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Intraday liquidity management in gross settlement system as a coordination game

Master's thesis submitted in partial fulfilment of the requirements for the degree of Master of Science in Technology

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ABSTRACT OF MASTER'S THESIS

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Abstract:

Real time gross settlement systems (RTGS-systems) operated by central banks are the predominant form of large value payment systems. They allow execution of fund transfers between central bank accounts of commercial banks and other financial institutions as a continuous process by settling payments individually, with their full value and with immediate finality.

Because individual payments are settled in gross, the amount of liquidity needed for smooth processing of payments in RTGS-system is large. Central banks mitigate this problem by offering intraday credit for RTGS-system participants (banks henceforth) against eligible collateral or against interest. Both of these options create expenses for banks either in form of opportunity cost of collateral or explicit interest payments. Since the liquidity obtained from incoming payments from counterparties is free for banks, they may have an incentive to delay payments in expectation of forthcoming payments from other banks, which would decrease the cost of liquidity.

This thesis theoretically analyzes the strategic behavior of banks that are processing stochastic payment orders from their customers in an RTGS-system. The purpose of the thesis is to (1) extend earlier theoretical studies into multi-player games and, especially, (2) analyze the possibility of emergence of a coordination game in which multiple equilibrium outcomes would be possible. Starting points for the thesis have been the models presented by Bech – Garratt (2002) and Angelini (1998).

In the formulated model intraday liquidity is assumed to be available against collateral, the amount of which can be dynamically managed by the banks. The game has two periods, and information possessed by banks is incomplete and imperfect. Banks are modelled as heterogeneous and risk neutral agents who minimize the total costs of payment processing. These consist of the cost of liquidity and the cost of delaying payments. The latter is used to describe reputation risk and possible loss of future revenues due to customer dissatisfaction. Strategy of a bank is describing decision of overall share of payment orders received from customers in the first period, which are processed immediately.

It was found that the formulated multi-player model did not have an analytical solution. Thus Bayesian equilibria for the game were solved numerically as a function of paremeters for cost structure and payment distribution in an example case with a quadratic form for cost functions. The resulting optimal responses of banks were shown to be inversely related with the average decision of counterparties especially when the cost of delaying is higher than the cost of liquidity. This results in a unique expected equilibrium point for the game. In cases where the liquidity cost is larger, the possibility of a coordination game and thus the possibility of inefficient market practices were shown to exist. In a case where cost of liquidity was heterogeneous it was proposed that the winners might be the banks who have higher liquidity costs because they are facing increased possibility for free riding.

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Keskuspankin ylläpitämä reaaliaikainen bruttomaksujärjestelmä (RTGS-järjestelmä) on yleisin suurten maksujen katteensiirtoihin käytetty järjestelmärakenne. Se mahdollistaa liikepankkien ja muiden rahoituslaitosten välisten rahasiirtojen toteuttamisen osapuolten keskuspankissa pitämien tilien välillä jatkuvana prosessina, tapahtumakohtaisesti, täysimääräisinä ja peruuttamattomasti.

Maksujen bruttomääräisestä toteuttamisen takio RTGS-järjestelmän viiveettömään toimintaan tarvitaan paljon likviditeettiä eli rahakatetta keskuspankkitilillä. Helpottaakseen tätä ongelmaa keskuspankit tarjoavat maksujärjestelmäosapuolille (jatkossa pankeille) päivänsisäistä luottoa pantattuja vakuuksia vastaan tai korolla hinnoiteltuna. Molemmat vaihtoehdot aiheuttavat pankeille kuluja, joko vakuuden vaihtoehtoiskustannuksien tai määrätyn koron muodossa. Koska muilta pankeilta saapuvien maksujen tuoma likviditeetti on pankeille ilmaista, pankeille voi syntyä kannustin viivyttää lähtevien maksujen toteuttamista saapuvien maksujen toivossa, mikä vähentäisi pankin omia likviditeettikustannuksia.

Tämä työ tarkastelee laskennallisesti pankkien strategista käyttäytymistä RTGS-järjestelmässä, jossa pankit toteuttavat asiakkailtaan saamiansa satunnaisia maksutoimeksiantoja. Työn tarkoituksena on (1) laajentaa aikaisemmin esitettyjä malleja usean toimijan peleiksi ja erityisesti (2) tarkastella onko asetelmasta mahdollista muodostua nk. koordinaatiopeli, jossa on useita tasapainoratkaisuja. Lähtökohtana on pidetty Bechin ja Garrattin (2002) sekä Angelinin (1998) esittämiä malleja.

Muodostetussa mallissa pankki voi nostaa päivänsisäistä luottoa vakuuksia vastaan dynaamisesti tarpeen mukaan. Peliasetelmassa on kaksi jaksoa ja pankeilla ei ole täydellistä tietoa vastapuolien kustannusrakenteesta tai pelin historiasta. Pankit kuvataan riskineutraaleiksi toimijoiksi, jotka minimoivat maksujen välityksestä aiheutuvia kustannuksia. Nämä kustannukset muodostuvat likviditeetin ylläpitämisestä ja maksujen viivästyttämisestä. Kustannusten osatekijöiden painoarvoissa voi olla eroja pankkien välillä. Jälkimmäinen kustannustekijä kuvaa maineriskiä eli mahdollista tulevien tuottojen menetystä asiakastyytyväisyyden laskemisen myötä. Pankin strategia pelissä kuvaa niiden maksutoimeksiantojen osuutta ensimmäisen jakson maksuista, jotka pankki päättää toteuttaa välittömästi.

Työssä esitettyä monen pelaajan peliasetelmaa ei voi ratkaista analyyttisesti. Bayesiläinen tasapainoratkaisu pelille laskettiin numeerisesti pankkien kustannusten rakennetta ja maksujen jakauma kuvaavien parametrien funktiona esimerkkitapaukselle, jossa kustannusfunktioille valittiin neliöllinen muoto. Pankkien reaktiofunktio oli kääntäen riippuvainen vastapuolipankkien keskimääräisestä strategiasta tilanteessa, jossa maksujen viivyttämisen kustannukset ovat likviditeettikustannuksia suuremmat. Tästä on seurauksena yksikäsitteinen tasapainoratkaisu pelille. Tapauksissa, jossa likviditeetin kustannukset ovat korkeammat, koordinaatiopelin muodostuminen ja siten tehottomaan tasapainoratkaisuun päätyminen osoitettiin mahdolliseksi. Eroja pankkien likviditeettikustannuksissa sisältävien pelien tarkastelun pohjalta osoitettiin, että tällaisessa tilanteessa hyötyjiä saattavat olla myös ne pankit, joilla likviditeettikustannukset ovat muita korkeammat. Tämä johtuu saapuvien maksujen lisääntyneestä mahdollisuudesta.

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Matti Hellqvist

Contents

1. Introduction

1.1. Background

Economic activity creates payments, compensations that have to be delivered from one participant to another. Processing these payments is one of the main tasks of banks in a modern economy. Payment systems are arrangements, which are used for transferring funds between different banks and financial institutions. Depending on the value and volume of payments, payment systems are usually divided to retail and large-value payment systems (LVPS). The latter is often maintained by central bank while retail systems can be operated by some group of banks or banking association. Existence of reliable and efficient payment systems has often been described as one of the prerequisite for modern economy.

Value and volume of transactions executed in large value payment systems have grown substantially during the last decades as a result of liberalization of financial market, economic growth and technical developments[1]. Also the numbers of counterparties banks have, and value of international payments in large-value payment systems has been increasing. These reasons were driving the change of dominant LVPS architecture to real-time gross settlement (RTGS) systems from previous design based on netting of payments and credit granted between the participants. In RTGS system, payment instructions are executed

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[1] For international statistics of payment systems see BIS (2005) and previous similar publications. For history and analysis of changes in payment systems see Pauli (2000).

continuously transaction by transaction in gross, i.e. with their full value, and with immediate finality.

Because of gross settlement, the required amount of funds needed for smooth processing of RTGS system is large. This problem has usually been solved by central banks, who are offering intraday liquidity for the participants of payment system against collateral, or for some low interest. The intermediation of central bank lowers the cost of intraday liquidity but some price can however be assumed to exist in form of explicit interest or implicit opportunity cost of collateral. This creates incentives for banks to postpone their payments in hope of incoming payments from other banks, and thus decrease their own amount of required liquidity. Such delays have shown to result in possible welfare losses on aggregate level and decrease of efficiency of payment system[2].

1.2. Objectives, scope and research methods

Strategic behaviour of participants in RTGS system under different intraday liquidity regimes has been analysed before mainly as two participant games with two or three periods[3]. Resulting from the variations existing in real systems of different countries, distinct assumptions have also been made in these studies of the structure of RTGS system and ingredients of costs of participants.

The objective of this study is to formulate the setup of intraday liquidity management as a multi participant game and analyse whether it forms a coordination game. This possibility has been outlined previously in both empirical[4] and theoretical[5] context. Formulation as a multi participant game would enable the analysis of effects of increased number of decision makers in the game and heterogeneity among the banks. If the setup is recognised as a co-

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[2] Angelini (1998), Kobayakawa (1997),

[3] Kobayakawa (1997), Angelini (1998), Bech – Garratt (2002), Buckle – Cample (2003)

[4] McAndrews – Rajan (2000)

[5] Bech – Garratt (2002)

ordination game, possibility for coordination failures i.e. inefficient equilibrium outcomes of the game can also be assessed.

