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Statistical estimation of price elasticity to support aftermarket pricing

Master's Thesis
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ABSTRACT OF MASTER'S THESIS

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Efficient pricing is a crucial factor in profitability, but especially in the aftermarket business the vast amount of sales items might drive companies to base pricing decisions solely on acquisition costs. Even though this might save time in the pricing process, such methods generally do not capture the true value the customers see in the item. Consequential over- and underpricing is likely to result in lost profit. Understanding the price sensitivity of the customers can help companies implement pricing that is satisfactory for both parties.

The aim of this master's thesis it to estimate the price elasticity of demand for several types of spare parts. In order to provide general guidelines to support aftermarket pricing, we divide items into segments based on predefined item attributes, and use price elasticity of demand to illustrate general behavior of demand for that segment. The sales items we analyze in this study are frequency converter spare parts and spare part bundles offered by a frequency converter manufacturer on global markets.

We discover that contrary to literature based hypothesis, our data does not show evidence that the criticality or company specificity of a spare part would significantly reduce the price sensitivity of the customers. This might indicate that there are other more significant underlying factors affecting the price response. With a more precise segmentation framework we find that for example the life cycle phase of the related core product is likely to affect the price elasticity of demand for some types of components. Generally, the achieved price elasticity estimates indicated relatively inelastic demand, which we suspect might well from the special characteristics of the aftermarket business. For the item segments for which the estimates indicated elastic demand, we identified possible alternatives that the customers might consider in purchase situation, thus resulting in more elastic demand.

Our model and analysis is based on multiple simplifications and the achieved results are rather indicative than the exact truth. It should be noted that the estimates are segment specific and in reality each segment is likely to contain items that react differently to price changes.

Keywords: Price elasticity of demand, aftermaket pricing, spare parts

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Hinnoittelulla on keskeinen merkitys kannattavassa liiketoiminnassa. Erityisesti jälkimyynnin myyntiartikkeleiden paljous saattaa kuitenkin kannustaa yhtiöitä perustamaan hinnoittelupäätöksensä kustannuspohjaisiin menetelmiin. Vaikka kyseiset menetelmät mahdollistavat yksinkertaiset ja nopeat hinnoittelupäätökset, eivät saavutetut hinnat yleensä vastaa asiakkaiden näkemystä tuotteen todellisesta arvosta.

Tämän diplomityön tavoitteena on tukea jälkimyynnin hinnoittelupäätöksiä tutkimalla tietynlaisten varaosien kysynnän hintajoustoa. Tarkoituksena on luoda yleiskäsitys asiakkaiden hintaherkkyydestä, minkä vuoksi analyysi kohdistetaan eri ominaisuuksien perusteella jaoteltuihin varaosarykelmiin. Työssä analysoidaan taajuusmuuttajavalmistajan tarjoamia tuotteisiin liittyviä varaosia maailmanlaajuisilla markkinoilla.

Vastoin kirjallisuuteen pohjautuvia hypoteeseja analyysimme tulokset eivät tue ajatusta, että varaosan kriittisyys tai rajoittunut kaupallinen saatavuus erityisesti vähentäisi asiakkaiden hintaherkkyyttä. Syynä tähän saattaa olla, että kysynnän taustalla on muita tekijöitä, joiden vaikutus hintajoustoon on merkittävämpi. Yksityiskohtaisemman varaosien jaottelun avulla huomaamme, että esimerkiksi varaosaan liittyvän taajuusmuuttajan elinkaarivaiheella vaikuttaa olevan merkitystä tietyntyyppisten komponenttien hintajoustoon. Työssä laskettujen hintajoustoestimaattien perusteella varaosien kysyntä on suhteellisen joustamatonta. Tämän arvellaan johtuvan jälkimyyntimarkkinoiden erityisestä luonteesta esimerkiksi tavallisiin kulutustuotteisiin verrattuna. Varaosaryhmittymille, joiden estimaatit viittaavat joustavaan kysyntään, tunnistettiin vaihtoehtoisia ratkaisuja, joita asiakas mahdollisesti ostotilanteessa harkitsee.

Rakennettu malli ja siten koko analyysi pohjautuu useisiin yksinkertaistuksiin ja siten on tärkeää ymmärtää että saavutetut tulokset ovat suuntaa-antavia. Työssä saavutetut estimaatit on laskettu varaosarykelmille. Tuloksia tulkitessa on olennaista muistaa, että määritellyt rykelmät todennäköisesti sisältävät tuotteita, joiden kysyntä reagoi hinnan muutoksin varsin eri tavoin.

Asiasanat:	Kysynnän hintajousto, jälkimyynti, varaosat, hinnoittelu
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Espoo, September 8, 2015

Tiina Kangasniemi

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Chapter 1

Introduction

1.1 Background

Price — the combination of numbers in a tag attached to a product or service that makes us question if the money we are about to pay is worthy of the utility received. Not only does the price reflect the value of the product but it also is an important tool for marketing and competition. Selecting a suitable number combination for the price tag, say for an industrial product, is no black-and-white task: the manufacturer needs to balance between production costs and evaluating how the customers value the product, naturally considering competition, marketing expenses, promotions and so on.

Price is a crucial factor in profitability. Too small prices fail to cover for production, storage, and transportation costs whereas too high prices risk alienating the customer and dropping the demand. The role of pricing as a competitive tool is quite intuitive from the viewpoint of the consumer: when comparing two similar products with equivalent features and quality, the price difference is likely to affect the selection. Consider manufacturers of expensive durables or industrial products: products are relatively pricey and acquired by consumers, other manufacturers or service providers at frequency of one product maybe in ten or twenty years. Whereas competing in price, features and quality cannot go on forever, other instruments of competition are bound to surface. As a potential source of additional profit and competitive advantage, in many industries the equipment manufacturers have started to extend from pure manufacturing to offering after-sales services for their customers.

In the aftermarkets manufacturers and service suppliers provide spare parts and services to create additional value for the customers. The significance of after-sales services and spare parts is emphasized with expensive durables in business-to-business markets. In this situation, the core product supplied by the manufacturer is often a tool for the customer to produce their own product or service. Consequently, an unexpected and long downtime might turn out to be costly, not because the equipment is expensive and repair costs money, but because in some industries every downtime second leads to lost profit.

Compared to pricing in general, the pricing of services and especially pricing of spare parts has received relatively little academic attention. We believe that understanding the price sensitivity of the customers and the underlying determinants affecting the price sensitivity have the potential of providing support for aftermarket pricing decisions.

1.2 Objectives and scope

This thesis focuses on aftermaket pricing, more precisely in pricing of spare parts and spare part bundles. The aim is to understand how price changes affect the demand for spare parts and how this information can be used to support aftermarket pricing decisions.

When the price of a product increases, the economically rational response is that the demand decreases. The underlying reason is that some of the customers are likely to be displeased with the new price and either decide not to purchase the item from the specific vendor or reduce the amount of items purchased. This implies the customers have some sensitivity to price and the price plays a role in a purchase decision. The most common parameter to measure the price sensitivity of the customers is the price elasticity of demand (Simon, 1989).

In this study we use the price elasticity of demand to express the relationship between the changes in price and changes in demand. In the aftermarkets the portfolio of sales items can be rather wide with plenty of different types of electrical and mechanical components with diversity of features. This is the case especially when the offering of the main products is broad and diverse. Assigning and updating prices for vast amount of spare parts is laborious and it might not be possible to address the pricing of a single item with desired precision. Consequently, it is not necessarily interesting — or even possible with restricted time and data — to study the price elasticities of single items, but to provide general guidelines and understanding of customer behavior. In order to fulfill this, we aim our price elasticity estimation for clusters of similar spare parts instead of single sales items.

In this thesis we derive a price elasticity estimate for item clusters and experiment different spare part features as determinants to segment sales items into suitable clusters. We study how our data responds to a segmentation framework introduced in related literature and in addition to that we also introduce our own segmentation framework.

Due to the time consuming nature of customer surveys, the analysis is based solely on sales data. We focus on studying the connection between price changes and changes in demand. Other explanatory variables, such as marketing investments, are not studied in this thesis. Even though understanding cost structure is an important aspect in pricing, it is not discussed in this study. Thus for example inventory management or understanding total cost of a product are not studied.

1.3 Structure

The rest of this thesis is structured as follows: in Chapter 2 pricing and aftermarket business are introduced, including presentation of a common pricing strategy categorization and a framework for spare part segmentation. In Chapter 3 we familiarize ourselves with the mathematical features of the price elasticity of demand and derive an estimate for segment specific price elasticity of demand. We introduce two separate segmentation frameworks to divide aftermarket sales items into segments and estimate the segment specific price elasticities of demand. When applicable, these estimates are compared to theories in related literature. In Chapter 4 we discuss how the results of our analysis reflect the aftermarket business, and speculate how they can support aftermarket pricing decisions. Finally a summarizing discussion of the findings is conducted in Chapter 5.

Chapter 2

Pricing and Aftermarket Business

2.1 Pricing

In this Section we discuss pricing in general and provide a short summary of three common perspectives for pricing: cost-based, competition-based and customer value-based pricing.

Simon (1989) defines price in the following manner: "the price of a product is the number of monetary units a customer has to pay to receive one unit of that product or service". In order to profit from their sales, product suppliers must price their products to account for all the expenses the product causes, but also to include some extra to make the business profitable. However, usually the customers have certain price sensitivity, and products priced too high end up just wasting storage space. Companies set pricing objectives based on what they wish to accomplish with product pricing: the target might be for example to grow market share or maximize short-term profits. To pursue these objectives, the companies implement pricing strategies. (Noble and Gruca, 1999)

One rather common pricing strategy categorization framework divides pricing strategies into three types: cost-based, competition-based and customer value-based pricing strategies (employed for example by Hinterhuber (2008a)). Basically this categorization divides pricing strategies based on the main information used for price determination. Of these categories, cost-based strategies are widely used because the information needed is usually readily available and the implementation is easy. Value-based strategies on the other hand are the most difficult to evaluate but are considered very effective (Doyle, 2009). Even though we discuss these categories separately, it is important to recognize that managers seldom base their pricing decisions on a single pricing strategy but effective pricing decisions usually take all of the specified aspects into account (Phillips, 2005). Also other pricing strategy categorization frameworks can be found from literature. For example Noble and Gruca (1999) categorize pricing strategies based on the pricing situation they are commonly linked to (e.g. new product pricing situation).

In this thesis we focus on pricing in aftermarket business where all products are, one way or another, connected to a main product. Nevertheless, in this Section we aim at providing more general view on pricing that is not linked to any specific industry or field.

2.1.1 Cost-based pricing

Companies executing cost-based pricing strategies base their prices on the total cost of the product. For example in cost-plus pricing the price of the product is set by defining the total cost of providing the product and adding a margin to earn profit for each sale (Meehan et al., 2011). Cost-plus pricing has historically been the most widely used pricing procedure (Nagle et al., 2011). Noble and Gruca (1999) discuss multiple studies regarding the extent of usage of cost-based pricing strategies. The findings of their study state that the cost-based pricing methods are still commonly used in industrial goods pricing.

One of the most important advantages of cost-based pricing strategies is that they are simple to use. However, in many industries it is difficult to define the true cost of a product before knowing the production volume and, consequently, the unit cost. The assumption that the price would not affect the demand is often oversimplifying. Moreover, executing cost-based pricing strategies might result in overpricing if demand reduces and thus the unit cost rises. On the other hand, when demand growth results unit cost to drop, this pricing strategy might lead to underpricing. (Nagle et al., 2011)

According to theoretical pricing research on industrial goods by Noble and Gruca (1999), cost-plus pricing strategies are used by companies most likely in situations where estimating demand is difficult. However, the approach is troubling since, as noted for example by Meehan et al. (2011), cost-based strategies fail to take market influence into account. The disadvantages of cost-based pricing have been widely highlighted for example by Nagle et al. (2011), Hinterhuber (2008a) and Phillips (2005), and the consensus seems to be that the prices should never be based solely on cost. Nevertheless, Nagle et al. (2011) emphasize that cost should not be completely ignored when pricing strategies are formulated as profitable pricing decisions require that managers understand how demand and production volume affect the total cost of the product.

2.1.2 Competition-based pricing

In competition-based (sometimes also called market-based (Phillips, 2005)) pricing strategies a company sets its prices primarily according to competitors prices, either perceived or anticipated. Competition-based pricing strategies are among the most used methods next to the cost-based strategies. According to the literature review covering multiple empirical studies from different industries from 1983 to 2006, competition-based pricing strategies had a stronger influence on pricing decisions than cost-based or value-based strategies (Hinterhuber, 2008b).

In certain circumstances competition-based pricing approaches have advantages compared to others. Phillips (2005) and Hinterhuber (2008a) assert that competition-based pricing strategies are sub-optimal approaches with commodities, provided that there is no notable distinction between competing products. Noble and Gruca (1999) find that the competition-based pricing strategies are used for industrial goods especially in mature markets with existing competition. A study of Simon (1979) suggests that penetration pricing might be an optimal strategy for new brands brought to markets with existing substitutes. Penetration pricing is a form of competition-based pricing, where the price is originally set relatively low compared to competing products and then increased gradually when product gains market share (Noble and Gruca, 1999).

One advantage of competition-based pricing approaches is that the required information can often be relatively easily acquired (Hinterhuber, 2008a). However, the market price does not always correspond to what the customers would be willing to pay for the product, resulting in lost profit. (Phillips, 2005)

2.1.3 Value-based pricing

Hinterhuber (2008a) defines that the main factor for prices in customer value-based pricing is the value that a product delivers to a certain customer segment. As pointed out by Hinterhuber (2008b), the definitions of value used in literature are not unanimous. Researchers often see customer value through benefits received by the buyer versus the cost of acquiring those benefits. However, there is no clear consensus of the attributes that should be considered as components of value or what the role of price in customer value is.

Apart from slightly different definitions of customer value, the researchers do concur that the value-based pricing strategies are the overall best approaches to pricing and offer an opportunity for profit maximization, regardless the challenges in application (Meehan et al., 2011). However, Hinterhuber (2008b) states that even though many researchers and marketing scholars argue for the superiority of value-based pricing, there is so far rather little empirical evidence to support that value-based pricing strategies increase profitability. Ingenbleek et al. (2003) examined how cost, competition and value information were used in new product pricing of 77 industrial products and whether the used information had effect on new product success. The results of this study suggest that using customer value information in new product pricing has a positive effect on new product performance.

Regardless of the recommendations of researchers and marketing scholars, value-based pricing methods are infrequently used. According to Hinterhuber (2008b), customer value-based pricing approaches have an adoption rate of 17%, which is relatively little compared to that of competition-based (44%) or cost-based (37%) approaches. Hinterhuber (2008a) states that the difficulties in value assessment and value communication are the two most common reasons to why value-based approaches are not used.

How can this customer value information be obtained? To the task researchers suggest for example such methods as expert interviews, focus groups and conjoint analysis (Hinterhuber, 2008a).

Expert interviews are generally conducted inside the company, which makes assessing the information easier. The experts consulted are usually working with the customers and are thus assumed to have insight into the features the customers value. Since the input does not come from actual customers but from company personnel, the evaluations might be distorted and consequently not as reliable. (Hinterhuber, 2008a), (Meehan et al., 2011)

Methods to gain value information directly from customers might produce more reliable results. Probably the most used method to measure customer value for a set of attributes is the conjoint analysis (Hinterhuber, 2008a). In conjoint analysis the customers are asked to express their purchase preference on alternatives with different attribute levels. Statistical analysis can then be used on these answers to extract how customers value single attributes. The conjoint analysis has the ability to reveal customer value for attributes even when the customer is not able to provide a reliable answer when directly questioned. (Hinterhuber, 2008a)

2.2 Aftermarket business

Cohen et al. (2006) specify that companies can produce customer value and thus enhance their competitiveness in three phases of product life-cycle: design, production and after-sales services. Of these phases the after-sales services is the only one generating revenue years after the actual product is sold even if the demand for the original product starts to decline. Customers in this context are not necessarily consumers; in fact, the role of the aftermarket business is particularly emphasized in business-to-business (B2B) marketing (Roy et al., 2009). This thesis focuses especially on industrial markets and therefore the customer usually is another company or industry that utilizes the products of the manufacturer in their line of business. In this Section we discuss the characteristics, benefits and challenges of offering after-sales services and spare parts.

Kotler (1994) defines services as follows: "A service is any act or performance that one party can offer to another that is essentially intangible and does not result in the ownership of anything. Its production may or may not be tied to a physical product". In this thesis we confine ourselves in the aftermarket business and discuss services solely from the product manufacturer's point of view. We therefore omit the analysis of traditional, not product-related, service business. It should be noted that a company providing services in the aftermarket is not necessarily the original equipment manufacturer, but also third-party suppliers exist (Lay et al., 2010). In this section, however, we focus on services provided by manufacturers, even though many discussed aspects apply to third-party suppliers as well. To further clarify the concept of after-sales services we should make the following corrective: a service might be delivered via a physical product, for example a replacement module or a product upgrade. Service does not result in the ownership of anything *new*, but it might for example enable the usage of something the customer already owns or extend its functionalities or lifetime.

