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Capabilities, structural change and climate policy

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In order to hold the global warming below 2 degrees Celsius, also developing countries are required to urgently limit their greenhouse gas emissions. However, developing countries tend to fear that climate policy measures could harm their development.

Assessing whether or not such worries are justified requires deep understanding of the drivers of economic development. On the one hand, it is currently widely accepted that societal capabilities, including aspects like institutions, various infrastructure, human capital and social capital, are the determinants of long-term economic performance. On the other hand, development is known to be accompanied by structural change. Recent empirical evidence provides support to the hypothesis that capabilities and structural change are fundamentally linked. It seems that the expansion and the diversification of production contribute to capabilities which then enable further diversification into productive activities that require these capabilities.

This thesis seeks to explain how societal capabilities, structural change and development interact, and what this interaction implies for climate policy. A multi-sector endogenous growth model driven by the interplay of capabilities and structural change is proposed. The model implies that if climate policy interferes with structural change by harming the operation of some sectors, economic growth may temporarily slow down or even permanently stagnate. Hence, climate policy can threat development, whereby poorer countries are more severely affected. To avoid adverse effects, this theory suggests a focus on capabilities in policy design.

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Jotta ilmaston lämpeneminen voitaisiin rajoittaa kahteen asteeseen, myös kehittyvien maiden on pikaisesti rajoitettava kasvihuonekaasupäästöjään. Useat kehittyvät maat kuitenkin pelkäävät, että ilmastopolitiikka voi vahingoittaa niiden taloudellista kehitystä.

Jotta voitaisiin arvioida, onko huoli perusteltu, on ymmärrettävä taloudellisen kehityksen ajureita. On yleisesti tunnustettua, että yhteiskunnalliset kyvykkyydet, sisältäen muun muassa instituutiot, erilaisen infrastruktuurin sekä inhimillisen ja sosiaalisen pääoman, määrittävät talouden kehityksen pitkällä aikavälillä. Toisaalta talouden rakennemuutoksen tiedetään lähes poikkeuksetta esiintyvän talouden kehityksen yhteydessä. Viimeisimmät empiiriset tutkimustulokset tukevat hypoteesia, jonka mukaan kyvykkyydet ja rakennemuutos liittyvät perustavanlaatuisesti toisiinsa. Vaikuttaa siltä, että tuotannon laajentaminen ja monipuolistaminen kehittävät kyvykkyyksiä. Uudet kyvykkyydet taas mahdollistavat uudenlaiset tuotantoaktiviteetit, joille juuri nämä kyvykkyydet ovat toimintaedellytys.

Tämä diplomityö pyrkii selittämään yhteiskunnallisten kyvykkyyksien, rakennemuutoksen ja taloudellisen kehityksen vuorovaikutusta ja sen seurauksia ilmastopolitiikan kannalta. Työssä esitetään monisektorinen endogeeninen talouskasvumalli, jossa kyvykkyyksien ja rakennemuutoksen vuorovaikutus on kasvun ajuri. Mallin valossa talouskasvu saattaa väliaikaisesti hidastua tai jopa pysyvästi lamaantua, jos ilmastopolitiikka puuttuu rakennemuutokseen häiritsemällä joidenkin sektorien toimintaa. Täten ilmastopolitiikka voi uhata kehitystä. Vaikutukset vähemmän kehittyneisiin maihin ovat suurempia. Jotta vahingolliset seuraukset voidaan välttää, tämä teoria kehottaa huomioimaan kyvykkyydet politiikan suunnittelussa.

Asiasanat:	Kyvykkyydet, rakennemuutos, ilmastopolitiikka, kehitysmaat, monisektorinen endogeeninen talouskasvumalli
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Chapter 1

Introduction

Climate change is undoubtedly the greatest threat mankind currently faces (World Economic Forum, 2016). In December 2015 195 countries negotiated in Paris in the 21st Conference of the Parties of the UNFCCC (COP21), where they agreed to hold the average temperature increase “well below” two degrees Celsius in comparison to the pre-industrial levels. To achieve this, ambitious policy measures to reduce greenhouse gas emissions will be necessary. Many, first and foremost the developing countries, have concerns about the economic effects of climate policy measures. Indian Prime minister Narendra Modi said famously at the opening of the COP21 that “climate justice demands that with the little carbon space we still have, developing countries should have enough room to grow”. But exactly how would climate policy hamper economic development?

To answer, we must first understand development. It is currently a widely accepted hypothesis that fundamental *capabilities*, including aspects like institutions, various infrastructure, human capital and social capital, set the framework for growth and determine the long-run performance of economies. These capabilities go under different names in the literature, such as “social overhead capital” (Hirschman, 1958), “social capabilities” (Abramovitz, 1986), “technological capabilities” (Lall, 1992) and “social infrastructure” (Hall and Jones, 1999). Many studies have found that capabilities limit the possibilities of developing countries to adapt advanced technology developed elsewhere (Borensztein et al., 1998; Murphy, 2001; Gallagher, 2006).

Another central insight in development economics is that development entails structural change, i.e. the reallocation of economic activity from agriculture to manufacturing and later services. Historically, industrialization has been the key enabler of rapid and sustained economic growth rates and has permitted a number of countries, such as Japan, South Korea and Taiwan, to catch up with the Western nations. Rodrik (2016) describes three features that make manufacturing activities instrumental in the growth process: the unconditional labor productivity convergence that manufacturing sectors exhibit, the capability to absorb significant quantities of relatively unskilled labor, and the tradable nature which sets manufacturing free of the demand constraints of the domestic market.

However, not all countries manage to move significant fractions of labor to manufacturing and industrialize their economies. Structural change seems to be vulnerable to outside influences: McMillan et al. (2014) documented that since 1990 labor has moved to lower-productivity activities in Latin America and Africa (although things seem to have turned around for Africa after 2000). Rodrik (2016) found a significant premature deindustrialization trend also outside the developed economies. They suspect that this is a result of globalization and exposure to foreign competition.

The understanding of structural change has greatly improved as a result of recent empirical findings. Hausmann and Klinger (2006) studied the network relatedness of exported products using international trade data to build a network they named the "product space", where nodes are different products and links are pairwise probabilities that a country that effectively exports one also effectively exports the other. This probability is also taken to be a measure of "proximity", which is assumed to signal how similar the capabilities needed to produce these products are. Hausmann and Klinger (2006) showed that the product space is not smooth but heterogeneous, some areas being dense and others sparse. They found that the speed of structural transformation depends on the density of the product space near the products which the country currently exports. Hidalgo et al. (2007) found that the product space has a core-periphery structure, with most upscale products located to the densely connected core while lower income products occupy a less connected periphery. They also concluded that economies grow by upgrading the products they export, and that countries tend to move to products close to what they are currently producing. This suggests that the capabilities needed to make upgraded products are more easily adapted from some products than others.

Radebach et al. (2016) studied structural change on a disaggregated sectoral level with production data from 57 sectors. Their results replicate the broad pattern of aggregate structural change, but in addition they identified robust patterns on a disaggregated level. They found what they call "bridging sectors", classified as light manufacturing, which seem to act as bottlenecks in the transition from agriculture to an industrialized economy. A possible explanation is, as Radebach et al. (2016) hypothesize, that while building the production in these sectors, the economy develops also various capabilities, which are necessary to upgrade production.

It seems probable that capabilities and structural change are fundamentally connected: production in certain sectors contributes to capabilities, which then enable the production in some other sectors. This raises a question: what if climate policy interferes with structural change? If it hinders production in the vital bridging sectors, the affected economy may not be able to build the capabilities it needs to industrialize. It is not difficult to think of ways how climate policy could impede certain sectors; emission taxes could harm the competitiveness of a sector against similar goods produced somewhere else with cleaner technology, or performance standards could raise the minimum level of capabilities that are required to operate the bridging sectors themselves.

The economic literature provides little help in studying this issue. Analyses of structural change tend to be based on the aggregated sectors of agriculture, manufacturing and services. The shifts of activity are explained either through differing rates of total factor productivity growth between these sectors, i.e. faster advancements of technology in manufacturing and services compared to agriculture, or through non-homothetic preferences of the consumers, i.e. the desire to consume relatively more manufactured goods and later services as income grows (Herrendorf et al., 2014). Generally the technology advancements are taken as given and not modeled endogenously. To consider the effect of climate policy on this process, a different kind of a theory is needed.

This leads us to the research questions of this thesis: How can the interaction between societal capabilities, structural change and economic development be explained? What does it imply for climate policy?

The objectives of this thesis are to to construct a theory of economic growth that is consistent with the literature on capabilities and the main observed features of economic development as well as the newest empirical findings regarding structural change, and to use the attained framework to provide insights on the possible implications of the introduction of climate policy to developing economies.

This thesis presents a multi-sector endogenous growth model that enables the analysis of economic growth and structural change on a more granular level than the traditional three aggregated sectors, on which the literature mostly focuses. The model incorporates capabilities as a key driver of structural change and therefore economic development. The central hypothesis is that capabilities can be acquired by building up production in a string of sectors in a given order. The acquisition of additional capabilities enables production in more complex sectors and productivity growth in sectors that already operate.

The main result of this thesis is that given our assumptions, climate policy can pose a threat to development by temporarily slowing down growth or even by driving economies to stagnation, and that poorer countries are especially in danger. To our knowledge, this is the first effort to theoretically show that climate policy can have an effect on long term development. To avoid adverse effects, this theory suggests a focus on capabilities in policy design.

This thesis is structured as follows: Chapter 2 introduces the relevant literature on developing countries and climate policy as well as capabilities and structural change. Chapter 3 presents the growth model and its implications to climate policy. Chapter 4 discusses the model and its insights, and Chapter 5 concludes.

Chapter 2

Literature review

This chapter introduces the relevant past research and serves as a basis for the model developed in the following chapter. The first section establishes the relevance of climate policy to developing countries and introduces the risks associated with it in the past literature. The second section overviews the different suggested concepts of capabilities, their empirically found relevance to growth and how capabilities are thought to accumulate. The third section elaborates on the role of structural change in development and on the interaction between structural change and capabilities. The fourth section presents how capabilities and structural change have been modeled previously.

2.1 Developing countries and climate policy

The anthropogenic emissions of greenhouse gases (GHGs), such as carbon dioxide (CO_2), increase the concentration of GHGs in the atmosphere and therefore raise surface temperatures and acidify oceans. According to the fifth assessment report of the Intergovernmental Panel on Climate Change (IPCC), rising surface temperatures are expected to cause increasing extreme weather phenomena, such as more intense and more frequent heat waves and extreme precipitation events, as well as warming oceans, rising sea levels and changes in the precipitation patterns. In many mid-latitude dry regions precipitation will likely decrease and in many mid-latitude wet regions precipitation will likely increase. Climate change is expected to undermine food security, decrease the amount of renewable surface water and groundwater in most subtropical dry regions and lead to increases in ill-health in many regions, mostly in developing countries. Developing countries are generally recognized to be especially vulnerable, both because of their location in the most impacted areas near the equator and their lower ability to adapt to the changes (IPCC, 2014).

IPCC (2014) estimates that to limit the global warming likely under 2 degrees Celsius, the GHG emissions reductions need to reach 40 to 70% by 2050 compared to 2010 and by 2100 the emissions need to be near zero or even negative, depending on the scenario. There are little signs of movements in this direction. On the global scale, GHG emissions in CO_2 equivalents have been accelerating: between 1970 and 2000 their growth averaged 1.3% per year, whereas between 2000 and 2010 the growth was already 2.2% per year (IPCC, 2014).

To better understand the rise of CO_2 emissions, we can look at the Kaya identity that decomposes four drivers of emissions: population growth, economic growth, the energy intensity of the economy and the carbon intensity of energy: $CO_2 = P \text{ x } \frac{GDP}{P} \text{ x } \frac{E}{GDP} \text{ x } \frac{CO_2}{E}$, where P stands for population, GDP for gross domestic product and E for energy (Kaya et al., 1990). Raupach et al. (2007) found that currently the economic growth in developing countries is the main driver for increasing the global CO_2 emissions. In 2004, developing and the least developed countries accounted for 73% of the global emissions growth. (However, they also accounted for 80% of the population and only 41% of the global emissions and 23% of the cumulative emissions). Some of the emissions growth is driven by the rising carbon intensity of energy in poor, fast-growing countries that use coal to satisfy their energy demand (Steckel et al., 2015). Jakob et al. (2012) found that developing countries that economically converge to the world average are also converging to the energy use patterns of developed countries, whereas the economic growth in high income countries is partially decoupling from energy use, but clearly at an unsustainable level.

Considering the growing role of developing countries in the production of emissions, it is obvious that they also are required to reduce their emissions. The possibilities for emissions control policies are broad, including both incentive based policies and direct regulatory instruments. Goulder and Parry (2008) listed incentive based policies to include emissions taxes, subsidies to emissions abatement, taxes on goods associated with emissions and auctioned or freely allocated tradable emissions allowances, or a hybrid of these. Direct regulatory instruments include mandated abatement technologies and performance standards. In an ideal world where the policy makers would know everything and there would be no uncertainties, all of the instruments would yield the same emission reductions with the same costs, which are in the end borne by the consumers that purchase emissions-intensive goods or services. However, in reality the best policy choice depends on various things. Goulder and Parry (2008) named economic efficiency, cost-effectiveness, the distribution of benefits and costs, the ability to address uncertainties and political feasibility as evaluation criteria.

Even though studies have identified significant synergies with climate policy measures, such as improved energy security and decreased air pollution and related illnesses (McCollum et al., 2013), it is clear that in developing countries climate policy will incur costs relative to the business-as-usual. This could tie funds that could otherwise be used for poverty eradication. As it also recognized that developing countries have contributed little to the historical emissions that have caused climate change, the United Nations Framework Convention on Climate Change has agreed that developed countries will cover "the agreed full costs incurred by developing country Parties in complying with their obligations" (United Nations, 1992).

However, despite their greater vulnerability to climate damages and the promised compensations, in climate negotiations developing countries often oppose binding emission reduction targets and strongly advocate for their rights to continue the use of fossil fuels. Illustratively the then Indian environmental minister Jairam Ramesh declared in an interview that India, China, South Africa and Brazil had "protected their right to continued economic growth" by torpedoing the attempts to impose binding targets of global emissions reductions in COP15 (ABC News, 2009).

It is undisputed that economic growth needs to continue for developing countries to reach higher development levels (Rodrik, 2014) and to be able to absorb the climate stress that also a 2 degree warming brings (Bowen et al., 2012).

Goldemberg et al. (1998) argue that continued economic growth in developing countries is possible in compliance with the emission targets if the developing economies “leapfrog” the emission-intensive development steps that the current industrialized countries have taken and instead incorporate modern efficient technologies early in the development process. They provide examples, such as the use of solar panels and efficient light bulbs to provide lighting in rural areas instead of kerosene lanterns, thus leapfrogging over the phase of building an electrical grid.

In fact, leapfrogging is exactly what the developing countries are expected to do. Steckel et al. (2013) assessed the data from different integrated assessment models used by the IPCC to produce mitigation scenarios for its assessment reports and found that developing countries are projected to undertake substantial energy efficiency and carbon intensity improvements compared to the business-as-usual scenario, while continuing to grow. Particularly in ambitious mitigation scenarios, developing countries are projected to not significantly increase their energy use from current levels, implying radical decoupling of energy use and economic growth already at low levels of development.

However, several scholars have found that the lack of various *capabilities* limits the possibilities for leapfrogging. Steckel et al. (2013) argue for the existence of minimum energy thresholds that need to be fulfilled in order to enable the economy to develop past a certain level. Murphy (2001) examined three technologies targeted at rural households in East Africa (conventional grid expansion, renewable energy technologies supplying electricity, and improved cookstoves) and concluded that “technological capabilities” with technical, institutional and organizational components limit the ability of the people to switch into using or supplying these technologies. Gallagher (2006) found that the introduction of US automotive technology through Sino-US joint ventures in passenger cars failed to induce leapfrogging in the Chinese automobile industry. He emphasizes that often developing countries do not have the technological capabilities to produce or integrate advanced energy technologies themselves. On a more general level, Borensztein et al. (1998) discovered that foreign direct investment only contributes to economic growth when a sufficient “absorptive capability of advanced technologies”, which they associate with human capital, is available in the host economy.

Marcotullio and Schulz (2007) in fact found evidence of some leapfrogging: the developing world experiences energy-related transitions faster and at lower levels of income and therefore has a lower systematic environmental impact per capita than the United States. However, they also found that these transitions are limited by inadequate financial revenues to provide for the simultaneous changes in infrastructure, social and environmental needs, i.e. the necessary capabilities.

Moreover, climate policy has also been associated with possible adverse effects on capability building. Jakob et al. (2015) claim that the possibly sizeable inflows of climate finance could create a "climate finance curse" (as oppose to a resource curse, see e.g. Van der Ploeg (2011)). If the climate finance scheme would take the form of international emissions trading where the developing countries would be allocated more emission permits than they need themselves, the fluctuation of the emission permit price could cause increasing volatility in the economy. Moreover, the inflow of finance could result in a "Dutch disease", where the currency of the recipient country appreciates and causes the export sectors to contract. On the domestic level climate finance transfers through the government's budget could promote "rent-seeking", i.e. private agents seeking to get disproportionate amounts of funding in relation to their mitigation efforts. If government officials are entrusted with selecting the funded projects, this could lead to increasing corruption. However, Jakob et al. (2015) argue that most serious problems could be prevented by appropriately designed policies.

Also domestic climate policy poses risks to capabilities. Climate policy is likely to result in rising energy prices, as the lifetime cost of energy for many renewable energy technologies is still higher than current energy prices, and for instance a carbon tax would raise the prices of fossil fuels (Edenhofer et al., 2011). Jakob and Steckel (2014) argue that this could undermine the energy access of the poor and discourage investments in energy intensive capital goods, such as infrastructure. This change in investment patterns could delay structural change, particularly industrialization. Historically, industrialization has enabled countries to reach high living standards (Rodrik, 2014).

The next two sections will dig deeper into the concepts, roles and drivers of capabilities and structural change, and the interaction between them.

2.2 Capabilities

2.2.1 Concept

The concept of capabilities is best known as an approach to welfare and social justice, developed by Amartya Sen. He first published his theory in 1979 (Sen, 1979) and has since elaborated it in many publications. Sen (1989) explains that individuals' capabilities to function, i.e. capabilities to convert the same resources into valuable "functionings" can differ greatly, and therefore an evaluation of well-being focusing only on means without considering what particular people can do with them is insufficient. Sen defined functionings as states of "being and doing", such as being well-nourished, having shelter. They should be distinguished from the commodities employed to achieve them - as bicycling is distinguishable from possessing a bike. Sen understood poverty as a deprivation of the capability to live a good life and development as capability expansion. He refused to provide a list of necessary capabilities and emphasized freedom.

Sen's capability approach has a counterpart in development economics to explain the shortcomings of the neo-classical growth theory, which identified technological development as the source of sustained economic growth (Solow, 1956). The neo-classical theory considered technology to be a kind of a public good: non-rival and non-excludable, something that everyone could utilize with no incurred costs. Hence, the theory implied a convergence hypothesis: at some stage, all countries should converge to the same income level, known also as unconditional convergence. However, empirical research could not find evidence of it. Instead, so-called conditional convergence is well supported in the data. Regions within countries and similar developed countries do seem to converge (Barro and Sala-i-Martin, 2004). Analogously to Sen's approach for individuals, it seems that countries greatly differ in their abilities to utilize technology. Currently a widely recognized hypothesis is that fundamental capabilities, such as institutional quality, macroeconomic stability and human capital set the framework for growth, and ultimately long-term growth depends on the accumulation of these capabilities (Rodrik, 2014).

But as Abramowitz (1994) points out, capabilities are a poorly defined and a vague subject of matters, few of which can be clearly defined or measured. Numerous scholars have suggested similar and variably overlapping concepts under the same of different names. The following provides a brief overview of some of them.

Hirschman (1958) introduced the concept of "social overhead capital", which he defined to comprise of basic services without which primary, secondary and tertiary productive activities cannot function. These include, for instance, law and order, education, public health, transportation, communications, power and water supply as well as agricultural overhead capital, such as irrigation and drainage systems. He named three conditions for inclusion: first, the service needs to facilitate or be basic to carrying on of a great variety of economic activities. Second, the services should be provided by public agencies or by private agencies subject to some public control. Third, these services cannot be imported. Hirschman theorizes that investment in social overhead capital both permits and invites directly productive activities to come in.

Rosovsky et al. (1973) first introduced the term "social capability" to describe factors contributing to country's ability to import or engage in technological and organizational progress. They argue that a low level of social capabilities could limit the possibilities to introduce advanced foreign technology. Later Abramovitz (1986) adopted the term and defined it as technical competence, along with political, commercial, financial and industrial institutions. By these he explained to mean the stability and the effectiveness of the government, as well as the experience with the organization and management of a large scale enterprise, and with financial institutions and markets capable of mobilizing capital for individual firms. In a later work he expanded the definition to include social attitudes and issues with incentives and opportunities (Abramovitz, 1989).

A Korean development scholar Kim (1980) used the term "technological capabilities" to describe features that firms needed in order to be competitive. He later defined technological capabilities to include production capabilities, investment capabilities and innovation capabilities, and also used the term on an aggregate economy-wide level in addition to the original firm-level (Kim, 1997).

Lall (1992) defined technological capabilities a little differently. He sees that firm-level technological capabilities, including production capabilities, investment capabilities and linkage capabilities (with other local actors), form the base for national technological capabilities, which can be characterized as the common element of response of firms to the policy, market and institutional framework. Lall groups national technological capabilities into human capital, physical investment and technological effort. All of these are needed, as economic growth rises from the interplay of all these different capabilities and incentives within an institutional framework.

North (1990) argues that institutions are the underlying determinant of the long run performance of economics. He characterizes institutions as constructs of human mind, which cannot be measured. Institutions exist due to uncertainties involved in human interaction, and are essentially constraints to structure interaction. North stresses that institutions determine the kind of economic activities that will be profitable and viable, and shape the adaptive efficiency of the internal structures of firms and other organizations by, for instance, regulating entry to market, governance structures and the flexibility of organizations. North also believes that institutions exhibit increasing returns, i.e. that it will always be economically profitable to improve institutions.