Method of the analysis is noncooperative game theory. Strategies of banks are used to describe the decided share of immediately processed payments. Solutions are searched in form of Nash equilibriums for a game where participants are minimizing expected costs of processing the payments in environment including stochastic variables for demand of payment processing and uncertainty of the strategies of counterparties. The complexity of multi participant environment is simplified by forcing the banks in the model to treat all counterparties equally. Thus for each bank only one overall strategy decision is needed.

Cost structure of participants will be assumed to consist of liquidity and delaying costs. Numerical method for evaluating the expected value of costs will be presented based on Monte Carlo simulations because stating the expected value of cost in closed form was noticed to be out of reach. Based on the numeric approach best responses and equilibrium outcomes for the intraday liquidity management game will be solved and discussed.

1.3. Structure

The study is structured as follows. Payment systems, different approaches for intraday liquidity management and review of earlier published studies on the field of incentives and strategic interaction in RTGS-systems are presented in section 2. Required concepts of noncooperative game theory and coordination games are presented in section 3. The model for intraday liquidity management as multi participant game is constructed and required assumptions for solving the game are presented in section 4. Section 5 includes the implications of solving the game numerically and section 6 the conclusions and discussion of the study.

2. Payment systems and intraday liquidity

2.1. Large value payment systems

For a long time the most popular design of large value payment systems (LVPS) was deferred net settlement system (DNS). In DNS structure participants send payment instructions to the system during the day, and only the accumulated net value of payments is covered at the end of day either with bilateral payments between individual participants or multilaterally trough payments sent to central clearing agent e.g. a central bank. If some of the participants is unable to meet its payment obligations at the end of the day, all of its payments are removed from the netting in a process called unwinding. This will create changes in payment obligations of other participants and pass the effect forward. Possibility of default of a participant causing cascading defaults of other participants in payment system is called systemic risk. For DNS systems the possibility of systemic risk has been analysed in several studies[6] starting from Humphrey (1986). Although the risks of DNS can be clearly decreased by introducing additional rules like increased number of netting runs or limits for bilateral of multilateral open positions, the increase of transaction numbers and volumes in LVPS during 1990 highlighted the problems of DNS structure. This was leading to development where many large value payment systems were changed to real time gross settlement (RTGS) system architecture.

[6] See e.g. Van den Bergh (1994), Angelini et al (1996) and Soramäki – Bech (2005)

RTGS systems[7] are currently the most common system structure in large value payment systems[8]. In basic RTGS individual payments are settled gross, without netting and the information of incoming payments is delivered to receiver together with the corresponding payment. Processing individual payments with immediate finality will remove many risks observed in DNS structure, but raises another question: much more funds are needed to enable smooth processing of payments in the gross settlement process.

2.2. Intraday liquidity and different RTGS structures

Central banks can solve the problem of increased need of settlement funds observed in gross settlement systems by offering intraday liquidity for participants. Currently at least two ways for doing this are implemented.

The most common policy of central bank is to give participants of payment system free liquidity up to certain given limit value, but require that the limit is backed up fully or partially by collateral. Depending on the legislation of the country hosting a RTGS system the collateral can be required in form of repurchasing agreements of assets (REPO) or as assets pledged for the central bank. Collateralized intraday credit is currently used in all countries of the euro area and in many other RTGS systems. The implementation of collateralized intraday liquidity facility can have different forms. If cost of changing the amount of collateral is low and process for making the changes is rapid and automated, banks can change the pledged amount dynamically according to their needs. This kind of implementation is in place e.g. in ARTIS, the RTGS system of Austria[9]. This will be later referred as dynamic collateral management. If the process of collateral management is having manual processing or other sources of delay, which will make changing of pledged amount slower, the amount of pledged collateral can be assumed to be more stable and represent precautionary

[7] For basic concepts and structures of RTGS systems see report by Bank of international settlements (1997).

[8] Danmarks Nationalbank (2005), ch 3.

[9] ECB, Blue book (2001) p. 379

estimates of liquidity need. This in the contrast will be called delayed collateral management.

Another way for providing intraday liquidity is implemented in Fedwire, the RTGS system in United States maintained by the central bank. There daylight overdrafts, negative balances on central bank account, are allowed for participants up to certain given level, and a small interest is charged for the used overdrafts. The charging is calculated based on the amount of used overdraft at the end of each minute[10]. Maximum value of intraday credit which a participant can obtain is constrained by limit called net debit cap.

2.3. Studies on incentives in RTGS

A recent review of the status and trends of RTGS systems has been published by Bank of Finland[11]. According to it, one recent change in the focus of payment system design has been the increased attention paid on social efficiency of the systems instead of emphasising reduction of risks.

Important aspects affecting the efficiency of a system are the incentives which system and cost structure create to participants. Among the first analyses of the efficiency and incentives in RTGS systems is article by Furfine and Stehm[12], who discuss the effect of alternative intraday credit policies of central banks. They present comparison of free, collateralized and priced intraday credit policies and give necessary conditions for optimal choice of intraday credit policy as a function of cost structures of the payment system participants. Possible implications of reactions of system participants on other participants, i.e. strategic interaction, were shortly mentioned but not analysed in their paper.

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[10] Federal Reserve system (2005)

[11] Iivarinen (2004)

[12] Furfine – Stehm (1997)

Game theoretical approach to intraday liquidity management problem was presented in more detail by Kobayakawa[13] in comparison of collateralised and priced intraday liquidity policies in RTGS. The model in this paper consisted of two participants or banks and two periods. The used costs structure included an element of cancelling costs. These were caused by dissatisfaction of banks customers after payments became rejected in LVPS due to banks lack of liquidity. This model structure is suitable for analysis of some RTGS systems which do not have a queuing facility like BoJ-NET in Japan. For systems which have queue implemented the cancelling costs fail to indicate the increase of dissatisfaction of the customers when delay and queuing time are increased.

Another similar analysis in more general framework was presented by Angelini[14]. He also considered setup of two participants and two periods with stochastic payments but only explicitly priced intraday credit. The model of Angelini included possibility of heterogeneous participants in terms of cost structures, possibility of select the delayed value of payments continuously and also concept of increasing the cost of delaying payments as the delayed value is increasing. He showed that the resulting behaviour of delaying payments is socially ineffective. Also he noted that delaying payments causes a negative externality in payment system because the quality of information available for participants in LVPS is decreasing. Besides that Angelini predicted the rise of daylight market of funds in a system where intraday liquidity provided by central bank is costly. The model structure used by Angelini included an assumption that banks are facing the liquidity costs only based on the net value of liquidity required on each period[15]. This is correct if restrictions on the amount of used

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[13] Kobayakawa (1997)

[14] Angelini (1998)

[15] In Angelini (1998) the liquidity cost is calculated as function of banks liquidity position after each period. This is defined as $D_i^t \equiv \sum_{s=t_0}^{t} \left[z_s^i - a_s^i - y_s^i - v_s^i \right] - R_{t_0}^i$, where z_s^i stands for value of payments available for processing, a_s^i for value of delayed and y_s^i for value of received payments and v_s^i for operation in inter-bank market during period s. Thus the value of D_i^t after some period is affected by the net flow of payments during that period.

intraday credit are controlled or the charged interest is calculated only periodically e.g. based on balance at the end of each minute like in Fedwire-system[16]. However, if the limits for used intraday liquidity are strict and controlled continuously, this model fails to describe RTGS system and is instead a model of DNS structure.

More recent publications of incentives in RTGS systems include a paper by Bech and Garratt[17] who presented a Bayesian game setup for free, priced and collateralised intraday liquidity regimes. Their model consisted of two players and tree periods: morning, afternoon and end of day. The efficiency of equilibrium outcome of the game was analysed in one-shot and also in repeated setups. In this approach the amount of collateral was modelled as variable cost for the participants i.e. according to dynamic collateral management. For describing the priced intraday credit opportunity, also this study was mimicking Fedwire overdraft calculation rule and thus considered only the net value of payments during each period relevant for cost of liquidity. In the game banks were making their strategy decisions on the fist period after receiving private information of first period payments of their customers. The two last periods contain only processing of the payments according to given strategy and do not include new decisions by the players. Payments were described as undividable unit sized transfers. Besides standard risk neutrality assumption, also impact of risk aversion was examined in this paper.

Another recent theoretical analysis was conducted by Buckle and Campell[18]. This paper concentrated on presenting rationale for the troughput rules, which have been implemented in CHAPS, the LVPS at Bank of England. These are rules on the value of payments participating banks must submit before some given time limits, and are intended to encourage immediate processing of payments in stead of delaying. The model included deterministic payments and fixed equal cost of collateral for the participants. One of the key assumptions in the study was that banks participating in payment system would consider imbalances in the payment flows as costs. This, according to Buckle and Campell, can be justified by ex-

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[16] Federal Reserve system (2005)

[17] Bech – Garratt(2002)

[18] Buckle – Campell (2003)

istence of bilateral limits for net value of sent payments, which are observed in some payments systems, in this case especially the CHAPS. It could be argued, however, whether these limits are really describing something, which creates possible costs in everyday processing of payments. Their role as limitations for risk of liquidity effects in case of default of some participant is more evident but seems rather distant as a motivator of strategic decisions in intraday processing of individual payments.

Besides theoretical approaches McAndrews and Rajan[19] have conducted statistical analysis of payments on data collected from Fedwire. They measured to which extent participants are able to fund outgoing payments with incoming payments and to which extent there exists synchronization of payments, which would create joint gains for participants. The market driven synchronisation of payments would be practical evidence of existing incentives for coordination and management of intraday liquidity and payment activity in LVPS.

