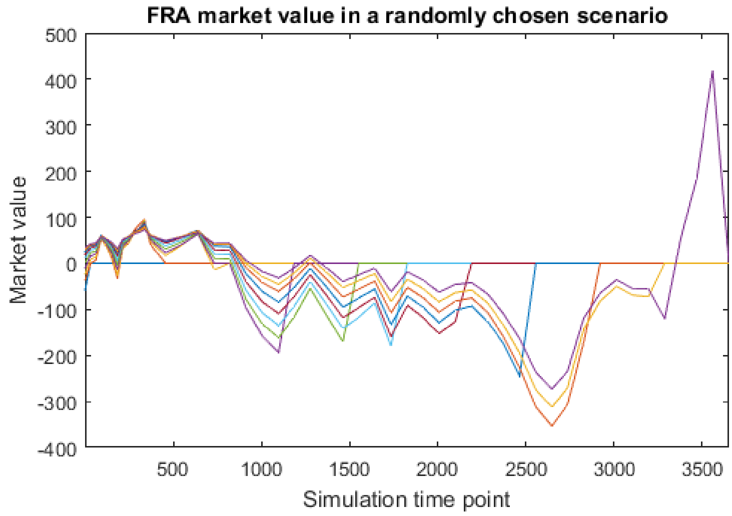

Figure 5.4: Simulated FRA market values in a randomly chosen scenario. The Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$.

Figure 5.4 illustrates how the swap is decomposed into single FRAs which mature every time the swap is fixed. Figure 5.5 shows the simulated market values for the interest rate swap.

For each simulation date, the instantaneous forward rate until the maturity of the FRA is used an an explanatory variable for the regression. The regressed market values for the swap are in Figure 5.6. We use linear regression

$$F(\omega; t_k) = a_0 + a_1 X(t_{k-1}), \tag{5.2}$$

where $X(t)$ is the instantaneous forward rate until the maturity of the FRA and a_0, a_1 are constants. Parameters are estimated by the least-squares approach. The regressed values for each simulation date and scenario are

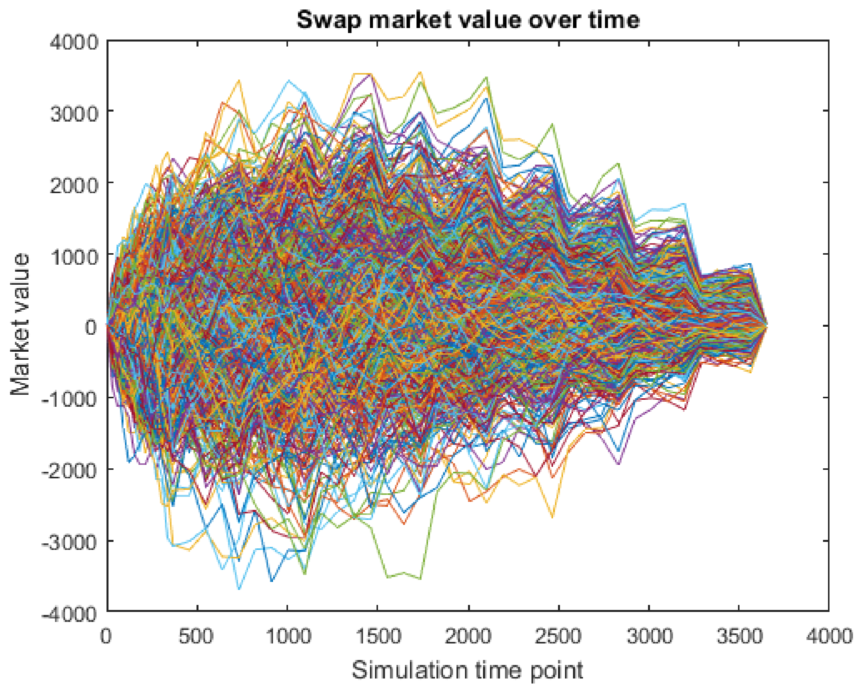

Figure 5.5: Simulated swap market values from $t = 0$ to $t = 10$(years) in 1000 scenarios.