In the aftermarkets companies offer spare parts and after-sales services for some core products in order to provide additional value to the customers. The spare parts and services are directly related to a product the customer is utilizing, and the additional value could be for example minimized risk of downtime or necessary tasks to ensure the operation of the product. Often manufacturers choose to support their own products only, but it is not unheard of that a company offers a "one-stop service" for their customers by supporting competing products as well (Cohen et al., 2006). The companies that have extended to the aftermarket therefore no longer provide mere products, but rather solutions where the combination of the core product and specific set of services are aiming to fulfill the needs of the customer (Roy et al., 2009). Vandermerwe and Rada (1988) coin the phenomenon — companies migrating from providing either goods or services to providing both — as *servitization*.

What kinds of after-sales service products do companies offer their customers? The possibilities of the aftermarket are broad and some types of services work better with some industries than the others. Customers have differing requirements, and at the end the service portfolio should be broad enough to meet the various needs. Service offerings might comprise of for example providing financial security through warranties, low cost maintenance with spare parts or speedy delivery of exchange modules to reduce downtime. Gebauer et al. (2005) categorize services into product-related (for example repair, spare parts, documentation) and customer support services (for example process oriented engineering). As the product-related services aim to keep the product operating, the customer support services are more sophisticated and customized service products.

Cohen et al. (2006) specify that companies should offer their customers services on the range from “platinum to silver”. The more demanding customers should be offered more expensive services that secure fast response (platinum services). Producing top-level services is costly: the required maintenance personnel, replacement parts or even complete products must be readily near the customer. On the other hand, more economical option should be available for customers who are not ready to pay extra for short response time. This could mean for example delivering spare parts when the breakdown occurs. Since it is not economically sensible to maintain extensive spare part inventories in numerous locations and near every customer, the spare part delivery might require more time. However, as only the defective parts are replaced, the maintenance is cheaper.

Incentives for expanding to aftermarket business

What are the incentives for companies to transform from manufacturing companies to product-service providers? Cohen et al. (2006) state that the aftermarket is a potential source of substantial revenues and profits, which requires smaller investments compared to production. In addition to that, functioning after-sales services and customer support boost brand image and customer loyalty. Especially in B2B markets, the availability and high quality of services has been seen to influence the purchase decision of the core product (Roy et al., 2009).

The above-mentioned aspects are often mentioned in aftermarket literature. As one of their core findings, Roy et al. (2009) summarize that in literature there are three commonly mentioned motives for companies to servitize: financial, strategic and marketing.

One of the main financial reasons for companies to invest in service business is the desire for a higher profit margin. Cohen et al. (2006) cite studies performed by Accenture and Aftermarket Research (AMR) providing examples of how aftermarket revenues have produced relatively higher profits than the actual manufacturing. Knecht et al. (1993) claim that in industrial companies the after-sales business accounts a higher percentage of contribution margin than of revenue.

Another financial incentive for services is the relative stability. The demand for after-sales services does not seem to suffer from economic cycles and thus aftermarket may help companies stabilize their income. For products with long life-cycles the need for services remains even if the demand for the actual product has started to decline. (Roy et al., 2009)

Strategically speaking services provide competitive advantage. As the industries develop, it gets increasingly difficult to create distinction to competitors' products by product development. Since competing with the price cannot continue endlessly, offering services has the ability to provide a company competitive advantage in the eyes of the customer.

As already pointed out, especially in B2B marketing the availability of services might affect the decision to purchase the core product. In addition to the advantage against competitors, the servitization of a company can be seen as an advantage in marketing. As after-sales service business leads to establishing and maintaining a relationship with the customer, it also enables the manufacturer to better understand the customers' industry, thus providing valuable insight into future service development needs. (Roy et al., 2009)

Challenges impeding aftermarket success

Despite the expected profit potential, the aftermarket business is often neglected or handled reluctantly (Cohen et al., 2006). Gebauer et al. (2005) sum up the lack of managerial motivation to servitize into three aspects. First of all, managers seem to consider tangible products and their features as better tools for competition than intangible services. In addition to that, the economic potential of services is overlooked. The prices of services are usually relatively small compared to the prices of the core products and thus recognizing the potential revenues is more difficult. However, the most significant obstacle to extending to service business is avoiding the perceived risk. It is seen safer to invest into core business instead of expanding to a new field with no prior experience.

Cohen et al. (2006) state that aftermarket business generally is no easy business to master. The demand for services is irregular and unpredictable, and services cannot be produced beforehand and stored like actual products. Those service items that can, such as spare parts and replacement units, are numerous compared to what pure manufacturing companies have to manage. The service business often supports products not only currently in production but also those produced in the past. The amount of knowledge and stock keeping units aftermarket business must handle is therefore much greater than that of actual production. (Cohen et al., 2006)

Even when manufacturing companies make the decision to become a product-service system provider, studies indicate that the results do not always fulfill the expectations of increased profits (Lay et al., 2010), (Neely, 2008), (Gebauer et al., 2005). The results of quantitative studies by Neely (2008) and Lay et al. (2010) support the claim of Cohen et al. (2006) about the profit potential of aftermarkets being often squandered.

The share of servitized manufacturers is difficult to estimate, and the estimates presented in studies vary together with the variety of industries and countries considered. However, the findings of Neely (2008) propose that the percentage of companies providing services was higher for larger firms than for smaller ones. Even though the firms providing services had on average higher revenues than pure manufacturing companies, the proportional profits were lower due to for example higher labor costs. However, Neely (2008) does not specify whether there was any significant variation in profitability and level of servitization in different fields of manufacturing included in the study. One could speculate for example tobacco products manufacturing having smaller markets for services than industrial and commercial machinery and computer equipment, both industries being included in the study. Further investigation should be aimed to ensure that combining variety of industries does not induce bias to the results.

Lay et al. (2010) estimate that for the studied set of almost 2000 European companies, on average only 16% of the value of sales was from services. This result is similar to what can be estimated from the results of Gebauer et al. (2005): focusing on almost 200 German and Swiss machinery and equipment manufacturers, the results state that more than 35% of the companies earned less than 10% of their revenues from services. Most of the studied manufacturing companies expected higher returns for their service investments. The situation where investments made to extend to service business do not yield in expected returns is termed in literature as the *service paradox* (Gebauer et al., 2005).

Not every company struggles with making service business profitable (Gebauer et al., 2005), but what are the functions separating the successfully servitized firms from those that drift into service paradox?

For one thing, strong emphasis is set on service strategy (Roy et al., 2009). According to the study of Gebauer et al. (2005), all the successfully, and none of the non-successfully servitized companies had defined their service strategy clearly. Goals are one part of the strategic aspect. Setting appropriate goals help boost the employee efficiency, and goals are seen as an important part of service success. However, Gebauer et al. (2005) recognized that inappropriately high goals, inevitably leading to failed expectations, result in unmotivated employees and thus corrode the profitability of the service business.

For a strategy to be effective, the organizational aspects must be in order. On some level the difficulties of transiting from manufacturing to services are attitudinal. For a service organization to work, the employees and managers must accept the service culture (Gebauer et al., 2005), (Neely, 2008). On the other hand, the organizational structure should support providing services, enabling feedback loops and going all the way down to optimizing the locations for warehouses and trained service personnel. Cohen et al. (2006) and Gebauer et al. (2005) support that having a separate service organization helps the company succeed in aftermarket business.

The third major component of aftermarket success is market orientation. The demand for services wells from customer needs, which should therefore be the foundation for service development. This changes the spirit of customer interaction from transactional into creating a lasting relationship with the customer. The active customer-supplier relationship not only offers the manufacturer a unique understanding of the needs of the customer, but also benefits the customer by enabling the development of more customized service products. (Neely, 2008), (Gebauer et al., 2005)

2.2.1 Spare parts

So far we have discussed the benefits and challenges of the aftermarket business, but we have mostly focused on services. Even though many discussed aspects apply to both services and spare parts, it is important to distinguish the differences of spare parts and services. Succeeding in service development requires understanding the needs of the customers, and services aim at providing additional value to the customer. Managing spare part business on the other hand requires understanding the technical requirements of the core products, and spare parts offer a relatively economical solution for the customer in case of a break down.

In literature, the spare parts are often discussed from the angle of inventory management or logistics (for example Eaves and Kingsman (2004), Altay and Litteral (2011)). This is no surprise: the sporadic demand for services also concerns spare parts, but unlike services, the spare parts can and must be stored. Companies often choose to provide spare parts for older product generations as well and the older products generally use different components from different vendors than the current products. This results in a vast number of stock keeping units and with the varying demand the required stock levels are not easy to forecast (Cohen et al., 2006). Optimizing stock levels for vast amount of spare parts having hard-to-predict demand comprises a real challenge for aftermarket organizations. Regardless, inventory control as such is out of scope for this thesis.

Pricing of spare parts, however, has received relatively little academic attention. Knecht et al. (1993) recommend value-based pricing for spare parts instead of cost based pricing. The value of a spare part for the customer comprises from two viewpoints: how easy it is to find a corresponding part elsewhere and how serious the financial consequences of a failure are. In some industries the faulty part might cause entire production facility shutdown and the cost of the spare part is next to nothing when compared to the downtime cost. In this case a fast delivery is more important to the customer than the price tag. On the other hand, if the acquisition of a spare part is easy, say one can find it in the local electrical supply shop, the customer is unlikely to pay a premium price just to get the part delivered by the service provider. (Knecht et al., 1993)

However, the large amount of items to be priced might drive the companies apply cost-based pricing strategies and consequently squander some of the profit potential from their spare parts. For example Gallagher et al. (2005) report a transportation manufacturer improving gross margin of company's spare parts with 30% by considering the competition and criticality when pricing their spare parts.

The objective of this thesis is to approach the task of spare part pricing through analyzing how price changes have affected spare part demand. We aim our analysis at certain types of sales items at a time, thus dividing the data into segments and estimating price elasticity of demand for each of these item segments separately. Hence, our study problem consists of two subobjectives: in addition to estimating the segment specific price elasticities we also pursue finding relevant item attributes for segmentation. In the course of this thesis, terms "price elasticity" and "elasticity" always refer to the price elasticity of demand.

Identifying potential segmentation attributes for spare part segments is a prerequisite for successful segmentation and reliability of price elasticity estimates. Next we specify some classification attributes used in literature and discuss their usability in the analysis for this thesis. We also briefly discuss other, spare part and aftermarket related criteria that are seen as potential segmentation attributes.

Segmentation attributes

As mentioned earlier, Knecht et al. (1993) encourage evaluating the value of spare parts in two dimensions: criticality and the amount of competing suppliers (or availability risk (Paakki et al., 2011)). Multiple suppliers effectively mean easier acquisition for the customer and also allows the customers to be more price sensitive. Paakki et al. (2011) use this aspect to categorize spare parts into key parts, industry specific parts and commercial parts for distribution chain performance improvement. Parts in the commercial group are easy to acquire, and have multiple suppliers. Key parts and industry specific parts are generally customized for the needs of the equipment manufacturer and thus only have one or few suppliers. Of these the industry specific parts are easier to manufacture than the key parts (for example mechanics), and thus might have more competition than key parts.

If a failure of a specific part causes the entire production facility to shut down, it is likely that the customer evaluates this part as critical. On some level the criticality varies from customer to customer, since the core product might be used in different applications and thus have different downtime cost.

The segmentation based on criticality and existence of competing suppliers is a premise for one segmentation framework for the analysis. Because spare part items studied in this thesis are mainly electrical components of different types, we are additionally interested in studying if the customers have different price sensitivity for different types of components. To reveal variation within component categories, we also consider the possibility of component categories containing sales items of significantly different scale and complexity. Since in the aftermarket business the spare parts are connected to some main product used by the customer, we extend our segmentation to account for core product life cycle phase as well.

2.3 Discussion

In this Chapter we have presented a brief exploration in the vast topic of pricing and aftermarket business, and discussed spare part pricing and segmentation. A common pricing strategy categorization framework was presented, and the main features of cost-based, competition-based and value-based pricing strategies were discussed. Regardless of emerging interest and recommendations from marketing scholars, value-based pricing is less applied than the other two. One reason to this is likely to be the difficulties in obtaining necessary information, which for cost and competition based methods is usually more easily accessible.

The aftermarket business aims at providing customers additional value by offering after-sales services and spare parts. The recognized profit potential is an incentive for a manufacturer to expand to providing services as well. In this study we concentrate on the spare part side of aftermarket business. The pricing of spare parts has received a little attention, and often the large number of spare part items might push companies to apply cost-based pricing strategies. To benefit from the potential of spare parts, Knecht et al. (1993) recommend value-based pricing.

In order to reflect how the customers value spare parts, we aim our study to understand the price sensitivity of the customers. In the following Chapter we familiarize ourselves with the mathematical definition of the price elasticity of demand and conduct a statistical analysis to estimate the price elasticity for above discussed segmentation frameworks.

Chapter 3

Statistical Analysis

3.1 Problem overview

Price elasticity of demand measures how changes in the price of an item affect the demand. Simon (1989) itemizes four methods to collect data for modeling the price response of the customers and consequently price elasticity estimation: expert judgment, customer survey, price experiment and collection of market data. The analysis in this thesis is based on the last data source, market data.

The aim of the analysis it to estimate the price elasticity of demand for several types of sales items. More precisely, we aim to segment items based on pre-defined item attributes, and use price elasticity to illustrate the general behavior of demand for items within that segment. The analysis is conducted at segment level in order to provide general guidelines to support aftermarket pricing decisions, instead of studying a large amount of items individually for which adequate amounts of data might not even be available. The sales items we analyze in this study are frequency converter spare parts and spare part bundles offered by a frequency converter manufacturer on global markets.

Sales items are segmented based on several item attributes. Segmentation based on criticality and existence of competing suppliers as discussed earlier in Section 2.2.1 is studied and the results are compared to theoretical hypotheses. In addition to that, more specific categorization attributes are taken into consideration, namely the type of the component (segmentation for example into fuses, capacitors, etc.), the life cycle phase of the related core product and the estimated relative complexity of the item compared to other items within the same component type. We examine how the price elasticity varies across component categories and study if additional segmentation attributes provide sub-segments with significantly different price elasticity within component category. In order to reduce the number of segments, we also aim to find clusters of segments within which the items get similar price response from the customers.

In each segment s we have N_s sales items i and each sales item i has n_i observed price-demand pairs. This type of data is sometimes called clustered data (Cameron et al., 2008) or multi-stage samples (Bell and McCaffrey, 2002), since clusters of data pairs are related to specific items. This will influence the analysis as all the data pairs are not directly comparable. For example, segments contain items with different average prices and levels of demand, an aspect which must be taken into account in the analysis.

To estimate the segment specific price elasticity of demand, we assume each item within the segment has the same price elasticity. This assumption, however, is never completely truthful since each item is likely to have their own price elasticity. The motivation to this assumption is that we believe the price elasticities within a well-selected segment lie rather close to each other and items with notably different price elasticity are by chance. What we are interested in is the general behavior of customers towards the segment and not the exact estimate for an item. As long as the share of items with notably different price elasticity in a segment is small, the segment specific price elasticity estimate should fulfill its purpose. Consequently, it is important to recognize that the resulting elasticity estimates should be interpreted as a measure which describes the general behavior of demand for the specified segment but not necessarily the behavior of each and every of the items within the segment.

As discussed in Section 2.2.1, the demand for spare parts is sporadic and the level of demand varies even with constant price. In other words, the change in demand is not purely produced by the price change. This variation is handled as an error term. As already noted, each item has its own demand level. Intuition suggests that also the natural variation of demand can be item specific, which implies that all the error terms for the segment do not have equal variance. The situation where error terms have unequal variance is called heteroscedasticity.

To avoid correlation of the error terms within a segment, any knowingly dependent items are excluded from the analysis. This is justified since the price changes of mutually dependent items are likely to affect each other's demand. As this cross-price effect is not taken into account in this study, the mutually dependent items might induce bias to the price elasticity estimates. Respectively, any inference about the price response of items with within segment dependencies cannot be made based on the segment specific price elasticity estimates. Because of this restriction to the data, we assume the error terms of separate items within the segment are independent. However, correlation between item specific error terms might occur.

As any real world data, the data used in this analysis is not immune to errors. We have identified some noteworthy sources of error: single data point errors, incorrectly defined attribute values and items with special characteristics compared to other similar items. The possible biasing effect of any of these error types is assumed to diminish when the size of the cluster grows.

We have now discussed the main features of the research problem at hand that should be taken into account when the methods for analysis are chosen. The rest of this Chapter is ordered as follows: we start by discussing the statistical methods used in the analysis, namely regression analysis (Section 3.2.1) and bootstrap methods (Section 3.2.2). In Section 3.3 we familiarize ourselves with the mathematical definition of price elasticity and derive the price elasticity estimate for an item segment. Data pre-processing and used segmentation framework for the analysis are presented in Section 3.4 and the results of the analysis in Section 3.5. Further discussion about the analysis is conducted in Section 3.6.