Cohen and Levinthal (1990) introduced the term "absorptive capacity" to describe the ability of a firm to recognize the value of new external information, assimilate it, and apply it to commercial ends. They argue that absorptive capacity is crucial for the firm's innovative capabilities. They did not take the concept to national level.

"Social capital" also describes an important form of capabilities. There is a large literature and differing views on this concept alone, but many are willing to accept the approach employed by Woolcock (1998). He defined social capital as the "nature and extent of a community's personal and institutional relationships". High levels of social capital nurture a sustainable, equitable and participatory economic development. Rather than defining conditions which increase social capital, Woolcock proposes a set of conditions which undermine it. These include, for instance, widespread inequalities of any kind, endemic poverty, weak or unjust laws, lack of democracy and situations which undermine the basic sense of order and predictability, such as war.

Hall and Jones (1999) introduced a new term, "social infrastructure", which they used to describe institutions and government policies, which determine the economic environment within which individuals accumulate skills, and firms accumulate capital and produce output. They name social institutions which protect the output of individual productive units from diversion (such as thievery or mafia protection) an integral part of good social infrastructure. Overall a good social infrastructure should provide an environment that supports productive activities and encourages capital accumulation, skill acquisition, invention and technology transfer.

Furman et al. (2002) suggested the term "national innovative capacity" to describe the ability of a country to produce and commercialize a flow of innovative technology over the long term. They claim that national innovative capacity is determined by the strength of a country's common innovation infrastructure, the environment for innovation in a country's industrial clusters, and the strength of linkages between these two. The innovation infrastructure includes things like the country's overall science and technology policy environment, the mechanisms in place for supporting basic research and higher education, and the cumulative stock of technological knowledge upon which new ideas are developed and commercialized.

Acemoglu and Robinson (2010) suggest that the main determinant of differences in prosperity across countries are differences in "economic institutions", which are collective choices and an outcome of a political process. Robinson and Acemoglu (2012) claim that poverty traps are a result of self-enforcing low quality institutions.

Based on the preceding literature, Fagerberg and Srholec (2008) attempted to create an integrated framework of the various concepts of capabilities. This framework is presented in Figure 2.1. All of the overviewed concepts are not included with their original names, but due to overlapping formulations the framework manages to catch the essence of most of them. Social capital and social infrastructure can be thought to be included in broadly defined institutions, Hirschman's social overhead capital is spread in institutions, social capabilities and technological capabilities. National innovative capacity is included in the technological capabilities.

It is noteworthy that literature uses these terms often broadly and inter-

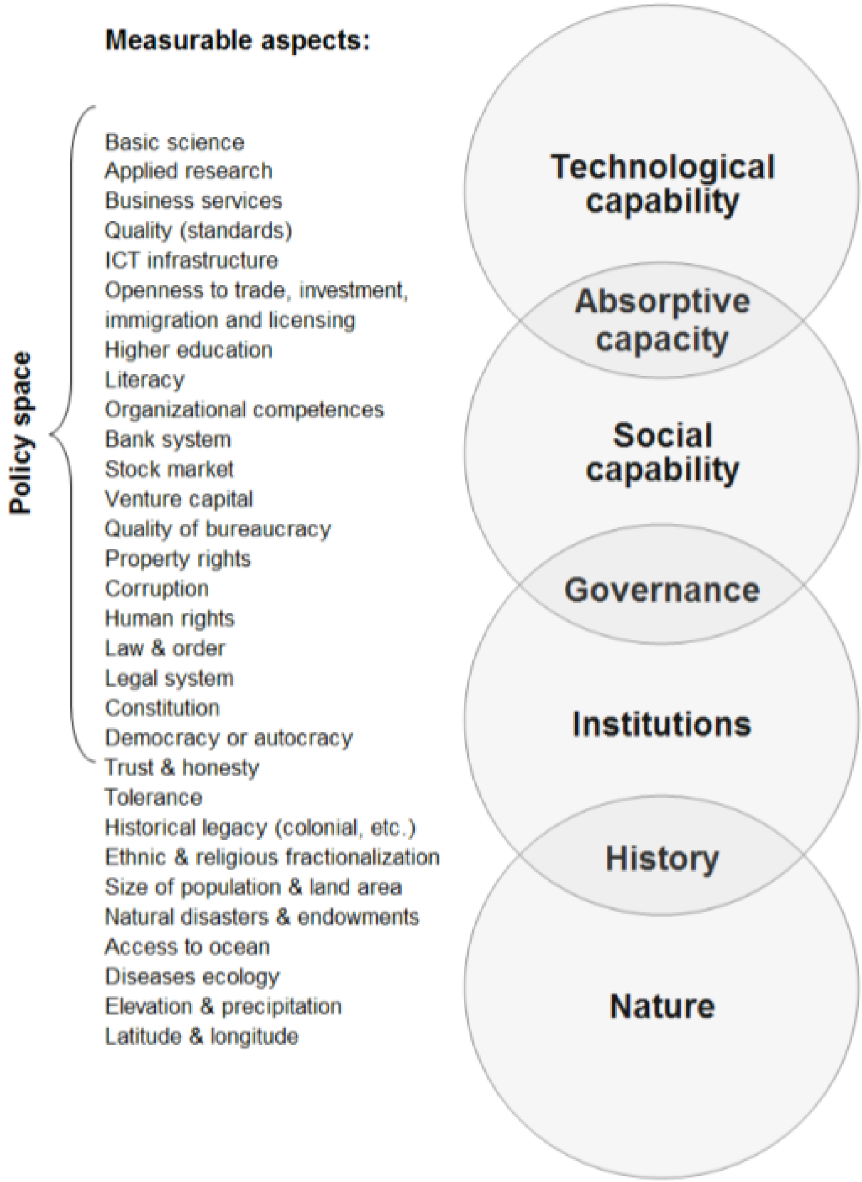

Figure 2.1: An integrated framework of capabilities by Fagerberg and Srholec (2008).

changeably. To distinguish from the above-given definitions, we shall adopt yet another new term, *societal capabilities*, to cover all of the introduced aspects.

2.2.2 Empirical results on capabilities and growth

The abstract nature of many aspects of capabilities pose challenges for measuring, but there is a large empirical literature that shows societal capabilities to be important for growth.

As Figure 2.1 presents, Fagerberg and Srholec (2008) offer also a selection of aspects of capabilities that can be measured and used as indicators. They did a a factor analysis on a number of indicators for 115 countries and identified four dimensions, for which the indicators within the dimension are strongly correlated. Based on the indicators that are included in these synthetic dimensions, they called them the innovation system, the quality of governance, the character of the political system and the degree of openness of the economy. Innovation systems and good governance were shown to be closely correlated with economic growth, while political system and openness had little effect. Later Fagerberg and Srholec (2016) repeated the exercise for regional level in Europe and showed that the close connection between capability building and economic performance also holds for regions. This time technological capabilities, education, access to ICT and good governance emerged as dimensions from the factor analysis.

Also institutions and social capital have been shown to correlate with economic growth (Easterly et al., 2006; Guiso et al., 2004). Formal institutions and social capital have in fact been shown to substitute each other to a certain extent: Easterly et al. (2006) found that good institutions are most necessary and beneficial where there are ethnolinguistic divisions and social capital is in shorter supply, and Guiso et al. (2004) showed that social capital matters more for financial development when education levels are low and law enforcement is weak. However, formal institutions and social capital are also complements and reinforce each other: Knack and Keefer (1997) discovered that trust and civic norms, used as a proxy for social capital, are stronger in nations with higher and more equal incomes, with good institutions, and with better-educated and ethnically homogeneous populations. Easterly et al. (2006) found that income equality and ethnic homogeneity endogenously determine institutional quality, which in turn determines economic growth.

Dawson (1998) studied two different mechanisms, through which capabilities could have an effect: a direct effect on total factor productivity and an indirect effect through increasing investment. They showed that economic freedom enhances growth through both channels. However, political and civil liberties seemed to have no direct effect on growth and there was mixed evidence that they may affect investment. Hall and Jones (1999) documented that the cross-country differences in capital accumulation, productivity, and therefore output per worker can be explained by differences in institutions and government policies.

"Physical" capabilities have been shown to be of importance as well. Demurger (2001) found that the geographical location and infrastructure endowment account significantly for observed differences in growth performance across Chinese provinces. Lipscomb et al. (2013) documented large positive effects of electrification on development, and concluded that broad-based improvement in labor productivity across sectors and regions appeared to be the likely mechanism by which the development gains were realized. Ozturk (2010) found that electricity seems to be a limiting factor to economic growth and, hence, shocks to energy supply can be expected have a negative impact on economic growth.

2.2.3 Accumulation of capabilities

The literature strongly agrees that capabilities are endogenously determined and therefore path-dependent.

North (1990) argues that institutions determine the structure of an economy, which fosters the accumulation of certain kills and knowledge, which will shape the direction of change and gradually alter the institutional framework. Acemoglu and Robinson (2010) see that the economic institutions of a society depend on the nature of political institutions and the distribution of political power in the society, which in turn are dependent on economic outcomes. Because the politically powerful have incentives to support such political institutions and through them such economic institutions that keep them in power, reforming institutions can be difficult.

Empirical research supports this. Glaeser et al. (2004) showed that good policies (e.g. secure rights to private property) are a driver of growth in economies, which causes growth in education, and subsequently improves political institutions. Dawson (1998) found that economic and civil liberties increase human capital investment. Moreover, economic performance affects institutional change: economic growth promotes economic freedom. Knack and Keefer (1997) showed that secondary education is associated with trust and therefore social capital, although they could not determine the direction of causality.

Hirschman (1958) theorizes that development consists of subsequent investments in social overhead capital (SOC) and directly productive activities (DPA). He claims that there is a minimum level of SOC required for a certain level of DPA, and development can happen via shortage or via excess capacity of SOC. Development via shortage means that investments in DPA create a demand for increased SOC to push down the costs of production. Development via excess means that investment in SOC invites new DPA. Either way, it only makes sense to relatively incrementally increase either investment to keep the investment costs at bay, resulting in gradual, path-dependent increases of capabilities and production.

Lall (1992) suggests that the building of national capabilities starts at the firm-level: demand for new firm-level technological capabilities is simply a result of the need for new skills to put new technologies into production. Of course, external factors that affect the perceived return on capabilities investment, such as the macroeconomic and institutional environment, are seen to affect this demand. The ability to acquire new capabilities depends on the firm's access to skills, technical goods, information and support. Cohen and Levinthal (1990) advocate that the absorptive capacity is largely a function of the firm's level of prior related knowledge, which creates path dependence. They argue that a lack of investment in an area of expertise early on may foreclose the future development of a technical capability in that area.

Fagerberg and Srholec (2016) empirically found specialization in knowledge-intensive activities to be positively associated with both regional economic performance and the capabilities that underpin it (technological capabilities, education, access to ICT and good governance). But from their data they could not tell if the capabilities came first and enabled the advanced production activities, or if the capabilities were built as a result of the expansion of advanced production activities, or a bit of both.

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2.3 Structural change

Structural change generally refers to the reallocation of economic activity across the aggregate sectors of agriculture, manufacturing and services. It is a robust ongoing phenomenon accompanying economic growth. The stylized facts include a decrease in both the employment share and the (nominal and real) value added share of agriculture, an increase in the employment and the value added share of services and a hump-shaped pattern of first increasing and then decreasing employment and value added share of manufacturing as the GDP grows. In addition, the employment share and the value added share of services are bounded away from zero, and for poor countries the employment share of agriculture is larger than the value added share, i.e. most of the population is employed in agriculture even though it has the lowest total factor productivity (Herrendorf et al., 2014).

The stylized facts of economic growth in general include a roughly constant growth rate of output per worker over time, a roughly constant capital/output ratio, a roughly constant rate of return to capital and roughly constant shares of capital and labor in net income. These are also known as the Kaldor facts of growth (Herrendorf et al., 2014).

2.3.1 The importance of industrialization

Industrialization refers to the phase in structural change when the manufacturing sector absorbs large quantities of labor from agriculture and the value added share of manufacturing rises. Historically industrialization has been the key element that has enabled rapid and sustained economic growth rates, with the exception of some economies that are very rich in natural resources (Rodrik, 2014).

Rodrik (2014) attributes the importance of industrialization to two things. First, manufacturing has the possibility of absorbing large quantities of relatively unskilled labor, partly due to the tradable nature of the goods, which frees manufacturing from the constraints of the domestic demand. Even if there would be no total factor productivity growth in manufacturing, simply the relocation of labor to these activities would raise the average productivity of the economy, as the productivity of agriculture tends to be lower than that of manufacturing. Second, the total factor productivity growth in manufacturing is in fact significant. Manufacturing industries exhibit strong unconditional convergence in labor productivity, i.e. the productivity of specific manufacturing sectors in different countries converges despite various differences in circumstances. Rodrik (2013) documented this unconditional convergence at various levels of disaggregation for a large sample covering more than 100 countries over recent decades. This is a remarkable finding, because like we mentioned in section 2.2.1, economies as a whole converge only conditionally.

Moreover, Rodrik (2014) remarks that many manufacturing industries can operate even in the presence of a low level of societal capabilities, such as bad governance and lousy policies. He argues that these properties suggest that "formal manufacturing industries are natural 'escalator' industries that tend to propel an economy forward". However, he stresses this does not denigrate the role of good policies or favorable external circumstances, as countries with better institutions and policies will experience faster convergence.

Nevertheless, Rodrik (2014) emphasizes that the industrialization process is not automatic but fraught with government and market failures. The labor productivity convergence is robust and happens in all manufacturing industries, but all economies do not manage rapid industrialization, i.e. the moving of significant shares of labor to manufacturing. There is even evidence of "premature deindustrialization", i.e. structural change that is going "backwards". It is well known that developed countries deindustrialize as the share of services rises, but for the recent decades Rodrik (2016) documented a significant deindustrialization trend also outside the post-industrial economies, with Asian countries being an exception. McMillan et al. (2014) showed that since 1990 labor has been moving from high- to low-productivity sectors in Latin America and until 2000 also in Africa. They attribute these trends to international trade and globalization, which indicates that structural change is vulnerable to outside influences.

2.3.2 Structural change and capabilities

Rodrik (2014) handled the development of capabilities and structural change as largely separate phenomena, but recognized that the social value of investments directed to expanding manufacturing activities greatly exceeds the private value. This is because the expansion of manufacturing activities in low-income environments induces various externalities and spillovers, including demonstration effects for prospective entrants, training labor that can be employed elsewhere, providing inputs and demand for other activities that may not have otherwise started, and perhaps most importantly, generating technical learning that spills over to other actors. We could call this building capabilities.

The idea that the expansion of productive activities provides inputs and demand for other activities resonates with Hirschman (1958), who introduced a famous theory of backward and forward linkages between industries. According to him, every activity that does not by its nature cater exclusively to final demands will induce attempts to utilize its outputs as inputs in some new activities, creating a forward linkage, and other attempts to fulfill its demands for inputs, creating a backward linkage. He did not explicitly formulate how the linkage theory and his vision of social overhead capital interact, but we could imagine that the linkages describe the development of directly productive activities, given that the necessary investments in social overhead capital are made. Hausmann and Hidalgo (2010) suggest that we could also understand the backward and forward linkages through capabilities. A forward linkage would be a provision of capability that promotes the development of a new product or industry. A backward linkage would be a demand for a new capability that emerges when a producer attempts to make a new product that needs it.

Others too believe that capabilities are crucial enablers of diversification into new productive activities and that learning from new productive activities builds more capabilities. Bell and Pavitt (1995) argue that the accumulation of technological capabilities opens up opportunities for diversification into related products and new industries. They discuss that historically the technological development paths of today's developed countries were based on cumulative knowledge and experience. Improving personal and organizational skills along with related institutional structures enabled countries gradually to adopt and develop process and product technologies of increasing complexity. Over time the learning processes laid the groundwork for production in other sectors. They also argue that the accumulated knowledge and expertise could be transferred to enhance the productivity in other firms and sectors. However, they stress that without government policy the economy will underinvest in capabilities.

This interaction idea is supported by empirical research, which has recently investigated structural change at a much more detailed level than the traditional three sectors of agriculture, manufacturing and services.

Hausmann and Klinger (2006) approached structural change through export data. They measured the relatedness between pairs of products based on the probability that countries in the world effectively export both, and used these measures of "proximity" to build a network of products, which they call the "product space". The proximity of products is assumed to signal how similar the capabilities needed to produce these products are. Hausmann and Klinger showed that the product space is very heterogeneous, with some areas being very dense and others quite sparse. They also found robust evidence that the speed of structural transformation will depend on the density of the product space near the area where a country is currently a strong exporter. Hidalgo et al. (2007) also studied the product space and found that most upscale products are located in a densely connected core while lower income products occupy a less connected periphery. This implies that higher income countries are able to upgrade their exports basket more quickly, while low-income countries have difficulties adapting the technology, capital, institutions and skills needed to make new products from the products they are currently producing.

Hidalgo and Hausmann (2009) constructed measures describing the complexity of a country's economy based on how diversified and how "ubiquitous" its exports are. They defined ubiquity by how many other countries export the same products. They showed that their measures of complexity correlate with the country's income and that deviations from this relationship are predictive of future growth. They theorize the complexity to in fact measure the availability of capabilities, or "non-tradable inputs", in the country. These ideas are in line with the findings of Imbs and Wacziarg (2003), who discovered robust patterns in the evolution of sectoral concentration, showing that countries diversify their production as they develop until they reach approximately the development level of current Ireland, which is when they start to specialize again.

Radebach et al. (2016) analyzed structural change on a disaggregated level using value-added data from 57 sectors. They also used a network approach, building on the concept of the product space. Their network is based on inter-sector similarities, i.e. if two sectors are simultaneously relatively strong in the same group of regions and relatively weak in another group, they are considered to be positively related. They found "communities" that can be characterized as the well-known aggregated sectors of agriculture, industry and resource extraction, but in addition they discovered what they call "directed bridging sectors", mainly light manufacturing, that connect the agriculture community to the industrial one. They showed that countries are only likely to move to sectors that are close to what they are already producing, effectively making the bridges bottlenecks in the transition from an agrarian economy to an industrialized one. Radebach et al. hypothesize that the bridges foster the building of capabilities that are necessary for engaging in more complex activities.

2.3.3 Structural change, capabilities and climate policy

It is seems worthwhile to consider the possibility that economic development arises from the interplay of structural change and growing societal capabilities, two widely documented phenomena to accompany growth.

This raises a question. Could climate policy interfere with structural change? Because if it did, developing countries might have difficulties in building the capabilities that they need in order to successfully adopt more advanced, "cleaner" technology and to continue expanding their productive activities to more complex goods. This could undermine their development.

The bridging sectors identified by Radebach et al. (2016) offer a likely candidate for a place where a disturbance to structural change could have detrimental effects to development. If light manufacturing truly is a bottleneck and no other alternative routes to industrialization naturally emerge, developing countries could be in trouble.

Climate policy could harm the bridges in different ways. Rising energy prices (due to emissions taxes, expensive emission permits or a shift to low-carbon energy sources) or taxes on emission-intensive goods could harm their competitiveness against similar products produced in more advanced countries with cleaner energy or technology. Performance standards and mandated abatement technologies could raise the capabilities that are required to operate the bridging sectors themselves. Appreciation of the national currency due to the inflow of international climate finance could harm the competitiveness of exports (Jakob et al., 2015). If the capabilities that the bridging sectors need or normally develop involve energy intensive capital goods like infrastructure, rising energy prices could divert investments away from them (Jakob and Steckel, 2014).

The next section examines economic models that incorporate some aspects of capabilities or structural change.

2.4 Related economic models

2.4.1 Endogenous growth theory

The neoclassical model of Solow (1956), also known as the most popular example of the exogenous growth theory, allows for long-term growth only via exogenously determined technological progress. As opposed to this, endogenous growth theory, known also as the new growth theory, understands technological progress as an endogenous phenomenon. Generally endogenous growth models create growth through externalities on human capital or knowledge, which we can understand as the development of capabilities.

One of the most famous models in this area is the one by Romer (1986).

In his model, growth is driven by the accumulation of knowledge via two key features: knowledge spillovers and increasing returns to knowledge. The creation of new knowledge by one firm is assumed to have a positive external effect on the production possibilities of other firms, as knowledge cannot be perfectly patented or kept secret. To describe this, the production function of the model includes both firm-specific knowledge k_i and the aggregate knowledge in the economy $K = \sum_i k_i$ as input factors. The production function takes the form $F(k_i, K, \mathbf{x_i})$, where $\mathbf{x_i}$ describes all the other input factors, which for simplicity are assumed to be in fixed supply. Most importantly, the production function as a whole exhibits increasing returns. For any $\Psi > 1$

$$F(\Psi k_i, \Psi K, \Psi \mathbf{x_i}) > F(\Psi k_i, K, \Psi \mathbf{x_i}) = \Psi F(k_i, K, \mathbf{x_i}).$$

This model features only one state variable, knowledge, and one control variable, the invested share of production. The stock of knowledge can be increased by forgoing some consumption and investing in research, which produces new knowledge as a function of investment and the existing stock of knowledge. However, the growth rate of knowledge is bounded from above and the investors only take into account their private gain. These features allow for a competitive equilibrium solution, which is suboptimal from the social point of view due to the presence of externalities.

Another famous endogenous growth model is that of Lucas (1988). In his formulations, the accumulation of human capital drives growth. Lucas examines two alternative models, representing the extremes for human capital accumulation. The first and more famous model features the possibility to invest in human capital by withdrawing time from production, like by going to school instead of working. New human capital is formed as function of the fraction of time devoted to education, and the existing human capital in the economy. Like in the Romer (1986) model, there is a maximum growth rate for human capital, but this is only reached when all available time is devoted to increasing human capital. The productivity of capital and labor are enhanced by human capital - both the human capital employed by the particular producer as well as the general average level of human capital in the economy, which induces increasing returns. Because the producers gain advantages from the general level of human capital but only the private gains are considered in the investment decisions, the competitive equilibrium is suboptimal. This model is rather similar to that of Romer (1986), except that it includes two state variables, physical capital and human capital, and two control variables, the time devoted to education instead of production and the share of production invested in capital instead of consuming it.