RTGS systems in different countries have many similarities but also distinguishing country or system specific features. As a summary of the previous research on efficiency and incentives in RTGS systems it can be noted that the results from many of the analyses are bound to the system setup of corresponding country. Therefore the outcome from previous customised or general level research can not be taken for granted in other RTGS setups. Also, all the studies so far have eventually considered the simplified setup of two participants and, in practice, two periods. Possibilities for generalisations in the analysis are therefore evident.

[19] McAndrews – Rajan (2000)

3. Game theoretic framework

Game theory analyses setups with multiple utility optimizing decision makers, where the utility received by each participant is dependent on the actions taken by the others together with the decision made by the participant itself. Game theory can be divided into cooperative and noncooperative games of which this study is mainly focusing on the noncooperative side. Solutions in noncooperative games are called equilibriums, or *Nash equilibriums*, and are formed by such a set of decisions that none of the participants would benefit from changing their decision alone.

Another concept which used in this study in classifying the outcomes in games is *pareto optimality*. From pareto optimal set of decisions it is not possible to move away without decreasing the utility of some participants. Generally the sets of pareto optimal solutions and Nash equilibriums in a game are not overlapping, since reaching the former would usually require some cooperation between the participants. The idea of pareto optimality can however be used also within the subset of noncooperative equilibrium outcomes in comparisons of possible equilibrium points.

For games with uncertainty the solution can also be solved as Bayesian equilibrium. In this case decisions of players are made conditional on observations made during the game and changes in assumptions of probabilities of different strategies and actions of counterparties are performed according to Bayesian rule.

3.1. Coordination games

Game setup can be called coordination game if there are several possible Nash equilibriums available. Most simple example of a coordination game is the stag hunt: two players can choose to hunt either stag or hare. The bigger pray, stag, can be caught only if both players are choosing to hunt a stag where hunting for hare alone offers a secure but smaller result. The rewards for both players in the possible outcomes of the game are presented in

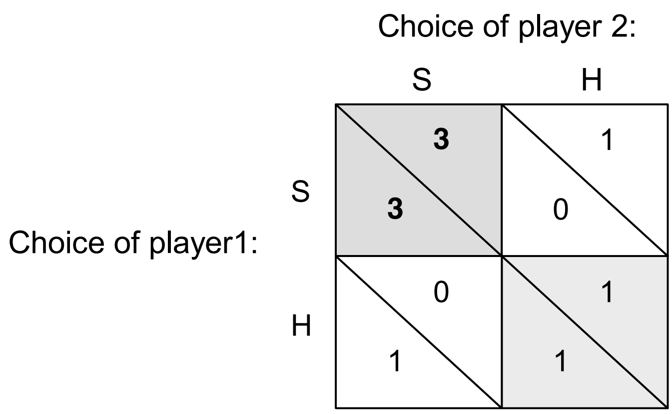

Figure 1 Utilities received by the two players in different outcomes of stag hunt game. Upper triangle shows the reward for player 2 and lower triangle for player 1. Nash equilibriums are shaded and the pareto optimal solution is emphasized with bold letters.

This game has two Nash equilibriums: either both are hunting stag together or both choose to hunt hare alone. Selecting to hunt the stag together is preferred by both players, but since the decision has to be made alone without knowing the action of the other, this choice includes a risk of being left alone in the stag hunt, which is the worst possible outcome in the game. Possibility to negotiate and coordinate the actions would remove trivially the multiplicity of equilibriums.

Besides the utility optimization, additional criteria have been presented to describe the player's rational choice among multiple equilibrium solutions in coordination games. Some of these assume that similar game is played repeatedly, while some are also applicable in one shot games. The set of proposed criteria includes precedence i.e. selecting the equilibrium that was played before, "playing for sure" i.e. maximin-strategy, payoff-dominance i.e. selecting among the pareto-optimal solutions and risk dominance i.e. avoiding equilibriums, in which the payoffs are sensitive to deviations. Also players can be assumed to act as if the other players were only limitedly rational. Conclusions gathered from empirical investigations of different types of coordination games suspect that no single additional selection rule can be defined to cope with all games[20]. In stead a statistical mixture of selection rules seems to be present, and the composition of these rules depends on the type of the game.

Similar setup as in stag hunt game has been analysed empirically in multi participant environment by Van Huyck et al[21]. In their game $e_1,...,e_n$ represent the selection of n players and payoffs for each participant are defined by

$$\pi(e_i, e_j) = a \min(e_i, e_j) - be_i,$$
$$a > b > 0$$

where $e_j = \min(e_1, e_2, ..., e_{i-1}, e_{i+1}, ..., e_n)$. The selections are bound to positive integers below given maximum value $\bar{e}$, and the payoff and strategy space is known to be known by all. In this game everyone benefits if all decide to choose large values for their e_i. Best off is however the one who selected smallest value, and thus public and individual interest are conflicting. In the game all possible strategy options are Nash equilibriums: given any set of actions by the others player is best off by selecting the same value as the minimum among the others. Payoff dominant strategy would be to choose largest possible option, but according to empirical results selections converged to pareto inferior "playing for sure" solution $e_i = 1$. Reason behind this observation was shown to be player's uncertainty of the strategies and rationality of other players. Increase in the number of players was noticed to make the inefficient outcome even more probable.

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[20] Camerer (2003), chapter 7.

[21] Van Huyck – Battalio – Beil (1990)

In coordination game, this kind of outcome where pareto inferior equilibrium point is chosen from the possible Nash equilibriums is called a coordination failure. Thus in coordination failure situation it would be possible to make some participant or participants better off while none would need to lose. Coordination failures can also be called inefficient solutions. Empirical results from different coordination games have shown that coordination failures are common – the actions are not always evidently converging to efficient solutions[22]. Therefore analysis of the prevailing practices in setups, which can be identified as coordination games, can reveal possible inefficiencies at participant and also on the aggregate level.

[22] Camerer (2003), chapter 7, page 403.

4. Intraday liquidity management as multi participant game

The payment system under study is considered to have n participants, henceforth banks. As an abstraction the opening hours of the system are divided to two periods: morning and afternoon. Banks are collecting payment orders from their customers during the day and settling these orders through the payment system in the period they choose.

Settling the payments is costly for the banks in two ways: first, processing the payments requires liquidity, which creates expenses in form of opportunity costs for the bound collateral or price of the money borrowed from intraday market. Secondly, if bank chooses to delay the settlement of a payment, it is also facing a cost. This can be seen as an immediate fine imposed to the bank or as discounted value for loss of future demand caused by customer dissatisfaction. On the revenue side, the price bank is charging for payment procession is considered to be fixed and independent of the settlement process. Therefore it is sufficient to minimize payment procession costs in order to understand the incentives in the model. It is assumed that the banks are risk neutral, i.e. minimizing the expectation value of costs.

In a real LVPS, many forthcoming payments are known to the banks in advance. Such are for instance payments, which are related to clearing and settlement of securities or foreign exchange trades. This is because clearing and settlement typically takes place two or three days after the actual trade. On the other hand some part of the demand of payment processing is revealed to the banks only during the ongoing day. These payments form the stochastic side of payment flows.

In LVPS working with gross settlement of payments the liquidity is reserved and transferred immediately when payments take place. The amount of liquidity a participant needs in such a system can vary substantially if the order of payments is changed. Thus the information of actual timing of payments is crucially important for determining the amount of needed liquidity. Some payments have strict time limits[23], but for most cases the decision of timing of outgoing payments can be taken by each bank by it self. For incoming payments the exact timing is respectively decided by the counterparties. Because of this the entire amount of processed payments has to be considered relevant in the intraday liquidity management – not only the stochastic payments. This reasoning differs from the one presented in some earlier studies[24] and increases the importance of intraday liquidity management. The model used in this study is however based on the same assumption of having only stochastic payments in the incoming side, because the information structure of the game would become significantly more complicated if some of the payments were assumed to be known by the participants.

The effect of timely order of payments is represented in this model by the possibility of delaying: during each period the payments have to be sent and liquidity gathered before incoming payments are received. Thus the incoming liquidity can only be utilized later on the next period.

Demand for payment processing is considered as an exogenous variable in the model. Thus payment arrivals are independent from the actions taken by the banks during intraday period. This can be justified by assuming that changes in customer behaviour are slow enough not to show up during the same day. The effect of changes on demand has been taken care of in the form of delay cost, the expected value of lost future demand.

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[23] For example payments related to CLS, the international clearing system for foreign exchange trades, have strict schedule.

[24] Angelini (1998)

4.1. Information structure

The aggregate statistical properties of payment flows between the banks are assumed to be known by all participants. Each bank can observe the demand of payment processing created by its own customers and the flow of incoming payments. These can be used to identify distribution for value of outgoing payments to be processed and incoming payments received from counterparties for both the morning and afternoon periods. These observations constitute the view of each individual bank of its surrounding environment and affect the decisions made by banks in the game by partially defining the minimized expected cost functions. In the current model the market structure is assumed to reach a stable state so that the distributions for customer and counterparty behaviour do not need to be changed and updated.

Process behind the incoming payments is assumed to stay hidden to banks. Thus it is not possible to deduct from values received from counterparties, which part was actually delayed and which just came later from the customers of the counterparty. In reality the cards are more open, at least if all the payments processed in LVPS would be considered relevant, since the total amount of payments includes also known transactions, which are known by the recipient in advance, such as payments related to securities settlement.

Cost structure of each bank, the price of intraday liquidity and the cost of delaying payments, are assumed to be private information of each participant.