3.2 Statistical methods

3.2.1 Regression analysis

Simon (1989) summarizes that when market data is available, regression is the most popular method to estimate price elasticity among other parameters that characterize the relationship between price and demand.

Regression analysis aims to model the relationship between the dependent variable y and the independent variables $\mathbf{x}$, in this thesis between demand and price, based on observations. This relationship is defined by a function $y = f(\mathbf{x})$. In order to fit such a model, i.e. find the form and parameters for function f, we start by considering two main questions:

1. What is the functional form of the underlying relationship of the variables? Is it linear or non-linear? Which independent variables should be taken into account in the model?
2. Which estimation method is suitable, taking into account the features of the data and the selected functional form? What are the underlying assumptions that justify the selected method?

To answer the first question, the features of the data and the research problem must be addressed. In order to find a suitable functional form for the analysis, we should first familiarize ourselves with the definition of the price elasticity of demand. This will be done in more detail in Section 3.3. For simplicity let us for now assume a linear relationship between the dependent variable y_i and a single independent variable x_i:

$$y_i = \beta_1 x_i + \beta_0 + \epsilon_i$$

where β_0 and β_1 are unknown model parameters and ϵ_i is the random error. Parameters for linear function are estimated with linear regression.

A common approach to linear regression is the least squares regression (Chatterjee and Simonoff, 2013). With *ordinary least squares* (OLS) the parameter estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ are estimated in such way that the sum of squared residuals (i.e. the difference between the observed value of y_i and the value predicted by the model $\hat{y}_i = \hat{\beta}_1 x_i + \hat{\beta}_0$) is minimized:

$$\underset{\hat{\beta}_0, \hat{\beta}_1}{\arg\min} \sum_i \left(y_i - (\hat{\beta}_1 x_i + \hat{\beta}_0) \right)^2 .$$

Alternative estimation methods include for example weighted least squares and minimizing the sum of absolute values of residuals.

In this thesis we apply OLS to estimate regression parameters. However, OLS and linear regression are based on several assumptions. In order to ensure the validity of the results, these assumptions should be met or the violation should be justified with appropriate methods: the selected functional form should reflect the actual underlying relationship (for example, if the true form is $y = \alpha \sin x$, using linear regression is likely to give poor results), and $\epsilon_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$ (Chatterjee and Simonoff, 2013). In other words, following assumptions about the error terms ϵ_i are made:

1. The expected value of the error terms is zero
2. The variance of error terms is constant
3. The error terms do not correlate with each other
4. The error terms are normally distributed.

For the analysis it is necessary to understand how violating these assumptions might affect the regression results. For now we assume the linearity assumption holds.

The first assumption is that the expected value of the error terms is zero. Effectively this means we assume there is no systematic error in our data. Systematic error could be produced for example by faulty measuring equipment calibration. We are confident our data is not prone to systematic error and thus the first assumption holds.

The situation where the assumption about the constant variance of the error terms fails is called heteroscedasticity. If heteroscedasticity is present, the regression parameter estimates become more unstable and this should be taken into consideration when the results are interpreted. Situation where the variance grows together with the independent variable is an example of heteroscedastic variance. Especially in situations where the independent variable varies a lot, it is intuitive to question if the absolute variance can be the same for the largest and the smallest values of the independent variable. As discussed in Section 3.1, heteroscedasticity might be present in the segment specific errors. Davison and Hinkley (1997) point out that OLS is no longer effective for parameter estimation if errors are heteroscedastic. We however continue using the OLS method for parameter estimation despite the suspected heteroscedasticity. This is motivated by the clustered structure of the data. Similar decision is done for example by Bell and McCaffrey (2002) and Cameron et al. (2008). To account for the possible instability of the estimate, we take heteroscedasticity of the errors into account in confidence interval estimation.

Correlation between error terms occurs for example if time series data is analyzed based on external variables only, even though the previous values of the dependent variable affect the future values. For example, modeling temperature with only external variables is likely to produce correlated error terms. Correlating error terms might lead to biased assessment of how well the model explains the underlying phenomena (Chatterjee and Simonoff, 2013). In this study we allow for error correlation within items, but not across items. This will be taken into account in confidence interval estimation.

The assumption of normally distributed error terms is generally used for confidence interval construction and hypothesis testing (Chatterjee and Simonoff, 2013). If uncertainty about error term distribution occurs, alternative methods that do not assume certain distribution can be applied. In this thesis we consider the normality assumption too strict. In addition to that, heteroscedasticity and possible within item error correlation require more flexible methods for confidence interval construction. Therefore we resort to Bootstrap methods for hypothesis testing and confidence interval construction.

3.2.2 Bootstrap methods

Using a variety of statistical methods can produce a variety of statistical quantities. In real world situations probabilities are always present, which raises the question: how confident are we that the estimated measure is reliable? Different tests and confidence interval estimations can be used to express the reliability of the estimates. As discussed above, typical methods for confidence interval estimation and test statistics require an assumption of the underlying distribution. Finding a suitable distribution is not always easy and incorrect assumptions could produce misleading results.

In this thesis we analyze real sales data from multiple sales items. In many situations it is difficult to estimate the true distribution behind the parameters estimated. Therefore we turn to Bootstrap methods when it comes to confidence interval estimation and testing the statistical significance of difference in estimator values.

The basic idea of Bootstrap methods is to estimate statistical properties for a measure of interest, for example confidence intervals for a mean, by re-estimating the measure multiple times with data achieved by re-sampling from the original data. Sampling can be conducted directly from the actual data points or through a fitted model. Bootstrap methods are particularly useful in situations in which we are not fully confident of which probability distribution to use. There are also non-parametric Bootstrap methods with which no assumption of an underlying distribution function is made but that it exists. All in all, the variety of bootstrap methods for different types of applications is wide. (Davison and Hinkley, 1997)

Next we introduce the resampling procedure used in this thesis and the bootstrap methods used for hypothesis testing and confidence interval construction. Lastly we present a clustering procedure for segment clustering.

Resampling

Consider true segment S of which we have a sample s that contains N_s items. Each item $i = 1, ..., N$ consists of n_i data pairs x_{ij}, y_{ij}, $j = 1, ..., n_i$. In order to calculate the measure of interest, we form R bootstrap samples. To preserve item specific features, each bootstrap sample r is formed by resampling N_s items with replacement from the original sample instead of resampling separate data pairs. This resampling method is non-parametric and it is chosen for the analysis in this thesis because it in a way resembles the original data gathering: sample s is formed by items from the true segment S. In literature this resampling method has been called for example pairs cluster bootstrap, cluster bootstrap and case bootstrap, because instead of resampling individual data pairs, we sample clusters of data pairs (Cameron et al., 2008).

We use resampling quantity $R = 1999$ for all bootstrap estimates in this thesis. This should offer sufficient accuracy for example for 90–95% confidence intervals (Carpenter and Bithell, 2000).

Hypothesis testing

In this thesis we will face situations in which comparing achieved price elasticity estimates with each other or with theory based hypotheses will be of interest. Firstly, in order to evaluate the relevance of segmentation, we need to test if price elasticity estimates for two item segments are significantly different or is it possible that they come from the same distribution. Secondly, to compare our results and data to the underlying theory and assumptions about price elasticity, we test if our estimates support the literature based hypotheses.

Consider the following situation: we have item segments a and b for which we have estimated price elasticities $\hat{\varepsilon}_a$ and $\hat{\varepsilon}_b$ respectively. We are interested to know if the difference between the estimates is by chance, implying that the items in segments a and b have similar price elasticity, or if the difference in estimates is significant at some level α. If the items in segments have similar price elasticity, the two estimates $\hat{\varepsilon}_a$ and $\hat{\varepsilon}_b$ come from the same distribution. Thus, the difference between true segment elasticities ε_a and ε_b is zero. If we choose the test statistic to $T = \varepsilon_a - \varepsilon_b$, the null hypothesis is $H_0 : T = 0$ and the test statistic value is $t = \hat{\varepsilon}_a - \hat{\varepsilon}_b$.

If we have no prior assumption of the price elasticities of the item clusters, we set the alternative hypothesis $H_A : T \neq 0$ and use the two-tailed test. If we on the other hand test the significance of a theory suggesting that customers are more price sensitive for items in segment b than for items in segment a leading to $\varepsilon_a > \varepsilon_b$, we choose the one-tailed test with alternative hypothesis $H_A : T > 0$.

The p-value indicates the probability of the test statistic T getting value equal to or even more extreme than t by chance, when the null hypothesis holds. If the p-value gets smaller values than some pre-defined significance level α, we reject the null hypothesis and accept the alternative hypothesis. For one-tailed test $p = \Pr(T \geq t|H_0)$. As mentioned earlier, we have no assumptions regarding the underlying distribution, but we can approximate the probabilities by using the Monte Carlo approach as presented by Davison and Hinkley (1997):

$$p_{mc} = \frac{1 + \#\{t_r^* \geq t\}}{R + 1} \tag{3.1}$$

where R is the count of simulated bootstrap samples, t_r^* is the test statistic value for a simulated sample r and $\#\{A\}$ is the number of times event A occurs. The p-value for two-tailed test can be calculated with (3.1):

$$
\begin{aligned}
p_2 &= 2 \cdot \min \left\{ \frac{1 + \#\{t_r^* \geq t\}}{R+1}, \frac{1 + \#\{t_r^* < t\}}{R+1} \right\} \\
&= 2 \cdot \min \left\{ p_{mc}, \frac{1 + R - \#\{t_r^* \geq t\}}{R+1} \right\} \\
&= 2 \cdot \min \left\{ p_{mc}, \frac{2 + R - (1 + \#\{t_r^* \geq t\})}{R+1} \right\} \\
\Rightarrow \quad p_2 &= 2 \cdot \min \left\{ p_{mc}, \frac{R+2}{R+1} - p_{mc} \right\} \qquad (3.2)
\end{aligned}
$$

The simulated test statistics t_r^* are calculated in the following manner:

1. Concatenate datasets of segments a and b into a set s
2. Collect n_a items randomly with replacement from set s, where n_a is the amount of items in segment a and calculate $\hat{\varepsilon}_{a_r^*}$ based on sampled items
3. Collect n_b items randomly with replacement from set s, where n_b is the amount of items in segment b and calculate $\hat{\varepsilon}_{b_r^*}$ based in sampled items
4. Calculate test statistic $t_r^* = \hat{\varepsilon}_{a_r^*} - \hat{\varepsilon}_{b_r^*}$
5. Repeat steps $2 - 4$ R times.

Confidence intervals

As we have already discussed, heteroscedasticity in model error might induce instability to the parameter estimate. We select the confidence interval construction method to be relatively simple but also allow heteroscedastic errors and possibly even within item error correlation. As presented by Cameron et al. (2008), using bootstrap-t procedure might lead to improved accuracy of the confidence intervals when heteroscedastic error is present. Bootstrap-t, also known as the studentized bootstrap (Cameron et al., 2008) uses following Wald statistic for each bootstrap sample r to compute confidence intervals:

$$w_r^* = \frac{\hat{\beta}_r^* - \hat{\beta}}{s_{\hat{\beta}_r^*}}$$

where $\hat{\beta}_r^*$ is the parameter estimate calculated for the sample r, $\hat{\beta}$ is the parameter estimate calculated from the original sample and $s_{\hat{\beta}_r^*}$ is a standard error for $\hat{\beta}_r^*$ (Cameron et al., 2008). The confidence intervals can be estimated from the bootstrapping statistics for confidence level $1 - \alpha$ as

$$\begin{aligned}\hat{\beta}_{\alpha/2} &= \hat{\beta} - s_{\hat{\beta}} w^*_{(R+1)(1-\alpha/2)} \\ \hat{\beta}_{1-\alpha/2} &= \hat{\beta} - s_{\hat{\beta}} w^*_{(R+1)\alpha/2}\end{aligned} \tag{3.3}$$

where $s_{\hat{\beta}}$ is the standard error for original statistic $\hat{\beta}$ and w^*_a is the ath largest value of all the resampled statistics w^*_r, provided that a is an integer. This can be arranged by selecting the amount of bootstrap samples R conveniently, for example $R = 999$ or $R = 1999$.

One important feature of the achieved confidence intervals should be noted. Due to for example the resampling method used, the confidence interval narrows when the item count in the segment increases. This is because the effect of single items to the estimates gets smaller, which then stabilizes the variation. Comparing the estimate variances for two segments of different sizes should be done with caution. When the size of the segment is small, the possible special case items might have a strong biasing influence on the estimate due to bootstrap sampling. We refer to this situation as bootstrap error.

Segment clustering

As a final step of the analysis we wish to find clusters of segments within which the items have similar price elasticity. The clustering algorithm used operates as follows:

1. Assign each segment to a separate cluster index $i = 1, ..., n$
2. Estimate the p-value $p_{i,j}$ (3.2) for the two-tailed test for each cluster pair $i, j, \quad i = 1, \ldots, n-1, \quad j = i+1, \ldots, n$
3. Combine such clusters K and L that $K, L = \arg\max_{i,j} p_{i,j}$, and assign the new cluster to cluster index $n+1$
4. Estimate the p-values between the new cluster and the existing clusters $p_{i,n+1}$ with Equation (3.2), $i = 1, ..., n$
5. Update $p_{K,j} = 0$, $j = K+1, ..., n+1$ and $p_{i,L} = 0$, $i = 1, ..., L-1$
6. Update $n = n + 1$
7. Repeat steps 3–6 until there is only one cluster left containing all the original segments, or the desired significance level α is achieved, i.e. $\max_{i,j} p_{i,j} \leq \alpha$, $i = 1, ..., N-1$, $j = i+1, ..., N$ where N is the index of the most recently combined cluster.

3.3 Price Elasticity of Demand

Price elasticity of demand ε is defined as the relation between the proportional change in demand D and the corresponding proportional change in price P: (Simon, 1989)

$$\begin{aligned}\varepsilon &= \frac{\Delta D}{D} \Big/ \frac{\Delta P}{P} \\ &= \frac{\Delta D}{\Delta P}\frac{P}{D}.\end{aligned}$$

In the case of an infinitesimal change in demand and price the formula gets form

$$\varepsilon = \frac{\partial D}{\partial P}\frac{P}{D}. \tag{3.4}$$

Mathematical features and interpretation

Consider a situation where the price increases. The rational economical response to this is that the demand decreases since less customers are ready to pay the new price. Consequently the ε is negative, i.e. the price and the demand are inversely related. Because the price elasticities are usually negative, they are sometimes discussed as absolute values (Meehan et al., 2011). We however keep the negative values negative in order to distinguish possible positive price elasticity estimates.

It is not economically rational that the demand for some product would increase as a result of a price increase. Positive price elasticity estimates generally indicate that the model ignores one or more price independent variables that have significant effect on the demand. For example disregarding investments to marketing and advertising might produce this type of a situation (Simon, 1989). This thesis focuses on spare part items in aftermarket business where marketing does not seem like a common method to increase demand. Nevertheless, for example the age structure of the installed base for the main product might have a significant effect on the demand, and it should be acknowledged that omitting it from the analysis might affect the results.

Another important feature of the price elasticity of demand is the proportionality: the percentual increase in price results in percentual decrease in demand and vice versa. This enables comparing products with varying prices and different levels of demand. Especially in situations that involve money, proportional measures are often easier to interpret than absolute ones (Chatterjee and Simonoff, 2013). For example, if we consider a situation where the price of an item increases with five units, we are unable to estimate if the change is notable or not unless we know the starting price. If we instead report that the price has increased by 50%, corresponding a price change from 10 to 15 units, the significance of the price change is easier to evaluate. However, even though the proportionality gives better perspective on the significance of the change, one can argue that for a typical decision maker a 50% increase is not necessarily equally significant regardless the absolute price, but also the absolute prices matter.

Even though proportionality enables comparing different priced items, it should be taken into account that especially low levels of demand make elasticity estimation more difficult. The demand is often measured in units, which is not a continuous scale but can only have integer values. Consider the following example: the original demand of a product is four units. The smallest change in demand that can occur is either increase or decrease by one unit, a change of $\pm 25\%$. Say the decrease from four units to three units happens if the price increases at least with 5% and a decrease of two units happens only if price increase is more than 15%. Depending on the realized price change the resulting price elasticity estimate is between -1.67 and -5. This span is wide: the meta-analysis of Bijmolt et al. (2005) gathered nearly 2000 price elasticity estimates across literature, of which approximately 9.1% were ≤ -5, implying extremely elastic demand, whilst the overall mean was -2.62. Extrapolating this type of estimates to products with substantially greater demand levels will not necessary produce reliable results.

If we examine the Equation (3.4), we notice that the price elasticity of demand has a singularity at $D = 0$. In other words, with the current definition, the price elasticity of demand cannot be defined for situations where the original demand is zero. For example if the demand rises from 0 to 10 units, the percentual growth of demand is infinite which results in infinite price elasticity.