The second model by Lucas (1988) produces human capital through learning-by-doing. For this model economy Lucas introduced two differing goods, which can be produced as a function of good-specific human capital and the share of labor time devoted this good (Lucas omits physical capital for this model for simplicity). The production of a good i accumulates the good-specific human capital as a spillover. The difference between the goods is how efficiently they accumulate human capital. This model only features a static optimization problem, the labor time allocation for either good, and the choice depends on the substitutability of the goods in the consumer's preferences. In the absence of capital accumulation and with purely external human capital accumulation, the consumer has no intertemporal tradeoffs to decide on. Due to the external human capital accumulation, the equilibrium is again suboptimal.

In order for the model to feature endogenous growth, learning-by-doing cannot have diminishing returns, which would make human capital lose its status as the engine for growth. Lucas (1988) recognizes that this seems to violate the diminishing productivity growth of particular products that studies have observed. To solve the issue, he interprets the dynamics to stand for two industries, in a situation where new products are constantly introduced within an industry, the diminishing returns to learning apply to each of them separately, and human capital specialized to old products is inherited by new products. Consumers only have demand for the industry output in general. This model in fact features a kind of structural change between these two industries. The direction of change depends on the preferences of the consumer. If the goods are substitutes, more and more time will be devoted to the good with more efficient human capital accumulation abilities. If the goods are complements, the opposite is true.

Endogenous growth theory offers good aggregate-level dynamics, where capabilities are the engine for growth. Some of the features are especially appealing from our point of view: increasing returns to capabilities, the ability of other producers to utilize the capabilities that others have created and the creation of capabilities through learning-by-doing. However, these models do not consider the interaction between structural change and capabilities.

2.4.2 Models of structural change and capabilities

Herrendorf et al. (2014) overviewed the recent advances in the research on structural change and concluded that models of structural change are mostly concerned with replicating the stylized facts of structural change and delivering "generalized balanced growth", i.e. that all aggregate variables, like the GDP, the capital stock and the wages, grow at the same constant rate - which means that the Kaldor facts hold. The models usually include the three aggregate sectors that produce differentiated goods: agriculture, manufacturing and services. Structural change in these models is driven either by differing externally given rates of total factor productivity growth or by non-homothetic preferences of consumers, i.e. that the preferences change when consumers get wealthier. Other than incorporating more sectors, they are usually based on the one-sector neoclassical model of Solow (1956) and do not exhibit endogenous growth. Examples of these models include those of Herrendorf et al. (2014), Kongsamut et al. (2001) and Ngai and Pissarides (2007), except that Ngai and Pissarides (2007) modeled an arbitrary number of sectors.

There are however some models that incorporate some aspects of both capabilities and structural change.

Tamura (2002) provided a model where human capital accumulation causes the economy to switch from "agriculture" to "industry", which both produce the same good, but with different technologies. Agricultural technology utilizes land and human capital and exhibits decreasing returns. Industry uses only human capital as an input and features constant returns to scale. Human capital can be accumulated by parents investing time in their children, who receive human capital in proportion to the parent's. Eventually when enough human capital is accumulated, it becomes profitable to start producing with the industrial technology.

Rodrik (2007) took a different approach to sectors: he classified them as "importables" (industries that competes with imports), "non-traditional exportables" (industries that can compete in the world market), "traditional exportables" (e.g. raw materials) and "non-tradables" (like most services). Each sector uses labor as the only input, and all except non-tradables exhibit decreasing returns. The central idea behind the model is that growth is driven through learning-by-doing in the "modern" importables and non-traditional exportables sectors. The production in these modern sectors raises their own productivity, which drives growth. The other sectors experience no productivity growth.

Parente and Prescott (1994) focused on technology adoption and barriers to such adoptions. They assumed that to adopt a new technology, the firm must make an investment. The required investment depends on the level of general knowledge in the world and the barriers to adoption in the country where the firm is located. The larger these barriers, the greater the investment the firm must make to adopt a more advanced technology, and the more knowledge there is, the smaller the required investment. However, the growth of knowledge and the size of the barriers are given exogenously and they do not interact with development.

Jovanovic and Nyarko (1996) provided a one-agent Bayesian model of learning-by-doing and technology choice. The more the agent uses a technology, the better he learns to use it, and the more productive he gets. Any given technology has bounded productivity, which means that the agent needs to keep switching to better technologies in order to keep his productivity growing. But a switch of technologies results in temporary reduction of expertise: the bigger is the technological leap, the bigger the loss in expertise.

Hausmann and Hidalgo (2010) developed a "binomial" model that assumes each product to require a potentially large number of non-tradable inputs, i.e. capabilities, and that a country can only make the products for which it has all the requisite capabilities. The more capabilities a country has, the more products it can produce and therefore the more diversified it is. Because they assume that products need on average a lot of capabilities, countries with few capabilities will gain access to few if any new products when they acquire a new capability. Countries in possession of already many capabilities will likely unlock many new products with one additional capability. This model is static, i.e. includes no dynamics and provides no explanations to how capabilities are accumulated. When considering their results, we should keep in mind that their assumptions are based on *export* data. On a domestic level the returns to new capabilities may not be as negligible for poor countries. Another shortcoming of this approach is that it doesn't consider that some capabilities can substitute each other to some extent, as documented for social capital and formal institutions by Easterly et al. (2006) and for social capital, education and law enforcement by Guiso et al. (2004).

Ferrarini and Scaramozzino (2016) developed an endogenous growth model where increased complexity raises the rate of economic growth through enhanced human capital accumulation. They assume a uniformly distributed continuum of sectors that differ in their complexity. Each country has an exogenously given distribution of activities in these sectors, represented by a fixed average complexity parameter. Human capital is developed as a function of time devoted to education, the existing stock of human capital and the given average complexity of the economy. Accumulating human capital will increase productivity, but the economy will not move to different activities as it develops.

To summarize, the literature simply lacks an endogenous growth model that is (1) consistent with the stylized facts of growth, and (2) driven by the interplay of the accumulation of capabilities and the ongoing structural change and diversification of the economy. There are interesting approaches and ideas, including the less-advanced sector enabling the move to the next sector through the accumulation of some form of human capital, the barriers to adaptation making investments in advanced technology more expensive, the bounded productivities of single sectors, the products' prerequisites of capabilities and the economic complexity enhancing human capital accumulation. However, the specific modeling techniques used to execute these ideas are not very compelling for our purpose.

Chapter 3

Multi-sector endogenous growth model

Mathematical models are an integral part of economics and the most common way of presenting economic theories. Models are an important thinking aid, because they allow one to examine complicated chains of cause and effect, and provide a template for logical experiments to produce different scenarios and to evaluate the effect of different policies.

This chapter introduces a multi-sector growth model, in which growth is driven by the interplay of endogenous diversification, structural change and the accumulation of capabilities. This chapter starts by introducing the "building blocks" of the model in relation to existing literature. It then presents the model, analyzes its behaviour and considers what the framework implies for the introduction of climate policy.

3.1 Building blocks

This model utilizes well established modeling assumptions from the economic literature. These include (see e.g. Barro and Sala-i-Martin, 2004):

1. The division of the economic actors into producers and households, which

are represented by so-called *representative firms* and *representative households*. The concept of a representative firm assumes that all firms represented by the firm have access to the same production function and can convert production factors to final goods with the same efficiency. The concept of a representative household assumes that the demand side of the economy can be represented as if there was a single household making the aggregate consumption decisions.

2. *Perfect competition*, essentially meaning that all producers are price-takers. They cannot affect the market prices of their inputs or the market price of their output, which equals the unit cost of production. This follows from the assumptions that all firms have a small market share and there are no barriers to entry or exit.

3. Households own all "private" production factors, labor and capital, and rent them to firms, for which they receive compensation in the form of wages and interest, i.e. rental income on capital. This is equivalent to assuming that firms own the capital and households own the stock of the firms, only simpler.

4. Demand and investment decisions are taken by a representative household that maximizes its utility over an *infinite time horizon*. While the life of an individual might be finite, individuals are assumed to equally care about the well-being of their descendants. These optimization problems are solved with dynamic optimization tools.

4. Existence of a *public good*, which is something that is non-rivalrous, i.e. it can be used without reducing its availability to others, and non-excludable, i.e. nobody can be prevented from using it. In our model, capabilities are a public good.

5. Existence of *positive externalities*, which occur when an action creates benefits for others who cannot be charged for receiving the benefit. A *positive spillover* is a related term, but does not imply that the spillover is not compensated for. In our model, we assume capabilities to accumulate as a spillover from investment in new production sectors. Capabilities are also an externality of investment, because other producers can utilize the created capabilities and do not pay for them.

The solution to a dynamic model at every point in time is called the *competitive equilibrium.* It is reached when the households maximize their utility over an infinite horizon and firms maximize their profits at any given time. An *optimal* or *Pareto optimal* solution is reached when nobody's utility can be improved without decreasing someone else's. In the presence of externalities, these solutions will not coincide. In economic models, the Pareto optimal solution is found as a solution the optimization problem of social planner, who takes into account all dynamics, including externalities.

The behavior of the model over time is usually of most interest. *A balanced growth path* is the goal of most models, meaning that all variables grow at the same constant rate, except for the interest rate, which stays constant. A balanced growth path is consistent with the Kaldor facts of economic growth. *A generalized balanced growth path* is a term used for models that include structural change. It means that the aggregate variables grow at a constant rate, but the variables describing specific sectors do not. When the model economies are not initially on a balanced growth path, how they get there is described by *transitional dynamics.*

This model borrows its general spirit from Romer (1986), in the sense that capabilities are included in the production function and they cause the production function to exhibit increasing returns, which enables endogenous growth. The model uses a Cobb-Douglas production function like most other economic models. Unlike most models, this model includes multiple sectors, one producing investment goods and all others producing consumption goods, following Jensen and Larsen (2005) and Ngai and Pissarides (2007). However, the sectors differ not in terms of exogenous productivity growth rates but in terms of subsistence levels of capabilities. Not all of them produce at all times; the accumulation of capabilities opens up possibilities for diversification. This is a unique feature. Another unique feature is how the accumulation of capabilities is modeled as a spillover from investment in new sectors, discussed in detail in section 3.2.3.

Capabilities are measured with a one-dimensional index. This is obviously a departure from the key feature of variety, but it is how capabilities are measured to track and compare the performance of countries. Archibugi and Coco (2005) compared the recently developed measures of capabilities by the World Economic Forum, the UN Development Program, the UN Industrial

Development Organization and the RAND Corporation. All of them measure capabilities by summing together various statistics describing the different aspects of capabilities.

It is worth noting that economists tend to define "sectors" according to whatever is relevant for their objectives, which was also seen in the models in section 2.4. So does this model, as it differentiates between the sectors in terms of their requirements for capabilities. This implies that in the model some distinguished real-world sectors that produce different products but have the same requirements for capabilities are aggregated as one. This might for instance be the case for manufacturing food-products and manufacturing beverages, which are classified as separate sectors according to the International Standard Industrial Classifications of the United Nations, but perhaps require similar underlying capabilities.

The demand side of the model is more conventional. The representative household has constant elasticity of substitution (CES) preferences for different goods and a logarithmic time preference, following Acemoglu (2008) in chapter 13.4, except that in our model goods are discrete instead of a continuum. These preferences imply that the consumption decisions depend only on *relative* prices. All prices are therefore normalized using the investment good as the *numeraire*, i.e. all prices are expressed in terms of this good, following Herrendorf et al. (2014) and Ngai and Pissarides (2007).

This model features two state variables, the capital stock and the level of capabilities, which uniquely describe the "state" of the system. They can be guided with one control variable, consumption. Mulligan and Sala-i Martin (1993) provide help for analyzing the transitional dynamics of such a system with two state variables.

3.2 Construction of the model economy

3.2.1 Production

We consider an economy consisting of N sectors. Firms operating in each sector have constant returns to scale in terms of the private inputs, labor L and capital K. In addition each firm requires various societal capabilities, represented by a capability index G, for production, leading to overall increasing returns. Each sector i admits a representative firm with a Cobb-Douglas production function

$$\begin{aligned} Y_i(t) &= F_i(L_i(t), K_i(t), A_i(t), G(t)) \\ &= A_i(t)L_i(t)^{1-\alpha_i}K_i(t)^{\alpha_i}(G(t) - G_{0i})^{\beta_i} \quad \forall i \in [1, N]. \end{aligned} \tag{3.1}$$

Here $Y_i(t) \geq 0$ is the amount of units of output, not to be mixed with revenue, which is represented by the product of the output unit price and the produced units of output, $P_i(t)Y_i(t)$. $A_i(t) \geq 0$ is a sector-specific coefficient that augments the need of either labor or capital to produce a unit of output, also referred to as the (sector-specific) level of technology. $L_i(t) \geq 0$ is the employed labor and $K_i(t) \geq 0$ is the employed capital in sector i at time t. $0 < \alpha_i < 1$ is the sectoral output elasticity of capital and $0 < \beta_i < 1$ is the sectoral output elasticity of societal capabilities $G(t) \geq 0$ above the minimum necessary level of $G_{0i} \geq 0$. If $G(t) \leq G_{0i}$, the sector is not producing and $Y_i(t) = L_i(t) = K_i(t) = 0$.

The representative firms are assumed to operate under perfect competition, and therefore take the unit input prices for labor and capital $(W(t), R(t))$ as given and sell their output at unit cost $P_i(t)$. The societal capabilities are considered to be a public good, and therefore the firm does not pay to use them. The firms maximize their profits at each timepoint, and consequently choose the employed amounts of capital and labor so that the marginal productivity of labor $(MP_{L_i(t)} = \partial Y_i(t)/\partial L_i(t))$ multiplied by the output price equals the wage rate, and the marginal productivity of capital $(MP_{K_i(t)} = \partial Y_i(t)/\partial K_i(t))$ multiplied by the output price equals the capital rental rate

$$W(t) = P_i(t)MP_{L_i(t)}, \tag{3.2}$$

$$R(t) = P_i(t) MP_{K_i(t)}. \tag{3.3}$$

Because the output is sold at unit cost, the revenue of the firm equals the total costs,

$$P_i(t)Y_i(t) = W(t)L_i(t) + R(t)K_i(t). \tag{3.4}$$

For calculations see appendix A.1.

Following Herrendorf et al. (2014), one of the sectors is assumed to produce investment goods for all the rest of the economy while all the other sectors are assumed to produce differentiated consumption goods which fall into all of the broad categories of agricultural, manufacturing and service goods. The amount of consumption good sectors producing at time t is denoted by $n(t)$, and the investment sector is denoted by x. Formally,

$$n(t) = \#\{i \mid G(t) > G_{0i} \; \& \; i \neq x\}. \tag{3.5}$$

The investment sector is assumed to always be producing, so we set $G_{0x} = 0$. All in all $n(t)+1$ sectors operate at time t. All of the output will be utilized, so that we have

$$Y_i(t) = C_i(t) \quad \forall i \in [1, n(t)], \tag{3.6}$$

$$Y_x(t) = X(t), \tag{3.7}$$

where $C_i(t)$ is the total consumption of good i at time t. The demands for the consumption goods $C_i(t)$ and the investment goods $X(t)$ are determined by households, who can spend their income either on consumption or on investment in capital through the investment goods they buy from sector x. Therefore the households own all of the capital and rent it out to the firms at the rental rate $R(t)$. Investment goods accumulate capital according to

$$\dot{K}(t) = X(t) - \delta K(t), \tag{3.8}$$

where δ is the depreciation rate of capital.

Because only the relative prices matter, following Herrendorf et al. (2014) the investment good is chosen as the numeraire and all prices are obtained relative to that. The price of good i relative to the investment good is defined as

$$p_i(t) = \frac{P_i(t)}{P_x(t)} \quad \forall i \in [1, N]. \tag{3.9}$$

Using equations (3.2) and (3.3) we get the relative prices as the relations between the marginal productivities of labor and capital,

$$p_i(t) = \frac{MP_{K_x(t)}}{MP_{K_i(t)}} = \frac{MP_{L_x(t)}}{MP_{L_i(t)}} \quad \forall i \in [1, N]. \tag{3.10}$$

The relative price of the investment good is therefore always 1. The wage rate and the capital rental rate are also denoted in terms of investment goods,

$$w(t) = \frac{W(t)}{P_x(t)} = p_i(t)MP_{L_i(t)} = MP_{L_x(t)}, \tag{3.11}$$

$$r(t) = \frac{R(t)}{P_x(t)} = p_i(t)MP_{K_i(t)} = MP_{K_x(t)}. \tag{3.12}$$

3.2.2 Household preferences

We consider the households to be infinitely lived dynasties, and the population to be a constant L. We adopt a discrete-good version of the CES preferences of Acemoglu (2008), chapter 13.4:

$$U = \int_0^\infty \mathrm{e}^{-\rho t} \ln C(t) dt, \tag{3.13}$$

where U is the utility of the household, ρ is the discount factor and $C(t)$ is the consumption index,

$$C(t) = \Big(\sum_{i=1}^{n(t)} c_i(t)^{\frac{\epsilon-1}{\epsilon}} \Big)^{\frac{\epsilon}{\epsilon-1}}, \tag{3.14}$$

where $c_i(t) = C_i(t)/L$ is the per capita consumption of good i at time t. This is also known as the love-for-variety utility function when $\epsilon > 1$, as the utility of the household is then increased by the possibility to consume a larger variety of goods. We will assume that $\epsilon > 1$. The budget constraint for the household is

$$\dot{k}(t) = (r(t) - \delta)k(t) + w(t) - \sum_{i=1}^{n(t)} p_i(t)c_i(t), \tag{3.15}$$

where $k(t) = K(t)/L$, the capital stock per capita. The left hand side of the equation shows the savings, i.e. new investment, and the right side shows the net income from capital and wages minus the consumption expenditure at time t. The per capita total consumption expenditure is denoted by

$$e(t) = \sum_{i=1}^{n(t)} e_i(t) = \sum_{i=1}^{n(t)} p_i(t)c_i(t). \tag{3.16}$$

Using equations (3.15) and (3.16) we get the accumulation rule for per capita capital

$$\dot{k}(t) = (r(t) - \delta)k(t) + w(t) - e(t). \tag{3.17}$$

Appendix A.2 solves the dynamic optimization problem of dividing income into consumption and investment and yields the Euler equation

$$\frac{\dot{e}(t)}{e(t)} = r(t) - \delta - \rho. \tag{3.18}$$

Appendix A.2 also solves the static optimization problem of deciding how much of each consumption good to consume and produces the rule

$$c_i(t) = \frac{p_i(t)^{-\epsilon}}{P(t)^{1-\epsilon}} e(t), \tag{3.19}$$

where $P(t)$ is an ideal price index, as called by Acemoglu (2008), and defined by equation (A.14) as

$$P(t) = \Big(\sum_{i=1}^{n(t)} p_i(t)^{1-\epsilon} \Big)^{\frac{1}{1-\epsilon}}. \tag{3.20}$$

To guarantee the optimality of the dynamic solution, this infinite horizon optimization problem also requires a transversality condition

$$\lim_{t \to \infty} \nu(t)k(t) = 0, \tag{3.21}$$

where $\nu(t)$ is the costate variable.

3.2.3 Accumulation of capabilities

We represent societal capabilities with a capability index G, and assume capabilities to accumulate as spillover effect from investment, especially investment in new production sectors. This could be considered to work through a kind of a learning-by-doing mechanism, where the expansion of production leads to the creation of productivity-enhancing knowledge, which then spills over to other producers. Alternatively, we could think that the expansion of production creates demand for new capabilities, like new infrastructure or new institutions. In both cases it is natural to assume that investment in new sectors is more efficient in producing this knowledge or demand - the learning curve tends to be steeper when there is little experience in a particular task, and a new sector that has just the sufficient capabilities to operate has a strong demand for more to be able to increase its productivity.

We assume that over relatively long time periods Δt the accumulation of capabilities can be described as a Cobb-Douglas type of function

$$\Delta G = (\Delta n)^{\eta} \Big(\frac{\Delta k}{B}\Big)^{1-\eta}, \tag{3.22}$$

where $\frac{1}{B} > 0$ describes the effective share of investments that contributes towards capabilities. We assume B to be constant for this analysis. The elasticity parameter η measures the responsiveness of the newly accumulated capabilities to a change in the amount of new sectors and new investment. Note that in the formulation of equation (3.22) we use per capita investments Δk instead of total investments exceeding the investments to replace depreciated capital ΔK. It is quite intuitive to think that to produce equivalent capabilities, economies with differing population sizes need to invest relatively similar amounts. Furthermore, the use of a per capita variable allows us to avoid so-called scale effects, i.e. that an economy with a larger population would exhibit a larger per capita growth rate than an economy with a smaller population, all other things held equal, which is not supported by empirical observations (see e.g. Barro and Sala-i-Martin, 2004).

Equation (3.22) displays reasonable properties: the returns to increasing either factor are positive but diminishing and the "returns to scale" are constant. These features are in line with our assumption that investments in new sectors create capabilities more efficiently than investments in the more mature sectors. The more new sectors are opened, the larger is the share of the investments likely directed to them. On the other hand, increasing the amount of new sectors for a certain amount of investment is likely to have diminishing returns in terms of capabilities, as the production becomes increasingly marginal when the investment is divided between the sectors.

In the extreme situation, when no new investments are made or no new sectors are opened, the capabilities do not grow according to equation (3.22). In the case of zero investments, this seems intuitively reasonable. If new sectors are absent, we can consider this to be acceptable only as an approximation over quite long periods of time, when the existing sectors have had time to mature, and their learning curves and their demand for new capabilities have flattened. For smaller time scales equation (3.22) cannot be used due to the discrete nature of the number of operating sectors. To describe the development of $G(t)$ in continuous time, we need to somehow approximate $\dot{n}(t)$ in continuous time.

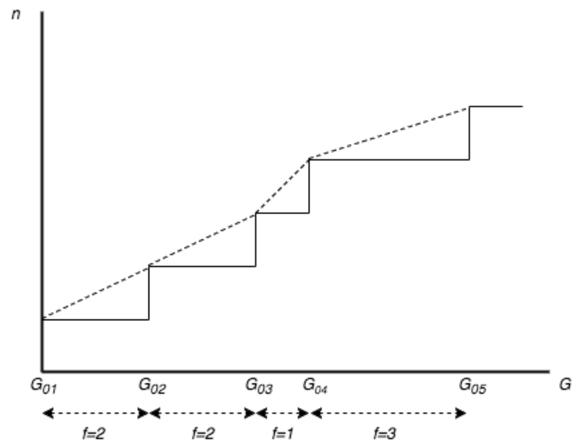


Figure 3.1: The number of sectors $n(t)$ as a function of $G(t)$.