4.2. Intraday liquidity

Banks in the model are allowed a dynamic collateral management. In the morning period they can first observe the realization of demand of payment processing for the morning, decide how much of this demand they process immediately and only after this pledge the required collateral for intraday liquidity at central bank. Similarly in the afternoon banks observe the amount of new demand and received payments before they pledge required extra collateral. The pledged amount belongs also to the private information of each bank.

The selected structure of intraday liquidity can also be seen as a simplified presentation of general setup, where participants can flexibly adjust the amount of liquidity they keep in certain LVPS. In practice this liquidity can inter alia consist of balance kept for minimum reserve requirements, intraday limit granted by central bank, funding from interbank intraday market and amounts of liquidity, which are moved between accounts which the bank may have in different LVPS. It is actually irrelevant, what is the source of intraday liquidity, once there is enough of it available. Marginal cost of liquidity for a bank can then be assumed to increase together with the required amount of liquidity, which describes the shifting from basic funding methods towards more ad hoc solutions.

As a more strict interpretation of intraday liquidity, when only the balance and limit granted by central bank are counted with, the flexibility of intraday liquidity arrangement could be reduced. In Finland, for example, the process of pledging collateral is currently not conducted dynamically in real time which makes the amounts of pledged collateral more or less stable. In a case like this the decision of how much liquidity to hold by pledging collateral and holding a balance has to be made in advance before any realizations of demand of payment processing are observed. The model of such case should include a liquidity constraint based on the amount of pledged collateral decided earlier, and also extra costs for payment which are delayed until the end of the day because of this constraint. This extra cost would describe the price premium of required market funding or overnight credit by central bank. By comparing these two models of intraday liquidity the benefit of dynamic collateralization possibility could be measured. This is, however, out of the scope of this study.

4.3. Actions of the participants in the game

As a illustration of the game structure presented, the actions of the participants during the two periods are described below in Figure 2. Here the game is presented as interaction between an individual bank and the environment or nature, which includes in this case all the counterparties and customers: the sources for all external variables which the individual bank is observing in the game. Thus actions of all separate banks are supposed to be happening simultaneously and the information of them is passed forward only when the nature takes its move.

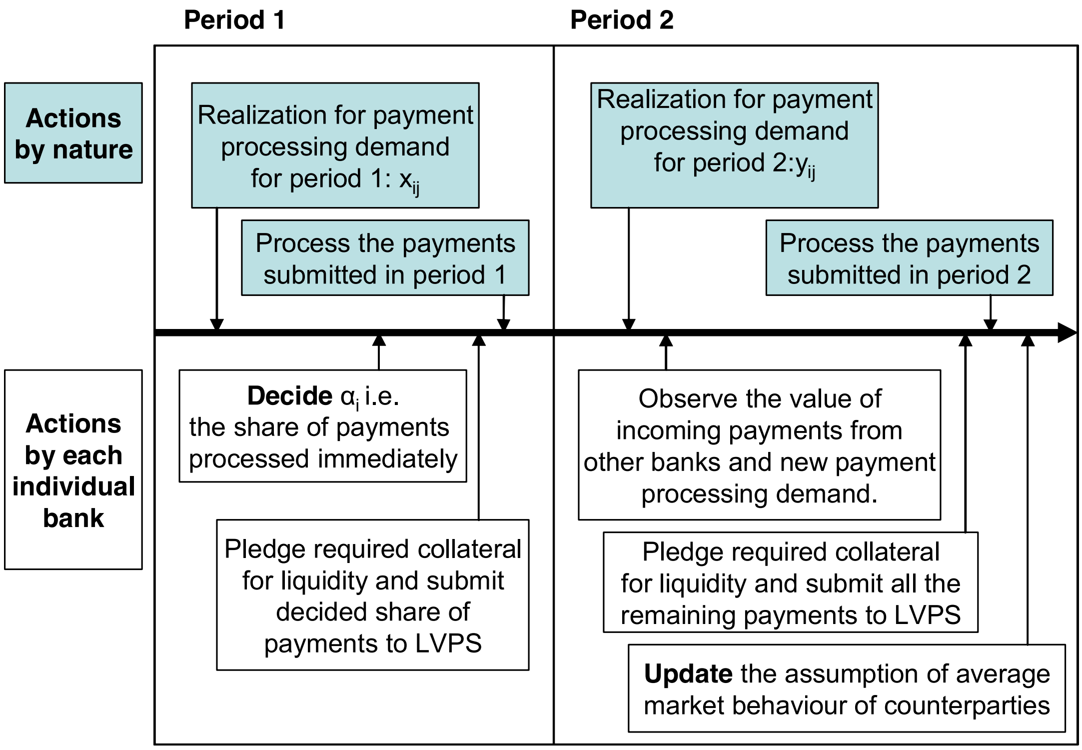

Figure 2 Process flow of the game from the viewpoint of individual bank.

In the game as it is formulated here, there is actually only one moment of real decision making by the participating bank: the selection of the share of demand of payment processing collected in the morning period to be processed immediately. The second action described in Figure 2, updating the assumption of counterparty behaviour is shown for clarity. In the analysis presented in this study the market structure and average behaviour of the others is assumed to reach stable state and therefore no fluctuation or corrections in the banks view of their environment is required.

4.4. Analytical framework

Let $B = \{1,...,n\}$ be set of banks in the payment system under study. Let $X_{ij}, Y_{ij} \in \Re^+$ be random variables for demand of payment processing from bank i to bank j in morning and afternoon periods and x_{ij}, y_{ij} their realizations correspondingly. For simplifying the notations, following sums of these random variables are in the marked as

$$\sum_{j=1}^{n} x_{ij} = \hat{x}_i \quad \sum_{j=1}^{n} y_{ij} = \hat{y}_i$$

These denote total of outgoing payments of bank i for morning and afternoon periods. In the model all payment orders sent to the banks from their customers are assumed, for simplicity to be of unit size i.e. Bernoulli distributed. Banks can however decide to split these orders if they wish to postpone submission of some proportion part of the payment order. This setup can be understood as means to describe a real process where banks are receiving a large number of individual transactions, and selecting some of these to be processed. Different magnitudes of real payments can be modelled by different Bernoulli probabilities of getting the unit sized payment requests. If market situation is assumed to be stable, it is natural to require that expectation of net value of payments received and sent by each bank equals to zero.

For bank i, let $c_i(z)$ mark the unit cost of liquidity, with total level z of liquidity required during the day and $w_i(d)$ the unit cost of delaying payments similarly. For both cost functions following assumptions are made

$$\begin{matrix} c_i(z), c_i'(z) > 0 \forall z \geq 0 \\ c_i''(z) \geq 0 \forall z \geq 0 \end{matrix} \quad \text{and} \quad \begin{matrix} w_i(d), w_i'(d) > 0 \forall d \geq 0 \\ w_i''(d) \geq 0 \forall d \geq 0 \end{matrix} \tag{1}$$

Thus both of the unit cost functions are assumed to be positive and increasing functions of total level of required liquidity. The more liquidity is needed, the more expensive it will be for the bank. Similarly the small delayed amounts cause smaller dissatisfaction but with the size of delayed amount also loss of future revenues is increasing. Third assumption, translates to convex form of cost functions, which is assumed for convenience in the formulation of optimization task of individual bank.

Delaying payments includes the possibility that liquidity needed for covering these payments can be received as incoming payments from other banks, and thus the own liquidity costs might decrease. Decision problem of a bank is a trade-off between liquidity and delaying costs. It can be described by defining a payment processing strategy for each bank:

$$\alpha_i(X_i, c_i, w_i) : \Re^{n+2} \rightarrow [0,1] \in \Re$$

It represents the overall proportion of demand of payment processing collected in the morning period from bank *i*:s customers, which is also processed in the morning. This is again a fairly simplified choice of a model. Possibilities for using more complicated strategies are discussed in paragraph 4.5.4.

The objective function of individual bank consists of expected cost of liquidity required for payment processing during morning and afternoon periods and known cost of delayed payments from morning. Let M_i and A_i denote the liquidity collected by bank i for payment processing purposes in the morning period and in the afternoon respectively, and D_i the delayed amount. The deterministic ones of these can be written as

$$\begin{aligned} M_i &= \alpha_i \sum_{j=1}^{n} x_{ij} = \alpha_i \hat{x}_i \\ D_i &= (1-\alpha_i) \sum_{j=1}^{n} x_{ij} = (1-\alpha_i) \hat{x}_i \end{aligned} \tag{2}$$

In the afternoon, banks will have to settle the payments postponed from the morning together with the new demand of payment processing collected in the afternoon. Liquidity needs can be covered with payments received from other banks during morning period. Amount of required liquidity can have also negative values if the bank is receiving more payments than it is due to pay. In such case no extra liquidity needs to be collected by pledging collateral or other means and liquidity costs in the afternoon are zero. This is because no interest is usually being paid to excess liquidity held in the banks account in the central bank operating the RTGS systems. Thus the actual value for required liquidity in the afternoon A_i is given by:

$$A_i = \max\left(0, (1-\alpha_i)\hat{x}_i + \hat{y}_i - \sum_{j=1}^{n} \alpha_j x_{ji}\right) \tag{3}$$

Here the maximization represents the cut off of values below zero. In the later nonzero expression, first term is the amount of postponed own payments from morning period and second is new payment orders received in the afternoon. Third sum term represents the value of incoming payments received from other banks in the morning period.