Economical interpretation

We have now familiarized ourselves with the basic mathematical features of the price elasticity of demand. But why is such measure interesting, and how should different values of price elasticity of demand be interpreted? As we already discussed, the price elasticity of demand is generally negative. This means that in an economically rational situation the demand for a product decreases if the price increases and vice versa.

The magnitude of the price elasticity of demand describes how elastic or inelastic the demand is. If $-1 < \varepsilon < 0$, the demand is *inelastic*. This denotes that a price change has a smaller relative effect on demand, i.e. the customers are not that price sensitive. Generally, products that are seen as essentials and do not have available substitutes have inelastic demand, for example some pharmaceutical products (Ingenbleek et al., 2003). If in contrast to that $\varepsilon < -1$, the demand is elastic and price changes induce larger changes in demand. The customers in this situation are more price sensitive and consider the price to be an important determinant of the purchase decision. For instance, elastic demand is common for groceries that are not seen as essentials, for example soft drinks (Hoch et al., 1995).

Simon (1989) highlights the distinction between short-term and long-term price elasticity. For certain types of sales items it is possible that the immediate response to price change is rather radical but that the permanent change in demand level is significantly less. The short-term and long-term elasticity estimates aim at providing distinction between these situations. This study uses data that is aggregated on yearly bases in order to smooth the possible short term effect and therefore the resulting elasticity measures can be interpreted as long-term rather than short-term.

3.3.1 Demand modeling

As mentioned previously in this section, using regression to estimate the price elasticity of demand from market data requires selecting a suitable functional form for the relationship between price and demand. Let us refer to this function as the *price response function* $D = f(P)$. Unlike for example with some physical phenomena, there is no "true" functional form for the relationship between price and demand. Simon (1989) presents two relatively simple and widely used price response models for monopoly situation: linear price response function and multiplicative price response function.

The linear price response function in monopoly assumes that the price and the demand are linearly dependent:

$$D = a_l - b_l P$$

where D is demand, P is price and a_l, b_l are model parameters. The advantage of the linear model is that it is easy to interpret and there is only two parameters to be estimated. As demonstrated in Figure 3.1, if the price is set to zero, we get the maximum sales $D = a$. On the other hand, no sales is made if $P = \frac{a}{b}$. Parameter b quantifies the absolute change in demand if the price changes by one unit. However, the assumption that the absolute response to price change is equal at any price is rather unrealistic (Simon, 1989).

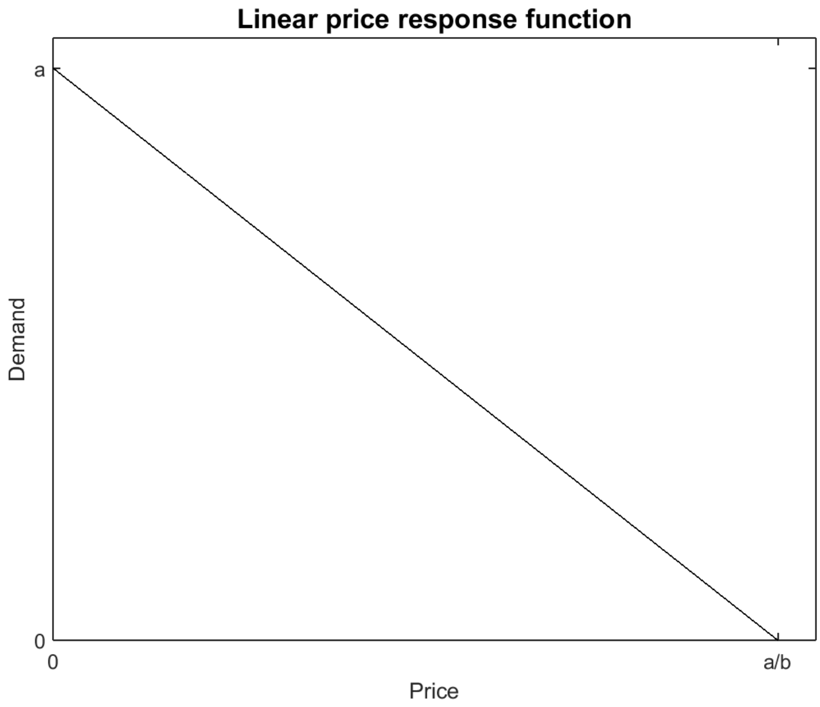

Figure 3.1: Graphical illustration of the linear price response function.

The multiplicative price response function relates price and demand non-linearly:

$$D = a_m P^{b_m}, \qquad a_m > 0, b_m < 0 \tag{3.5}$$

where D is demand, P is price and a_m, b_m are model parameters. The function is illustrated in Figure 3.2. It should be acknowledged that with the multiplicative price response function the absolute change in demand depends on the price level: a price change of one unit produces larger absolute change in demand for products with lower price. Like the linear function, the multiplicative function has only two parameters to be estimated, but as mentioned by Simon (1989), the theoretical foundation behind the multiplicative form is better than that behind the linear one.

The situation studied in this thesis is not necessarily an actual monopoly situation. In order to be able to use price response functions modified for competitive situations, additional information about markets, for example mean market prices for studied items, is needed (Simon, 1989). Such information is not available for this study, and thus the situation is modeled as a monopoly situation.

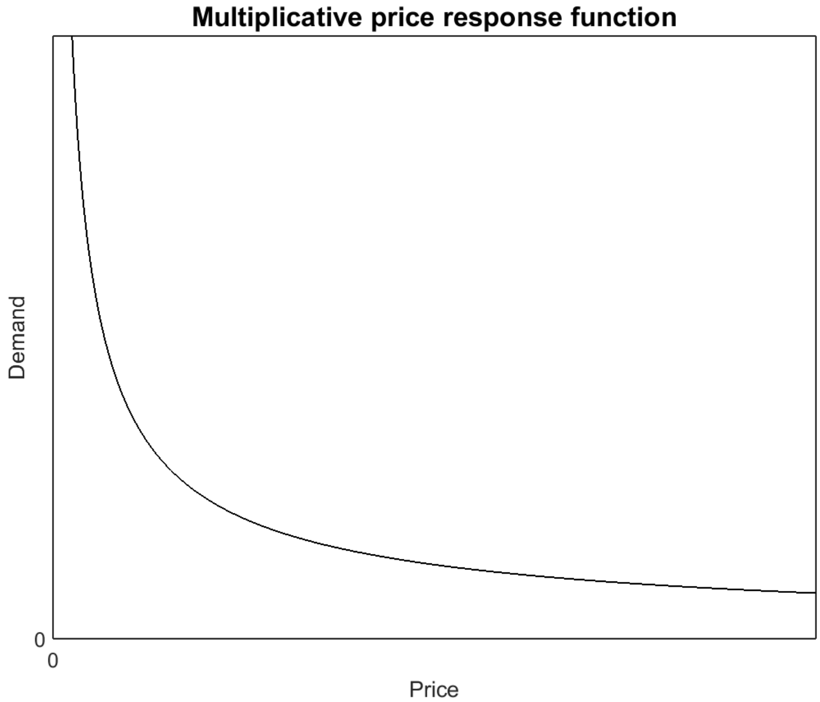

Figure 3.2: Graphical illustration of the multiplicative price response function.

There is no clear best practice for which functional form to use for price response function (Simon, 1989). A common way for model evaluation and selection is to select the model which produces the best fit for the data. However, in this study the data available is not numerous enough to produce reliable or noticeable difference between the two models. Therefore we now approach the model determination from another angle: which one is of the most favorable form considering the problem at hand?

The aim of this study is to estimate the price elasticity for segments of items with varying prices. Therefore we begin with the hypothesis that the price elasticity for the specified cluster is a constant that does not depend on price and is common for all the items in the cluster. The assumption, however, is not problem-free. The major downside is that eventually it becomes economically questionable: are the customers likely to have similar response to a proportional price change regardless of the original price? If we consider a 50% price increase for a product that costs one monetary unit and another product that costs thousands of monetary units, we suspect the customers do not find the price changes equally significant.

On the other hand, when the prices increase and the demand decreases, at some point even one item decrease in demand is proportionally so large that the assumption of constant price elasticity suggests that very high price increase is allowed before the decrease in demand happens. As a result, it is important to note that the estimated model is likely to behave unrealistically if it is applied to situations with extremely low levels of demand or significantly large price variation.

In this study we assume that only the price of the studied item affects the demand for that item and that other possible independent variables are fixed through the study period. Thus we can derive the price response function that fulfills the hypothesis of a constant ε from Equation (3.4) with variable separation:

$$\begin{aligned} \varepsilon &= \frac{dD}{dP}\frac{P}{D} \\ \frac{1}{P}\varepsilon dP &= \frac{1}{D}dD \\ \int \frac{\varepsilon}{P}dP &= \int \frac{1}{D}dD \\ \varepsilon \log P + c_1 &= \log D + c_2 \\ \log(P^\varepsilon) + (\underbrace{c_1 - c_2}_{=:c_3}) &= \log D \\ e^{\log(P^\varepsilon)+c_3} &= e^{\log D} \\ D &= e^{\log(P^\varepsilon)} \underbrace{e^{c_3}}_{=:c} \\ D &= cP^\varepsilon. \end{aligned} \tag{3.6}$$

The result is of the same form as the multiplicative price response function for monopoly situation (3.5). Notice that the parameter value earlier denoted as b_m can be interpreted as the price elasticity ε. Note that when the multiplicative price response function was presented, we made the requirement $b_m < 0$. A positive parameter b_m would indicate growth in demand, when the price increases, which is not economically plausible. However, for further analysis we do not restrict the price elasticity to be negative.

The meta-analyses of Tellis (1988) and Bijmolt et al. (2005) did not find significant effect on price elasticity estimations due to the functional form of demand used. Therefore we assume that using the multiplicative model — and consequently hypothesizing a constant price elasticity for an item segment — is justified as long as the price variations for separate items are adequate.

The statistical estimates of the price response function parameters are usually achieved with regression. Even though Equation (3.6) is non-linear, it is linearizable (Chatterjee and Simonoff, 2013). Assume both D and P are greater than 0 at all times. If we apply logarithm on both sides of Equation (3.6), we get

$$\begin{aligned} D &= cP^{\varepsilon} \\ \log D &= \log(cP^{\varepsilon}) \\ \log D &= \log c + \log(P^{\varepsilon}) \\ \log D &= \varepsilon \log P + C \end{aligned} \tag{3.7}$$

which is indeed a linear dependency between the logarithm of price and the logarithm of demand with model parameters C and ε. This type of models are sometimes called log-log models or constant elasticity models (Chatterjee and Simonoff, 2013). Due to the linearization, the theoretical form supports using linear regression for parameter estimation. As assumed above, the demand and price must be greater than zero at all times.

3.3.2 Segment specific price elasticity estimation

Keeping the pricing objective in mind, the item specific price elasticities are not necessarily interesting but rather the characteristics of a larger segment of products. On the other hand, calculating reliable estimates for item specific price elasticity would require notable amount of data collected for a long time period, which is not available for this thesis.

To support pricing decisions as well as possible, we segment the data based on pre-defined product attributes. Next we derive an estimate for segment specific price elasticity of demand using OLS. The estimate is based on the multiplicative price response function (3.7).

Let S be the set of items in the segment s that is under study. Each item $i \in S$ is associated with a price vector $\mathbf{P}_i$ of $n_i \times 1$ and a demand vector $\mathbf{D}_i$ of $n_i \times 1$. Then, P_{ij}, is the price of the item i in the jth period and D_{ij} is the corresponding demand ($j = 1, \ldots, n_i$).

The leading assumption of our analysis is that the price elasticity ε is common for all the items i in the segment. Let us now refer to the studied segment specific price elasticity of demand as ε and its estimate as $\hat{\varepsilon}$.

Since the baseline demand and price are item specific, all items cannot be characterized with the exactly same price response function. The multiplicative model chosen for the analysis only contains two parameters for each price response function and we already stated that the elasticity parameter is common for items within the cluster. Consequently, the constant parameter C_i (corresponding estimate $\hat{C}_i$) is separate for each item i. A vector consisting of all the constant parameters in the segment is denoted as C and the corresponding estimate is $\hat{C}$.

The model for single data point can be expressed with the theoretical model (3.7) with an error term ϵ_{ij} added:

$$\log D_{ij} = \varepsilon \log P_{ij} + C_i + \epsilon_{ij}, \qquad i \in S, \quad j = 1, \ldots, n_i \tag{3.8}$$

where ε is the price elasticity, and C_i is the constant parameter specific for item i and ϵ_{ij} is the random error with expected value of zero: $\mathrm{E}(\epsilon_{ij}) = 0$.

The total number of parameters that must be estimated for a segment of N_s items is $N_s + 1$: N_s regression constants C_i and the price elasticity of demand ε. For parameter estimation we use OLS as discussed earlier. The optimization problem to solve $\hat{\varepsilon}$ and $\hat{C}$ is

$$\underset{C,\,\varepsilon}{\arg\min} \sum_{i \in S} \sum_{j=1}^{n_i} \left(\varepsilon \log P_{ij} + C_i - \log D_{ij}\right)^2 . \tag{3.9}$$

The objective function is quadratic. The extreme points of a continuous unbounded function lie in the stationary points, i.e. where the derivative is zero. Quadratic functions have only one such point and it is the minimum if and only if the second derivative of the quadratic function is positive.

Since the objective function has multiple variables, the extreme point lies where the partial derivatives with respect to each variable are zero. First consider the partial derivative of the objective function in (3.9) with respect to the kth constant parameter C_k:

$$\begin{aligned}
&\frac{\partial}{\partial C_k} \sum_{i \in S} \sum_{j=1}^{n_i} \left(\varepsilon \log P_{ij} + C_i - \log D_{ij}\right)^2 \\
&= \sum_{i=k} \sum_{j=1}^{n_i} 2\left(\varepsilon \log P_{ij} + C_i - \log D_{ij}\right) \\
&= \sum_{j=1}^{n_i} 2\left(\varepsilon \log P_{kj} + C_k - \log D_{kj}\right)
\end{aligned}$$

Set the derivative to zero to calculate the optimal $\hat{C}_k$:

$$\sum_{j=1}^{n_k} 2\left(\varepsilon \log P_{kj} + \hat{C}_k - \log D_{kj}\right) = 0$$

$$n_k \hat{C}_k + \sum_{j=1}^{n_k} (\varepsilon \log P_{kj} - \log D_{kj}) = 0$$

The optimal $\hat{C}_k$ that minimizes the objective function is

$$\begin{aligned}
\hat{C}_k &= \frac{1}{n_k} \sum_{j=1}^{n_k} (\log D_{kj} - \varepsilon \log P_{kj}) \\
&= \frac{1}{n_k} \sum_{j=1}^{n_k} \log D_{kj} - \frac{1}{n_k} \sum_{j=1:n_k} \varepsilon \log P_{kj} \\
&= \overline{\log D_k} - \varepsilon \overline{\log P_k}
\end{aligned}$$

where $\overline{\log D_k}$ is the arithmetic mean of the logarithmic demand data for item k and $\overline{\log P_k}$ is the arithmetic mean of the logarithmic price data for the same item. More generally: the optimal value for variable C_i is $\hat{C}_i = \overline{\log D_i} - \varepsilon \overline{\log P_i}$.

If we substitute the variable C_i in Equation (3.9) with $\hat{C}_i$, the optimization problem reduces to a single-variable problem:

$$\min_{\varepsilon} \sum_{i \in S} \sum_{j=1}^{n_i} \left(\varepsilon \log P_{ij} + \overline{\log D_i} - \varepsilon \overline{\log P_i} - \log D_{ij}\right)^2$$

$$\min_{\varepsilon} \sum_{i \in S} \sum_{j=1}^{n_i} \left[\varepsilon \left(\log P_{ij} - \overline{\log P_i}\right) - \left(\log D_{ij} - \overline{\log D_i}\right)\right]^2$$

The first order partial derivative of the objective function with respect to ε is

$$\begin{aligned}
&\frac{\partial}{\partial \varepsilon} \sum_{i \in S} \sum_{j=1}^{n_i} \left[\varepsilon \left(\log P_{ij} - \overline{\log P_i}\right) - \left(\log D_{ij} - \overline{\log D_i}\right)\right]^2 \\
&= \sum_{i \in S} \sum_{j=1}^{n_i} \left\{2\left[\varepsilon \left(\log P_{ij} - \overline{\log P_i}\right) - \left(\log D_{ij} - \overline{\log D_i}\right)\right] \left(\log P_{ij} - \overline{\log P_i}\right)\right\}.
\end{aligned}$$

In order to solve $\hat{\varepsilon}$ that minimizes the objective function of (3.9), set the derivative to zero:

$$\hat{\varepsilon} = \frac{\sum_{i \in S} \sum_{j=1}^{n_i} \left(\log D_{ij} - \overline{\log D_i}\right) \left(\log P_{ij} - \overline{\log P_i}\right)}{\sum_{i \in S} \sum_{j=1}^{n_i} \left(\log P_{ij} - \overline{\log P_i}\right)^2}. \tag{3.10}$$

The objective function is in fact minimized, since

$$\begin{aligned}
&\frac{\partial^2}{\partial \varepsilon^2} \sum_{i \in S} \sum_{j=1}^{n_i} \left[\varepsilon \left(\log P_{ij} - \overline{\log P_i}\right) - \left(\log D_{ij} - \overline{\log D_i}\right)\right]^2 \\
&= \sum_{i \in S} \sum_{j=1}^{n_i} \left(\log P_{ij} - \overline{\log P_i}\right)^2 \geq 0.
\end{aligned}$$

In order to estimate the confidence interval for the segment specific price elasticity estimate using bootstrap-t method (3.3), we must calculate an estimate for the standard error of $\hat{\varepsilon}$:

$$\hat{s}_{\hat{\varepsilon}} = \frac{\sqrt{\frac{N}{N-1} \sum_{i \in S} \sum_{j,k=1}^{n_i} r_{ij} r_{ik} \left(p_{ij} - \overline{p_i}\right) \left(p_{ik} - \overline{p_i}\right)}}{\sum_{i \in S} \sum_{j=1}^{n_i} \left(p_{ij} - \overline{p_i}\right)^2}. \tag{3.11}$$

where r_{ij} are model residuals and N is the number of items within the segment. See appendix A for in-detail derivation. The standard error estimate (3.11) assumes that the error terms of separate items do not correlate, but allows for heteroscedasticity and within item correlation.