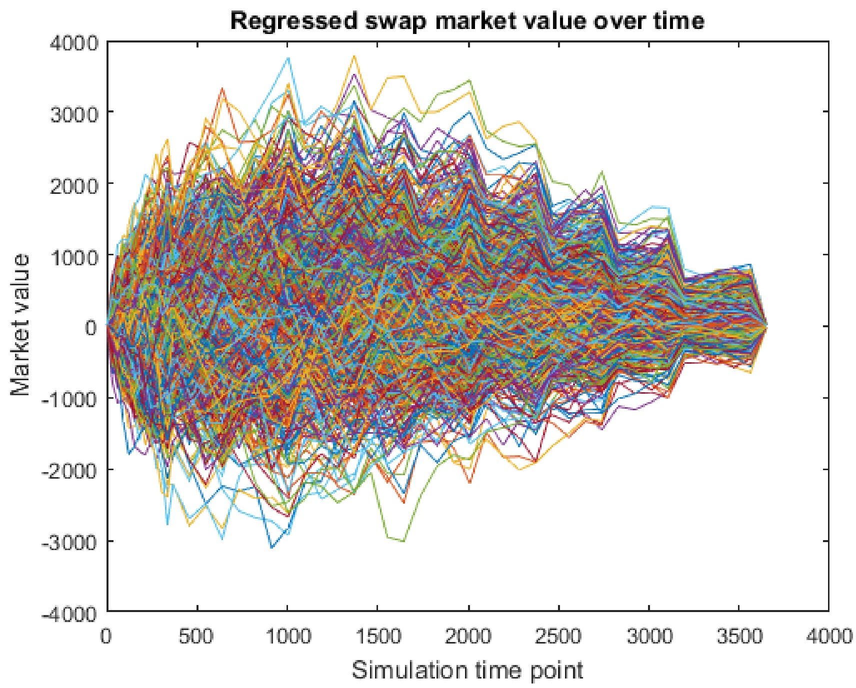

Figure 5.6: Estimated swap market values from $t = 0$ to $t = 10$(years) in 1000 scenarios. Simulated market values are explained with $Y = a + bx$,

shown against the simulated values in Figure 5.7. The EE profile for the swap is shown in Figure 5.8.

The spot capital charge for all simulation dates is shown in 5.9. The numerical integration techniques discussed Chapter 4 give $KVA \approx -17$ units or -17bp of the swap's notional. This means that in order to cover the capital costs resulting from entering into the swap, the bank must offer the counterparty such a fixed rate, that right after inception the market value is $+17$ - and this is only for the default risk capital charge. The result is of similar magnitude to the results in Green et al. [2014]. The 10-year credit default swap (CDS) spreads for sovereigns from January 2006 to September 2008 were between 19.70bp and 73.20bp [European Central Bank, 2010]. Typical CDS spreads for investment grade companies in August 2017 ranged from 80.01bp (industrials) to 211.15bp (high volatility) [Markit, 2017]. By comparing the magnitude of KVA to the typical credit-based premia, is it clear that the impact having to hold regulatory capital should be considered in OTC derivatives pricing.

According to empirical experiments, the results are sensitive to the selection of the volatility parameter σ in the Hull-White model. For example, using $\sigma = 0.15$ gives $KVA \approx -143$. However, the realizations for even the shortest rate reached levels above 20%, which does not seem reasonable given the development of Libor rates during the past 30 years [Macrotrends, 2017]. The spot capital profile with $\sigma = 0.15$ is shown in Figure 5.10.

Due to the selection of the American Monte Carlo algorithm, the computation time for the selected settings was approximately 5 seconds with a normal PC, which makes it very likely that the computational performance of the model is sufficient for pricing purposes.