Figure 3.1 illustrates the development of $n(t)$ as a function of $G(t)$. A linear approximation of $n(t)$ between the opening of successive sectors is indicated by a dashed line and seems reasonable enough for our purpose: we could assume that once we have reached the halfway between G_{0i} and $G_{0,i+1}$ we are also halfway into starting production in sector $i+1$, and even though the sector is not yet operating, we are going towards it. Therefore we approximate

$$\dot{n}(t) \approx \frac{\dot{G}(t)}{f(t)}, \tag{3.23}$$

where $f(t) > 0$ is the distance between the successive G_{0i}s in terms of G. We can now write equation (3.22) for continuous time as

$$\dot{G}(t) = \Big(\frac{\dot{G}(t)}{f(t)}\Big)^{\eta}\Big(\frac{\dot{k}(t)}{B}\Big)^{1-\eta} = \frac{\dot{k}(t)}{Bf(t)^{\frac{\eta}{1-\eta}}} = \frac{\dot{k}(t)}{d(t)}, \tag{3.24}$$

where $d(t) = Bf(t)^{\frac{\eta}{1-\eta}}$.

3.2.4 Aggregate economy

This section combines the production and consumption sides of the economy and derives the necessary equations to characterize the aggregate economy and factor allocations following the framework of Jensen and Larsen (2005).

All labor and capital in the economy must be utilized. Sectoral allocations of labor and capital satisfy

$$\sum_{i=1}^{n(t)+1} L_i(t) = L, \qquad \sum_{i=1}^{n(t)+1} K_i(t) = K(t). \tag{3.25}$$

National income $Y(t)$ is defined as the total of sectoral producer revenues. Following from equations (3.4), (3.11), (3.12) and (3.25), it is equivalent to total factor income

$$Y(t) = \sum_{i=1}^{n(t)} p_i(t)Y_i(t) + X(t) = w(t)L + r(t)K(t). \tag{3.26}$$

The by far mostly used measure of economic development, the gross domestic product (GDP) per capita, is in this economy defined as

$$y(t) = \frac{Y(t)}{L} = w(t) + r(t)k(t). \tag{3.27}$$

Marginal rates of technical substitution in each sector are by definition

$$\omega_i(t) = \frac{MP_{L_i(t)}}{MP_{K_i(t)}} = \frac{1-\alpha_i}{\alpha_i} k_i(t) = \omega_i(k_i(t)), \tag{3.28}$$

which are positive monotonic functions in sectoral capital-labor ratio $k_i(t) = K_i(t)/L_i(t)$. There is free mobility of labor and capital between the sectors, which imposes a common marginal rate of technical substitution for all sectors,

$$\omega_i(t) = \omega(t). \tag{3.29}$$

We define the factor allocation fractions

$$\lambda_{L_i}(t) = \frac{L_i(t)}{L} \quad \forall i \in [1, n(t)+1], \quad \sum_{i=1}^{n(t)+1} \lambda_{L_i}(t) = 1, \tag{3.30}$$

$$\lambda_{K_i}(t) = \frac{K_i(t)}{K(t)} \quad \forall i \in [1, n(t)+1], \quad \sum_{i=1}^{n(t)+1} \lambda_{K_i}(t) = 1. \tag{3.31}$$

As shown in the appendix A.3 the factor allocation fractions are given by

$$\lambda_{L_i}(t) = \frac{(1-\alpha_i)e_i(t)}{w(t)} \quad \forall i \in [1, n(t)], \quad \lambda_{L_x}(t) = \frac{(1-\alpha_x)(y(t)-e(t))}{w(t)}, \tag{3.32}$$

$$\lambda_{K_i}(t) = \frac{\alpha_i e_i(t)}{r(t)k(t)} \quad \forall i \in [1, n(t)], \quad \lambda_{K_x}(t) = \frac{\alpha_x(y(t)-e(t))}{r(t)k(t)}. \tag{3.33}$$

3.2.5 Assumptions

This section introduces assumptions which we make to make the model tractable and to focus on the role of capabilities.

Assumption 1. *There are no sector-specific technology differences or advancements over time, i.e.* $A_i(t) = A$. *We normalize* $A = 1$.

We abstract from sectoral technological development to focus the analysis on the role of capabilities.

Assumption 2. *The production functions exhibit constant returns to scale in terms of capital and capabilities, i.e.* $\beta_i = 1 - \alpha_i$.

Because we are interested in a model that exhibits balanced endogenous growth, we focus on the special case where $\beta_i = 1 - \alpha_i$, implying that the production functions exhibit constant returns to scale for $G(t) - G_{0i}$ and $K_i(t)$ for a constant labor input $L_i(t)$. If we had $\beta_i < 1 - \alpha_i$, the production functions would exhibit diminishing returns and endogenous growth would not be possible in the long term. If $\beta_i > 1 - \alpha_i$, we would have increasing returns to scale and the growth rates would tend to rise over time.

Following Herrendorf et al. (2014) and several others we make also the following assumptions:

Assumption 3. *The households can be represented as a unit mass;* $L = 1\ \forall t$.

This implies conveniently that the per capita variables equal the total variables, i.e. that $Y(t) = y(t), K(t) = k(t), Y_i(t) = C_i(t) = c_i(t), p_i(t)Y_i(t) = e_i(t)$ and $\sum_{i=1}^{n(t)} p_i(t)Y_i(t) = e(t)$. Also the labor allocation equals the labor allocation fraction, $L_i(t) = \lambda_{L_i}(t)$.

Assumption 4. *All sectors have the same sectoral output elasticities of capital, labor and capabilities, i.e.* $\alpha_i = \alpha \quad \forall i \in [1, N]$.

From equations (3.28) and (3.29) we have now that $k_i(t) = k(t)$ and therefore all sectors have the same capital-labor ratio equalling the capital-labor ratio of the whole economy. Noting that $L_i(t)^{1-\alpha}K_i(t)^{\alpha} = L_i(t)k_i(t)^{\alpha} = \lambda_{L_i}(t)k(t)^{\alpha}$ the production functions can now be simplified to

$$Y_i(t) = \lambda_{L_i}(t)k(t)^{\alpha}(G(t) - G_{0i})^{1-\alpha} \quad \forall i \in [1, N]. \tag{3.34}$$

Finally, to be able to model balanced growth, we assume:

Assumption 5. *In the absence of external influences the distance between the subsistence levels of capabilities of successive sectors is constant, i.e.* $f(t) = f\ \forall t$.

This implies also a constant $d = Bf^{\frac{\eta}{1-\eta}}$.

3.3 Deriving the solution

3.3.1 Competitive equilibrium

The competitive equilibrium of this economy is a sequence of prices ($w(t), r(t)$, $p_i(t)\ \forall i \in [1, n(t)]$) and quantities ($L_i(t), K_i(t), c_i(t)\ \forall i \in [1, n(t)]$) determined uniquely for given $k(0)$ and $G(0)$ by the differential equations (3.17), (3.18) and (3.24), the transversality condition (3.21), the equations (3.5), (3.10), (3.11), (3.12), (3.19), (3.20), (3.30) and (3.32), and the assumptions 1-5. We collect those equations simplified by the assumptions here for a clearer synthesis. We omit the time dependency from the notation at this point.

Given $k(0)$ and $G(0)$, the behavior of the economy is described by the following differential equations

$$\frac{\dot{e}}{e} = r - \delta - \rho,$$

$$\dot{k} = (r - \delta)k + w - e,$$

$$\dot{G} = \frac{\dot{k}}{d},$$

where, given the equations (3.11), (3.12) and assumptions 1,2 and 4

$$w = (1 - \alpha)k^{\alpha}G^{1-\alpha}, \tag{3.35}$$

$$r = \alpha\Big(\frac{G}{k}\Big)^{1-\alpha}. \tag{3.36}$$

The GDP per capita is therefore

$$y = w + rk = k^{\alpha}G^{1-\alpha}. \tag{3.37}$$

Given the equation (3.10) and the assumptions 1,2 and 4, the price sequence is obtained by

$$p_i = \Big(\frac{G}{G - G_{0i}}\Big)^{1-\alpha} \quad \forall i \in [1, n]. \tag{3.38}$$

The demand for each consumption good is

$$c_i = \frac{p_i^{-\epsilon}}{P^{1-\epsilon}} e,$$

where the price index P is defined by equation (3.20).

Given the assumptions 3 and 4 along with equations (3.30), (3.32), (3.35) and (3.37), the labor allocations can be expressed by

$$L_i = \lambda_{L_i} = \frac{e_i}{y} \ \forall i \in [1, n], \ L_x = \lambda_{L_x} = \frac{y - e}{y}. \tag{3.39}$$

Given the assumption 3 and solving for K_i from the previously noted $L_i^{1-\alpha} K_i^{\alpha} = \lambda_{L_i} k^{\alpha}$ gives the capital allocations

$$K_i = \lambda_{L_i} k \quad \forall i \in [1, n+1]. \tag{3.40}$$

3.3.2 Generalized balanced growth path

We now consider whether the equilibrium conditions are consistent with the existence of a generalized balanced growth path (GBGP), defined following Herrendorf et al. (2014) such that the real interest rate, i.e. in our case the capital rental price r, is constant.

Proposition 1. *A GBGP exists, if the growth rate of the societal capabilities γ_G equals the growth rate of the capital stock γ_k, i.e. $\gamma_G = \gamma_k$.*

Proof. For the capital rental price $r = \alpha (G/k)^{1-\alpha}$ to be constant, we need the ratio G/k to be constant. Let us define a new variable

$$z = \frac{G}{k}, \tag{3.41}$$

which is constant along the GBGP. If we now take a logarithm and the time derivative of both sides of (3.41), we get

$$\frac{\dot{z}}{z} = \frac{\dot{G}}{G} - \frac{\dot{k}}{k}, \tag{3.42}$$

which is equivalent to

$$\gamma_z = \gamma_G - \gamma_k. \tag{3.43}$$

On the GBGP we need $\gamma_z = 0$, which is the case when $\gamma_G = \gamma_k$.

□

The next result shows that along a GBGP of our multi-sector model other Kaldor facts also hold, i.e. the GDP per capita y and the capital stock per capita k grow at constant rates, the capital-output ratio k/y is constant and the shares of GDP received by labor (w/y) and capital (rk/y) are constant. This is a standard feature of structrural change models that feature a GBGP, like the one of Herrendorf et al. (2014).

Proposition 2. *The Kaldor facts hold along the GBGP.*

Proof. We have defined the GBGP so that r is constant, so it suffices to show that k, w and y grow at the same constant rate γ.

Given a constant r and $\gamma_G = \gamma_k$, we have

$$\gamma_w = \frac{\dot{w}}{w} = \frac{(1-\alpha)\alpha k^{\alpha-1}\dot{k}G^{1-\alpha} + (1-\alpha)^2 k^{\alpha}G^{-\alpha}\dot{G}}{(1-\alpha)k^{\alpha}G^{1-\alpha}} = \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{G}}{G} = \gamma_k \tag{3.44}$$

and

$$\gamma_y = \frac{\dot{y}}{y} = \frac{\dot{w} + r\dot{k}}{w + rk} = \frac{\gamma_w w + r\gamma_k k}{w + rk} = \gamma_k. \tag{3.45}$$

The growth rate of capital can be expressed as

$$\gamma_k = \frac{\dot{k}}{k} = r - \delta + \frac{w}{k} + \frac{e}{k}, \tag{3.46}$$

which implies that a constant growth rate of capital can only be achieved if the capital stock grows at the same rate as consumption expenditure e. The growth rate of consumption expenditure is

$$\gamma_e = \frac{\dot{e}}{e} = r - \delta - \rho, \tag{3.47}$$

which is constant. Let us denote

$$\gamma = r - \delta - \rho, \tag{3.48}$$

the rate for balanced growth. We then have $\gamma_y = \gamma_w = \gamma_e = \gamma_k = \gamma_G = \gamma$. □

Next we characterize the GBGPs of the economy.

Proposition 3. *There are two equilibria, where growth is constant. These are characterized by interest rates $r = \delta + \rho$ and $r = \alpha\left(\frac{1}{d}\right)^{1-\alpha}$ for zero growth and non-zero growth, respectively.*

Proof. Using the terms of Mulligan and Sala-i Martin (1993), let us define a "control-like" variable which is also constant in the equilibrium,

$$a = \frac{e}{k}, \tag{3.49}$$

and use it in addition to our "state-like" variable z to characterize the equilibria, defined by $\gamma_a = \gamma_z = 0$. After some algebra we get

$$\gamma_z = \gamma_G - \gamma_k = \left(\frac{1}{dz} - 1\right)(z^{1-\alpha} - \delta - a), \tag{3.50}$$

$$\gamma_a = \gamma_e - \gamma_k = (\alpha - 1)z^{1-\alpha} - \rho + a. \tag{3.51}$$

We can immediately see that γ_z equals zero at two different points. Setting both growth rates equal to zero and solving gives the two equilibria

$$z^* = \frac{1}{d},\ a^* = (1-\alpha)\left(\frac{1}{d}\right)^{1-\alpha} + \rho \ \vee \ z^* = \left(\frac{\delta+\rho}{\alpha}\right)^{\frac{1}{1-\alpha}},\ a^* = \frac{(1-\alpha)\delta + \rho}{\alpha}.$$

For $z^* = \frac{1}{d}, r = \alpha\left(\frac{1}{d}\right)^{1-\alpha}$. According to (3.48), this responds to $\gamma = \alpha\left(\frac{1}{d}\right)^{1-\alpha} - \delta - \rho$, which can be either negative or positive depending on the parameter values. If $d < \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}, \gamma > 0$, and if $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}, \gamma < 0$.

For $z^* = \left(\frac{\delta+\rho}{\alpha}\right)^{\frac{1}{1-\alpha}}, r = \delta + \rho$. This responds to $\gamma = 0$. □

Note that two equilibria is a specialty of our model and not a common feature in growth models. The equilibrium at $r = \delta - \rho$ responds to a situation where the economy invests only to cover the depreciation of capital, and no new capital is accumulated. This equilibrium could be called a poverty trap, compared to the other possible equilibrium, if its growth is positive.

3.3.3 Transitional dynamics

To understand what happens if there are initial imbalances between the capital stock and the societal capabilities, or some type of shock alters some parameters or stocks of capital or capabilities, we look at the transitional dynamics of the system. First we look into the stability properties of the equilibria to prove that transitional dynamics exist, and then proceed to sketching three phase diagrams of the model, responding to different parameter value relations. Note that the transitional dynamics of this model are not typical due to the presence of two equilibria instead of just one. Most growth models that feature transitional dynamics feature one saddle-path stable equilibrium.

Proposition 4. *If $d < \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, the equilibrium responding to $r = \alpha\left(\frac{1}{d}\right)^{1-\alpha}$ is saddle-path stable and the equilibrium responding to $r = \delta + \rho$ is unstable. If $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, the equilibrium responding to $r = \alpha\left(\frac{1}{d}\right)^{1-\alpha}$ is unstable and the equilibrium responding to $r = \delta + \rho$ is saddle-path stable.*

Proof. See appendix A.4. □

We consider separately the three cases, $d < \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$ and $d = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$. Figure 3.2 sketches the phase portrait for the first. The two $\dot{z} = 0$ loci and the $\dot{a} = 0$ locus divide the area into seven regions, and the arrows show the direction of motion in all regions based on equations (3.50) and (3.51). The two points where the $\dot{a} = 0$ locus crosses either of the $\dot{z} = 0$ loci are the equilibrium points of the economy. In this case the stagnant equilibrium on the left is an unstable node, and unless the economy starts precisely at this point, it will never go there. The other equilibrium exhibits positive growth and is saddle-path stable, i.e. there exists one optimal trajectory, shown as the thicker line with arrows, along

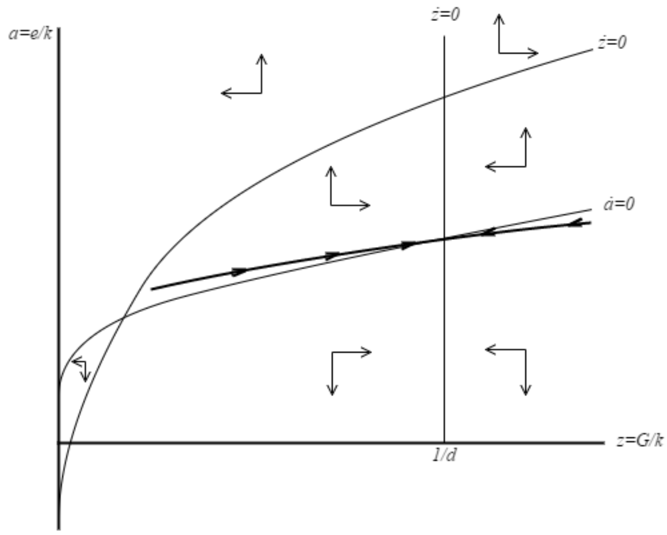


Figure 3.2: Transitional dynamics of the model when d is relatively small.

which the economy will move to the equilibrium by adjusting the control-like variable a in accordance with the initial value $z(0)$. It is notable that if $z(0) < \left(\frac{\delta+\rho}{\alpha}\right)^{\frac{1}{1-\alpha}}$, i.e. if $z(0)$, the initial ratio of capabilities to capital, is "too small", the economy is not able to reach either equilibrium and ends up at $z = 0$.

We then consider the case when $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, presented in Figure 3.3. Now the stagnant equilibrium on the right is saddle-path stable, and the thicker line with arrows again shows the optimal trajectory. The other equilibrium responds to negative growth, i.e. to the shrinking of the economy, and is an unstable node. If $z(0) < 1/d$, the economy will not reach any equilibria and again ends up at $z = 0$.

Lastly we consider the special case when $d = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, presented in Figure 3.4. The stability analysis regarding this case in appendix A.4 was inconclusive. We can however conclude by looking at the behavior of equations (3.50) and (3.51) in different regions that in this case the equilibrium seems to be "semi-saddle-path stable", i.e. the equilibrium exhibits saddle-path stability when approached from the right, but it is unstable when approached from the left. Again we conclude that if the economy starts from $z(0) < 1/d = \left(\frac{\delta+\rho}{\alpha}\right)^{\frac{1}{1-\alpha}}$, it will not be able to reach the equilibrium.

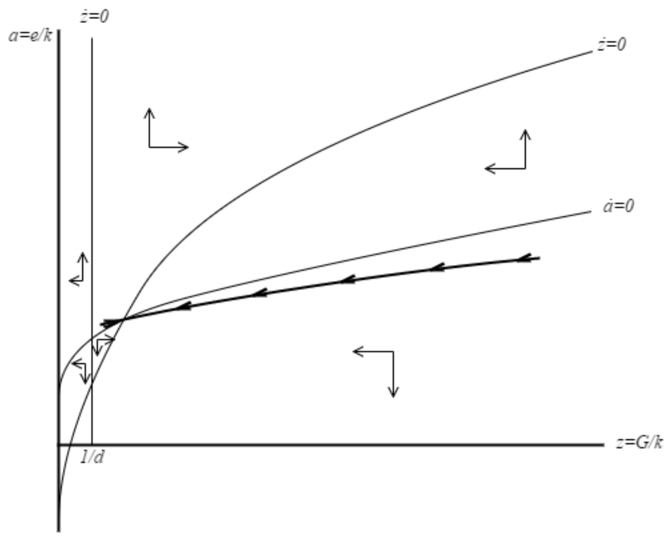


Figure 3.3: Transitional dynamics of the model when d is large.

Let us now briefly consider what these dynamics mean for our actual variables G, k and e. According to equation (3.24) the growth rates of G and k are always of the same sign. The imbalances in the two stocks will be corrected, because the growth *rate* of the variable that is relatively too low will be larger than the growth rate of the of the variable that is relatively too large. Equation (3.24) implies that the growth rates will asymptotically approach each other and therefore the ratio G/k will asymptotically approach the equilibrium value.

The total consumption expenditure is a so-called jump variable, which will be adjusted according to the initial values $k(0)$ and $G(0)$ to set the economy on the stable arm so that the transversality condition is not violated. After this the policy function for the consumption expenditure is the usual $\frac{\dot{e}}{e} = \alpha\left(\frac{G}{k}\right)^{1-\alpha} - \delta - \rho$, which is increasing in $\frac{G}{k}$. This means that if G is relatively low compared to k, the growth rate of e will start below the equilibrium value γ and increase along the transition. If k is relatively too low, the opposite is true and the growth rate of e will start above the equilibrium value and decrease as the economy moves to the equilibrium.

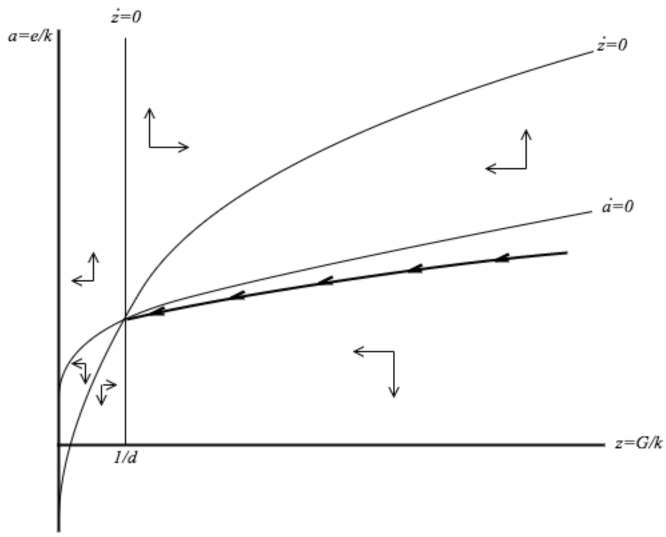


Figure 3.4: Transitional dynamics of the model when the two equilibria have merged into one.

3.4 Insights

3.4.1 Optimality

We now consider whether the decentralized solution, i.e. the competitive equilibrium of this economy (presented in section 3.3.1) is Pareto optimal by comparing it with a solution of a "social planner" that maximizes the same utility function as the representative household but takes into account the "social return" to investment, which in our case differs from the "private return" to investment, because investment creates a spillover effect and grows societal capabilities, which benefit all of the producers.