3.4 Data

The analysis in this thesis is based on spare part sales data from years 2006 to 2014. The sales items are spare parts and spare part bundles for a variety of frequency converters. The independent variable is the list price of an item, referred to as price P and the dependent variable is the sum of yearly sales quantity for the product, referred to as demand D. The geographic variable is not included in the analysis and the price considered is always the list price which might differ from the realized end user price.

Next we discuss the most important data pre-processing phases that are taken to ensure more reliable price elasticity estimates. Thereafter in Section 3.4.2 we introduce the segmentation frameworks used for analysis.

3.4.1 Pre-processing

As we have already noted earlier in this chapter, there are several issues in price elasticity estimation from real sales data that must be taken into account in the analysis. Following actions have been taken to pre-process the available data.

Demand aggregation. The demand is aggregated on yearly basis (12 months) to correspond to the pricing periods. Only data points that have constant price during the aggregation period are taken into account in the analysis to avoid uncertainty in the price variable.

Too few data points for regression. Parameter estimation with regression analysis is a common method in data analysis and the estimates gained with only a few data points are not reliable. Since the estimates in this thesis are calculated for segments rather than single items, the amount of data for a single item is not assumed to play as big a role, provided that the amount of items in a segment is sufficient. The length of the time series available for the analysis is originally rather short (up to nine years). We rule out items for which less than five years of pre-processing requirements fulfilling data is available.

Proportionality of the price elasticity of demand. As already pointed out, achieving reliable price elasticity estimates is more difficult with small levels of demand. In order to avoid bias to the price elasticity estimates, we rule out items with constantly low demand. We consider at least five years of total demand of more than 10 units sufficient enough for the analysis.

Logarithm in regression. Earlier in Section 3.3 we discussed regression analysis for price elasticity estimation and selected multiplicative model for demand. In order to justify linear regression for the data, a natural logarithm is applied for both price and demand. Logarithm is only defined for positive numbers. Thus possible zero-demand years are treated as missing values and not included in the analysis.

3.4.2 Segmentation and structure of the analysis

We have selected five attributes of interest to segment the available data. Segmentation attributes and their values are listed in Table 3.1. The analysis in this thesis is conducted in two parts for two different segmentation frameworks.

Segmentation framework A

First part of the analysis is based on a segmentation framework proposed by Knecht et al. (1993) that was discussed in Section 2.2.1. We divide our data into critical and non-critical items and estimate the existence of competing suppliers by considering spare part significance. The latter categorization

Table 3.1: Segmentation attributes and attribute values.

Segmentation Attribute	**Attribute Value**	**Explanation**
Criticality	Critical	Malfunction of the part could lead to unexpected down-time.
	Non-critical	Malfunction is not likely to result in down-time.
Significance status	Key parts	High value company proprietary parts with own design for company specific products.
	Industry specific parts	Company proprietary parts similar to Key parts, but contains simpler components.
	Commercial parts	General, not company specific parts that are easier to acquire.
	Standard parts	Similar to commercial parts, but more commonly available.
Component category	Mechanics	Non-electrical components, such as insulates and core elements.
	Fuses	
	Power Semiconductors	
	Switches, Relays and Contactors	
	Capacitors	
	Resistors	
	Fans and Air Filters	
	Boards	
	Wires	
Life cycle phase	A	In active production.
	B	Not in active production, full product support.
	C	Not in active production, limited product support.
Relative complexity	Simple	Acquisition cost less than 50% of the average.
	Normal	Acquisition cost between 50% and 150% of the average.
	Complex	Acquisition cost more than 150% of the average.
	N/A	Acquisition cost not available.

is similar to that of Paakki et al. (2011) and we divide data into four categories: key parts, industry specific parts, commercial parts and standard parts. Standard parts is our own extension to the categorization of Paakki et al. (2011) to separate commercially available parts into more and less common parts. More detailed descriptions of the attribute values are provided in Table 3.1.

Following notation is used for segment specific price elasticities in framework A: the price elasticity for a segment where the attribute i has value j is denoted by ε_{ij}. If a segment is determined by two attributes i and k with values j and l respectively, the price elasticity for the segment is denoted by $\varepsilon_{ij,kj}$. The attributes considered are Criticality (1) and Significance status (2) and the indices of attribute values for both attributes follow the order of Table 3.1. For example, ε_{23} is the price elasticity for commercial parts and $\varepsilon_{11,23}$ for the critical commercial parts.

Theoretical premises suggest that the customer is less interested in the price of the spare part if the part is essential for the product to operate. In other words, the customers are less price sensitive when it comes to critical parts. In terms of price elasticity, this means that the price elasticity for critical spares ε_{11} is less negative than that for non-critical spares ε_{12}:

H1: $\quad H_0 : \varepsilon_{11} = \varepsilon_{12}, \quad H_A : \varepsilon_{11} > \varepsilon_{12}$

The idea of Paakki et al. (2011) is that the customer is more sensitive to price changes when it is easier to find alternative suppliers for that item. This is the motivation for the second segmentation attribute. Of these, the segment Key parts is not likely to have any suppliers outside the company as the spare parts within the segment are company proprietary items with own design and manufacturing. Key parts are seen as the most valuable parts. The theory would suggest that the price elasticity is the most inelastic and therefore closest to zero. Industry specific parts are similar to Key parts: they too are company's own design but the parts are simpler and easier to manufacture. Thus some alternative suppliers might exists. We hypothesize that the price elasticity for key parts ε_{21} is less elastic than the price elasticity for industry specific parts ε_{22}:

H2: $\quad H_0 : \varepsilon_{21} = \varepsilon_{22}, \quad H_A : \varepsilon_{21} > \varepsilon_{22}$

Both Commercial parts and Standard parts are likely to be commercially available, and thus finding alternative suppliers is assumed to be easier than for key parts or industry specific parts. Further, the standard parts are estimated to be even easier to find commercially (thus more elastic demand) than the commercial parts.

H3: $H_0 : \varepsilon_{22} = \varepsilon_{23}, \quad H_A : \varepsilon_{22} > \varepsilon_{23}$

H4: $H_0 : \varepsilon_{23} = \varepsilon_{24}, \quad H_A : \varepsilon_{23} > \varepsilon_{24}$

The determination of criticality and significance status for studied items is received from the company. It should be noted that the values for both of these segmentation attributes are not set for single items in the data, but for clusters of items based on the type of the component. Thus, the segmentation is subject to misclassification on item level if the criticality and the extent of alternative supply for the item cluster in general differs from that of the item or if the component type of the item is incorrectly defined. In addition to that, the significance status is not exactly intended to measure the extent of alternative suppliers, but we believe the division into company proprietary items and commercially available items has a strong connection to number of alternative suppliers.

Segmentation framework B

The second part of the analysis is based on the idea that the price sensitivity of the customers might differ for different types of components. The component categories used are based on a categorization framework used by the company. The categories included in the analysis are selected so that data for at least 50 items is available for each category. This is to provide stability and reliability to the results and enable possible sub-segmentation. The standard error estimator (3.11) used for the price elasticity estimate is biased if the quantity of items in a segment is too low (Bell and McCaffrey, 2002). Based on the simulations of Cameron et al. (2008) and the availability of our data, we set a final 20 item limit for segment size. In other words, price elasticity estimates for sub-segments with less than 20 items are not calculated to avoid misleading and possibly erroneous price sensitivity perceptions. The component categories included in the analysis are listed in Table 3.1.

In order to study if the component categories are sufficient to describe the price sensitivity of the customers, we examine the diversity within component categories with additional segmentation. The price elasticity estimation is based on the assumption that the price elasticity of demand is the same for all the items within the segment and thus the achieved elasticity should be interpreted as an aggregate of the true price elasticities in the segment. However, if the segment contains items with notably different price elasticities, the achieved estimate might not succeed in reflecting the nature of the underlying price elasticity. For example, if we mix items with notably elastic demand, say $\varepsilon_1 = -2.5$ and items with very inelastic demand $\varepsilon_2 = -0.2$, the estimated elasticity for the entire item cluster lays somewhere between these two and is not likely to describe the true nature of any of the items within the segment. We aim to reveal possible sub-segments for different component categories by introducing two more segmentation attributes: the relative complexity of the component and the life cycle phase of the core product the component is primarily related to.

The first additional segmentation attribute is motivated by the idea that some component categories might contain products of significantly different scale, and thus one price elasticity estimate might not be sufficient to reflect the customers' price sensitivity for that component category. Because there is no direct attribute to reveal the relative complexity of the items within the category, we estimate relative complexity based on the relative acquisition cost compared to other components in the same component category. We hypothesize this is a relatively good measure to approximate the complexity of the components. It should be noted that our intention is not to distinguish small variation in item complexity but to see the large scale picture. Thus the roughness of the segmentation and possibly misclassified items are not seen as a remarkable problem for this segmentation.

Note that in theory the relative complexity determination is not connected to the price used in the analysis, but in practice some correlation is likely to occur. The segmentation based on this attribute can thus on some level also support or contradict the assumption that the price level does not affect the price elasticity within the segment. In order to apply the attribute for segmentation, the different values for relative complexity are divided into three segments: values less than 50% of the category mean, values more than 150% of the category mean and values between 50% and 150% of the category mean. A fourth segment "N/A" is established for items for which the acquisition cost was not available for some reason. The segmentation thresholds are arbitrary and might not provide the best possible results but should be enough to reveal if there is some significant underlying variation within component category due to differences in component complexity.

The latter additional segmentation attribute, the life cycle phase of the main product, might affect for example the general availability of the spare parts and the customers' motivation to repair the main product. Therefore one might hypothesize that at least in some component categories the life cycle phase of the main product might affect the price elasticity of demand. We divide the items based on life cycle phase into three segments with different status of production and availability of aftermarket support. The segments and the segmentation rules are listed in Table 3.1.

3.5 Results

Next we present the results of the analysis for the two separate segmentation frameworks. In the following analysis, if not specified in any other way, the segment specific price elasticity is estimated with Equation (3.10) and the confidence intervals are 95% two-tailed confidence intervals achieved with (3.3). For segmentation framework A, the p-values are p-values for one-tailed test (3.1) to test the corresponding hypothesis presented earlier in Section 3.4.2. For segmentation framework B, the p-values are for two-tailed test (3.2) of type $H_0 : \varepsilon_i = \varepsilon_j$, $H_A = \varepsilon_i \neq \varepsilon_j$. The default significance level selected for the analyses is $\alpha = 0.05$. We also mention separately if the results indicate significant differences at 0.01 level, but it should be noted that the test results with 1999 bootstrap samples are not necessarily accurate enough to provide certainty for the improved significance level.

3.5.1 Segmentation framework A

In the first part of our analysis, we divide the data based on two segmentation attributes: the criticality and the existence of competing suppliers. Total of 1323 items available for analysis are divided into segments. The amount of items in each segment is listed in Table 3.2.

Table 3.2: Item count in each segment for segmentation framework A.

	Critical	Non-critical
Key parts	136	222
Industry specific parts	3	294
Commercial parts	0	567
Standard parts	0	101

We notice that only key parts have sufficient amount of items in both critical and non-critical segments. Because there is not enough items labeled critical in any other segment and we have hypothesized that the significance status affects the price sensitivity of the customers, we restrict our hypothesis *H1* to key parts only. The price elasticity estimates and the related 95% confidence intervals (in parentheses) for the non-critical and critical segments of key parts are listed in Table 3.3.

Table 3.3: Price elasticity estimates (corresponding 95% confidence intervals in parenthesis) for critical and non-critical key parts.

	Critical	Non-critical
Key parts	-0.801	-0.913
	(-1.121, -0.433)	(-1.157, -0.685)

H1: The p-value for one-tailed test is $p = 0.293$, which is not enough to reject the null hypothesis at 0.05 or even at 0.1 level. Thus we conclude that our data does not show statistically significant evidence that the criticality of a key part would induce less price sensitive behavior among customers.

Since no significant effect was detected that the criticality would affect the price elasticity, the rest of the hypotheses are tested so that the criticality dimension is dismissed and the data is divided solely based on the significance status. The estimated price elasticities with corresponding confidence intervals for the segments are listed Table 3.4.

Table 3.4: Price elasticity estimates and corresponding 95% confidence intervals for significance based segmentation.

	$\hat{\varepsilon}$	Wald Confidence interval
Key parts	-0.875	(-1.062, -0.691)
Industry specific parts	-0.421	(-0.662, -0.184)
Commercial parts	-0.426	(-0.588, -0.246)
Standard parts	-0.172	(-0.583, 0.213)

H2: The price elasticity estimates in Table 3.4 imply that the demand for key parts would be more elastic than for industry specific parts, contrary to the theoretical hypothesis. The p-value for one-tailed test is $p = 0.998$, and we fail to reject the null hypothesis.

H3: The p-value for one-tailed test is $p = 0.479$. The null hypothesis is not rejected.

H4: Again, the price elasticity estimates achieved imply opposite than the theoretical hypothesis. The p-value for one-tailed test is $p = 0.906$, and we fail to reject the null hypothesis.

Rather surprisingly, our data did not support any of the theoretical assumptions on how the availability or criticality of an item affects the price elasticity of demand, and in some situations even completely opposite behavior was observed. Further speculation on possible reasons for such behavior is left to Section 3.6.

3.5.2 Segmentation framework B

In the second part of our analysis we study the price elasticity within a variety of component categories. As discussed earlier, the component categories studied are selected so that each segment contains at least 50 items. The elasticity estimates, related confidence intervals and item counts for each segment are listed in Table 3.5. The total count of items in the second part of the analysis is 1220.

Table 3.5: Price elasticity estimates, corresponding 95% confidence intervals and total item count for studied component categories.

	$\hat{\varepsilon}$	Wald Confidence Interval	Count
Mechanics	-0.611	(-0.882, -0.363)	200
Fuses	-0.356	(-0.586, -0.062)	182
Power Semiconductors	-0.370	(-0.695, -0.067)	162
Switches, Relays and Contactors	-0.505	(-0.786, -0.251)	89
Capacitors	-0.687	(-1.085, -0.259)	71
Resistors	-0.115	(-0.280, 0.114)	60
Fans and Air Filters	-1.748	(-2.076, -1.314)	61
Boards	-0.724	(-0.903, -0.541)	319
Wires	-0.261	(-0.555, 0.111)	76

We notice that most categories show more or less inelastic behavior, the category for fans and air filters being the only one with evidence of elastic behavior. To uncover if there is significant variation within the categories that can be explained by the relative complexity or the life cycle phase of the main product, further segmentation is conducted.

Let us start by studying the relative complexity measure. First the segmentation attribute is inspected independently from the component category. The estimated elasticities together with confidence intervals and segment item counts are presented in Table 3.6.

The estimate for the segment with undefined complexity differs strongly from the others by implying rather highly elastic demand. We conduct series of two-tailed tests to see if the segment specific price elasticity estimates differ significantly on 0.05 level. The p-values are presented in Table 3.7.

Table 3.6: Price elasticity estimates, corresponding 95% confidence intervals and total item count for relative complexity based segmentation.

	$\hat{\varepsilon}$	Wald Confidence Interval	Count
Simple	-0.473	(-0.622, -0.328)	652
Normal	-0.664	(-0.888, -0.432)	309
Complex	-0.839	(-1.117, -0.589)	206
N/A	-2.610	(-3.519, -1.420)	53

Table 3.7: p-values for two-tailed tests to compare price elasticity estimates for relative complexity based segmentation. Values in bold indicate rejection of the null hypothesis.