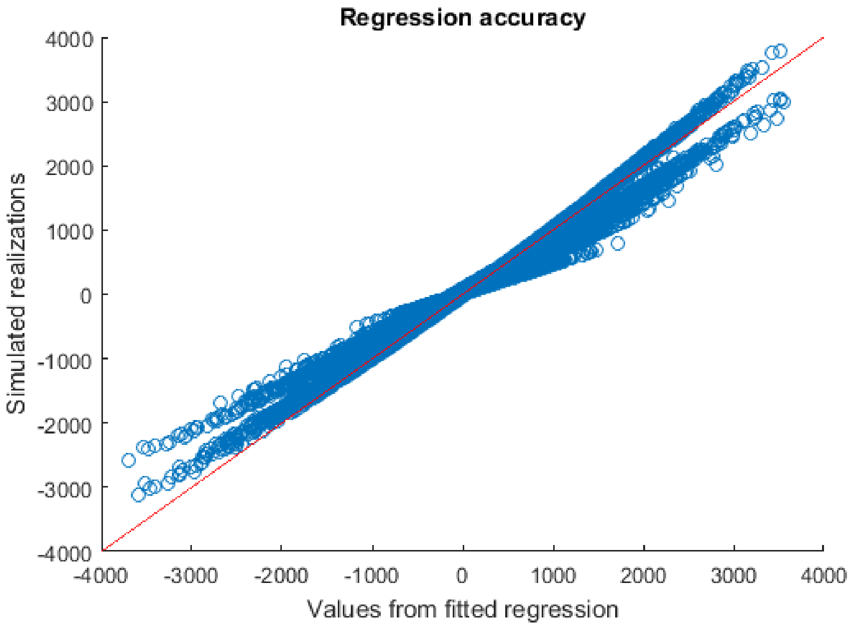

Figure 5.7: Fitted regression of swap market values

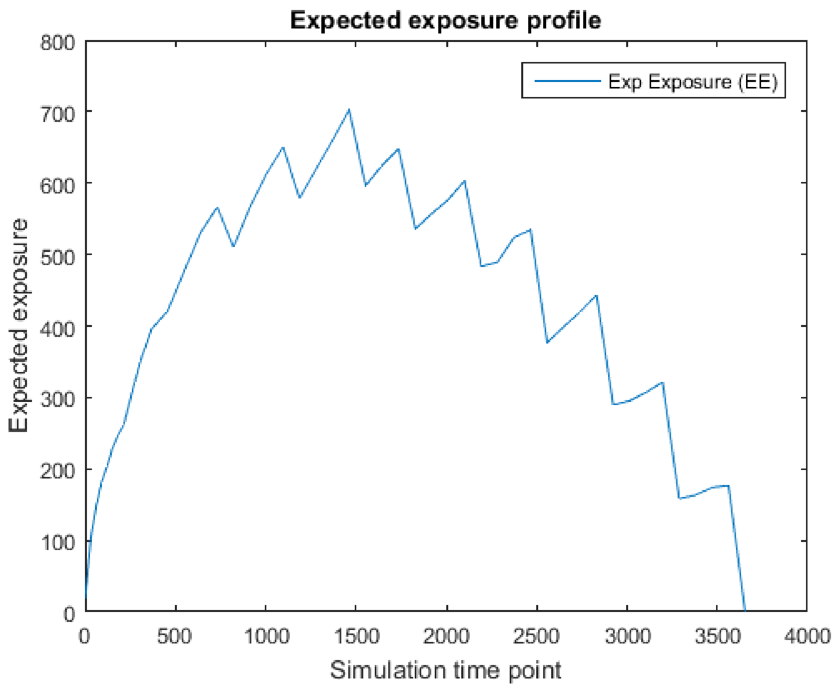

Figure 5.8: Expected exposure for the 10 year lifetime of the trade

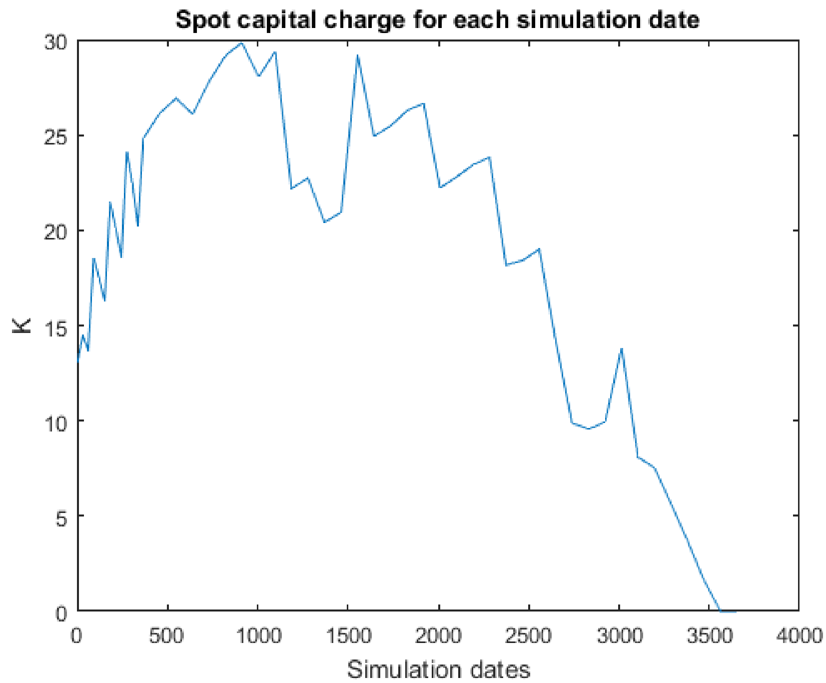

Figure 5.9: Spot capital charge profile when the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$.

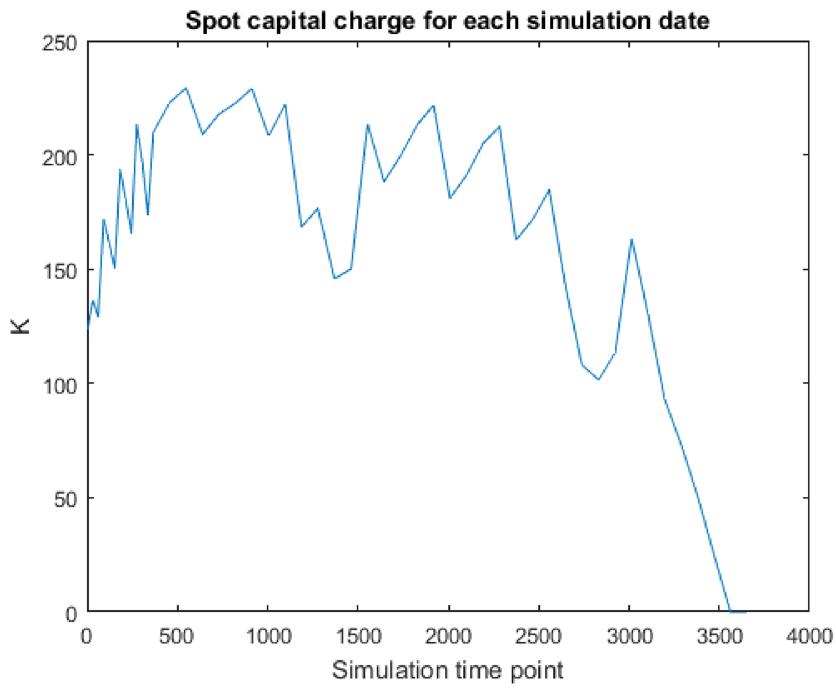

Figure 5.10: Spot capital charge profile when the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.15$.

5.3 Capital valuation adjustment for an in-the-money interest rate swap

Next, we show how the KVA could look like after the inception of a derivative, if the contract is in-the-money, i.e., the market value from the bank's point of view is positive. We assume similar conditions to Chapter 5.2, except that the interest rates have increased. The yield curve at base date is shown in Figure 5.11. Intuitively, the expected exposure profile in Figure 5.12 starts above zero, thus increasing EAD. The spot capital charge is show in Figure 5.13.

Table 5.2: Details of the in-the-money interest rate swap

Swap	Notional	Tenor (years)	Pay leg rate	Receive leg rate
1	10000	10	Fixed 3.95%	Floating

This setting results in $KVA \approx -20$, i.e., approximately an increase on 18% compared to the at-the-money swap. This shows that regular estimation of KVA for existing derivatives allows banks to understand their capital costs better and re-consider their pricing policy.