Total liquidity need of bank i is for short denoted with $L_i = M_i + A_i$ and the total cost function with C_i. Optimization problem of a single bank can be collected from definitions above and equations 2 and 3.

$$\begin{aligned}&\min_{\alpha_i} C_i = E[(M_i + A_i)c_i(L_i) + D_i w_i(D_i)] = \\ &\left(E\left[\left(\alpha_i\hat{x}_i + \max\left(0, (1-\alpha_i)\hat{x}_i + \hat{y}_i - \sum_{j=1}^{n} \alpha_j x_{ji}\right)\right)c_i(L_i)\right] + (1-\alpha_i)\hat{x}_i w_i(D_i)\right)\end{aligned} \tag{4}$$

4.5. Solving the game

Solving the game presented in analytical framework above will require some further assumptions to be made. It is necessary to at least fix values of the parameters describing the payment flows between the participants to proceed in solving the functional form of expected liquidity and delaying costs, best responses of participants and equilibrium outcomes of the game. This reduces the analysis into studying of some specific cases, which are tried be designed to give some insight as representative examples. Besides the required assumptions, also possibilities for more complex strategy alternatives and their implication to the solving of the game is discussed in paragraph 4.5.4.

4.5.1. Monte Carlo simulation for expected liquidity costs

Cost function of individual bank contains term for expected value of liquidity in the afternoon cost which is nonlinear function of sum of differently distributed Bernoulli random variables weighted by decision variables of the participants. Expression for this is repeated below from equation 4.

$$E\left[\left(\alpha_i \hat{x}_i + \max\left(0, (1-\alpha_i)\hat{x}_i + \hat{y}_i - \sum_{j=1}^{n} \alpha_j x_{ji}\right)\right) c_i(L_i)\right] \tag{5}$$

Remembering the information structure of the game, only morning period payment orders $\hat{x}_i$ and own decision variable, share of morning period payments processed in the morning α_i are known to the bank in question. Distribution for value of demand of payment processing collected in the afternoon $\hat{y}_i$ is known but value of incoming payments and the possible decisions behind these are hidden. Although α_j has been explicitly written in the equations for marking the decision parameters of counterparties, values of these are not directly identifiable to the banks. They can only observe the values of payments sent to them in morning and afternoon period, but not the original period when these payments arrived to the counterparty bank.

The methods for evaluating the expected value of this kind of random variable analytically or numerically have been discussed as an example case in another paper by the author[25]. In more complicated setups including nonlinearities or with larger number of participants, analytical method for solving the expectation value is not possible in practice. This leads to use of numerical approximations for required liquidity in afternoon.

As most versatile approach for solving the expected value numerically, Monte Carlo simulation was selected in this study. It can be used to estimate the expectation value of nonlinear liquidity requirement, and is usable also with more complex decision variable construc-

[25] Hellqvist (2005)

tions, such as bilateral credit limits or asymmetric behaviour patterns among participants. Monte Carlo approach mimics also nicely the real setup, in which the banks observe payment arrivals, and can record their own actions on order to estimate the expected value for liquidity need as their function.

The approximated nonlinear function for expected afternoon costs has two input variables: know aggregate value of morning period payment orders $\hat{x}_i$ and own decision of delayed payments α_i variables. Besides these the expectation is affected by $(n-1)$ probability parameters for both incoming payments of the morning period affected by the decisions of the counterparties and outgoing payments in the afternoon. Adding these together will give $2+2(n-1)=2n$ parameters or variables affecting the expectation value under study.

Monte Carlo method is in this case used by fixing some values for the argument variables and performing sampling for resulting stochastic function. From the samples, point estimates can be calculated for the descriptive statistics of stochastic function under study. This work can be performed with different combinations of argument values, say *m* points for each variable, and a grid of point estimates for the function is reached. Continuous numerical approximation of the whole function can then be created by fitting some appropriate approximating function into the estimated grid.

The problem of Monte Carlo approach is computational burden it creates. In the setup described above, with *r* Monte Carlo samples calculated in each grid point, the number of required samples *N* is

$$N = r\, m^{2n} \tag{6}$$

Required computing time is growing exponentially as a function of number of participants and polynomially – with degree *2n-* as function of the approximation accuracy. In practice large number of participants can not be analysed with this approach. As an example, with *r=1000* and *m=10*, the required number of samples is 10^{10} only with three participants and becomes 100 times larger after each new participant added.

As a solution, not all values of probability parameters are meaningful to be calculated. In stead the market structure under study can be decided and fixed to describe some theoretically interesting setup or to mimic a real market structure by estimating the probabilities for payment arrivals from real data. This is discussed in detail in paragraph 4.5.2 below.

When the set of probability parameters is dropped out from the list of required arguments, the number of sampled variables is dropping to constant value two – demand of payment processing from morning period and own decision variable. For describing different market structures two more variables, to be presented in following paragraph, are also varied. Thus amount of samples which is polynomial with degree four is reached. This can easily be calculated for any desired market structure.

$$N = r\, m^4 \tag{7}$$

Assuming the banks in the market under study are risk neutral, it will be sufficient approximating the expected value required liquidity and through it the expected value of liquidity costs. Unbiased estimator for this is calculated in each point of argument values as sample average of r individual samples. For purposes of sensitivity analysis also sample variance for the liquidity cost is estimated.

In approximation of the expected value function based on the sampled data cubic splines were selected in this study. These are piecewise polynomials of third degree, which can be fitted to the estimated values of expected value function in the n-dimensional grid of argument values. Resulting approximation function will be twice continuously differentiable and as a polynomial, easily handled also analytically if necessary. In comparison with other approximation methods, splines are also reported to have lesser tendency to oscillate, so called minimum curvature property[26], which is advantageous when fitting is done based on stochastic estimates and the purpose is to describe the expected value function, which is not noisy.

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[26] Ahlberg, Nilson & Walsh (1967)

Price of smoothness of spline interpolate is in increased size of group of equations which needs to be solved in fitting the spline. This is because locality of fitting the polynomial is lost when adjacent interpolants are tied together with smoothness requirements. With current model structure the fitting can easily be performed, since the number of equations is polynomial like the number of points in the estimated grid.

If number of variables in the expectation function would be increased or tied to the number of participants, the work required for calculating the spline approximation would also be increasing exponentially. It was noticed that fitting the spline approximation was the slowest part of numerical solving of expected value function. Thus it would not be applicable in general setups. By sacrificing the smoothness of interpolating function, simple linear approximations can be easily calculated for bigger numbers of argument variables. The search of more scalable smooth approximation methods is left for further studies.

4.5.2. Describing the market structure

In setup, where payment flows are modelled as Bernoulli distributed variables, different market structures can be described by giving different probability-parameters to random variables representing payments between participants. In this way it is possible to describe the market share of each participant and also structure of the payment flows i.e. how much each participant is sending payments to each other. Since decision variables of counterparties are unknown to banks in the game setup, the probabilities of incoming payments are assumed to describe the values of real arriving payments in each period.

For incoming payments these probabilities can be estimated from real payment system data, but for describing the demand of payment processing collected by each participant the observed payment flows will not suffice. Difficulty of setting the parameters in realistic way lies in the fact that real arrival times of payments from the customers to the banks are not known. Model of this study could however be used to calibrate the parameters of incoming demand of payment processing so that resulting behaviour by the bank makes the statistics of outgoing payments coincide with the desired distribution of outgoing payments. This requires, of course, that cost structures and other factors guiding the strategic behaviour would be correctly known in the model.

In the used model structure the banks have only one decision parameter affecting the outflow of payments. Also is was assumed that the market structure is stable, which in terms of payment flows means that the expected value of incoming and outgoing payments is equal on the aggregate level. These two facts together will make it unnecessary to model distinctively e.g. participants with different market share. All participants are, because of the simplified structure of the model, just exchanging payments with single counterparty consisted of aggregate of all the other players in the model.

To enable analysis of participant reactions to different market structures two additional parameters are included in the Monte Carlo sampling. Firs one of these is α_j, which is later denoting simply the average value of delaying decisions of the counterparties. Varying this value will shift the expected value of incoming payments between the periods similarly as the decisions of counterparties and allow comparison of best response of participant under study and the average of counterparty behaviour. Second new parameter is the overall share of demand of payment processing occurring in the morning period. It is denoted with θ and is used in the current model structure to scale Bernoulli probability parameters for morning and afternoon periods. Using the notation presented in paragraph 4.4 this means that random variable for demand of payment processing from bank *i* to *j* has distribution

$$x_{ij} \sim Bernoulli(\theta p_{ij})$$

and the afternoon period payments with the same bilateral pair of banks has distribution

$$y_{ij} \sim Bernoulli((1-\theta)p_{ij})$$

The parameters p_{ij} were given fixed value 0,8 in the Monte Carlo sampling. This contradicts the assumption of having a combination of differently distributed random variables describing different payment probabilities between individual participants. In terms of describing the market structure, the simplified model which is used in the current study, where each bank has only one decision parameter, makes it irrelevant what is the counterparty sending the payments and the selected probability parameters are therefore acceptable. On the other hand the argumentations, which lead into using of Monte Carlo sampling, can be reconsidered. The distribution of sum of Bernoulli variables with same probability parameter can be directly stated by binomial distribution. Since the nonlinearity of expected cost function by the maximization function in equation 3 still remains, the approach of Monte Carlo sampling is also still required.