The optimization problem of the social planner is

$$\max_{C(t)} \int_0^\infty \mathrm{e}^{-\rho t} \ln C(t) dt$$
$$\text{s.t.} \quad \dot{k}(t) = y(t) - \delta k(t) - P(t)C(t) = k(t)^\alpha G(t)^{1-\alpha} - \delta k(t) - P(t)C(t),$$
$$\dot{G}(t) = \frac{\dot{k}(t)}{d}. \quad (3.52)$$

As shown in the appendix A.5, this yields the Euler equation

$$\frac{\dot{e}}{e} = \alpha\Big(\frac{G}{k}\Big)^{1-\alpha} + \frac{1-\alpha}{d}\Big(\frac{G}{k}\Big)^{-\alpha} - \delta - \rho. \quad (3.53)$$

Remembering that the private return on investment is $r = \alpha\big(\frac{G}{k}\big)^{1-\alpha}$, we can see that this expression is the same as the consumer Euler equation except for the second term, which is positive. In fact, this term expresses the social return on investment, and the interest rate that the social planner uses is $r_{social} = \alpha\big(\frac{G}{k}\big)^{1-\alpha} + \frac{1-\alpha}{d}\big(\frac{G}{k}\big)^{-\alpha}$. If we recall that the Euler equation equals the growth rate of the economy on the GBGP, we can conclude that the growth is faster in the social planner solution and the market solution is not Pareto optimal. This is due to the positive externality of capital investment.

3.4.2 Behavior of the saving rate

The investment rate of the economy can be defined as the ratio of gross investment to GDP. Because the only way to save in this model economy is to invest, we can conclude that in our case the saving rate equals the investment rate,

$$s = \frac{X}{y} = 1 - \frac{e}{y}. \quad (3.54)$$

We can immediately see that along the GBGP the saving rate is constant. Noticing that $\frac{e}{y} = \frac{e}{k}/\frac{y}{k}$ and that according to equation (3.37) $\frac{y}{k} = \big(\frac{G}{k}\big)^{1-\alpha}$, we can write $s = 1 - \frac{a}{z^{1-\alpha}}$. Now we can find out the saving rates in the two equilibria by using the equilibrium values a^* and z^* from the proof of proposition 3. We get $s^* = \alpha - \rho d^{1-\alpha}$ for the non-stagnant equilibrium and $s^* = \frac{\alpha\delta}{\delta+\rho}$ for the stagnant equilibrium. These two are equal only when $d = \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$, which is the case only when the two equilibria have merged into one. Generally the two equilibria exhibit different saving rates.

To see how the saving rate behaves during the transition to an equilibrium, we write the dynamics of the system in terms of $b = \frac{e}{y}$ and z. After some algebra we get

$$\gamma_z = \Big(\frac{1}{dz} - 1\Big)((1-b)z^{1-\alpha} - \delta), \tag{3.55}$$

$$\gamma_b = z^{1-\alpha}\Big(\alpha b - \frac{1-\alpha}{dz}(1-b)\Big) + (1-\alpha)\delta\Big(\frac{1}{dz} - 1\Big) - \rho. \tag{3.56}$$

We can now draw the phase diagram for b and z with these equations. The two $\dot{z} = 0$ loci are $z = \frac{1}{d}$ and $b = 1 - \frac{\delta}{z^{1-\alpha}}$. The functional form for the $\dot{b} = 0$ locus is more complicated,

$$b = \frac{d\rho z + (1-\alpha)(z^{1-\alpha} + \delta(dz-1))}{(\alpha(dz-1)+1)z^{1-\alpha}}.$$

Numerical checks for feasible parameter values show that the general shape of the locus is always the same, and we proceed to sketching the phase diagram for two different situations, $d < \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$ and $d > \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$.

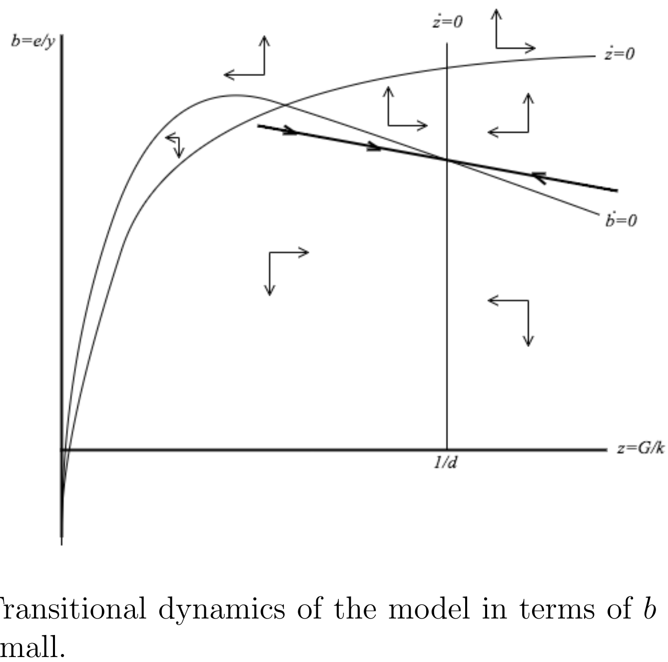


Figure 3.5: Transitional dynamics of the model in terms of b and z when d is relatively small.

Figure 3.5 shows the dynamics for $d < \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$. The stable arm is downward sloping when the equilibrium is approached from the left and upward sloping when the equilibrium is approached from the right. Because $s = 1 - b$, the behavior of the saving rate is exactly the opposite: the saving rate rises as z grows. When the equilibrium is approached from the left, i.e. when the initial value of G is relatively too low, the saving rate rises towards s^* as the economy approaches the equilibrium. In other words, if the initial capital stock is relatively too large, the economy adopts initially a lower saving rate, which leads to a lower growth rate of the capital stock than in the equilibrium or if the saving rate was a constant s^*. Furthermore, the lower growth rate of capital directly implies that also the growth rate of the capabilities will be lower than with a constant saving rate. Together these imply that the growth of GDP will unambiguously be slower. However, even if the saving rate was constant, the growth rate of the capital stock would be lower than in the equilibrium, because the relatively low level of capabilities harms the productivity of capital.

When the equilibrium is approached from the right, i.e. when the initial value of k is relatively too low, the economy adopts a higher saving rate, leading to a faster growth of the capital stock (and capabilities and GDP) than in the equilibrium or if the saving rate was a constant s^*. Again note that also a constant saving rate would produce faster growth than in the equilibrium, because the productivity of capital is larger due to a relatively higher level of capabilities.

This behavior is due to the behavior of the interest rate. If we remember that $r = \alpha\left(\frac{G}{k}\right)^{1-\alpha}$, we notice that also the interest rate increases as a function of z. A higher interest rate is a larger incentive for the households to save. As discussed in section 3.4.1, the interest rate only captures the private return on capital. If also the social return would be taken into account, the interest rate would be higher and therefore the saving rate would also be higher at all times, but especially when z is low.

Figure 3.6 shows the dynamics for $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$. Now the stagnant equilibrium is the saddle-path stable one. The behavior of the saving rate remains the same as in the previous case: the saving rate still increases as z increases. When the economic growth is stagnating, i.e. when the equilibrium is approached from the right, the saving rate decreases.

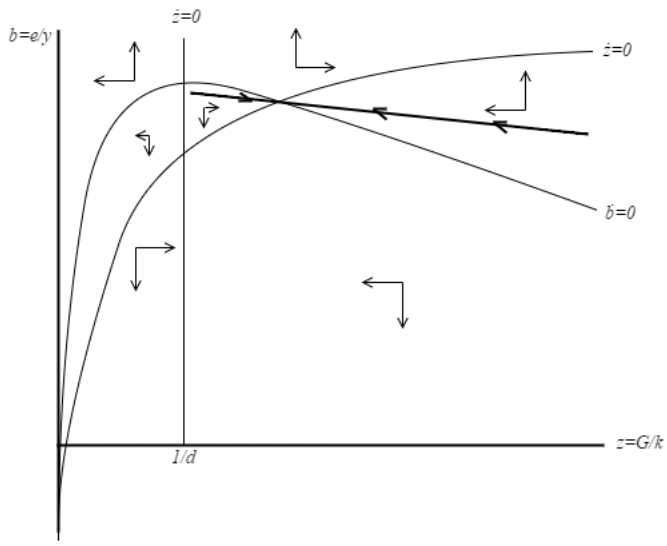


Figure 3.6: Transitional dynamics of the model in terms of b and z when d is relatively large.

3.4.3 Structural change

Following Ngai and Pissarides (2007) we define structural change as the state in which at least some of the labor shares, equalling in our case the labor fractions λ_{L_i}, are changing over time, i.e. $\dot{\lambda}_{L_i} \neq 0$.

Proposition 5. *The labor fraction in the investment sector remains constant along the GBGP.*

Proof. Equation (3.39) defines the labor fraction in the investment sector as $\lambda_{L_x} = \frac{y-e}{y}$. According to equation (3.26) $y = e+X$, so we can write $\lambda_{L_x} = \frac{X}{y}$. The rate of change is therefore

$$\frac{\dot{\lambda}_{L_x}}{\lambda_{L_x}} = \frac{\dot{X}}{X} - \frac{\dot{y}}{y}. \tag{3.57}$$

Along the GBGP we must have $\frac{\dot{X}}{X} = \frac{\dot{y}}{y}$, and therefore $\frac{\dot{\lambda}_{L_x}}{\lambda_{L_x}} = 0$. . □

This means that labor can only move to or away from the investment sector outside the GBGP, i.e. when the economy is in transition towards the GBGP, and when $\frac{\dot{X}}{X} \neq \frac{\dot{y}}{y}$. Had we imposed a constant saving rate, the labor fraction in the investment sector would always be constant.

Proposition 6. *In the consumption good sectors the relative labor fractions grow in proportion to relative prices, the factor of proportionality being one minus the elasticity of substitution.*

Proof. Equation (3.39) defines the labor fraction employed in the production of good i as $\lambda_{L_i} = e_i/y$, and equations (3.16) and (3.19) imply that $e_i = (p_i/P)^{1-\epsilon}e$. The relation between the labor fractions of good i and j is

$$\frac{\lambda_{L_i}}{\lambda_{L_j}} = \frac{e_i}{e_j} = \Big(\frac{p_i}{p_j}\Big)^{1-\epsilon}. \tag{3.58}$$

So the growth in relative labor fractions is

$$\frac{\dot{\lambda}_{L_i}}{\lambda_{L_i}} - \frac{\dot{\lambda}_{L_j}}{\lambda_{L_j}} = (1-\epsilon)\Big(\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}\Big). \tag{3.59}$$

□

Note that this is the same result that Ngai and Pissarides (2007) get for their model.

Proposition 7. *Structural change takes continuously place along the GBGP responding to $r = \alpha\big(\frac{1}{d}\big)^{1-\alpha}$, where growth is non-zero. There is no structural change in the stagnant equilibrium where $r = \delta + \rho$.*

Proof. Because we defined structural change as a state where $\dot{\lambda}_{L_i} \neq 0$, let us turn to the behavior of the individual consumption goods sectors. The dynamics of the individual labor fractions of consumption good sectors satisfy

$$\frac{\dot{\lambda}_{L_i}}{\lambda_{L_i}} = \frac{\dot{e}_i}{e_i} - \frac{\dot{y}}{y} = (1-\epsilon)\Big(\frac{\dot{p}_i}{p_i} - \frac{\dot{P}}{P}\Big) + \frac{\dot{e}}{e} - \frac{\dot{y}}{y} \quad \forall i \in [1,n], \tag{3.60}$$

where, following from the definition of P in (3.20) and the identity $e_i = (p_i/P)^{1-\epsilon}e$,

$$\frac{\dot{P}}{P} = \sum_i^n \frac{e_i}{e}\frac{\dot{p}_i}{p_i}, \tag{3.61}$$

which is a weighted average of the rates of price changes, the weight being each good's share of total consumption expenditure. We note that along the GBGP we have $\frac{\dot{e}}{e} = \frac{\dot{y}}{y}$, and the growth of the labor fractions are only dependent on the growth rates of the prices. This also is a similar result to that of Ngai and Pissarides (2007).

Let us now see how the individual and the average growth rates of prices develop over time. The similarities to Ngai and Pissarides (2007) end here, because their model features external total factor productivity growth rates and a constant number of operating sectors. According to equation (3.38)

$$p_i = \Big(\frac{G}{G - G_{0i}}\Big)^{1-\alpha} \quad \forall i \in [1, n].$$

After some algebra, we get

$$\frac{\dot{p}_i}{p_i} = -(1-\alpha)\frac{\dot{G}}{G}\frac{G_{0i}}{G - G_{0i}}. \tag{3.62}$$

We see immediately that if the capabilities do not grow ($\frac{\dot{G}}{G}$=0), which would be the case only when the economy is in the stagnant equilibrium, the prices do not change ($\frac{\dot{p}_i}{p_i} = 0 \ \forall i$) and, therefore, there cannot be structural change either according to equation (3.60).

If the growth of capabilities would be negative, which is the case in the equilibrium responding to $r = \alpha\big(\frac{1}{d}\big)^{1-\alpha}$ when $d > \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$, the prices would rise as G decreases, approaching infinity as $G \to G_{0i}$. When G reaches G_{0i}, the sector i stops operating. Looking at the equation (3.60) and remembering that $\epsilon > 1$, this responds to ever decreasing amount of labor in sector i until all labor is allocated to sectors with a lower G_0. We keep in mind that unless the economy starts from this equilibrium, it will never end up in this situation of shrinking.

The most interesting case is therefore the one where the growth of capabilities is positive, i.e. the equilibrium responding to $r = \alpha\big(\frac{1}{d}\big)^{1-\alpha}$ when $d < \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$ (or the transition towards this equilibrium). We note that the price change is always negative when there is positive capability growth, i.e. all consumption good prices decrease over time as a result of the increasing labor productivity due to the increasing societal capabilities. Along a GBGP

$\frac{\dot{G}}{G}$ is constant, and the absolute growth rate of p_i is decreasing in G. When the sector first starts to operate, $G - G_{0i} \approx 0$ and the price is close to infinity and decreasing infinitely fast, $p_i \approx \infty, \frac{\dot{p}_i}{p_i} \approx -\infty$. When the capabilities grow, G_{0i} becomes smaller and smaller compared to G, and the price asymptotically approaches one, while the growth rate of the price asymptotically approaches zero, $p_i \to 1, \frac{\dot{p}_i}{p_i} \to 0$.

Because $e_i = p_i c_i = \left(\frac{p_i}{P}\right)^{1-\epsilon} e$, we conclude that at the limit e_i and c_i will have to approach the same value, $e_i, c_i \to e/P^{1-\epsilon}$ as $p_i \to 1$. Also the growth rates of e_i and c_i must approach the same value, as $\frac{\dot{p}_i}{p_i} \to 0$. To find out which value, we look at the known relationships between growth rates. On the GBGP we have $\gamma_e = \gamma_G$. Remembering that in equation (3.24) we approximated $\dot{n} = \dot{G}/f$, we also must have that $\gamma_G = \gamma_n$ (with a little abuse of notation as n is actually not continuous). Because $e = \sum_i^n e_i = n\bar{e}_i$, where $\bar{e}_i$ is the average of e_i over $i \in [i, n]$, implies $\gamma_e = \gamma_n + \gamma_{\bar{e}_i}$, we must have $\gamma_{\bar{e}_i} = 0$, and there cannot be sustained growth of the expenditure on an individual good in the long run. We conclude that $\frac{\dot{e}_i}{e_i}, \frac{\dot{c}_i}{c_i} \to 0$.

As we noted previously, $e_i = \left(\frac{p_i}{P}\right)^{1-\epsilon} e$. Remembering that we have assumed $\epsilon > 1, e_i$ is decreasing in p_i. Because p_i starts at infinity, e_i starts at zero. The growth rate of e_i is given by

$$\frac{\dot{e}_i}{e_i} = (1-\epsilon)\Big(\frac{\dot{p}_i}{p_i} - \frac{\dot{P}}{P}\Big) + \frac{\dot{e}}{e}. \tag{3.63}$$

Because $\frac{\dot{p}_i}{p_i}$ starts near $-\infty$, $\frac{\dot{e}_i}{e_i}$ starts near ∞. We know that as $\frac{\dot{p}_i}{p_i} \to 0$, $\frac{\dot{e}_i}{e_i} \to 0$. We therefore get the result that

$$\frac{\dot{P}}{P} \to \frac{1}{1-\epsilon}\frac{\dot{e}}{e} \tag{3.64}$$

over time. And because $e = PC$, we get that in the long run

$$\frac{\dot{C}}{C} = \frac{\dot{e}}{e} - \frac{\dot{P}}{P} \to \frac{\epsilon}{\epsilon - 1}\frac{\dot{e}}{e}. \tag{3.65}$$

From equation (3.60) we can now conclude how the labor fraction behaves. At first the near infinitely fast growth of $\frac{\dot{e}_i}{e_i}$ dominates and the labor fraction grows very fast. Soon there will be a tipping point when $\frac{\dot{e}_i}{e_i} = \frac{\dot{y}}{y}$, and the labor fraction starts to fall. Over time

$$\frac{\dot{\lambda}_{L_i}}{\lambda_{L_i}} \to -\frac{\dot{y}}{y}, \tag{3.66}$$

and we conclude that the structural change is continuous also in the long run as labor steadily moves to newly opened sectors. The production in the earlier sectors however does not decrease due to increased labor productivity. □

It must be noted that when the number of sectors is low, the opening of a new sector has a relatively much larger effect on the ideal price index P and on the consumption index C and therefore on the momentary utility of the households. The resulting changes in the expenditure on other individual goods and in the labor fractions in different sectors are much larger and faster in the "early stages" of the economy. Momentarily the expenditure on some good i may also fall due to the disruption caused by the new sector.

3.5 Possible effects of climate policy

In this section we describe some possible effects of the introduction of climate policy in this framework. The aim of climate policy is to reduce greenhouse gas emissions. There are multiple ways to achieve this, e.g. taxing emissions, emissions trading or regulation. As mentioned in section 2.1, in an ideal world the choice of instrument does not make a difference.

We consider separately two possible effects on our economy, compared to a "business-as-usual" scenario. The first is an increased need of capabilities to operate a certain sector. The second is making some existing capital obsolete. For simplicity, we consider the new policies to be implemented instantly.

Let us start with the first effect. Consider an economy which is about to build an energy sector. In a business-as-usual scenario this sector would operate some relatively cheap coal power plants that do not have very high requirements for societal capabilities, e.g. in terms of education or financing. If a climate policy was introduced and the energy sector would be forced to use low-carbon technologies instead, the energy sector would suddenly need employees with higher education than before to design, build and operate the system. Furthermore, the sector would require to much better financing possibilities due to the higher capital intensity of low carbon energy technologies, such nuclear, photovoltaics and fossil plants with carbon capture and storage equipment (Hirth and Steckel, 2016). In our framework these requirements imply an increasing G_0 for the energy sector, i.e. an increase in f and therefore in $d = Bf^{\frac{\eta}{1-\eta}}$. Note that an increasing d implies that the economy does not only need to accumulate more capabilities before the energy sector can start, but a unit of capabilities becomes more difficult, or more expensive, to attain. As discussed in section 3.2.3, this is because the effectiveness of investment in creating capabilities is decreased when the opening of new sectors is slowed down. In other words, investments in more mature sectors do not create new capabilities as efficiently as investments in a new sector.

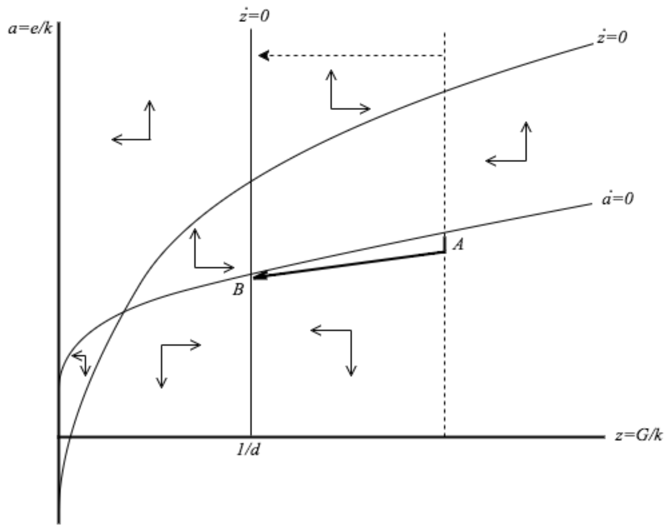


Figure 3.7: Behavior of the economy when d suddenly increases. Economy starts from point A and moves to point B along the stable arm.

Figure 3.7 illustrates how a sudden growth in d affects the economy. In terms of the phase diagram, the equilibrium is suddenly moved to the left. The households adjust their consumption to get the economy on the stable arm, and the economy starts to shift towards the new equilibrium, where the ratio of capabilities to capital stock is smaller. This will harm the productivity of capital, which slows down the economic growth. Furthermore, the interest rate and therefore the households' incentive to save will be reduced, which will decrease the investments and cause the growth to slow down even more.

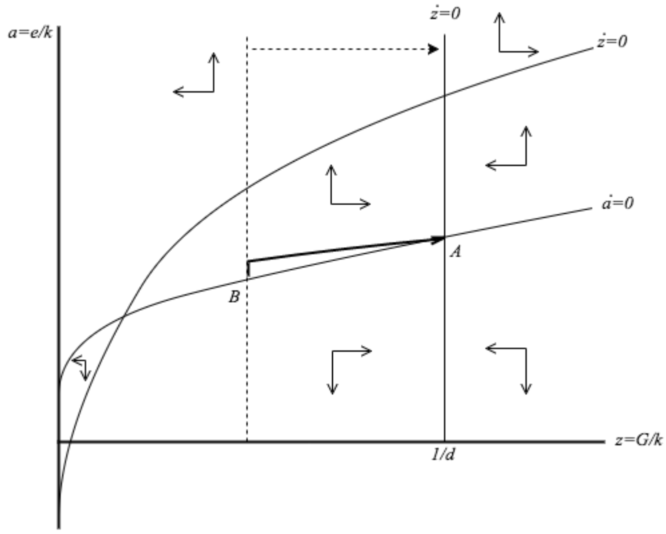


Figure 3.8: Behavior of the economy when d suddenly decreases.