Segment 1	Segment 2	p-value
simple	normal	0.152
simple	complex	**0.019**
simple	N/A	**0.001**
normal	complex	0.320
normal	N/A	**0.001**
complex	N/A	**0.001**

The results indicate that the price elasticity for the N/A segment is significantly different from all the other segments at 0.01 level. There is also statistically significant difference (at 0.05 level) between the price elasticities of the "simple" segment and the "complex" segment.

Let us next study the cross-sectional effect of the component category and relative complexity. The count of items within each sub-segment is listed in Table 3.8. Sub-segments with less than 20 items that are excluded from the cross-sectional study are marked in italics in the table.

Each component category that can be segmented into two or more sub-segments with sufficient amount of items are studied. Such categories are Mechanics, Fuses, Power Semiconductors, Boards and Wires. Other component categories are maintained as such for further analysis. The price elasticity estimates for sub-segments are presented in Table 3.9. The corresponding 95% confidence intervals are given in parenthesis below the estimate.

We test for statistically significant differences in sub-segment price elasticities within each component category with a two-tailed test. The p-values of each test are listed in Table 3.10, of which the ones that support rejecting the null hypothesis of equal price elasticity at significance level 0.05, are

Table 3.8: Count of items in each segment formed by component category and component relative complexity. Numbers in italics indicate the sub-segment is too small for further analysis.

	Simple	Normal	Complex	N/A	Total
Mechanics	132	37	27	*4*	200
Fuses	92	53	37	*0*	182
Power Semiconductors	80	50	31	*1*	162
Switches, Relays and Contactors	61	*15*	*12*	*1*	89
Capacitors	51	*8*	*11*	*1*	71
Resistors	29	*17*	*13*	*1*	60
Fans and Air Filters	39	*13*	*9*	*0*	61
Boards	130	94	50	45	319
Wires	38	22	*16*	*0*	76
Total	652	309	206	53	1220

Table 3.9: Price elasticity estimates (corresponding 95% confidence intervals in parenthesis) for relative complexity and component category based segmentation.

Attribute values	simple	normal	complex	N/A
Mechanics	-0.613 (-0.994, -0.304)	-0.527 (-1.434, 0.562)	-0.369 (-2.005, 1.423)	
Fuses	0.081 (-0.215, 0.336)	-0.922 (-1.741, -0.269)	-1.024 (-1.51, -0.466)	
Power Semiconductors	0.071 (-0.621, 0.605)	-0.838 (-1.492, -0.01)	-0.764 (-1.646, -0.155)	
Boards	-0.673 (-0.921, -0.423)	-0.766 (-1.184, -0.241)	-0.525 (-1.814, 0.153)	-2.544 (-3.6, -0.792)
Wires	-0.246 (-0.661, 0.404)	-0.278 (-0.849, 0.388)		

Table 3.10: p-values for two-tailed tests to compare price elasticity estimates for relative complexity based sub-segmentation on component categories. Values in bold indicate rejection of the null hypothesis.

Sub-segment 1	Sub-segment 2	p-value
Mechanics		
simple	normal	0.850
simple	complex	0.584
normal	complex	0.821
Fuses		
simple	normal	**0.004**
simple	complex	**0.005**
normal	complex	0.792
Power semiconductors		
simple	normal	**0.035**
simple	complex	0.072
normal	complex	0.851
simple	normal + complex	**0.025**
Boards		
simple	normal	0.692
simple	complex	0.588
simple	N/A	**0.001**
normal	complex	0.543
normal	N/A	**0.001**
complex	N/A	**0.002**
Wires		
simple	normal	0.963

marked in bold.

The results indicate that no significant variation in price elasticity due to component complexity was observed for Mechanics or Wires. The price elasticity estimate for Fuses categorized as simple differed from Fuses categorized normal or complex at the 0.01 significance level, whereas the categorization into normal and complex did not show statistically significant distinction. For Power Semiconductors the simple category was statistically different from the normal category at 0.05 level, but the null hypothesis of equal price elasticity for the simple category and the complex category, or the normal category and the complex category could not be rejected at 0.05

level. Therefore we conducted an additional test to study if the segmentation into two sub-segments: 1) simple and 2) normal + complex showed evidence of statistically significant difference in price elasticity. The p-value of 0.025 supports this sub-segmentation.

For the Boards component category the price elasticity for sub-segment "N/A" differed from all the other categories on 0.01 level, but no significant difference was detected between the other three sub-segments.

The distribution between component category sub-segments is illustrated in Figure 3.3. The histograms of sub-segment bootstrap samples used for confidence interval construction are drawn for each of the component categories. The sub-segments for mechanics have different variability, but the mean values lay relatively close. The histograms for fuses, power semiconductors and boards have two distinct peaks, consistent with the p-values achieved earlier. The histograms for the two sub-segments of wires support the conclusion separating wires based on relative complexity does not provide additional value to the analysis.

Based on these results we introduce following additional segmentation to the component categories:

1. Fuses_1: Fuses with simple relative complexity
2. Fuses_2: Fuses with normal or complex relative complexity
3. Power Semiconductors_1: Power semiconductors with simple relative complexity
4. Power Semiconductors_2: Power semiconductors with normal or complex relative complexity
5. Boards_1: Boards with simple, normal or complex relative complexity
6. Boards_2: Boards with undefined (N/A) relative complexity.

Let us next consider the second sub-segmentation attribute selected for this study, namely the life cycle phase of the main product. First, study the segmentation based on this attribute alone. The elasticity estimates $\hat{\varepsilon}$, corresponding confidence intervals and the item count in each segment are given in Table 3.11.

The results of two-tailed tests between segments are presented in Table 3.12. The segment B has price elasticity different than that of segments A or C at the 0.01 level. The difference between segments A and C however is not statistically significant at 0.05 level, even though the absolute values of the price elasticity estimates are relatively different. If we study the histograms

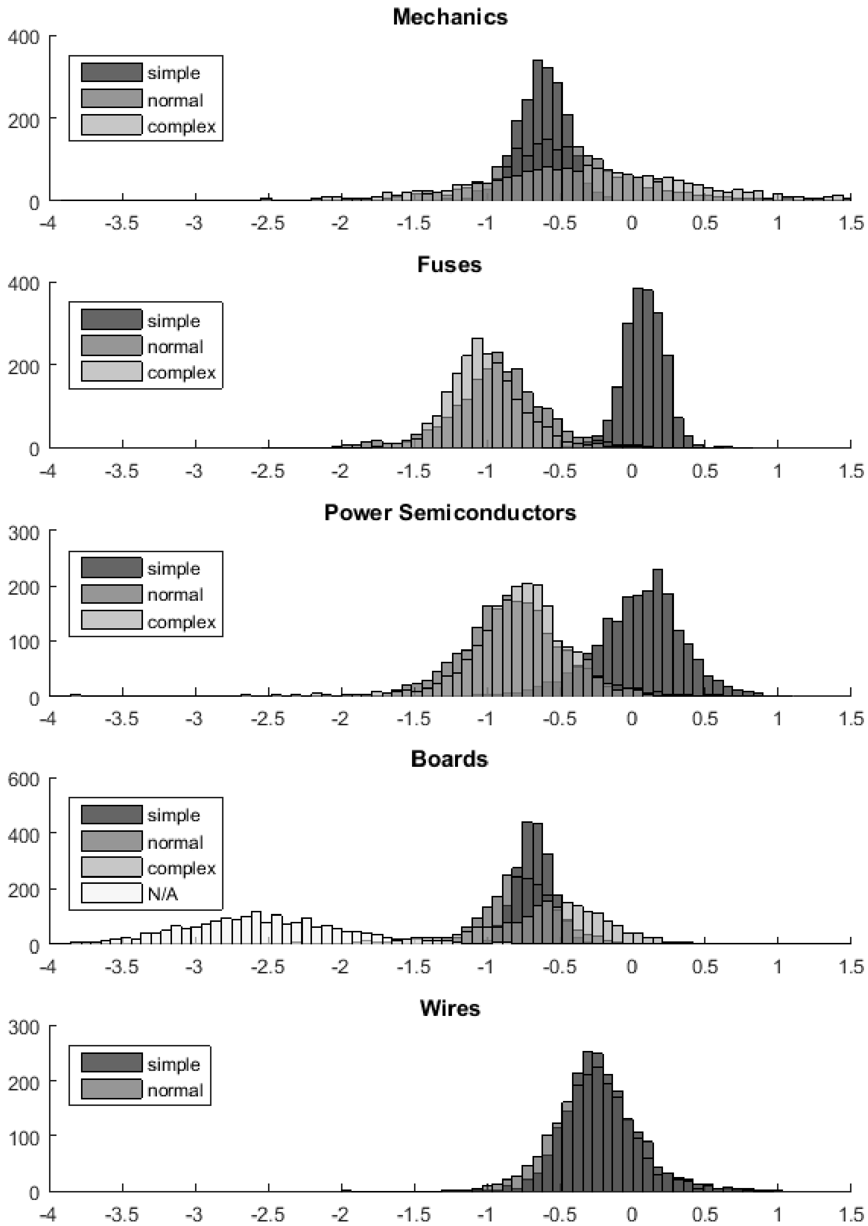

Figure 3.3: Bootstrap sample histograms for relative complexity sub-segmentation.

Table 3.11: Price elasticity estimates, corresponding 95% confidence intervals and total item count for main product life cycle phase based segmentation.

	$\hat{\varepsilon}$	Wald Confidence Interval	Count
A	-0.688	(-0.826, -0.524)	806
B	-0.172	(-0.370, 0.016)	282
C	-1.056	(-1.435, -0.660)	131

Table 3.12: p-values for two-tailed tests to compare price elasticity estimates for main product life cycle phase based segmentation.

Segment 1	Segment 2	p-value
A	B	0.001
A	C	0.070
B	C	0.001

of the bootstrap samples to calculate the confidence intervals (Figure 3.4), we notice that the price elasticity estimate for segment C has a lot more variation than that of A, which might explain why the null hypothesis cannot be rejected at 0.05 level. The difference in variation can be partially explained by the uneven amount of items within the segments, as the segment C has the least items.

Let us next study if additional segmentation based on the main product life cycle phase will reveal sub-segments in the updated component category segmentation with significant difference in estimated price elasticities. The item counts within each segment are listed in Table 3.13. Again, the sub-segments disregarded from the following analysis due to insufficient amount of data are marked in italics. Note that there is one less item in the analysis in total because the category Power Semiconductors was divided into two sub-segments, and there was not enough data to determine into which segment the item with relative complexity of N/A should belong to.

Component categories that have enough items for further analysis are Mechanics, Fuses$_1$, Power Semiconductors$_1$, Boards$_1$ and Wires. Sub-segment price elasticity estimates with 95% confidence intervals in parenthesis are listed in Table 3.14.

Again, a two-tailed test is used to determine if the price elasticities for segments are the same or not at 0.05 level. The results of the tests are presented in Table 3.15. The segmentation based on the core product life cycle phase has no statistically significant effect on the price elasticity estimates

Table 3.13: Count of items in each segment formed by component category and the life cycle phase of the main product. Numbers in italics indicate the sub-segment is too small for further analysis.

	A	B	C	Total
Mechanics	152	39	*9*	200
Fuses_1	64	28	*0*	92
Fuses_2	74	*15*	*1*	90
Power Semiconductors_1	55	21	*4*	80
Power Semiconductors_2	48	*19*	*14*	81
Switches, Relays and Contactors	78	*9*	*2*	89
Capacitors	38	*16*	*17*	71
Resistors	34	*17*	*9*	60
Fans and Air Filters	45	*16*	*0*	61
Boards_1	171	71	32	274
Boards_2	*0*	*7*	38	45
Wires	47	24	*5*	76
Total	806	282	131	1219

Table 3.14: Price elasticity estimates (corresponding 95% confidence intervals in parenthesis) for main product life cycle phase and component category based sub-segmentation.

Attribute values	A	B	C
Mechanics	-0.584	-0.466	
	(-1.002, -0.195)	(-0.914, 0.067)	
Fuses_1	0.099	0.008	
	(-0.22, 0.359)	(-1.136, 1.01)	
Power Semiconductors_1	0.075	-0.0003	
	(-0.703, 0.742)	(-2.13, 1.365)	
Boards_1	-0.811	-0.012	-1.479
	(-1.056, -0.561)	(-0.419, 0.505)	(-2.628, -0.897)
Wires	-0.852	0.388	
	(-1.184, -0.495)	(-0.513, 0.987)	

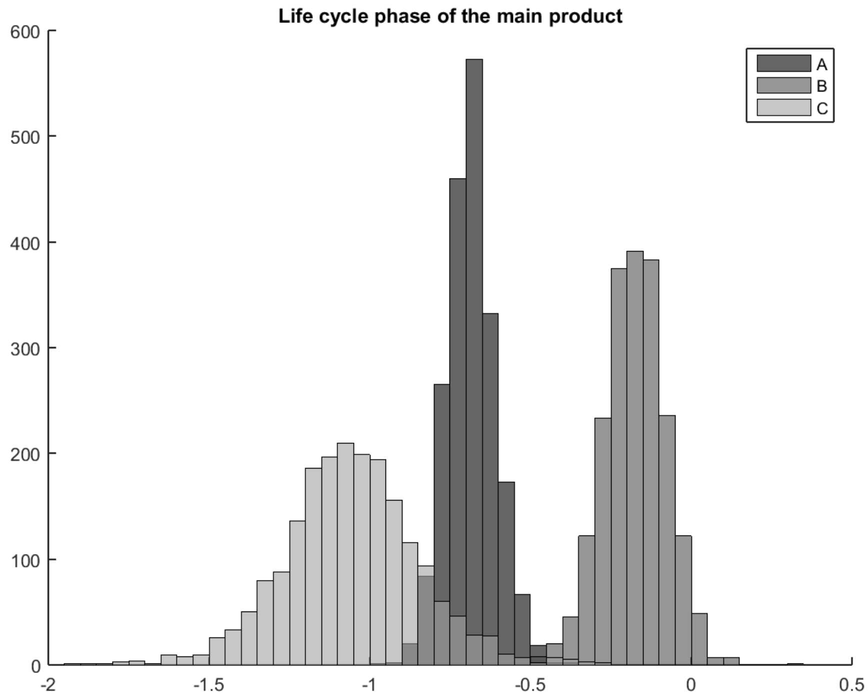

Figure 3.4: Bootstrap sample histogram for main product life cycle phase segmentation.

of component categories Mechanics, Fuses_1 or Power Semiconductors_1. The elasticity estimates for component category sub-segments of Boards_1 and Wires however were significantly different on 0.05 level.

Based on the results from two different sub-segmentation attributes, we propose the following update to the component category segmentation:

1. Fuses_1: Fuses with simple relative complexity
2. Fuses_2: Fuses with normal or complex relative complexity
3. Power Semiconductors_1: Power semiconductors with simple relative complexity
4. Power Semiconductors_2: Power semiconductors with normal or complex relative complexity
5. Boards_2: Boards with undefined (N/A) relative complexity
6. Boards_3: Boards with simple, normal or complex relative complexity, related to a product at life cycle phase A

Table 3.15: p-values for two-tailed tests to compare price elasticity estimates for main product life cycle phase based sub-segmentation on component categories. Values in bold indicate rejection of the null hypothesis.

Sub-segment 1	Sub-segment 2	p-value
Mechanics		
A	B	0.791
Fuses_1		
A	B	0.755
Power Semiconductors_1		
A	B	0.884
Boards_1		
A	B	**0.001**
A	C	**0.036**
B	C	**0.003**
Wires		
A	B	**0.001**

7. Boards_4: Boards with simple, normal or complex relative complexity, related to a product at life cycle phase B
8. Boards_5: Boards with simple, normal or complex relative complexity, related to a product at life cycle phase C
9. Wires_1: Wires related to a product at life cycle phase A
10. Wires_2: Wires related to a product at life cycle phase B.

For the updated segmentation we conduct an additional clustering procedure as described in Section 3.2.2. This is to reduce the amount of segments, and make it easier to compare which segments behave similarly and which do not. For the clustering we choose a more liberal significance level, namely 0.1. This is motivated by the characteristics of the analysis: joining segments that in reality do not behave similarly has more serious consequences than having two clusters with similar behavior. A dendrogram from the clustering procedure is presented in Figure 3.5. At each node the price elasticity estimate for the formed cluster is given, and the horizontal position of the node is determined by $1 - p$ where p is the p-value for a two-tailed test with $H_0 : \varepsilon_i = \varepsilon_j$, $H_A : \varepsilon_i \neq \varepsilon_j$.