4.5.3. Cost structure of participants

Form of liquidity and delaying cost functions have to be also fixed before the expected value of liquidity costs can be evaluated. For simplicity again the marginal cost functions are assumed to be linear functions of required liquidity or delayed amount of payments. Thus actual liquidity costs and actual delaying costs will thus be quadratic functions.

$$\begin{aligned} c_i(L_i) &= c_i L_i \\ w_i(D_i) &= w_i D_i \end{aligned} \tag{8}$$

In numerical sampling of expected value of liquidity costs the scalar coefficient of cost function, which now actually represents the modelling possibilities of different cost, can be brought out from the expression expected value. Same sampling will therefore suffice for all levels of liquidity cost structures.

$$E[L_i \ c_i(L_i)] = c_i E[L_i^{\ 2}]$$

Interesting reasoning can be made of the effects of different liquidity cost structures. If it is assumed that liquidity could be cheaper or more efficiently managed by the biggest participants there would be economics of scale in payment processing. This would alter the trade-off between delaying and liquidity costs of biggest participants and make them more eager to forward their customer demand into payment system immediately – assuming the delaying costs were similar for banks of all sizes. As a result smaller players, with higher liquidity cost, would also have increased possibility to receive incoming payments to cover their liquidity need, which could make delaying payments more profitable for them. Overall result from these assumptions and reasoning would be increased possibility for competition, since structural barriers for entry would become lower because smaller players would be able to indirectly benefit from decreased cost level of larger banks.

4.5.4. Alternative strategies

In the selected model structure each participant is given only one decision variable: the aggregate share of payments it is submitting directly in the morning period. As a result of this simplification behaviour of banks has to be symmetric towards all the other banks in the market. Having the possibility of asymmetric market shares and cost structures, this is likely to be a severe limitation of the model. Problem is that raising the number of decision variables will increase the number of required computations in numerical solution of expectation values both in Monte Carlo sampling and fitting the spline approximation. Some possibilities of setups where more complex strategies are allowed are discussed here. Implementation of these ideas is left for future studies however.

Division of counterparties

The smallest enlargement in the model could be allowing two decision variables for each participant in stead of one. This would enable participants to divide counterparties into two classes and have different throughput percentages to both groups. Division could be made based on e.g. the size or importance as a source of incoming payments. Classifying would also make possible the analysis of cooperation between players by dividing the banks to "own group" and "the others" and having different decision parameters for these. Resulting complexity of numerical calculations would be still polynomial, although number of possible divisions would increase exponentially with number of participants.

Individual decisions towards each counterparty

Giving the bank's individual decision parameter towards each other counterpart would increase the number of these variables in the model from one to $(n-1)$. Such a case can not be interpreted only as a set of parallel simple two player games, because the liquidity reservations made for payments toward one counterparty affect the availability of liquidity for other payments and the total liquidity costs are nonlinearly dependent of the total value used in all bilateral pairs of participants. This would be very interesting enlargement, but the amount of required computations would increase exponentially.

Bilateral limit interpretation of decision variables

Decision variables of banks can be interpreted also as risk management facilities towards other counterparties. In some real payment systems participants can have bilateral limits or bilateral caps, which in RTGS environment mean ceilings for net value of payments that can be sent towards specified counterparty. Existence of such limits can actually implicitly indicate management of intraday liquidity: possibility that others could be free riding with my liquidity is decreased if I set bilateral limits towards them. Under such conditions liquidity would not be sent out more than the limit value unless some liquidity also came back from the counterparty with active bilateral limit.

Using decision variables as bilateral limits would mean in the model that instead of indicating percentage, an absolute maximum value of payments that can be sent on first period would be defined by the variable. This would however not change the current equations of expected values. Thus it is also possible to interpret the results of this study to be caused by bilateral limits between participants instead of explicit decisions on the value of processed and delayed payments. Structure with single decision parameter translates in this framework into multilateral limit value: it describes maximum aggregate net value of liquidity outflow, which is accepted until the payments are suspended into queue.

5. Numerical results

5.1. Sampled liquidity costs

Monte Carlo sampling of expected liquidity costs was performed for market of ten banks as a function of four parameters: own decision of delaying α_i, observed amount of demand of payment processing on first period $\hat{x}_i$ aggregate decision of the others α_j and proportion of total demand expected to occur on the morning period θ. These all were given values between 0 and 1 with steps of 0,1 except $\hat{x}_i$ which ranged from 0 to 10 with unit steps. The presented sampling includes 100000 simulated cases per each point estimate of the expected liquidity costs.

Monte Carlo sampling was performed with Matlab code which is presented in the Annex1. The code is written to be compatible with parallel computing, since dividing the computational burden is well suited for Monte Carlo approach. In this study the calculations were performed with three PC:s having Intel Celeron 2.4 Ghz processors, 1 GB of main memory and Windows XP Professional operating system. The duration of sampling was approximately 4-5 hours for each PC.

Of the sampled variables, two first belong to the information set of a bank making its decision while the two last can be viewed as descriptions for the given environment. Performing the sampling also on these external variables, their effect on outcomes of the game can be analysed continuously in stead of using just point samples. Below is presented the expected liquidity cost as a series of figures, where the decision of other players has constant value $\alpha_j = 0{,}6$, and proportion of morning period of the demand of payment processing is varied from 30% to 80% i.e. $\theta = 0{,}3 \ldots \theta = 0{,}8$.

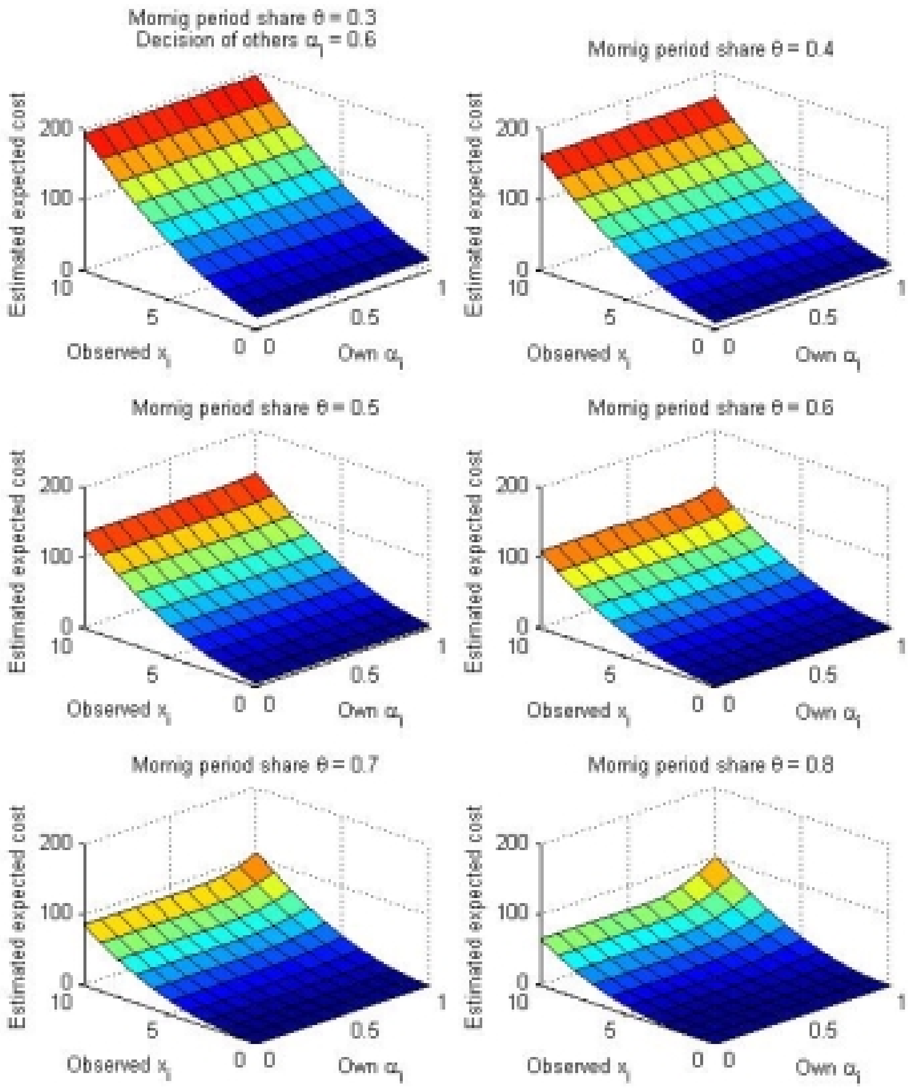

Figure 3 Estimated expected liquidity costs surfaces as function of α_i , $\hat{x}_i$ and the proportion of morning period θ. Average of decisions of other participants is given a constant value α_j = 0,6.

Observations of the structure of the expected liquidity cost function can be made based on these figures. First, the estimated expected cost is clearly a function of the observed demand of payment processing in the morning period $\hat{x}_i$ presented on y-axis. Secondly, the share of morning period of the expected value of payments is also having large impact. This is noticed by following the decrease in expected values from top left picture to the bottom right i.e. when the proportion of demand expected for the morning period is increased. Also a rice in the total level of costs can be seen by comparing the expected costs in level, where $\hat{x}_i$ is zero. When the morning period share of payments θ is large, all, or most of the payments in the afternoon can be covered by incoming payments, but when θ is close to zero, some costs are expected to occur anyway. Here it should be remembered, that in all cases 60% of their payment orders received in the morning period were assumed to be sent by counterparties in the morning.

Expected values change clearly more moderately when own decision variable α_i is varied. Delaying own outgoing payments is noticed to be more relevant when lots of incoming payments can be expected, namely when morning period activity is high in comparison to afternoon.

Figure 4 below depicts six cases in which the value of morning period share of payments θ is in turn frozen to 60% and average decision of counterparty banks α_j is varied.