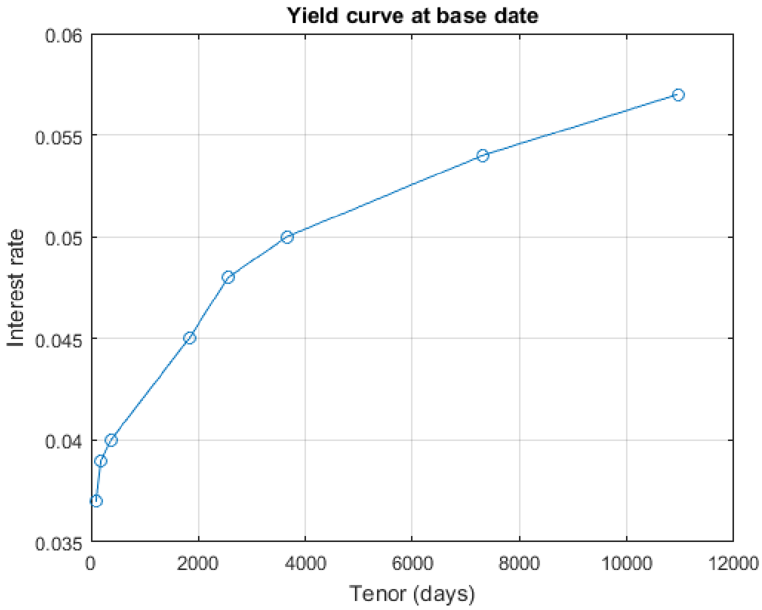

Figure 5.11: Yield curve at base date

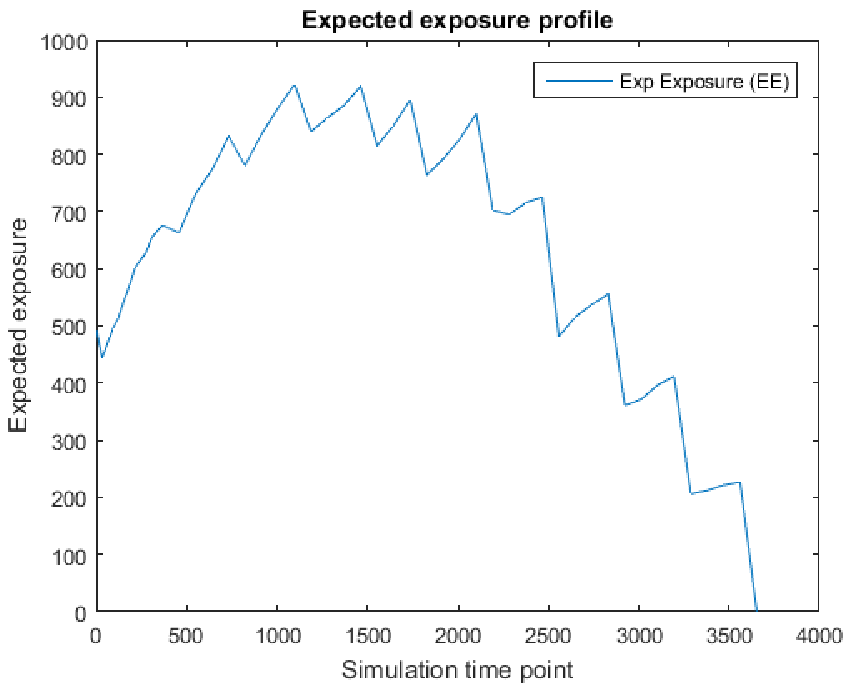

Figure 5.12: Expected exposure profile for the in-the-money swap when the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.015$.