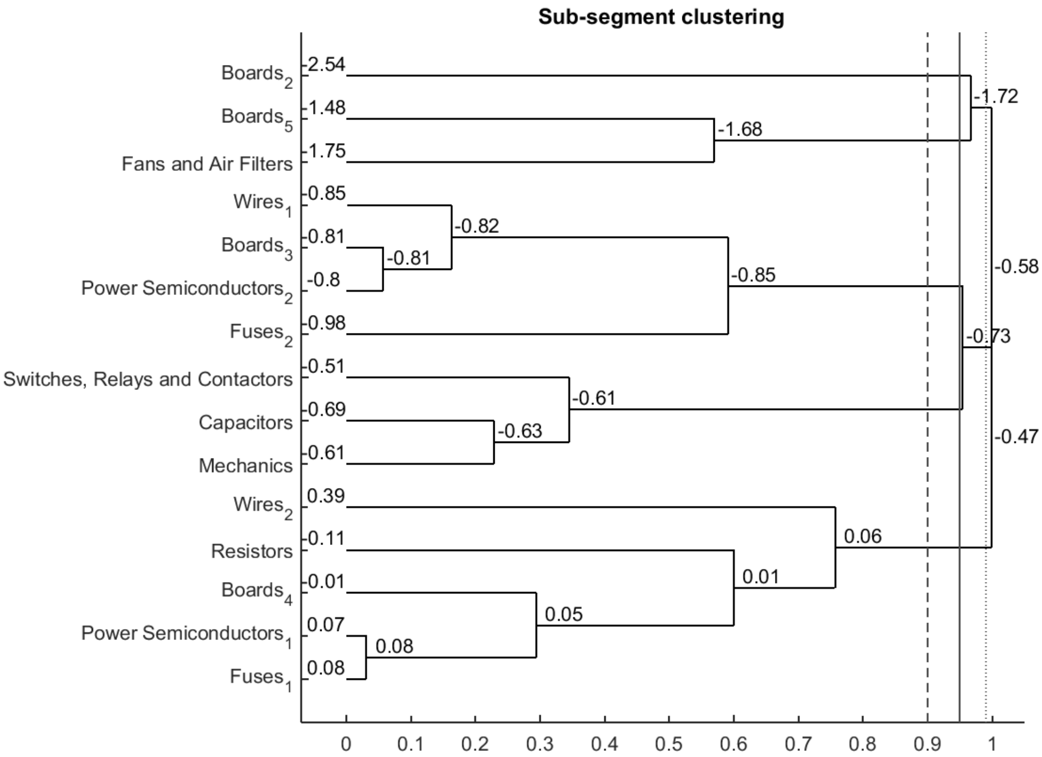

Figure 3.5: Dendrogram for final segment clustering. Dashed line marks 0.1 significance level, the solid line 0.05 level and the dotted line 0.01 level.

With threshold for clustering set to 0.1, five clusters are formed. From the Figure 3.5 we see that the exactly same clustering is achieved at the 0.05 level as well, yet just barely. The price elasticity estimates, 95% confidence intervals and segment item counts for these clusters and the segments included in each of them are presented in Table 3.16. Of these, Cluster 1 and Cluster 2 show signs of elastic demand and the rest indicate inelastic demand. The estimated price elasticity of Cluster 3 is positive, which might indicate that the price changes of items in this cluster in general have had little if any effect on demand.

Table 3.16: The price elasticity estimates and 95% confidence intervals for achieved clusters and the segments included in each of them.

	$\hat{\varepsilon}$	Wald Confidence Interval	Count
Cluster 1	**-2.544**	**(-3.634, -0.878)**	**45**
Boards$_2$	-2.544	(-3.634, -0.878)	
Cluster 2	**-1.682**	**(-2.035, -1.29)**	**93**
Boards$_5$	-1.479	(-2.668, -0.893)	
Fans and Air Filters	-1.748	(-2.17, -1.209)	
Cluster 3	**0.058**	**(-0.095, 0.21)**	**327**
Wires$_2$	0.388	(-0.41, 1.048)	
Boards$_4$	-0.012	(-0.379, 0.474)	
Power Semiconductors$_1$	0.071	(-0.621, 0.582)	
Fuses$_1$	0.081	(-0.209, 0.328)	
Resistors	-0.115	(-0.321, 0.179)	
Cluster 4	**-0.607**	**(-0.822, -0.406)**	**360**
Switches, Relays and Contactors	-0.506	(-0.863, -0.193)	
Capacitors	-0.687	(-1.165, -0.113)	
Mechanics	-0.611	(-0.915, -0.313)	
Cluster 5	**-0.855**	**(-1.007, -0.696)**	**389**
Fuses$_2$	-0.978	(-1.35, -0.629)	
Wires$_1$	-0.852	(-1.163, -0.452)	
Boards$_3$	-0.811	(-1.045, -0.544)	
Power Semiconductors$_2$	-0.799	(-1.237, -0.363)	

3.6 Discussion

In this Chapter we have familiarized ourselves with the mathematical definition of the price elasticity of demand and derived an estimate for segment specific price elasticity. With the derived estimate we studied the price elasticity of spare parts and spare part bundles by segmenting the available data based on item attributes of interest. Two different segmentation frameworks were studied in this thesis.

The first segmentation framework was adapted from literature and it segmented items based on criticality and existence of competing suppliers (estimated from significance status classification). Our results did not reflect the hypotheses adapted from literature and in some situations suggested completely opposite behavior. We suspect that the reason for this might be that the classification used was not precise enough, or that the significance status classification did not succeed in reflecting the availability risk. One possible source of error is misclassification of some items. Because these attribute values were assigned for specific component types, an item might get incorrect attribute values if the original component type categorization is wrong.

For the second segmentation framework we had no literature based hypotheses. We studied if variation in price elasticity of demand occurred when items were segmented based on component type. Some component categories used were likely to contain items of different scale and different relative complexity, which is why we determined a sub-segmentation attribute to further sub-segment the component categories. We also considered that the life cycle phase of the item related main product might affect the price sensitivity of the customers and thus another sub-segmentation was conducted. Through these sub-segmentation procedures, four component categories were divided into ten significantly separate sub-segments.

For the updated category segmentation a clustering algorithm was applied in order to unite segments with similar price elasticity. Five clusters were identified to be significantly separate at 0.1 (and 0.05) level. The two smallest clusters indicated possibly elastic behavior, containing boards with unavailable relative complexity and boards related to products no longer in production and with limited product support (life cycle phase C). Further inspection of Table 3.13 reveals that the majority of boards with unavailable relative complexity are related to products with life cycle phase C as well. One might speculate that the customers with such products are considering the alternatives to buying the spare part, for example updating the complete product instead of repairing the old one. Availability of viable alternatives might result in observed elastic demand.

Also Fans and air filters were included in a cluster with slightly elastic demand. The items were related mostly to products at life cycle phase A, so consideration of product update is not a potential explanation for elastic demand. However, elastic demand is often connected to products with available alternatives, which might explain the elastic demand for fans and air filters.

The rest of the achieved clusters showed different levels of inelastic demand. Considering that almost 90 percent of items in our study fell into inelastic clusters, it seems like the price is generally not that important factor to affect the demand for aftersales items. Compared to the meta-analysis of Bijmolt et al. (2005) where the mean price elasticity of nearly 2000 estimates was -2.62, our study shows generally rather price inelastic behavior among customers. This is not that surprising if we reflect the reasons why spare parts are acquired. Commonly spare parts are purchased to either replace a faulty part or to be prepared for part malfunction, i.e. the demand for spare parts is driven by compulsion. Especially if the faulty part causes an entire production facility to halt, the cost of the spare part is small compared to the cost of down-time. All in all, we believe it is possible that the customers in the aftermarkets are generally less price sensitive than with other purchase decisions since the circumstances where purchase decisions are made are often different. Further discussion on special features of the aftermarket demand is postponed to Chapter 4.

The most inelastic behavior (price elasticity estimate close to zero) was detected for Cluster 3, which contained wires and boards (not N/A) related to life cycle phase B, simple power semiconductors and fuses, and resistors. For these types of products the changes in prices did not generally explain the changes in demand. Reason to this kind of behavior could be that the item is extremely important or difficult to find elsewhere. On the other hand, if the price changes in the data are small, the effect of price sensitivity might stay undetected.

The second most inelastic behavior was detected for Cluster 4 ($\hat{\varepsilon} \approx -0.61$). The cluster contained switches, relays, contactors, capacitors and mechanics. The least inelastic behavior ($\hat{\varepsilon} \approx -0.86$) occurred for cluster with normal and complex fuses and power semiconductors, and boards (not N/A) and wires related to products at life cycle phase A. Items related to products at life cycle phase A might have a better overall availability, thus affecting the price sensitivity of the customers. On the other hand, as we hypothesized when the concept of relative complexity was introduced, the assumption of constant price elasticity for different priced items might not hold for some component categories. Based on the results of our analysis, studying if the price elasticity of demand varies with price would be of interest at least for fuses and power semiconductors.

In the analysis we only experimented two additional segmentation attributes that we found potential. As both attributes had effect on some component categories, examining other product related attributes for further segmentation would be an interesting continuum to this study. The major down-side of additional sub-segmentation is the growing need for data, since the elasticity estimates become more unreliable when the sample size gets smaller.

An interesting feature of the applied sub-segmentation was that the additional attributes did not affect all the component categories statistically significantly, but only few. However, the effect of sub-segmentation was similar across categories it had affected: items specified simple showed less elastic behavior than normal or complex items when significant difference was detected. Similarly, based on the price elasticity estimates for the life cycle phase segmentation, the demand for item segment B was less elastic than for item segment A, that again was less elastic than demand for item segment C. However, since the number of discovered sub-segments is rather small, we must consider the possibility of this being by chance. Additional study on the effects of sub-segmentation would possibly require including more component categories, which in turn would require more data.

When the achieved price elasticity estimates are studied and interpreted, we must also ponder the possibility that all relevant sub-segmentations were not found. Consider the component category for wires: the common price elasticity estimate for the segment is -0.261, but when the sub-segmentation based on life cycle phase of the main product into A and B was conducted, the estimates achieved were -0.852 and 0.388 respectively. Discussing the price sensitivity of the customers towards wires based on the general estimate would result in notable underestimation of customer price sensitivity for some wires (life cycle phase A) and overestimation for others (life cycle phase B). For the reliability of the estimates, it is crucial that the segmentation is successful. Further study to find a measure to estimate the goodness of segmentation would be of interest since the selected confidence interval estimations rely on the assumption that the items within the segment in fact have the same price elasticity.

In this study we conducted the two segmentation frameworks separately. We suspect it might be interesting to study the effect of life cycle phase and item complexity to significance status based segmentation. This is because we have noticed they have some significant effect on the component categories, which partially lay behind the significance status attribute value as well. We believe this additional segmentation might explain partially why the results of our analysis did not correspond to the hypothesis from literature.

The model we constructed for segment specific price elasticity of demand made some simplifying assumptions. Even though we attempted to minimize the effects of these assumptions by data restrictions and method selections, it is important to understand the most important simplifications. First of all, the model used for the price response function was continuous. However, technically the dependent variable (demand) is discrete. Because we use the logarithmic scale, the discreteness of the demand is emphasized on low levels of demand. Since we restricted our data based on general demand level in order to exclude items with constantly too low demand, we assume the effect of discreteness of demand is rather small.

We used the item specific price as the only variable to explain changes in demand. Yet, this might be misleading (Simon, 1989), especially if some external variable excluded from the analysis has a similar effect on the demand for multiple items within the segment. If the effect of the external variable correlates with the price changes, it is possible that the effect is absorbed into the price elasticity estimate and it seems like the price has affected the demand more (or less) than what it actually has. If the external variable affects only a few of the items within the segment, the biasing effect should be reduced when the segment size is sufficient.

Similar problems might occur if the assumption of independence between items fails. In this study we did not study how the prices of other items affect the demand, but we merely excluded the items we knew had dependencies within the segment. If there however are strong connections we were not able to identify, it is possible that the cross-price effect causes bias to the results.

When the data was pre-processed for price elasticity estimation, we disregarded items with too few data points for analysis. However, we did not rule out items based on how much the price varied in the data, but we merely assumed that if the number of items in each segment is large enough, the variability of single item prices is not as relevant. Nevertheless, if we consider a segment with very little price variation in almost all of the items, it is likely we would not be able to distinguish the price induced changes in demand. The resulting price elasticity estimate would not be reliable and would likely be close to zero.

As we pointed out when the model was built, OLS is no longer effective for situations with heteroscedastic errors. Nevertheless OLS was used for parameter estimation. With OLS it is possible to calculate the estimate for price elasticity analytically which in turn eased for example variation estimation. One might suspect different estimation method might produce slightly different price elasticity estimates. However, since the goal of the analysis was to understand the behavior of the customers on a general level, the exact numerical values of price elasticity are not as important as the relative position (e.g. greater or less than -1) and the relative order of different segments (e.g. segment a shows more elastic behavior than segment b).

Chapter 4

Application in aftermarket pricing

4.1 Pricing and price elasticity of demand

In this thesis we have discussed pricing and pricing strategies, and familiarized ourselves with the concept of aftermarket business. We have conducted statistical analysis to understand the price sensitivity of the customers for after-sales spare parts by estimating price elasticity of demand for different item segments. However, until now the connection between pricing and price sensitivity has been given little attention.

In order to model demand for price optimization, the diversity of underlying phenomena influencing demand should be considered: the price of a product is almost never, if ever, the only variable affecting the demand for that product. How strong an effect different factors have on the demand depends on the application: in some fields advertising expenses have a substantial role in sales promotion whereas for some products supplement and complement product price changes modify consumer preferences. Our study focuses solely on understanding the effect of item specific price changes on item specific demand, and therefore the demand model selected for price elasticity estimation is not directly applicable for example in actual price optimization.

But what is the connection between price elasticity and pricing, and how can we use price elasticity estimates to evaluate if the price adjustment is profitable or not?

To illustrate the relationship between price elasticity and profitability, let us consider the following simplified example. Assume a product with original price P and original demand D. The total cost of the product to the vendor is C. The profit of each item sold, i.e. the contribution margin the vendor profits is $M = P - C$, percentually $m = \frac{M}{P}$ (taxes etc. disregarded). The total profit therefore is MD. Say we are planning a price change of ΔP, thus the proportional change is $p = \frac{\Delta P}{P}$. Not only does the profitability of this decision depend on the price elasticity of the product but the contribution margin of the product.

Assume that the total cost C of a product is a constant that does not depend on demand D, and that price is the only independent variable that affects the demand. As a results, the price increase ΔP is transferred directly to contribution margin and the contribution margin increases percentually with $\frac{\Delta P}{M} = \frac{p}{m} = b$. The total profit does not decrease if the percentual change in demand $d = \frac{\Delta D}{D}$ is at most:

$$
\begin{aligned}
(1+b)M(1+d)D &\geq MD \\
(1+b)(1+d) &\geq 1 \\
(1+d) &\geq \frac{1}{(1+b)} \\
d &\geq \frac{1}{(1+b)} - 1 \\
-d &\leq 1 - \frac{1}{(1+b)} \\
-d &\leq 1 - \frac{m}{(m+p)}
\end{aligned}
$$

Thus, for a 1% price increase to be profitable for a product with for example 10% contribution margin, the decrease in demand must be less than 9.09% and with 60% contribution margin less than 1.64%. In other words, if the original contribution margin for a product is 60%, the price increase of 1% is profitable if the price elasticity for that product is ≥ -1.64. Respectively, if we are apply similar price decrease, it is profitable if the demand is more elastic than that.

The lower the contribution margin is, the greater the changes that are allowed in demand before the price increase becomes unprofitable. However, even if we make the theoretical assumption that the total cost of the product is zero, and consequently the price increase is the same as the marginal profit increase, the 1% price increase is profitable if demand decreases with less than 0.99%. This would imply that with inelastic demand the price increases are practically always profitable.

One of the main flaws of the constant elasticity model is that it suggests increasing prices for products with inelastic demand infinitely. Thus, it should only be applied to small price changes after which the price elasticity estimates must be updated. Furthermore, the price elasticity estimates calculated in this thesis are calculated for item segments and should therefore be interpreted as indicative. Applying them directly to estimate the profitability of a price change for a single item might produce incorrect evaluations. It is important to acknowledge that the clusters might include both over- and underpriced items, and thus the segment specific price elasticity should always be interpreted as an average price elasticity for products in that segment.

To conclude, we have noticed that price elasticity can be used to estimate if a relatively small price change is profitable or not. On some level, the price elasticity estimates always reflect the price and demand they were originally calculated at. When the prices change, the estimates should be updated to match the new situation in order to recognize if the price change has affected the price sensitivity of the customers.

4.2 Pricing in aftermarket business

Success in the aftermarket starts with strategy (Gebauer et al., 2005). Not only does this account for which services to offer and which products to support but also constructing and implementing an aftermarket pricing strategy that supports the long-term strategy of the company. This being said, interpreting the results of the analysis in this thesis, and discussing them for aftermarket price adjustments should always be accompanied with the question "What is our target, and will this decision bring us closer to that target?"

The results of our analysis in Chapter 3 suggest that most of the studied spare part segments have inelastic demand. The few segments for which we identified rather elastic demand contained items for which we were able to recognize some decision alternatives. We believe the existing alternatives explain the different price response compared to other item segments. Moreover, for the rest of the segments we identified varying levels of inelastic demand.