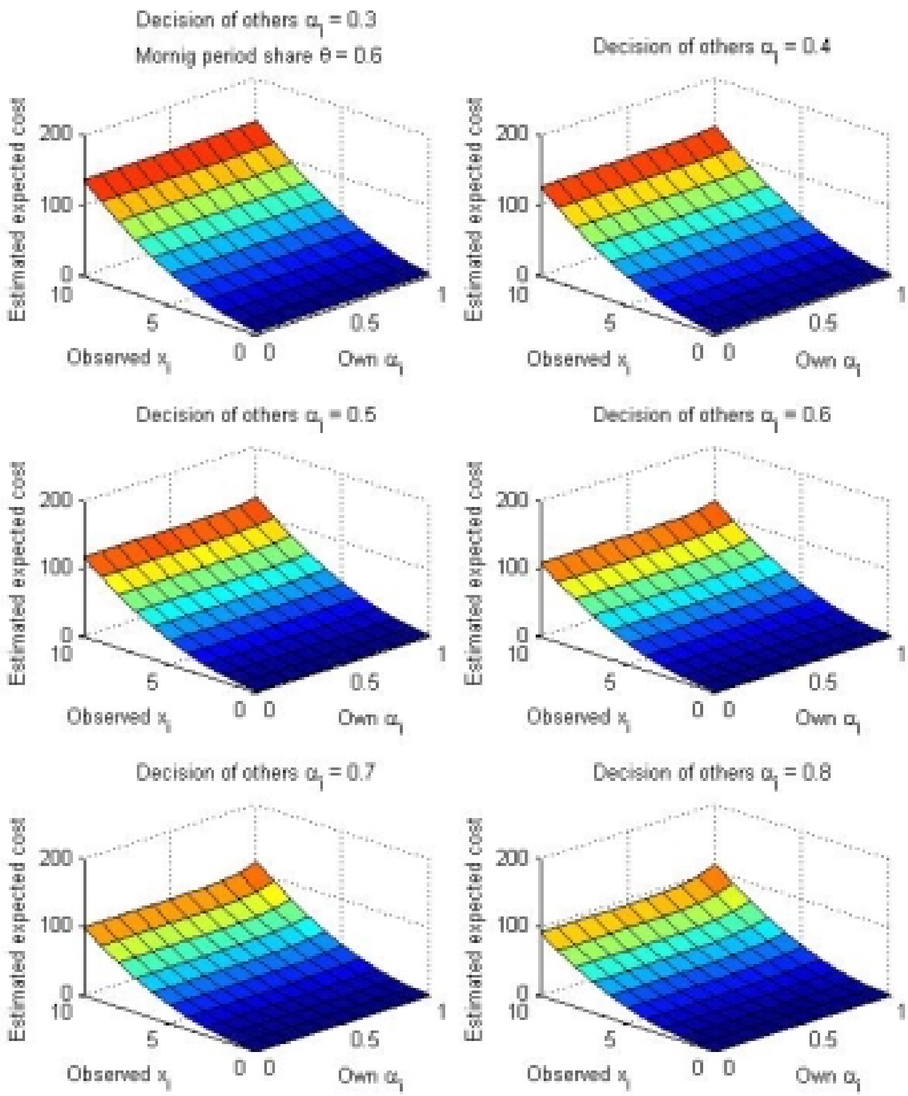

Figure 4. Change of the form of liquidity cost surface when the average value of decision variable of the other players α_j is varied. Estimated expected liquidity costs surfaces are presented as function of own decision of immediately settled payments α_i and observed demand of payment processing from morning period $\hat{x}_i$. Proportion of morning period payments is given a constant value $\theta = 0.6$.

Now an identical form of expected liquidity cost is observed as in Figure 3. Overall level of required liquidity decreases and importance of own decision α_i becomes more influential when the share of delayed payments is reduced by the counterparties. A comparison of

Figure 3 and Figure 4 shows that the effect of increasing α_j from 0.3 to 0.8 is smaller tan impact of similar change in θ.

Similarity of the impacts of the market structure parameter θ and the decision of counterparties α_j is evident considering the form of the cost function. Both of the environment variables affect to the possible and expected amount of incoming liquidity from other banks. Larger effect of θ can be explained by the effect it also has on the morning period value of own payments, which can be moved around more efficiently by α_i when θ is large.

Credibility of estimated liquidity cost values can be discussed after calculating confidence intervals for the estimates. These are easy to calculate, because based on central limit theorem the sample mean estimates follow normal distribution and also the sample variances were recorded in the Monte Carlo sampling. Denoting sample mean with μ, sample variance with s, c-quantile for two sided confidence interval of normal distribution with z_c and total number of samples with N the confidence interval can be stated as[27]

$$\mu \pm \frac{z_c s}{\sqrt{N}}. \tag{9}$$

Sample variance of liquidity cost was relatively large in the results. Its value was on the average 7.3 times the value of corresponding sample mean and over 17 times at largest. This makes the confidence intervals uncomfortably wide. With confidence 95% level the value of z_c is 1,96 and the widest individual confidence interval becomes $\mu \pm 0.106\mu$ i.e. more than 20 % of the estimated sample mean value. Average width of confidence interval from all sampled combinations of external variables was 9,0% of corresponding sample mean value. As an example of estimated expected liquidity cost and its confidence interval, the case with $\alpha_j = 0{,}6$ and $\theta = 0{,}6$ is presented below in Figure 5.

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[27] Se e.g. Laininen (1998).

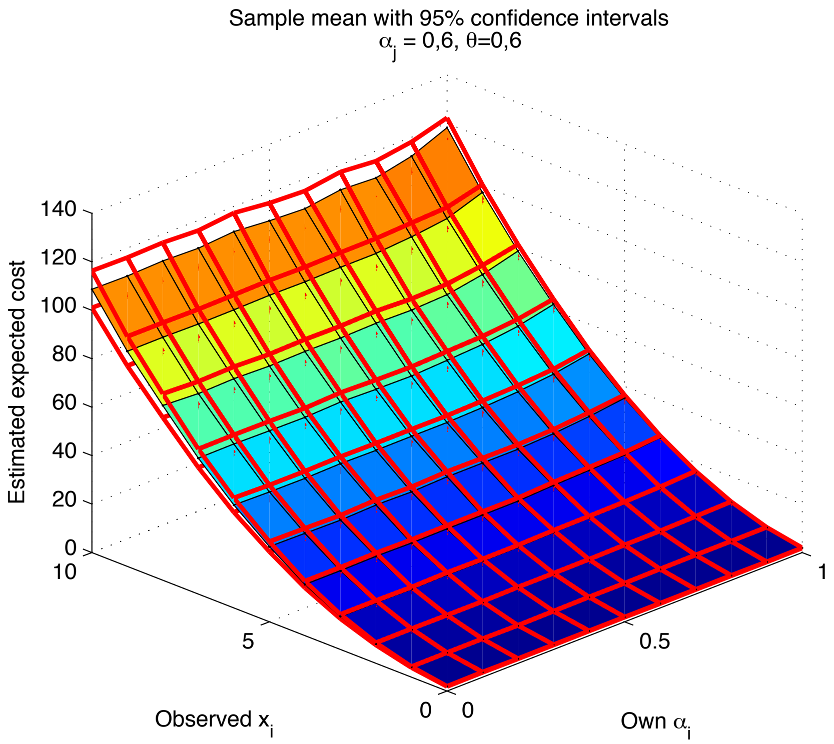

Figure 5. Example of sample mean of liquidity cost and its 95% confidence interval from sampling with 100 000 samples on each grid point. Confidence interval is presented with the red mesh above and below the estimated cost surface.

To make the confidence intervals narrower, it would be necessary to increase the sample size or implement more advanced Monte Carlo sampling with some variance reduction technique. For the purpose of current study these estimates of the artificial cost surfaces were, however, decided to be accurate enough as such. To justify this decision it can be noted that despite the uncomfortably wide confidence intervals there seems to be very little random noise in the estimated cost surfaces presented in Figure 3 and Figure 4. The form of estimated liquidity cost surfaces can therefore also be assumed to be close to correct and not distorted by random errors.

5.2. Best responses

Best responses can be reached by direct minimization of approximated cost function. After substituting the selected cost function structure from equation 8 to cost function of equation 4 and combining the two cost coefficients into one $K = w_i / c_i$ the minimization task can be written as

$$\min_{\alpha_i} \left(E\left[L_i^2\right] + K\hat{x}_i^3 (1-\alpha_i)^3 \right).$$

Result of the minimization is the best response $\alpha_i^*\left(K, \hat{x}_i, \alpha_j, \theta\right)$ resulting in lowest expected total cost of payment processing. Values for best responses with fixed value of morning period share of payments $\theta = 0{,}7$ are shown below in Figure 6 with different levels of average of decision of other banks α_j.

It can be observed that the best responses are very instable when $\hat{x}_i$, the collected value of payment order in the morning period approaches zero: different values are given with different levels of average counterparty behaviour. This is natural since, when $\hat{x}_i$ is small, also the amount of payments affected by the decision α_i^* becomes obsolete. On the other hand, with larger values of collected demand for payment processing, the optimal behaviour is almost alike: increase in $\hat{x}_i$ raises the best response as does increase in cost ratio *K*. Only a moderate increase in the overall level of best response can be observed, when average behaviour of counterparties is shifted towards increased delay in payment processing. The general structure of the game dynamics can be predicted already based on this observation: playing against average of counterparties drives individual banks also towards the average behaviour. This will be verified more clearly in next section by plotting and analysing reaction curves of banks.