Once the required capabilities are reached, we could expect d to decrease again. If we assume that climate policy does not change the distance between the G_0s of the energy sector and the next sector in line, d will return to its original value. Figure 3.8 illustrates how this affects the economy. The equilibrium is now moved back to its original position, and the consumption is immediately adjusted to get the economy on its way towards the new equilibrium. The growth rate of capabilities will now become greater than the growth rate of capital, which will start to improve the productivity of capital and raise interest rates. The households start to save more, which raises the growth rates of capital, capabilities and the GDP. Once the economy reaches the equilibrium, the growth rates will be the same as before the climate policy disrupted the business-as-usual. Because growth has been slower in between, the GDP per capita will be lower than if the climate policy was never introduced, but the long-term growth rate is not harmed.

Note that this analysis does not take into account possible effects of climate change itself - it may well be that some kind of disruption is caused in any case, and business-as-usual is simply not an option.

It is important to highlight here that depending on the existing stock of capital and capabilities, the economy might in fact not reach the new, lower growth equilibrium before it manages to reach the required G_0 and starts to move back to the original equilibrium. The richer, or the more developed the country is when it encounters a shift in G_0, the faster it will reach the G_0 and the smaller is the disruption to its economy. Poorer, or less developed countries will travel all the way to the new equilibrium and it will take them much longer to reach the elevated G_0 and then it will take them longer to travel back to the original equilibrium. In conclusion, the disruption to the economy will be much more severe for poorer countries. But nevertheless, the countries will not get stuck in poverty.

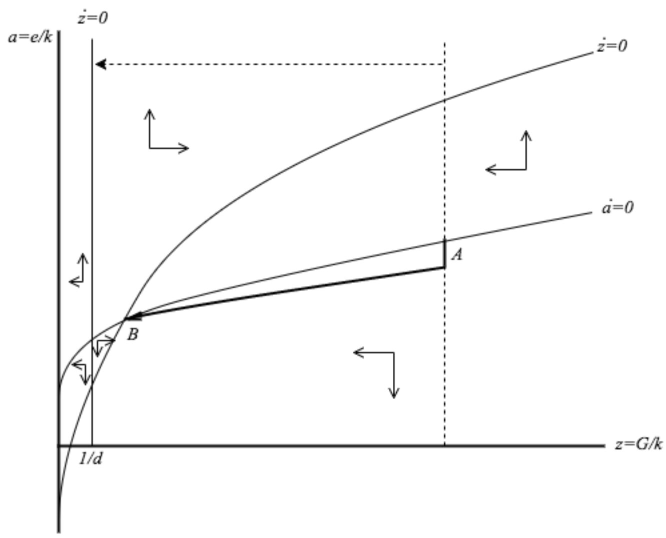


Figure 3.9: Behavior of the economy when d suddenly increases so much that the stagnant equilibrium becomes saddle-path stable.

However, if the increase in d is relatively large, the consequences for the economy are different. Figure 3.9 illustrates a scenario where the equilibrium is moved further left than the stagnant equilibrium. In this case the stagnant equilibrium becomes saddle-path stable, and the economy moves towards it along the new stable arm. If the economy does not manage to reach the next G_0 on the way to the new equilibrium and escape from the path, the growth will eventually stagnate. In this case the economy never manages to build the capabilities required to open the new sector and there is no way for the economy to get out of this stagnant equilibrium on its own. A government intervention would be needed to build the necessary capabilities and to get the economy out of this "poverty trap". This scenario is much more likely for the poorer countries, as the richer countries are more likely to be able to build the required capabilities before they reach stagnation.

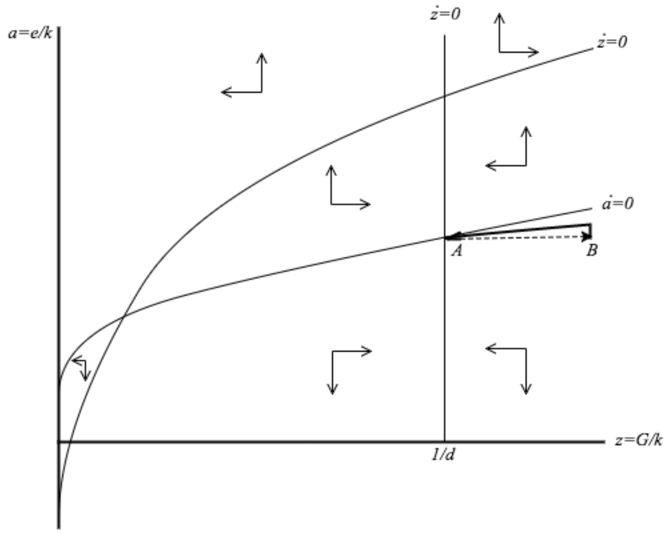


Figure 3.10: Behavior of the economy when the stock of capital suddenly decreases.

The second effect to be considered is climate policy making some capital obsolete. This could be a result of rising production standards in the industry, which would result in the abandoning of some old machines. Loss of capital would imply an immediate decrease of the GDP, but also an immediate increase of the capabilities-capital-ratio. The dynamics however remain unchanged. In the phase diagram, the economy will suddenly move to the right of the equilibrium. The consumption is again adjusted to get the economy on the stable arm. The productivity of the remaining capital will increase, which will increase the interest rate and the saving rate, and the households will start to invest more. The growth rates of capital, capabilities and the GDP will increase. Because the capital stock grows faster than capabilities, the productivity will start to decrease towards the equilibrium value, the interest rate will start to decrease again and eventually the economy will reach the equilibrium growth again. Only now it is ambiguous whether the level of GDP is lower or higher than it would have been without the disruption, because even though there was a sudden reduction of GDP, growth was boosted in between.

3.6 Numerical illustrations

This section illustrates the behavior of the economy with a set of standard benchmark parameter values. Because the model is not an initial value problem but a boundary value problem, simulating it as is is not very straightforward. Because our aim is not to perfectly calibrate the model but to visualize its behavior, we adopt a constant saving rate for the simulations, thus turning the model into a standard initial value problem. The qualitative behavior of the model is not changed as a result, as we discuss in appendix A.6.

We adopt the same standard parameter values for α, δ and ρ as Barro and Sala-i-Martin (2004), presented in table 3.1. Widely used estimates for the elasticity parameter of the constant elasticity of substitution preferences (ϵ) are harder to find. In our case the elasticity of substitution between the consumption goods has to be more than 1, or otherwise the households would be reluctant to give up some potential consumption of the older products to consume new ones. When $\epsilon > 1$, the utility function exhibits love-for-variety, and consuming more varieties will increase the utility of the household. As long as $\epsilon > 1$, the chosen value will in fact not affect the development of the economy on the aggregate level apart from the experienced utility of the households, so we simply choose a number. The parameter d obviously is a speciality of our model, and estimates for it do not exist elsewhere. We choose to set d so that the equilibrium GDP growth rate that the model produces somewhat corresponds to the actual measured long term growth rate of the US economy, as also the α, δ and ρ are measured for the US. If we set $d = 5$, corresponding to 20% capability-accumulation efficiency of capital investment, and the other parameters as described, the equilibrium growth rate of the model is $\gamma = \alpha\left(\frac{1}{d}\right)^{1-\alpha} - \delta - \rho = 2.7\%$ and the equilibrium saving rate is $s^* = \alpha - \rho d^{1-\alpha} = 24\%$, not too far from the actually measured growth rate of 2% and the saving rate of 19% (Barro and Sala-i-Martin, 2004).

To follow when the new sectors open, we also need to set the values for the components of $d = Bf^{\frac{\eta}{1-\eta}}$. Because this formulation is unique to us, we cannot refer to other literature for benchmark values, and for simplicity we set $\eta = 0.5, f = 1$ and $B = 5$. They respond to identical capability index elasticity of new sectors and new investments, i.e. that $\eta = 1 - \eta$, and to a unit distance between G_0s, and to a 20% capability-contributing share of investment, adding up to the 20% capability-accumulation efficiency of investment.

Table 3.1: Parameter values for simulations.

Parameter	Definition	Value
α	Sectoral output elasticity of capital	0.3
δ	Depreciation rate of capital	5%
ρ	Discount rate	2%
ϵ	Elasticity of substitution between goods	1.2
$\frac{1}{B}$	Capability-contributing share of investments	20%
f	Distance between G_0s	1
η	Capabilities elasticity of new sectors	0.5
s	Saving rate	24%

We first look at the behavior along the GBGP. Because the saving rate is constant along the GBGP, the simulation corresponds to the behavior of the model exactly. Figure 3.11 shows the development of the GDP, the capital stock, the consumption expenditure, the wages, the capability index, the consumption of unit goods and the interest rate over time for the initial values of $G(0) = 1, k(0) = 5$. The interest rate is stable, and the growth rates are equal for all other variables, except only approximately for the number of unit goods consumed. The number of consumed unit goods varies a little around the equilibrium growth, because new goods that are introduced are very expensive in comparison to others and when the total consumption expenditure stays balanced, the amount of goods fluctuates.

The behavior of the expenditures on and the prices of some single goods is shown in detail in Figure 3.12. As described in section 3.4.3, the expenditure on a single new good increases very quickly after the sector starts to operate. Then the growth trend slows down and starts to stagnate. The expenditure fluctuates around the trend, as the opening of new sectors causes the households to shift some of the consumption to those sectors. Looking at the expenditure curves, the dips occur exactly when a new sector is opened, as can be seen at the second dip on the blue e_3 line: as the sector 5 is opened and the expenditure on good 5 rises steeply (red line, e_5), e_3 falls. The less

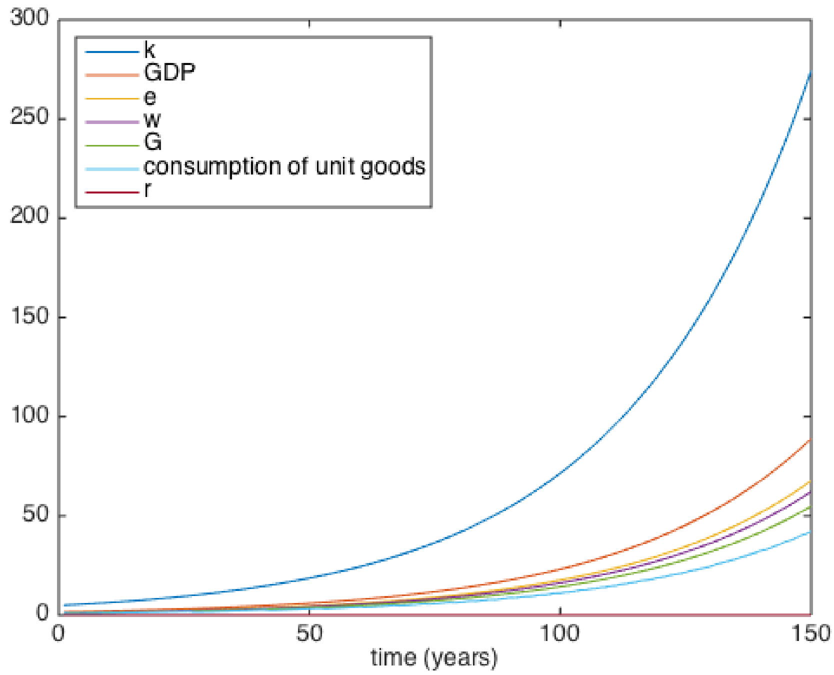
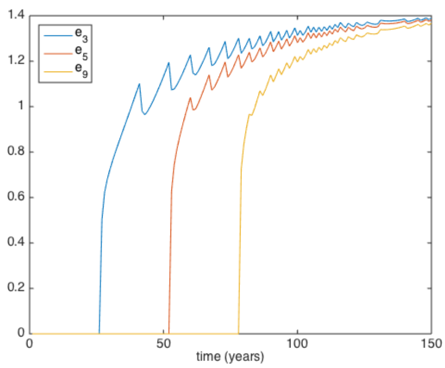
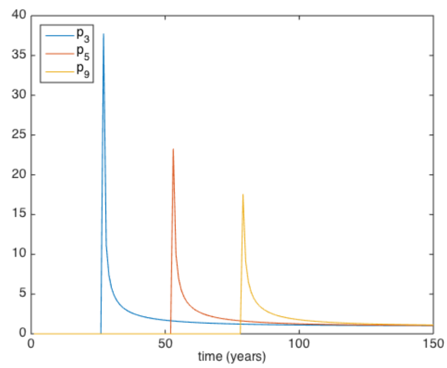


Figure 3.11: behavior along the GBGP.



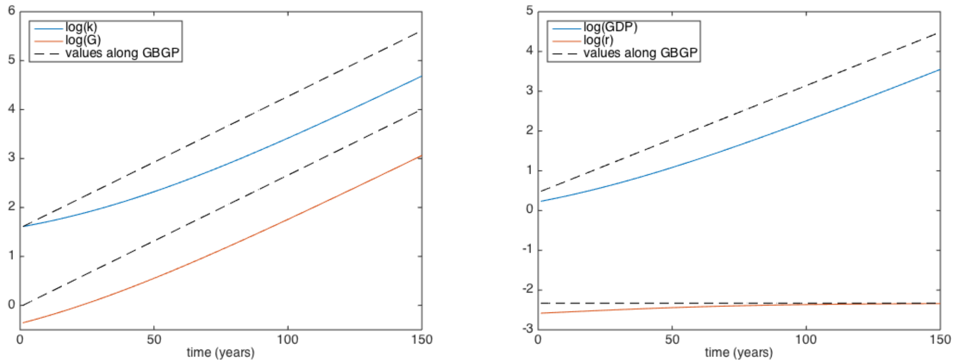
(a) Expenditure on goods.



(b) Price of goods.

Figure 3.12: Expenditure on and prices of goods produced by sectors 3, 5 and 9 over time.

operating sectors there are, the larger the effect; we see that the dips become smaller as time passes and more sectors already operate. Also the prices exhibit the expected behavior: they start very high but decrease very fast and approach 1 for each good.

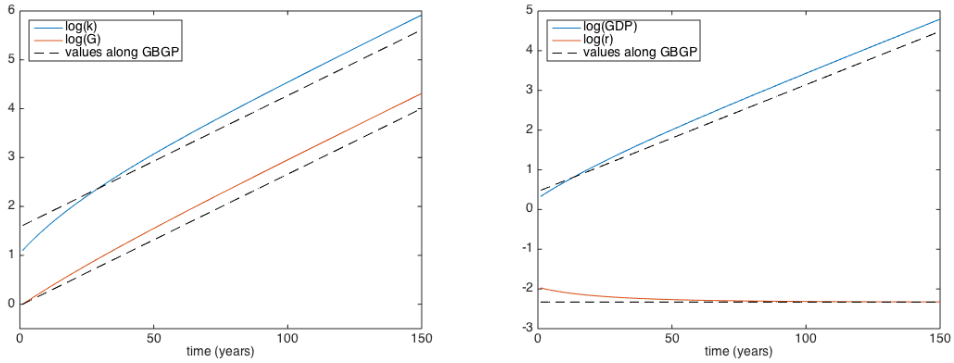


(a) Stocks of capital and capabilities. (b) GDP and the interest rate.

Figure 3.13: Behavior of the system on a logarithmic scale when the initial value of G is lower than on the GBGP. Dashed black lines indicate the behavior along the GBGP.

Let us then look at the transitional dynamics of the economy. The differences in the development of variables are best noticed when compared on a logarithmic scale. We first look at the behavior when the initial capabilities-capital-ratio is smaller than in the equilibrium, with starting values of $G(0) = 0.7$ and $k(0) = 5$. The behavior of the capital stock, capabilities, GDP and the interest rate are shown in Figure 3.13. The black dashed lines illustrate the behavior along the GBGP, shown in absolute values in Figure 3.11. Along the GBGP the growth rates of k, G and GDP are constant while r is stable. Now when the $G(0)$ is below the GBGP, the growth rates of k, G and GDP are much slower but start to approach the equilibrium growth rate. The interest rate starts below the equilibrium value but approaches it. The levels of k, G and GDP are permanently lower than they were when the economy was on the GBGP from the beginning. Note that even though this result is qualitatively the same as with consumer optimization, the households would in fact react to the low interest rate and further slow the transition down. A constant saving rate approaches the equilibrium faster than the market equilibrium.

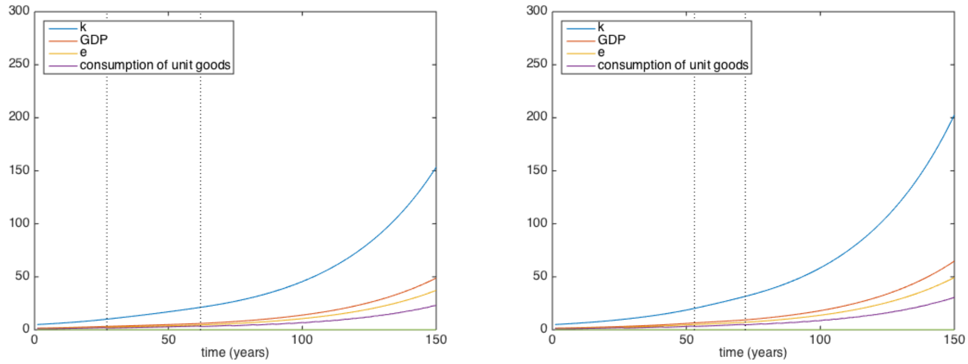
Let us then look at the transitional dynamics when the capabilities-capital-ratio is above the equilibrium. We set $G(0) = 1$ and $k(0) = 3$. Again the dashed lines represent the behavior along the GBGP for comparison. Now the growth rates of k, G and GDP start off larger than in the equilibrium before converging to it, and the variables actually end up on a higher level than they would have along the GBGP. The interest rate starts from a higher level before converging to the equilibrium value. Again note that if the consumers could optimize the saving rate, they would react to the higher interest rate and further boost the growth by investing more.



(a) Stocks of capital and capabilities. (b) GDP and the interest rate.

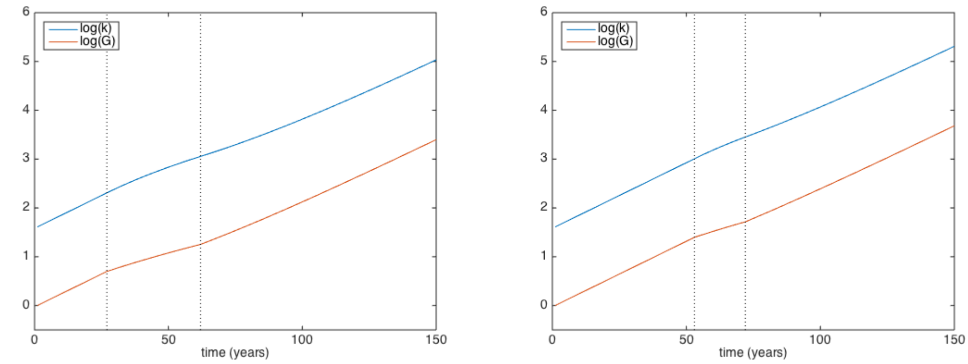
Figure 3.14: Behavior of the system on a logarithmic scale when the initial value of k is lower than on the GBGP. Dashed black lines indicate the behavior along the GBGP.

We now move on to simulate the different effects of climate policy described in section 3.5. We begin with the first effect and increase the G_0 of one sector so that the distance f to the previous sector grows 50% from 1 to 1.5. This responds to growing d from 5 to 7.5. and decreasing the capability accumulation efficiency of capital investment from 20% to 13%. After the necessary capabilities for this sector are built and the new G_0 is reached, we let f decrease back to its original value for the later sectors. To illustrate how the effect differs for economies at different development stages, we simulate the change for two different sectors, namely sectors number 4 and 6. Figure 3.15 compares the effects on the GDP, the capital stock, the consumption expenditure and the consumption of unit goods when the economy is affected at these different stages. In both cases the economies recover and approach the original growth rates, but if the disruption happens earlier, it lasts longer and the GDP and all the other variables lag more from the benchmark GBGP values presented in Figure 3.11. Note that if the saving rate was not held constant, the consumers would opt for a lower saving rate during the disruption, deepening its effects.



(a) G_{04} grows by 0.5 G. (b) G_{06} grows by 0.5 G.

Figure 3.15: Behavior of the capital stock, GDP, consumption expenditure and consumption of unit goods when f grows 50% between two sectors at different points in development. Dotted lines indicate the time when $f = 1.5$.

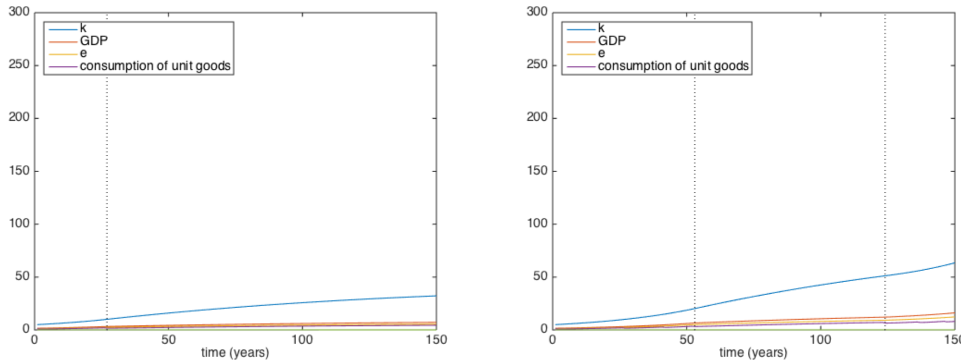


(a) G_{04} grows by 0.5 G. (b) G_{06} grows by 0.5 G.

Figure 3.16: Behavior of the capital stock and capabilities on a logarithmic scale when f grows 50% between two sectors at different points in development. Dotted lines indicate the time when $f = 1.5$.