Compared to the meta-analyses of Tellis (1988) and Bijmolt et al. (2005) on price elasticity estimates in general, our estimates for spare parts show more inelastic demand than sales items generally. The average price elasticity in our study, calculated as a cluster item count weighted mean from cluster elasticities, is -0.66 whereas the mean price elasticity in the study of Bijmolt et al. (2005) is -2.62 (total count of 1851 elasticities, median price elasticity -2.22). Based on this it seems like the customers are a lot less price sensitive in the aftermarkets than with purchase decisions in general.

In order to utilize this information in spare part pricing, it is crucial to discuss the possible underlying reasons for these results. The price elasticity estimates are achieved with a simplified statistical model, and we must consider the possibility the inelastic estimates might be produced by bias in our model. We discussed such aspects earlier in Section 3.6 and thus we now focus on the specialties of the aftermarket business that we believe might explain the inelastic demand.

The customer in the aftermarkets is almost always a quarter already owning the core product. Thus the pool of potential customers is limited, but since the customer is already familiar with the product manufacturer, the manufacturer has a competitive edge. In addition to that, the manufacturer usually possesses superior knowledge and understanding of the core product. Thus, even if the manufacturing company might have competition in the aftermarkets, it has an edge over the third party suppliers (Borenstein et al., 2000).

Another important special feature in the aftermarkets is that the demand for spare parts is often created by urge, and the spare part is acquired to replace a broken part. In some circumstances the broken part results in production facility down-time, which is likely to be costly to the customer. In these situations where the opportunity costs are high, the customer is not likely to be price sensitive and willing to spend time finding alternative suppliers to save a little in the price of a spare part.

It is not rare that some of the spare parts are proprietary and consequently not offered by competitors. This might limit the options the customer has in the case of a break-down. We must consider the possibility that the inelastic demand can sometimes be a result of absence of alternatives. If the customer has no other viable options but to purchase the part from the manufacturer, the demand seems inelastic. This however does not necessarily mean the customers would be happy with the prices they are paying. Especially for company proprietary products it might not be possible to make reliable conclusions on if the products are over- or underpriced based on price elasticity estimates alone.

We also believe that such aspects as convenience and quality might have an influence in the price sensitivity of the customers. In the analysis in this thesis we have ignored the possibility that our product segments might contain complement items. Complement items in this situation mean that the customers do not purchase just one spare part item but several items that are necessary to repair the main product. In addition to the unexplored effect on complement part price changes, one might hypothesize that the convenience of being able to purchase all the needed parts at once instead of searching for each part at a time from possibly multiple suppliers is a factor that might decrease the price sensitivity of the customers. We suspect this is likely especially for cheaper parts that are bough together with expansive parts, since the smaller price might be interpreted rather irrelevant next to the more expensive part that is needed.

As one may trust the manufacturer possesses the best possible understanding of its products, we believe the conveyed quality is another factor that might lower the price sensitivity of the customers in the aftermarkets. We often interpret the price as an indicator for quality. Even in situations in which there are third party suppliers offering alternatives with lower prices, the customers might prefer the product from the original manufacturer in order to ensure the quality and operation of the main product.

We have now discussed some aspects we believe are relevant factors in the aftermarkets and that might affect the price sensitivity of the customers. Even though the inelastic price elasticity estimates achieved in this thesis give room for possible price level increases, we want to emphasize that the estimates are merely indicative. It is necessary to understand the underlying factors of these estimates and the aftermarket business to comprehend the possible outcomes of price changes.

In this thesis we have mostly discussed the aftermarket business separately from the core business of the manufacturer. However, we must acknowledge that the aftermarket pricing decisions might have an effect on the actual market as well. In the case of a break-down the customers are likely to either fix the product or to replace the product with a new one. As we discuss expensive durables, the latter is more likely when the original product is old, and the availability and prices of spare parts and services do not satisfy the customer. Earlier we discussed that extending to aftermarket business has the ability to provide companies competitive advantage, but this requires careful execution. Correspondingly, we speculate that the customer is more likely to consider competitive manufacturers for main product replacement, if the quality and prices of after-sales services and spare parts have not pleased the customer.

Chapter 5

Conclusions

Efficient pricing is a crucial factor in profitability, and it should also be addressed by the companies that have extended to aftermarket business. The vast amount of aftermarket sales items might drive companies to apply cost-based strategies in spare part pricing which might lead to lost profit when the prices do not reflect the actual value the customer sees in the item. For example Knecht et al. (1993) recommend applying value-based pricing strategies for spare parts in order to improve profitability.

To provide support for aftermarket pricing, we studied the price sensitivity of customers for specific segments of spare part items and item bundles. We started with a compact literature based discussion on pricing, pricing strategies and familiarized ourselves with the concept of aftermarket business. In the aftermarkets companies pursue competitive edge by providing the customers additional value through services and spare parts. Studies have indicated that even though the profit potential of aftermarket business is acknowledged, a negligent implementation has caused companies to drift into service paradox: a situation in which the expected profits from the aftermarket business are not achieved.

In this thesis we have analyzed the price sensitivity of the aftermarket customers by studying the price elasticity of demand. We experimented two separate segmentation frameworks for the spare part items. The first framework was adapted from the literature and it segmented the data based on criticality and the extent of alternative suppliers. The latter was estimated by dividing the items into company proprietary items and commercial items, and based on the estimated value for the customer. Surprisingly enough, our data did not support any of the hypotheses adapted from the literature. For now we can only speculate whether the reason to this is possible categorization errors in the data, unsuitable estimation of extent of alternative supplies or some other more significant phenomena in the underlying data. Further study on the possible reasons for the unexpected results, perhaps with additional segmentation, would be an interesting continuum to this study.

In the second part of the analysis we applied our own segmentation framework to study if different types of components have different price elasticity of demand. The original component categories were first divided based on estimated relative component complexity to distinguish if some component categories contained items with significantly different price elasticity due to difference in component complexity. Our results suggest that such sub-segmentation provided significant separation at 0.05 level for Fuses and Power Semiconductors. With component category Boards, we found that items with unknown relative complexity had a different price elasticity than items with known complexity.

Aftermarket spare parts are generally connected to some core product produced by the company. This was the situation in our study as well. Because the spare parts are offered for core products at different stages of life, we proceeded to sub-segment the data based on related core product life cycle phase. The phases concerned were: (A) in active production and full product support, (B) no longer in production but full support, (C) no longer in production, limited product support. Within the limits of available data, we found evidence that the life cycle phase of the main product affected the price elasticities for boards with known complexity, and for wires. With boards the influence was detected for all life cycle phases and with wires for phases A and B. The behavior of wires connected to products at life cycle phase C was not studied due to insufficient amount of data.

At the end a clustering algorithm was applied to the achieved sub-segments. If we consider the nature of the study, we suspect that it is preferable to have multiple clusters with similar price elasticity than combining two clusters with truly different price elasticity. Thus the threshold to quit clustering was selected lower than usually, namely 0.1. Considering the application, even lower threshold might have been justified. At 0.1 level we found 5 clusters with significantly different price elasticity, for which common price elasticity estimates were calculated. One interesting feature of the results is that nearly 90 percent of items in the study belonged to clusters with relatively inelastic demand.

As in any statistical analysis on real data, the results are not exact. Even though the segment specific price elasticity estimates were calculated based on the assumption of a common price elasticity of demand for all the items, the emerged clusters are likely to contain items with varying elasticities. Nevertheless, the overall behavior shows signs of inelastic demand. We have suspected that this might be a result of the nature of the aftermarket business in general. As discussed in Chapter 4, the motivation to purchase spare parts is likely to differ from the motivation for other, not aftermarket related, purchases. The need to repair the product and high alternative costs might explain the less price sensitive behavior of the customers towards aftermarket prices. We also suspect that the unstudied complement product effect might reduce the overall sensitivity: if the customer needs to purchase multiple items at a time, the prices of the less expensive products might not be considered as significant compared to the more expensive products purchased. Also the convenience of acquiring all the items at once instead of searching and purchasing each of them separately might affect the elasticity of demand.

Mathematically, inelastic demand in general suggests increasing the prices to increase profits. Nevertheless, we want to emphasize that one should always consider what the company strategy is and whether the price change supports that strategy. Since the price elasticity estimates calculated in this thesis are estimates for item clusters, it is important to acknowledge that as the cluster is likely to contain items with different price elasticities, it is consequently likely that it contains both under- and overpriced items. The segment specific estimate is a measure to estimate the average price sensitivity of the customers towards the products within the cluster. The question of how well the estimate succeeds in representing the items within the item segment is strongly dependent on the goodness of segmentation. Understanding of the averaging nature of the price elasticity estimate is crucial when the results are interpreted.

The main limitation to our analysis is that we use price as the only independent variable. This might cause the price elasticity estimate to account for the effect of other demand affecting variables as well, and thus the estimate might be biased. However, as we have discussed, we believe that the sufficient amount of items within a cluster should reduce the biasing effect, as long as the omitted variables affect only few of the items within the cluster.

Using price as the only independent variable not only has the ability to induce bias but also fails to provide enough accuracy for price optimization. This being said, further studies including other marketing variables and thus estimating a more precise model for demand might produce a less biased price elasticity estimate and enable price optimization.

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Appendix A

Price elasticity standard error

Consider the price elasticity estimate $\hat{\varepsilon}$:

$$\hat{\varepsilon} = \frac{\sum_{i \in S} \sum_{j=1}^{n_i} \left(\log D_{ij} - \overline{\log D_i}\right)\left(\log P_{ij} - \overline{\log P_i}\right)}{\sum_{i \in S} \sum_{j=1}^{n_i} \left(\log P_{ij} - \overline{\log P_i}\right)^2}.$$

To simplify the notation, let us now denote $\log P_{ij} =: p_{ij}$ and $\log D_{ij} =: d_{ij}$. The standard error $s_{\hat{\varepsilon}} = \sqrt{\mathrm{Var}(\hat{\varepsilon})}$. Calculate the variance of the estimate $\hat{\varepsilon}$:

$$\mathrm{Var}(\hat{\varepsilon}) = \mathrm{Var}\left(\frac{\sum_{i \in S} \sum_{j=1}^{n_i} \left(d_{ij} - \overline{d_i}\right)\left(p_{ij} - \overline{p_i}\right)}{\sum_{i \in S} \sum_{j=1}^{n_i} \left(p_{ij} - \overline{p_i}\right)^2}\right)$$

The denominator only contains sums and products of the independent variable. The independent variable is not a random variable, and therefore the denominator is a constant. As $\mathrm{Var}(aX) = a^2\mathrm{Var}(X)$:

$$\begin{aligned}
\mathrm{Var}(\hat{\varepsilon}) &= \frac{\mathrm{Var}\left(\sum_{i \in S} \sum_{j=1}^{n_i} \left(d_{ij} - \overline{d_i}\right)\left(p_{ij} - \overline{p_i}\right)\right)}{\left(\sum_{i \in S} \sum_{j=1}^{n_i} \left(p_{ij} - \overline{p_i}\right)^2\right)^2} \\
&= \frac{\mathrm{Var}\left(\sum_{i \in S} \sum_{j=1}^{n_i} d_{ij}\left(p_{ij} - \overline{p_i}\right) - \sum_{i \in S} \sum_{j=1}^{n_i} \overline{d_i}\left(p_{ij} - \overline{p_i}\right)\right)}{\left(\sum_{i \in S} \sum_{j=1}^{n_i} \left(p_{ij} - \overline{p_i}\right)^2\right)^2}
\end{aligned}$$

Inspect the latter sum in the numerator:

$$\begin{aligned}
\sum_{i\in S}\sum_{j=1}^{n_i}\overline{d_i}\left(p_{ij}-\overline{p_i}\right) &= \sum_{i\in S}\overline{d_i}\sum_{j=1}^{n_i}\left(p_{ij}-\overline{p_i}\right)\\
&= \sum_{i\in S}\overline{d_i}\left(\sum_{j=1}^{n_i}p_{ij}-\sum_{j=1}^{n_i}\overline{p_i}\right)\\
&= \sum_{i\in S}\overline{d_i}\left(\sum_{j=1}^{n_i}p_{ij}-n_i\overline{p_i}\right)\\
&= \sum_{i\in S}\overline{d_i}\underbrace{\left(\sum_{j=1}^{n_i}p_{ij}-n_i\frac{\sum_{j=1}^{n_i}p_{ij}}{n_i}\right)}_{=0}\\
&= 0
\end{aligned}$$

And thus

$$\mathrm{Var}(\hat{\varepsilon}) = \frac{\mathrm{Var}\left(\sum_{i\in S}\sum_{j=1}^{n_i}d_{ij}\left(p_{ij}-\overline{p_i}\right)\right)}{\left(\sum_{i\in S}\sum_{j=1}^{n_i}\left(p_{ij}-\overline{p_i}\right)^2\right)^2}$$

Our model suggests that $d_{ij} = \varepsilon p_{ij} + C_i + \epsilon_{ij}$ 3.8

$$\begin{aligned}
\mathrm{Var}(\hat{\varepsilon}) &= \frac{\mathrm{Var}\left(\sum_{i\in S}\sum_{j=1}^{n_i}\left(\varepsilon p_{ij}+C_i+\epsilon_{ij}\right)\left(p_{ij}-\overline{p_i}\right)\right)}{\left(\sum_{i\in S}\sum_{j=1}^{n_i}\left(p_{ij}-\overline{p_i}\right)^2\right)^2}\\
&= \frac{\mathrm{Var}\left(\overbrace{\sum_{i\in S}\sum_{j=1}^{n_i}\left(\varepsilon p_{ij}+C_i\right)\left(p_{ij}-\overline{p_i}\right)}^{\text{No random variables}}+\sum_{i\in S}\sum_{j=1}^{n_i}\epsilon_{ij}\left(p_{ij}-\overline{p_i}\right)\right)}{\left(\sum_{i\in S}\sum_{j=1}^{n_i}\left(p_{ij}-\overline{p_i}\right)^2\right)^2}\\
&= \frac{\mathrm{Var}\left(\sum_{i\in S}\sum_{j=1}^{n_i}\epsilon_{ij}\left(p_{ij}-\overline{p_i}\right)\right)}{\left(\sum_{i\in S}\sum_{j=1}^{n_i}\left(p_{ij}-\overline{p_i}\right)^2\right)^2}
\end{aligned}$$

because $\mathrm{Var}(X + a) = \mathrm{Var}(X)$. Now, assume that $\mathrm{Cov}(\epsilon_{ij}, \epsilon_{kl}) = 0$ if $i \neq k$. This will allow us to calculate the variance of the outer sum as the sum of variances:

$$\begin{aligned}\mathrm{Var}(\hat{\varepsilon}) &= \frac{\sum_{i \in S} \mathrm{Var}\left(\sum_{j=1}^{n_i} \epsilon_{ij}\left(p_{ij} - \overline{p_i}\right)\right)}{\left(\sum_{i \in S}\sum_{j=1}^{n_i}\left(p_{ij} - \overline{p_i}\right)^2\right)^2} \\ &= \frac{\sum_{i \in S}\sum_{j,k=1}^{n_i} \mathrm{Cov}(\epsilon_{ij}, \epsilon_{ik})\left(p_{ij} - \overline{p_i}\right)\left(p_{ik} - \overline{p_i}\right)}{\left(\sum_{i \in S}\sum_{j=1}^{n_i}\left(p_{ij} - \overline{p_i}\right)^2\right)^2}\end{aligned}$$

The standard error of the price elasticity estimate is:

$$s_{\hat{\varepsilon}} = \frac{\sqrt{\sum_{i \in S}\sum_{j,k=1}^{n_i} \mathrm{Cov}(\epsilon_{ij}, \epsilon_{ik})\left(p_{ij} - \overline{p_i}\right)\left(p_{ik} - \overline{p_i}\right)}}{\sum_{i \in S}\sum_{j=1}^{n_i}\left(p_{ij} - \overline{p_i}\right)^2}.$$

Now the task reduces into estimating the unknown covariances of the error terms. In this thesis we use the linearization method discussed for example by Bell and McCaffrey (2002) and the covariance matrix is estimated with model residuals r_{ij}

$$\hat{s}_{\hat{\varepsilon}} = \frac{\sqrt{c\sum_{i \in S}\sum_{j,k=1}^{n_i} r_{ij} r_{ik}\left(p_{ij} - \overline{p_i}\right)\left(p_{ik} - \overline{p_i}\right)}}{\sum_{i \in S}\sum_{j=1}^{n_i}\left(p_{ij} - \overline{p_i}\right)^2}. \tag{A.1}$$

We use the typical $c = \frac{N}{N-1}$ (Bell and McCaffrey, 2002) where N is the count of items in the studied segment. However, the standard error estimate achieved might be downwards biased if the segment does not contain enough items (Bell and McCaffrey, 2002). The simulations of Cameron et al. (2008) show that with heteroscedastic clustered errors some over-rejection occurs even when segment contained 30 items when Wald statistic with standard error estimate similar to (A.1) was used.