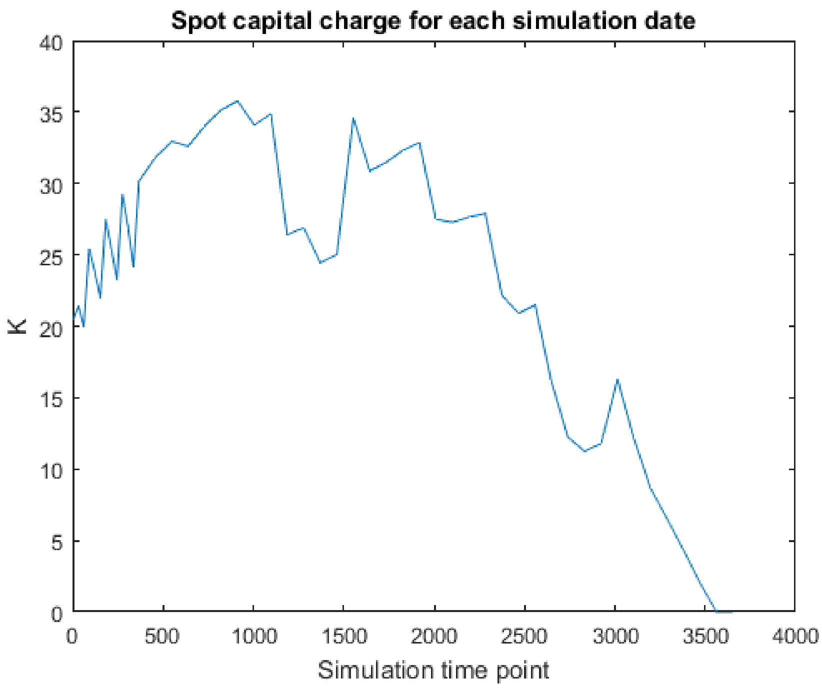

Figure 5.13: Spot capital charge profile for the in-the-money swap when the Hull-White short rate model is calibrated with $a = 0.2$ and $\sigma = 0.15$.

Chapter 6

Conclusions

The purpose of this study was to develop a method which can be used by a bank to incorporate cost of having to hold regulatory capital into over-the-counter (OTC) derivatives pricing. This capital valuation adjustment is known as KVA. We assumed that the bank has an approval for internal model method (IMM), and limited this study to regulatory default risk capital charge, which is calculated using simulation-based techniques.

First, we introduced the risks related to OTC derivatives and what mitigating actions banks may take to reduce them. Next, we introduced the key counterparty credit risk measures, i.e., current exposure (CE), probability of default (PD), loss given default (LGD), exposure at default (EAD), risk weight (RW) and risk weighted assets (RWA) and how they are translated into a regulatory capital requirement. Finally, we summarized the most common and important valuation adjustments to account for both parties' credit qualities, funding and capital in OTC derivatives pricing. We concluded that the classic Black-Scholes price for a derivative is problematic. This makes it increasingly important to incorporate credit valuation adjustment (CVA), debt valuation adjustment (DVA), funding valuation adjustment (FVA) and capital valuation adjustment (KVA) into OTC derivatives pricing in the environment of tightening regulation.

Then, we formulated a model that uses stochastic scenario generation to produce expected exposure (EE) curve for a portfolio of OTC derivatives and makes it possible to calculate exposure-at-default (EAD) for the portfolio. Together with regulatory risk weights and capital ratios, this gives the capital charge for default risk. Also, we briefly discussed other capital charges banks have to consider.

By extending the methods of Burgard and Kjaer [2013] and Green et al. [2014], we constructed a replicating portfolio for an OTC derivative which made it possible to derive an expression for KVA under the IMM methodology for regulatory default risk capital charge. While the problem could have been solved with brute force, to improve computational performance, we used American Monte Carlo techniques to calculate KVA. The selection of this method was critical in preserving the possibility to use the KVA model for pricing purposes.

Finally, we determined KVA for illustrative interest rate swaps. The results show that neglecting the cost of capital - even for only default risk - can lead to mispricing. In addition, a bank which considers other valuation adjustments, such as CVA, DVA, FVA and KVA for other capital charges, as well as operative costs, may have to charge a significant premium for a counterparty to make an OTC derivative profitable. Due to the careful selection of the simulation algorithm, the model's computational performance was sufficient for pricing.

Topics for further research include the modeling of KVA for other capital charges, primarily market risk charge and CVA risk charge. Also, the optimal choice of basis functions in the regression is left for further study. Possible basis functions for non-linear products include Hermite, Laguerre and Jacobi polynomials.

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