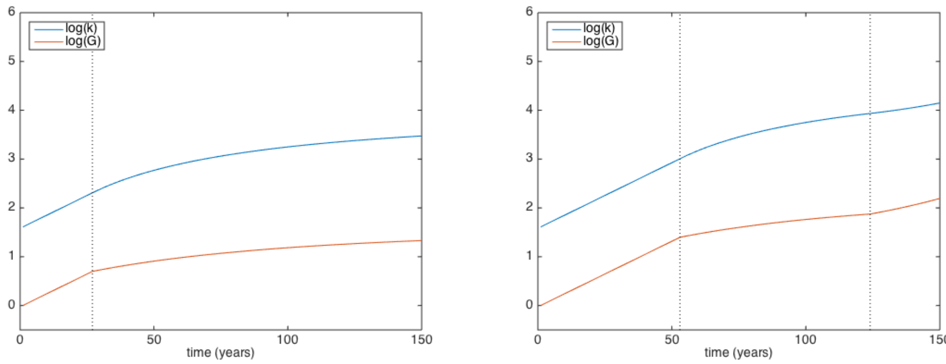
Figure 3.16 illustrates the effects on capital and capabilities in more detail on a logarithmic scale. As expected, the growth rate of capabilities reacts faster to changes than the growth rate of capital. Again we see that the period of slowing growth lasts much longer if the disruption happens earlier.

Let us then increase f by 150% from 1 to 2.5, responding to increasing d from 5 to 12.5 and decreasing the capability accumulation efficiency of capital investment from 20% to 8%. This will take the new equilibrium past the stagnant equilibrium, and the economies start to approach stagnation. Figure 3.17 compares the effects on the GDP, the capital stock, the consumption expenditure and the consumption of unit goods for the economies at two different development stages.



(a) G_{04} grows by 1.5 G. (b) G_{06} grows by 1.5 G.

Figure 3.17: Behavior of the capital stock, GDP, consumption expenditure and consumption of unit goods when f grows 150% for one sector at different points in development. Dotted lines indicate the time when $f = 2.5$.



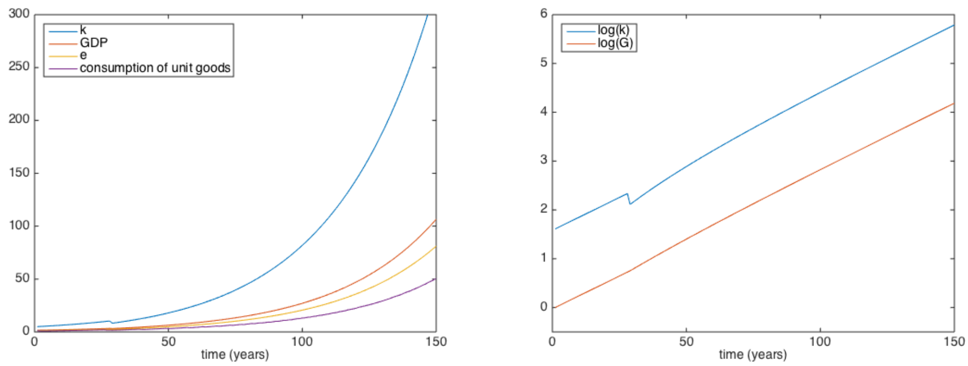
(a) G_{04} grows by 1.5 G.. (b) G_{06} grows by 1.5 G..

Figure 3.18: Behavior of the capital stock and capabilities on a logarithmic scale when f grows 150% for one sector at different points in development. Dotted lines indicate the time when $f = 2.5$.

We notice that while the less developed economy on the left cannot build the necessary capabilities for the new sector and asymptotically approaches stagnation, the little further developed economy on the right manages to escape the threat of stagnation and starts to move back to the original equilibrium. The poorer economy is therefore stuck in poverty, while the richer one is able to continue growing, eventually at the same growth rate as before the shock. Note again that if the saving rate was not held constant, a smaller decrease in f would be enough to produce this effect.

Figure 3.18 shows the behavior of k and G in more detail. Again the growth rate of G reacts faster to changes.

We then move to the second effect of making a share of capital obsolete. Figure 3.19 shows the development of k, GDP, e and the consumption of unit goods, as well as the growth of the stocks of capital and capabilities on a logarithmic scale, when 20% of the capital is suddenly destroyed. As the capital stock decreases at $t = 30$, the GDP drops a little, the growth rates are boosted and after a while the GDP and the other variables actually end up higher than along the benchmark GBGP. Again note that with consumer optimization this behavior would be further enhanced.



(a) GDP, k, e and $\sum c_i$.

(b) $\log(k)$ and $\log(G)$.

Figure 3.19: Behavior of the capital stock, GDP, consumption expenditure, consumption of unit goods and the capital and capabilities on a logarithmic scale when the the capital stock decreases 20% at $t = 30$.

Chapter 4

Discussion

4.1 Realisticity of the approach

The approach employed in this thesis is reasonably realistic in the sense how it reflects our knowledge of development, the accumulation of capabilities and structural change. Most importantly, the interplay of capabilities and structural change is the driver of growth. As the economy develops, it diversifies its production structure. However, the model does not account for the specialization in later stages of development, as documented by Imbs and Wacziarg (2003). An extension of the model that includes international trade could be an option to produce this effect.

The model considers a certain level of capabilities to be a prerequisite for a certain level of production, in line with the theories of Hirschman (1958) and Bell and Pavitt (1995). Growth takes place essentially through the backward and forward linkages of Hirschman (1958): investing in a new sector that can barely operate in the present conditions creates a strong demand for new capabilities to improve the productivity of the sector (a backward linkage), and as the capabilities accumulate, they enable and invite new production activities, that absolutely require these capabilities, to take place (a forward linkage). As Bell and Pavitt (1995) theorize, the competitive equilibrium exhibits suboptimal levels of investment; a government policy would be needed to reach the optimum. As documented by Hidalgo and Hausmann (2009), the available capabilities determine the complexity of production, which correlates with the country's income level, and deviations from this relationship predict future growth, as in the model higher than balanced capability-capital ratio temporarily boosts growth rates. In line with the findings of Hausmann and Klinger (2006), the speed at which the economy can diversify into new sectors, and therefore the aggregate growth rate, depend on the "proximity" of nearby goods that are increasingly complex.

The accumulation dynamic of capabilities is perhaps both the greatest strength and the greatest weakness of the model. It is a strength, because it captures the basic intuition in a very simple way that requires no tracking of when exactly the last sectors were opened, which would be analytically challenging. But the accumulation function does assume that if the distance between two successive sectors in terms of capability requirements is longer than the previous one, the accumulation of capabilities slows down immediately after the first sector of this pair is opened. Unless we assume that the proximity of the possibility to open new production activities acts as an encouraging force to develop capabilities (kind of a backward linkage extending further back than discussed before), this is not very intuitive. Nevertheless, we take it as a necessary approximation to reduce a significant amount of complexity.

Another question concerning the accumulation of capabilities is if they truly are a spillover, i.e. an unintended consequence of investing into new functions or rather something that requires deliberate investment from the government. The literature points to both directions, and it is of course intuitive to think that social capital accumulates as a spillover, whereas infrastructure obviously needs to be invested in, usually by the government. It might also be the case that the main accumulation channel of capabilities varies according to the development phase. However, like already mentioned in section 3.2.3, as long as we assume that the government-provided capabilities are accumulated as a response to the needs of new production sectors and do not need to be funded with taxes, it should not be problematic to model them with our dynamic. The government could respond to the demand with funds received from foreign aid or resource rents (profits from extracting natural resources). In this case, the "responsiveness" of the government to the capability demand could be described with the parameter B.

There is one peculiar detail about the model worth pointing out. In a situation where the economy is stuck in the stagnant equilibrium but the equilibrium is unstable, the model implies that the stagnant state could in fact be "escaped" by destroying some of the existing capital, because it is the *relative* lack of capabilities that causes trouble for future development. In the model, the falling share of capital raises the productivity of existing capital and therefore the interest rate, which induces growth in investments. This is due to the assumption that capital and labor can freely move across sectors. If the capital employed in one sector would somehow be lost, some existing capital from other sectors would be allocated to it to respond to the demand of the consumers. This is of course somewhat unrealistic. But if the capital loss would be spread across sectors and be only partial, the raising productivity of capital and interest rates could be true. We might still question that replacing the once existed capital produces the same learning effects and contributions to capabilities as investing in new types of productive activities - after all, we do not expect this to happen when replacing depreciated capital. Contributions to capabilities might perhaps take place if the loss of capital encourages the producers to rethink and reorganize their production and to acquire new technologies significantly faster than just through replacing depreciated capital. This might be the case with climate policy, as presented in section 3.5. If the producers would suddenly have to stop using old polluting capital, they most certainly would need to replan their production and would likely also create learning spillovers while doing it. However, this might only be true for high enough levels of capabilities, especially education. In general, the destruction of capital does not seem like a very reliable way of creating a growth boost.

4.2 Innovativeness of the approach

The model is mostly built on widely approved foundational assumptions of economic modeling, and uses well established functional forms to describe production and demand. Adopting multiple sectors, adding a subsistence level of capabilities for each sector and making the accumulation of capabilities dependent on the proximity of other sectors yields unique dynamics.

Relative to the previous efforts to describe the relationship of structural change and capabilities, presented in section 2.4, we could consider this approach rather innovative: it manages - with a simple setup - to describe the desired dynamics endogenously and in compliance with the Kaldor facts. Also the stylized facts of structural change can be replicated with an appropriate assumption about the order of complexity of the sectors; we assume the sectors that become available first to include agricultural production and some very basic services, then be followed by some less basic services but mostly manufacturing of increasing complexity, and finally be followed by mostly advanced services. This will cause the value added share and the labor share of agriculture to decline, those of services to rise and those of manufacturing to exhibit a hump-shape.

It is also worth reflecting on how different the capabilities are as a source of growth compared to the development of technology, which is the neoclassical idea. In a national context, not very. As capabilities can be thought to describe the ability of a country to adopt technology, we could assume that countries are only able to develop technology that they have the capability to use. In an international context, the views differ, as the available capabilities limit the adoption of relatively advanced technology developed somewhere else. In terms of this model, the used capabilities measure G can very well be thought to include the development of the "general" level of available technology. The most important differentiator between this model and the neoclassical models is that the neoclassical models tend to model the development of technology only exogenously, which means that they are not attempting to explain it at all, whereas our approach is endogenous and can in fact be considered to explain the development of technology.

4.3 Policy implications

The contributions of this thesis are the formulation of an economic theory where development is driven by capabilities and structural change, and the insight that according to this theory an external influence, such as climate policy or globalization, may pose a threat to development. This is important, because this theory provides an explanation for the fears of developing countries regarding climate policy and may therefore help design such a policy that does not jeopardize development. To our knowledge, this is the first effort to theoretically show that climate policy can affect long term development. Let us consider what can be learnt from the used approach, and what are the possible policy implications.

First of all, the very structure of the model implies the existence of a "poverty trap", i.e. a situation of economic stagnation of which the economy cannot get out on its own, without the help of a policy or some other external assistance. In our model, a *relative* lack of capabilities can cause the economy to get stuck in the trap. This can be considered to be in line with Robinson and Acemoglu (2012) who suggest that low-quality institutions cause poverty traps.

It is noteworthy that in the model an economy of any income level may face the risk of stagnation due to a strong external influence or a change in the economic environment. There are two possible situations that can cause the economy to move to the stagnant equilibrium. First, if existing capabilities are somehow destroyed to a large enough extent, the economy might move to the stagnant equilibrium. However, if the dynamics of the economy remain unchanged, the stagnant equilibrium is unstable and to get there, the economy needs to lose a very specific amount of capabilities. A loss any larger than this would cause the economy to collapse, and a loss any smaller than this would only slow growth down, but the economy would eventually rebuild the lost capabilities and return to the original growth equilibrium. Because the limit of "too much" is determined by the loss of capabilities relative to the capital stock, richer countries with higher capital stock and capabilities can afford to lose absolutely more capabilities before they are in danger of stagnation. The second possibility to get to the stagnant equilibrium is that the accumulation of capabilities suddenly becomes much more difficult, i.e. the dynamics of the economy change and the stagnant equilibrium becomes the stable one. If the change is permanent, richer countries have no advantage over the poorer countries and their economies too will eventually stagnate - but on a higher income level, naturally. However, if the change in the dynamics is temporary, i.e. the accumulation of capabilities is only harder for a time due to a disturbance in a specific sector, the richer countries are less likely to stagnate. Overall the model implies that more developed countries have more resilience to temporary disturbances of structural change and capability accumulation.

In section 3.5 we already assessed that climate policy could change the dynamics of the economy temporarily by disabling some sectors or by raising the capability requirements for certain sectors. As less developed countries are more severely affected, one could draw a conclusion that it is best to let developing countries grow now as fast as possible with the help of fossil fuels and then deal with the consequences of climate change later, when the economies are better equipped to handle the shock.

To evaluate this idea, we need to consider the damages and changes that climate change is likely to cause. Climate change might increase the capability requirements of sectors in terms of infrastructure and perhaps education and institutions, due to the changing conditions. This is also acknowledged by Bowen et al. (2012), who argues that investment in infrastructure and efforts to stimulate entrepreneurship and competitive markets must recognise climate risks and take more of a risk management perspective. In the model framework, the effect would then be similar as described for climate policy in section 3.5, i.e. a temporary - or perhaps permanent - spreading of G_0s further apart. However, losses of infrastructure and thus capabilities due to hardened weather conditions are still likely. Another significant issue are the risks that climate change poses to agriculture and in some areas to water supply, and therefore to food and water security. Loss of crops due to drought or excessive rain could quickly raise the prices of food and cause rising inequality and polarization of the society. Desertification could completely disable agriculture in some areas and the rising sea levels could force massive amounts of people to relocate, which can be expected to create tensions. These are not situations that foster high levels of social capital (Woolcock, 1998), and instability and risks of conflicts could severely harm the capabilities of an economy, up to the extent that the capabilities are low enough to cause the economy to effectively collapse. It seems unlikely that the developing countries could develop fast enough to gain enough resilience to counter these threats even with the continued help of fossil fuels.

Because on the one hand strict climate policy measures too early might drive developing countries to poverty traps and on the other hand too much delay could also lead to the same or worse, an optimal policy response could be a question of timing. Some level of climate stress is unavoidable, as the changes have already started to take place, and the resilience of developing countries must be strengthened. However, allowing developing countries to fuel their growth with further building of coal power plants will lead to *lock-in effects* for decades, and this alone is likely to make climate mitigation targets infeasible (Steckel et al., 2015). This seems to be much too large of a risk to take.

Therefore, developing countries might not be able to tackle this challenge on their own and the best response seems to be that developed countries support developing countries in accumulating the necessary capabilities for continued growth while limiting emissions. According to the model framework, this can be done in two ways: either by supporting the vitality of the crucial sectors that accumulate needed capabilities or by building the capabilities directly. It is likely that both are needed for an optimal outcome.

Further research is needed to determine the optimal policy, because the affected sectors and the capabilities they would have provided need to be identified. It should be assessed whether it makes more sense to support the sectors or to attempt to create the capabilities through different means. If climate policy harms the competitiveness of a sector, it could for example be supported by exempting it from climate policy or by rehearsing some type of protectionism or by granting subsidies. Another approach would be to support the sector in building the capabilities it needs to be competitive, e.g. through technology transfers, different kinds of education and consulting. This could be the right approach also if the required minimum capability level for the sector to operate has increased. In some cases it might make sense to abandon the sector and simply attempt to build the capabilities required for more advanced production through different means. A sector-specific assessment might ease the identification of the specific capabilities that are necessary at a specific time and in a specific situation.

The supporting of specific sectors could hopefully allow the economy to develop organically and to avoid the poverty trap. However, it might be useful to accelerate the development through the building of well-known important capabilities. Based on what we have learned in chapter 2, we can form an initial hypothesis that it seems crucial to help developing countries electrify their economies using low-carbon energy sources and to help them build low-carbon transport infrastructure, such as railways, along with helping them gain the capability to operate them. The development of social capital could be supported by reducing inequality, for instance through initiatives such as universal basic income, free education and access to financing, which could be expected to lead to improving institutional quality.

Chapter 5

Conclusion

The objectives of this thesis were to construct a theory of economic growth that is consistent with the literature on capabilities, the main observed features of economic development and the newest empirical findings regarding structural change, and to use the attained framework to provide insights on the possible implications of the introduction of climate policy to developing economies.

This thesis has presented an endogenous multi-sector growth model, in which growth is driven by the interplay of societal capabilities and ongoing structural change. The behavior of the model replicates the stylized facts of economic growth and structural change.

The model is also consistent with the empirical findings concerning disaggregated structural change. The available capabilities determine the complexity and the diversity of the economy, and the speed at which the economy can move to increasingly complex activities depends on the proximity of new potential sectors, in terms of the capabilities that they require.

There are two equilibria in the model, one of them representing stagnation and the other one representing balanced growth. An external influence that disables a production sector, raises the capabilities needed by a production sector, hinders the development of capabilities or decreases the available capabilities may cause the economy to move to the stagnant equilibrium. Less developed countries are especially in danger, and even if the disturbance would not drive them to the "poverty trap", they suffer more from these disturbances in terms of forgone GDP than more developed countries.

Hence, also the adoption of climate policy measures is recognized to pose a threat to development. To our knowledge, this is the first effort to theoretically show that climate policy can have an effect on long term development. This is important, because this theory provides a possible explanation for the fears of developing countries regarding climate policy and may therefore help design such a policy that does not jeopardize development. This theory suggests a focus on capabilities in policy design.

This result is of course limited by the assumptions from which it follows. Most crucially capabilities are assumed mainly to accumulate as a spillover from investing into new productive activities. If capabilities instead mainly accumulate as the government's deliberate investment decision independent of the needs of the current production, climate policy need not threaten development through the mechanism presented here.

Future research efforts should be directed to gain a deeper understanding of the accumulation mechanics of societal capabilities, and to recognize the capabilities that are necessary to successfully industrialize an economy. To mitigate the risks that climate policy poses to developing economies, it would be useful to identify the sectors that are likely to be affected by climate policy and to analyze the capabilities they would have provided. This knowledge can be used to plan an optimal policy to support the accumulation of necessary capabilities while limiting emissions.

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Appendix A

Calculations

A.1 Optimization problem of the firm

The optimization problem of the firm is

$$\max_{L_i(t),K_i(t)} \pi_i(t) = P_i(t)Y_i(t) - W(t)L_i(t) - R(t)K_i(t), \tag{A.1}$$

The first-order conditions for this problem are

$$\frac{\partial \pi_i(t)}{\partial L_i(t)} = P_i(t)MP_{L_i(t)} - W(t) = P_i(t)(1-\alpha_i)A_i(t)k_i(t)^{\alpha_i}(G(t)-G_{0i})^{\beta_i} - W(t) = 0 \tag{A.2}$$

$$\frac{\partial \pi_i(t)}{\partial K_i(t)} = P_i(t)MP_{K_i(t)} - R(t) = P_i(t)\alpha_i A_i(t)k_i(t)^{\alpha_i-1}(G(t)-G_{0i})^{\beta_i} - R(t) = 0, \tag{A.3}$$

which give

$$W(t) = P_i(t)MP_{L_i(t)} = P_i(t)(1-\alpha_i)A_i(t)k_i(t)^{\alpha_i}(G(t)-G_{0i})^{\beta_i} \tag{A.4}$$

$$R(t) = P_i(t)MP_{K_i(t)} = P_i(t)\alpha_i A_i(t)k_i(t)^{\alpha_i-1}(G(t)-G_{0i})^{\beta_i}, \tag{A.5}$$

where we have denoted $k_i(t) = K_i(t)/L_i(t)$, the capital-labor ratio in sector i at time t. Because the firms operate under perfect competition, the absolute output prices represent unit cost

$$P_i(t) = \frac{W(t)L_i(t) + R(t)K_i(t)}{Y_i(t)}. \tag{A.6}$$

By multiplying both sides with $Y_i(t)$, we notice that

$$P_i(t)Y_i(t) = W(t)L_i(t) + R(t)K_i(t). \tag{A.7}$$

Because revenues equal the costs, the firm's profits are always zero.

A.2 Optimization problem of the household

The optimization problem of the household is

$$\begin{aligned} &\max_{c_i(t)} \int_0^\infty \mathrm{e}^{-\rho t} \ln \left(\sum_{i=1}^{n(t)} c_i(t)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} dt \\ &\text{s.t.} \quad \dot{k}(t) = (r(t)-\delta)k(t) + w(t) - \sum_{i=1}^{n(t)} p_i(t)c_i(t). \end{aligned} \tag{A.8}$$

Let us first leave the intertemporal element out and just maximize utility for a given timepoint t. The problem then becomes

$$\begin{aligned} &\max_{c_i(t)} \ln \left(\sum_{i=1}^{n(t)} c_i(t)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \\ &\text{s.t.} \quad \dot{k}(t) = (r(t)-\delta)k(t) + w(t) - \sum_{i=1}^{n(t)} p_i(t)c_i(t). \end{aligned} \tag{A.9}$$

The Lagrangian is

$$\mathcal{L}(c_i(t), \lambda(t)) = \ln \left(\sum_{i=1}^{n(t)} c_i(t)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} + \lambda(t)((r(t)-\delta)k(t) + w(t) - \sum_{i=1}^{n(t)} p_i(t)c_i(t) - \dot{k}(t)). \tag{A.10}$$

The first order conditions yield

$$\Big(\frac{c_i(t)}{c_j(t)}\Big)^{-\frac{1}{\epsilon}} = \frac{p_i(t)}{p_j(t)} \quad \forall i \neq j \in [1, n(t)]. \tag{A.11}$$

Following Acemoglu (2008) on page 556 we define an ideal price index $P(t)$ so that

$$\Big(\frac{c_j(t)}{C(t)}\Big)^{-\frac{1}{\epsilon}} = \frac{p_j(t)}{P(t)}. \tag{A.12}$$

We now solve for $P(t)$:

$$P(t) = p_j(t)\Big(\frac{C(t)}{c_j(t)}\Big)^{-\frac{1}{\epsilon}} = p_j(t)\Big(\sum_{i=1}^{n(t)} \Big(\frac{c_i(t)}{c_j(t)}\Big)^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{1}{1-\epsilon}}. \tag{A.13}$$

We can now substitute (A.11) to get:

$$P(t) = p_j(t)\Big(\sum_{i=1}^{n(t)} \Big(\frac{p_i(t)}{p_j(t)}\Big)^{1-\epsilon}\Big)^{\frac{1}{1-\epsilon}} = \Big(\sum_{i=1}^{n(t)} p_i(t)^{1-\epsilon}\Big)^{\frac{1}{1-\epsilon}}. \tag{A.14}$$

We can now express consumption demand for good i in terms of its price $p_i(t)$, the price index $P(t)$ and the total consumption expenditure $e(t)$. Let us start by rearranging (A.11) to get

$$c_i(t) = \Big(\frac{p_i(t)}{p_j(t)}\Big)^{-\epsilon} c_j(t),$$

and continue by multiplying both sides with $p_i(t)$ and summing over all products i

$$\sum_{i=1}^{n(t)} p_i(t)c_i(t) = p_j(t)^{\epsilon} c_j(t) \sum_{i=1}^{n(t)} p_i(t)^{1-\epsilon}.$$

We can now see that the left hand side of the equation is the total consumption expenditure $e(t)$ and on the right hand side we have our price index $P(t)$. Rearranging and changing the notation of the index from j to i we get the demand

$$c_i(t) = \frac{p_i(t)^{-\epsilon}}{P(t)^{1-\epsilon}} e(t). \tag{A.15}$$

We can now substitute this into the definition of the consumption index

$$C(t) = \Big(\sum_{i=1}^{n(t)} \Big(\frac{p_i(t)^{-\epsilon}}{P(t)^{1-\epsilon}} e(t)\Big)^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{\epsilon}{\epsilon-1}},$$

and simplify, use the definition of price index (A.14) again and rearrange to get

$$e(t) = P(t)C(t). \tag{A.16}$$

Substituting these to the optimization problem yields

$$\max_{C(t)} \int_0^\infty \mathrm{e}^{-\rho t} \ln C(t) dt$$
$$\text{s.t.} \quad \dot{k}(t) = (r(t) - \delta)k(t) + w(t) - P(t)C(t). \tag{A.17}$$

The present-value Hamiltonian is

$$H = \mathrm{e}^{-\rho t} \ln C(t) + \nu(t)((r(t) - \delta)k(t) + w(t) - P(t)C(t)), \tag{A.18}$$

where $\nu(t)$ is a costate variable. The first-order conditions are

$$\frac{\partial H}{\partial C} = \frac{\mathrm{e}^{-\rho t}}{C(t)} - \nu(t)P(t) = 0, \tag{A.19}$$

$$\frac{\partial H}{\partial k} = \nu(t)((r(t) - \delta) = -\dot{\nu}(t), \tag{A.20}$$

and the standard transversality condition is

$$\lim_{t\to\infty} \nu(t)k(t) = 0. \tag{A.21}$$

We now solve the first order equations to attain the Euler equation. We start by solving for $\nu(t)$ in (A.19) and get

$$\nu(t) = \frac{\mathrm{e}^{-\rho t}}{P(t)C(t)} = \frac{\mathrm{e}^{-\rho t}}{e(t)}. \tag{A.22}$$

We now take the derivative with respect to time on both sides to get

$$\dot{\nu}(t) = -\frac{\rho \mathrm{e}^{-\rho t}}{e(t)} - \frac{\mathrm{e}^{-\rho t}}{e(t)^2}\dot{e}(t). \tag{A.23}$$

We now substitute (A.22) ja (A.23) into (A.20) and simplify to get

$$\frac{\dot{e}(t)}{e(t)} = r(t) - \delta - \rho, \tag{A.24}$$

which is the Euler equation.

A.3 Factor allocations

The sectoral output elasticities are

$$\zeta_{L_i} = \frac{\partial Y_i(t)}{\partial L_i(t)} \frac{L_i(t)}{Y_i(t)} = 1 - \alpha_i, \quad \zeta_{K_i} = \frac{\partial Y_i(t)}{\partial K_i(t)} \frac{K_i(t)}{Y_i(t)} = \alpha_i. \tag{A.25}$$

The define the GDP expenditure shares $s_i(t)$ as

$$s_i(t) = \frac{p_i(t) Y_i(t)}{Y(t)}, \quad \sum_{i=1}^{n(t)+1} s_i(t) = 1. \tag{A.26}$$

The expenditure shares for consumption goods are

$$s_i(t) = \frac{L e_i(t)}{Y(t)} \quad \forall i \in [1, n(t)], \tag{A.27}$$

And the expenditure share of investment is

$$s_x(t) = \frac{X(t)}{Y(t)} = \frac{Y(t) - Le(t)}{Y(t)}. \tag{A.28}$$

We define the macro factor income shares

$$\eta_L(t) = \frac{w(t) L}{Y(t)}; \quad \eta_K(t) = \frac{r(t) K(t)}{Y(t)}. \tag{A.29}$$

If we now note that based on equations (A.25),(3.11),(3.12) and (A.26)

$$\zeta_{L_i} = \frac{\partial Y_i(t)}{\partial L_i(t)} \frac{L_i(t)}{Y_i(t)} = \frac{w(t)}{p_i(t)} \frac{L_i(t)}{Y_i(t)} = \frac{w(t)L_i(t)}{s_i(t)Y(t)},$$
$$\zeta_{K_i} = \frac{\partial Y_i(t)}{\partial K_i(t)} \frac{K_i(t)}{Y_i(t)} = \frac{r(t)}{p_i(t)} \frac{K_i(t)}{Y_i(t)} = \frac{r(t)K_i(t)}{s_i(t)Y(t)}, \quad \text{(A.30)}$$

we can determine the factor allocation fractions in terms of expenditure shares, sectoral output elasticities and macro factor income shares based on equations (3.30), (3.31), (A.25), (A.29) and (A.30) as

$$\lambda_{L_i}(t) = \frac{L_i(t)}{L} = \frac{w(t)L_i(t)Y(t)}{w(t)LY(t)} = \frac{w(t)L_i(t)}{Y(t)} \frac{Y(t)}{w(t)L} = \frac{\zeta_{L_i} s_i(t)}{\eta_L(t)} = \frac{(1-\alpha_i)s_i(t)}{\eta_L(t)},$$
$$\lambda_{K_i}(t) = \frac{K_i(t)}{K(t)} = \frac{r(t)K_i(t)Y(t)}{r(t)K(t)Y(t)} = \frac{r(t)K_i(t)}{Y(t)} \frac{Y(t)}{r(t)K(t)} = \frac{\zeta_{K_i} s_i(t)}{\eta_K(t)} = \frac{\alpha_i s_i(t)}{\eta_K(t)}. \quad \text{(A.31)}$$

We can now substitute equations (A.27) and (A.28) back in to get

$$\lambda_{L_i}(t) = \frac{(1-\alpha_i)e_i(t)}{w(t)} \quad \forall i \in [1, n(t)], \quad \lambda_{L_x}(t) = \frac{(1-\alpha_x)(y(t)-e(t))}{w(t)}, \quad \text{(A.32)}$$
$$\lambda_{K_i}(t) = \frac{\alpha_i e_i(t)}{r(t)k(t)} \quad \forall i \in [1, n(t)], \quad \lambda_{K_x}(t) = \frac{\alpha_x(y(t)-e(t))}{r(t)k(t)}. \quad \text{(A.33)}$$

A.4 Stability of the equilibria

As our system is nonlinear, we linearize the system around the equilibria z^*, a^* and via the determinant and the trace of the Jacobian matrix of the linearized system we draw conclusions about the eigenvalues, which characterize the behavior of the system at least locally in the neighborhoods of the equilibria.

We define the system

$$\dot{z} = f(z, a) = \left(\frac{1}{d} - 1\right)(z^{1-\alpha} - \delta - a), \quad \text{(A.34)}$$

$$\dot{a} = g(z,a) = ((\alpha - 1)z^{1-\alpha} - \rho + \alpha)\alpha. \tag{A.35}$$

We then set new variables as the distances from the equilibrium points

$$u = z - z^*, v = a - a^*. \tag{A.36}$$

Note that $\dot{u} = \dot{z}$ and $\dot{v} = \dot{a}$. To linearize the system, we approximate it around the equilibria with a Taylor series expansion as follows:

$$\dot{u} \approx f(z^*, a^*) + \frac{\partial f}{\partial z}\Big|_{z^*,a^*} u + \frac{\partial f}{\partial a}\Big|_{z^*,a^*} v, \tag{A.37}$$

$$\dot{v} \approx g(z^*, a^*) + \frac{\partial g}{\partial z}\Big|_{z^*,a^*} u + \frac{\partial g}{\partial a}\Big|_{z^*,a^*} v. \tag{A.38}$$

Note that $f(z^*, a^*) = g(z^*, a^*) = 0$. We can write this as matrices

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f}{\partial z}\Big|_{z^*,a^*} & \frac{\partial f}{\partial a}\Big|_{z^*,a^*} \\ \frac{\partial g}{\partial z}\Big|_{z^*,a^*} & \frac{\partial g}{\partial a}\Big|_{z^*,a^*} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}, \tag{A.39}$$

where the coefficient matrix is the Jacobian matrix of the system. Let us denote it by J. The eigenvalues of J carry important information about the behavior of the system near the equilibrium points. If the eigenvalues are real and of opposite signs, the phase portrait is a saddle. If the real parts of eigenvalues are both negative, the equilibrium is stable and the phase portrait is a sink. If they are both positive, the equilibrium is unstable and the phase portrait is a source. Let

$$\tau = \mathrm{tr}J = \frac{\partial f}{\partial z}\Big|_{z^*,a^*} + \frac{\partial g}{\partial a}\Big|_{z^*,a^*} \tag{A.40}$$

be the trace and

$$D = \det J = \frac{\partial f}{\partial z}\Big|_{z^*,a^*} \frac{\partial g}{\partial a}\Big|_{z^*,a^*} - \frac{\partial f}{\partial a}\Big|_{z^*,a^*} \frac{\partial g}{\partial z}\Big|_{z^*,a^*} \tag{A.41}$$

be the determinant of the Jacobian matrix. The eigenvalues of the matrix are the roots of the characteristic polynomial, defined as $p(\lambda) = \det(J - \lambda I)$. We can now express the polynomial with the help of the trace and determinant as

$$p(\lambda) = \lambda^2 - \tau\lambda + D. \tag{A.42}$$

The roots of $p(\lambda)$ can be found with the help of the quadratic formula

$$\lambda = \frac{\tau \pm \sqrt{\tau^2 - 4D}}{2}. \tag{A.43}$$

Based on this equation we can draw conclusions about the eigenvalues with the help of the trace and the determinant. Clearly if $D < 0$, the eigenvalues are real and of opposite signs. If $D > 0$, The sign of the (real part of the) eigenvalues depends on the trace; if $\tau < 0$, the eigenvalues are negative, and if $\tau > 0$, the eigenvalues are positive.

The partial derivatives are

$$\frac{\partial f}{\partial z}\Big|_{z^*,a^*} = \frac{(1-\alpha)}{d} z^{*-\alpha} - (2-\alpha)z^{*1-\alpha} + \delta + a^*, \tag{A.44}$$

$$\frac{\partial f}{\partial a}\Big|_{z^*,a^*} = z^* - \frac{1}{d} \tag{A.45}$$

$$\frac{\partial g}{\partial z}\Big|_{z^*,a^*} = -(1-\alpha)^2 a^* z^{*-\alpha} \tag{A.46}$$

$$\frac{\partial g}{\partial a}\Big|_{z^*,a^*} = 2a^* - (1-\alpha)z^{*1-\alpha} - \rho \tag{A.47}$$

Let us then proceed to calculating the trace and the determinant at both equilibrium points. We start with $z^* = 1/d, a^* = (1-\alpha)(1/d)^{1-\alpha} + \rho$. As we can straight away see that $\frac{\partial f}{\partial a}\big|_{z^*,a^*} = 0$, we get

$$D = \frac{\partial f}{\partial z}\Big|_{z^*,a^*} \frac{\partial g}{\partial a}\Big|_{z^*,a^*} = (-\alpha\Big(\frac{1}{d}\Big)^{1-\alpha} + \delta + \rho)((1-\alpha)\Big(\frac{1}{d}\Big)^{1-\alpha} + \rho). \tag{A.48}$$

We can see that $\frac{\partial g}{\partial a}\big|_{z^*,a^*}$ is clearly always positive. Therefore the determinant is negative, when $\frac{\partial f}{\partial z}\big|_{z^*,a^*}$ is negative, i.e. when

$$d < \Big(\frac{\alpha}{\delta + \rho}\Big)^{\frac{1}{1-\alpha}}. \tag{A.49}$$

When this condition holds, this equilibrium is saddle-path stable. If $\frac{\partial f}{\partial z}\big|_{z^*,a^*}$ is positive, i.e. when $d > \big(\frac{\alpha}{\delta+\rho}\big)^{\frac{1}{1-\alpha}}$, the determinant is positive and we must look at the trace to determine the stability properties. Recall that the trace is

$$\tau = \frac{\partial f}{\partial z}\Big|_{z^*,a^*} + \frac{\partial g}{\partial a}\Big|_{z^*,a^*}, \tag{A.50}$$

which is clearly positive in our area of interest, defined by $\frac{\partial f}{\partial z}\big|_{z^*,a^*} > 0$. Therefore we can conclude that this equilibrium is unstable, when $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$. In the special case when $d = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, $D = 0$ and this determinant-trace-analysis is inconclusive. Note that this responds to the situation when the two equilibria of the system are at the exact same point.

Let us then look at the other equilibrium point, $z^* = \left(\frac{\delta+\rho}{\alpha}\right)^{\frac{1}{1-\alpha}}$, $a^* = \frac{(1-\alpha)\delta+\rho}{\alpha}$. Now

$$\begin{aligned} D = &\Big[(1-\alpha)\frac{\delta+\rho}{\alpha}\Big(\frac{1}{d}\Big(\frac{\delta+\rho}{\alpha}\Big)^{\frac{-1}{1-\alpha}} - 1\Big)\Big]\Big[\frac{(1-\alpha)\delta+\rho}{\alpha}\Big] - \\ &\Big[\Big(\frac{\delta+\rho}{\alpha}\Big)^{\frac{1}{1-\alpha}} - \frac{1}{d}\Big]\Big[-(1-\alpha)^2\Big(\frac{\delta+\rho}{\alpha}\Big)^{\frac{-\alpha}{1-\alpha}}\frac{(1-\alpha)\delta+\rho}{\alpha}\Big]. \end{aligned} \tag{A.51}$$

We can see that $\frac{\partial g}{\partial a}\big|_{z^*,a^*}$ is again always positive and that $\frac{\partial g}{\partial z}\big|_{z^*,a^*}$ is always negative. When $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$, then $\frac{\partial f}{\partial z}\big|_{z^*,a^*} < 0$ but $\frac{\partial f}{\partial a}\big|_{z^*,a^*} > 0$. This time we cannot draw conclusions about the sign of the determinant just by looking at the pieces. Let us rearrange the second term of the equation. We note that $\left(\frac{\delta+\rho}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}} = \frac{\delta+\rho}{\alpha}\left(\frac{\delta+\rho}{\alpha}\right)^{\frac{-1}{1-\alpha}}$ to get

$$\begin{aligned} D = &(1-\alpha)\frac{\delta+\rho}{\alpha}\Big(\frac{1}{d}\Big(\frac{\delta+\rho}{\alpha}\Big)^{\frac{-1}{1-\alpha}} - 1\Big)\frac{(1-\alpha)\delta+\rho}{\alpha} - \\ &(1-\alpha)^2\frac{\delta+\rho}{\alpha}\Big(\frac{1}{d}\Big(\frac{\delta+\rho}{\alpha}\Big)^{\frac{-1}{1-\alpha}} - 1\Big)\frac{(1-\alpha)\delta+\rho}{\alpha}. \end{aligned} \tag{A.52}$$

We can now see that the absolute value of the first term of the determinant is always larger than the absolute value of the second term, and therefore the sign of the determinant is determined by the first term, specifically by $\frac{\partial f}{\partial z}\big|_{z^*,a^*}$. We can conclude that the determinant is negative when $\frac{\partial f}{\partial z}\big|_{z^*,a^*}$ is negative, i.e. when $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$ and positive when $\frac{\partial f}{\partial z}\big|_{z^*,a^*}$ is positive, i.e. when $d < \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$. The trace is again positive whenever the determinant is positive. Therefore, the equilibrium is saddle-path stable when $d > \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$ and unstable when $d < \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$.

A.5 Optimization problem of the social planner

The optimization problem of the social planner is

$$\max_{C(t)} \int_0^\infty \mathrm{e}^{-\rho t} \ln C(t) dt$$

$$\text{s.t.} \quad \dot{k}(t) = k(t)^\alpha G(t)^{1-\alpha} - \delta k(t) - P(t)C(t),$$

$$\dot{G}(t) = \frac{\dot{k}(t)}{d} = \frac{1}{d}(k(t)^\alpha G(t)^{1-\alpha} - \delta k(t) - P(t)C(t)). \quad \text{(A.53)}$$

The present-value Hamiltonian is

$$H = \mathrm{e}^{-\rho t} \ln C(t) + (\nu(t) + \frac{\mu(t)}{d})(k(t)^\alpha G(t)^{1-\alpha} - \delta k(t) - P(t)C(t)), \quad \text{(A.54)}$$

where $\nu(t)$ and $\mu(t)$ are costate variables. The first-order conditions are

$$\frac{\partial H}{\partial C} = \frac{\mathrm{e}^{-\rho t}}{C(t)} - (\nu(t) + \frac{\mu(t)}{d})P(t) = 0, \quad \text{(A.55)}$$

$$\frac{\partial H}{\partial k} = (\nu(t) + \frac{\mu(t)}{d})(\alpha\Big(\frac{G}{k}\Big)^{1-\alpha} - \delta) = -\dot{\nu}(t), \quad \text{(A.56)}$$

$$\frac{\partial H}{\partial G} = (\nu(t) + \frac{\mu(t)}{d})(1-\alpha)\Big(\frac{G}{k}\Big)^{-\alpha} = -\dot{\mu}(t), \quad \text{(A.57)}$$

We can now use (A.55) to solve for

$$\nu(t) + \frac{\mu(t)}{d} = \frac{\mathrm{e}^{-\rho t}}{C(t)P(t)} = \frac{\mathrm{e}^{-\rho t}}{e(t)}, \quad \text{(A.58)}$$

and substitute this into equations (A.56) and (A.57). We now take the time derivative of (A.58) to get

$$\dot{\nu}(t) + \frac{\dot{\mu}(t)}{d} = -\frac{-\rho \mathrm{e}^{-\rho t}}{e(t)} - \frac{\mathrm{e}^{-\rho t}}{e(t)^2}\dot{e}(t). \quad \text{(A.59)}$$

We then substitute (A.56) and (A.57) here and simplify, which yields the

Euler equation

$$\frac{\dot{e}}{e} = \alpha\Big(\frac{G}{k}\Big)^{1-\alpha} + \frac{1-\alpha}{d}\Big(\frac{G}{k}\Big)^{-\alpha} - \delta - \rho. \tag{A.60}$$

A.6 Constant saving rate

In this section we briefly show that the behavior of the system is qualitatively similar with a constant saving rate and with consumer optimization.

Utilizing the saving rate $s = \frac{X}{y} = 1 - \frac{e}{y}$ we can write the capital accumulation rule as

$$\dot{k} = sy - \delta k = sk^{\alpha}G^{1-\alpha} - \delta k, \tag{A.61}$$

so the growth rate of the capital stock is

$$\frac{\dot{k}}{k} = s\Big(\frac{G}{k}\Big)^{1-\alpha} - \delta = sz^{1-\alpha} - \delta. \tag{A.62}$$

The growth rate of the capabilities is

$$\frac{\dot{G}}{G} = \frac{\dot{k}}{dG} = \frac{1}{dz}(sz^{1-\alpha} - \delta), \tag{A.63}$$

so the growth rate of z is

$$\frac{\dot{z}}{z} = \frac{\dot{G}}{G} - \frac{\dot{k}}{k} = \Big(\frac{1}{dz} - 1\Big)(sz^{1-\alpha} - \delta). \tag{A.64}$$

This equals zero at two points: at $z = \frac{1}{d}$ and at $z = \big(\frac{\delta}{s}\big)^{\frac{1}{1-\alpha}}$. The first one is not dependent on s and always coincides with the non-stagnant equilibrium of the system with consumer optimization. The second one only coincides with the stagnant equilibrium if s equals the stagnant equilibrium saving rate $s^* = \frac{\alpha\delta}{\delta+\rho}$. For the simulations in section 3.6 we will however use the non-stagnant equilibrium saving rate $s^* = \alpha - \rho d^{1-\alpha}$. As a result the location of the stagnant equilibrium will differ a little with the constant saving rate.

With the help of equation (A.64) we can now sketch the behavior of the growth rate of z as function of z for the two possible orders of the equilibria for some constant s. These are presented in Figures A.1 and A.2. The arrows indicate the direction of movement along the path. We can see that as with the consumer optimization, the equilibrium further on the left is always unstable and the equilibrium on the right is stable. The behavior of the growth rates of k and G is qualitatively similar to consumer optimization: when the positive-growth-equilibrium $z = \frac{1}{d}$ is approached from the left, G grows faster than k, and the vice versa when the equilibrium is approached from the right.

Section 3.4.2 already discussed the differences in the transitional dynamics between constant and consumer optimized saving rates. In summary, the behavior is qualitatively the same, as growth always slows down left of the positive growth equilibrium $z = \frac{1}{d}$ and speeds up right of the equilibrium. The optimization of the saving rate just enhances this behavior.

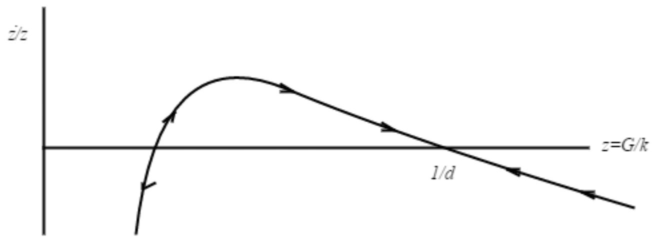

Figure A.1: Transitional dynamics of the model for a constant saving rate when d is relatively small.

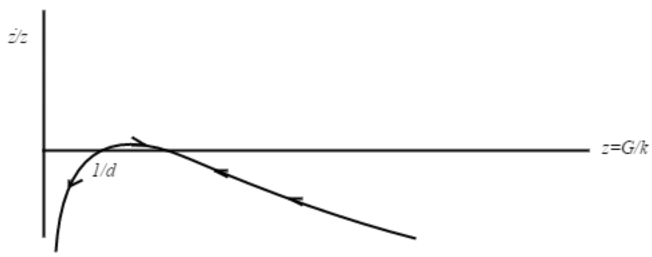

Figure A.2: Transitional dynamics of the model for a constant saving rate when d is relatively large.