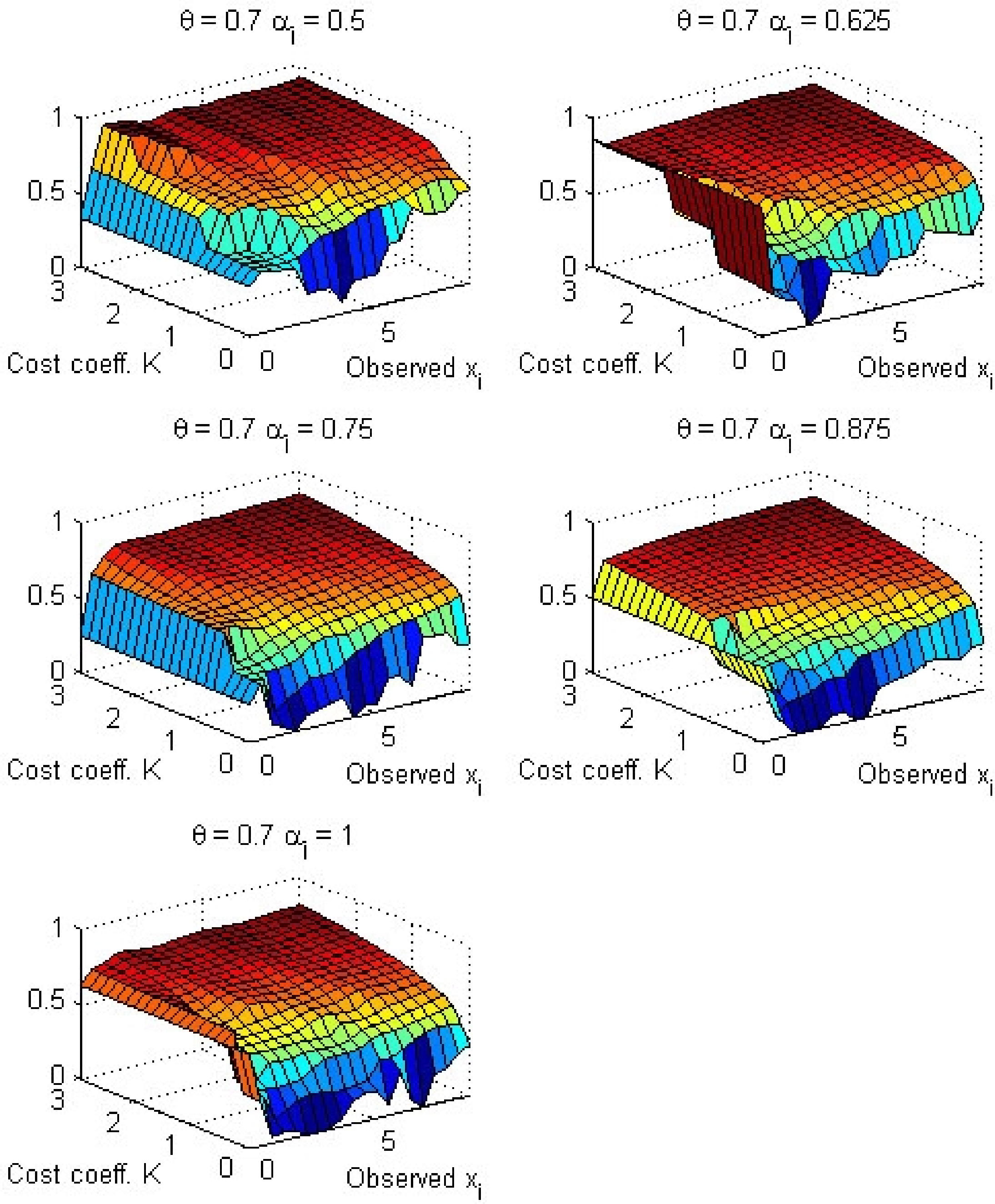

Figure 6 Best responses as function of cost ratio K and collected demand of payment processing from the morning period x_i in different levels of average counterparty behaviour and fixed percentage 70% of payments initiated in the morning period.

Best responses were also calculated during the study by solving the traditional necessary conditions for optimum point from equation 4. In practise this stands for finding zeros for differentiated spline approximation which was fitted on Monte Carlo estimates. This method was not used however, because compared to direct numerical minimization of objective function it was noticed to give numerically clearly more unstable solutions.

5.3. Equilibrium outcomes

Equilibrium outcomes can be calculated from best responses assuming that if best response of an individual bank and average behaviour of other banks are identical, there is no drift of average behaviour happening over time. Since cost structures of banks are not necessarily identical this is a somewhat restricting assumption: in reality banks with different costs can make opposite decisions resulting in unchanged equilibrium. The possibility of multiple equilibriums, which is interesting from the coordination game point of view, should however be revealed with a comparison of individual and average strategy.

Easiest way of locating equilibriums in this case is plotting the reaction function, $\alpha_i^*(\alpha_j)$ together with average behaviour as a function of individual choice $\alpha_j(\alpha_i^*)$. The latter is simply a line with unit slope, because the averaging does not include any behavioral elements.

The best responses were before calculated as a function of four variables: $\alpha_i^*(K, \hat{x}_i, \alpha_j, \theta)$. Since payment probabilities in Monte Carlo sampling were fixed as described in section 4.5.2, the value of morning period share of demand θ freezes the expected value of payment orders collected in the afternoon denoted with $\hat{y}_i$. Therefore the best response calculated after fixing θ and varying the collected morning period payment orders $\hat{x}_i$ will in practice also describe the expected behaviour of a bank in different levels of θ. The values of these two parameters can thus be connected with equation $\theta = \hat{x}_i / (\hat{x}_i + \hat{y}_i)$, and plotting of reaction functions can be done by first choosing either one of these e.g. the implicit value of θ. After calculating the corresponding value of $\hat{x}_i$ the only external variable remaining is K, the ratio of unit costs of delaying and unit cost of liquidity.

As an example of reaction functions, below is plotted a single case where 61% of demand of payment processing is occurring in the morning (θ=0,606) and unit cost of delaying is slightly smaller than unit cost of liquidity with ratio $K=0,80$.

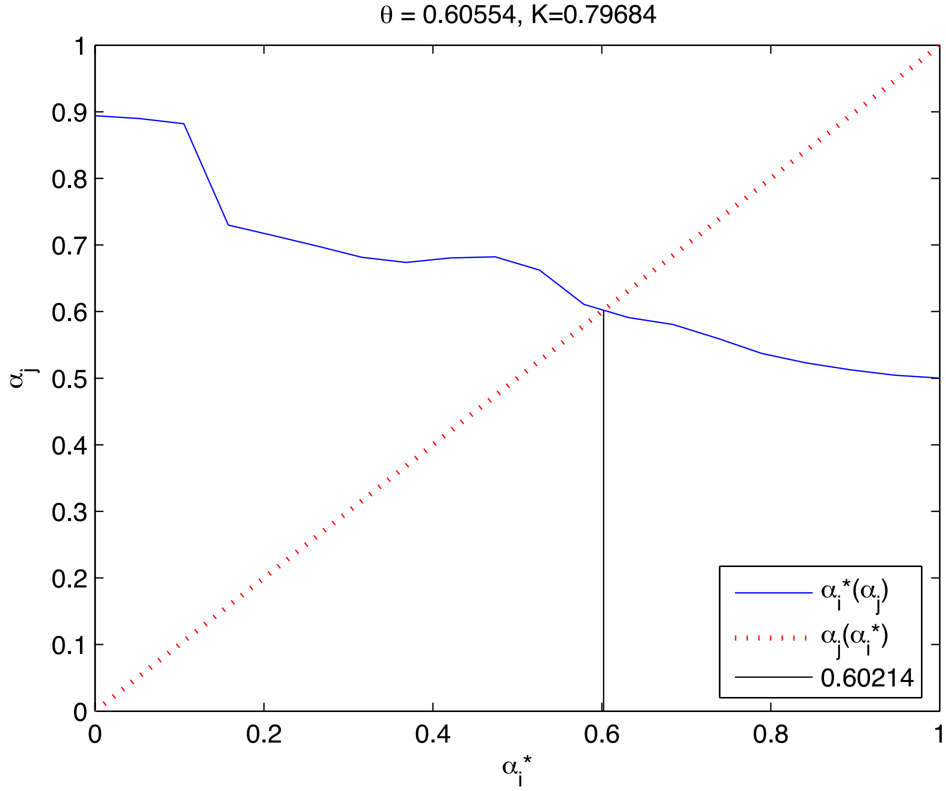

Figure 7 Example of reaction of individual bank as function of average behavior of counterparties (blue line). With red color is plotted the average as function of individual values to reveal such values where these lines are crossing. These represent possible equilibrium values of delaying decision.

The example selected here is a clear and simple case. There is one distinct equilibrium value where 60,2% of payments from morning period are settled immediately. The reaction function of individual participant is showing a downward sloping form which is in line with earlier observations: when the form of cost functions and best responses were examined, it was noticed that delaying becomes more profitable when the others are not delaying payments.

In this point the game under study differs from the pure coordination game of Van Huyck presented in paragraph 3.1, where it was advantageous for participant to follow the decisions, or in that case the minimum of decisions of the others. In the intraday liquidity management game the strategic setup leads to playing against the average, which could be assumed to lead into unique stable point. When reaction functions are studied further, it is noticed, however, that this is surprisingly not always the case. In stead sometimes there can be several equilibrium values or even a range of values where individual and average behaviour practically coincide. One example of such case is presented below in Figure 8.

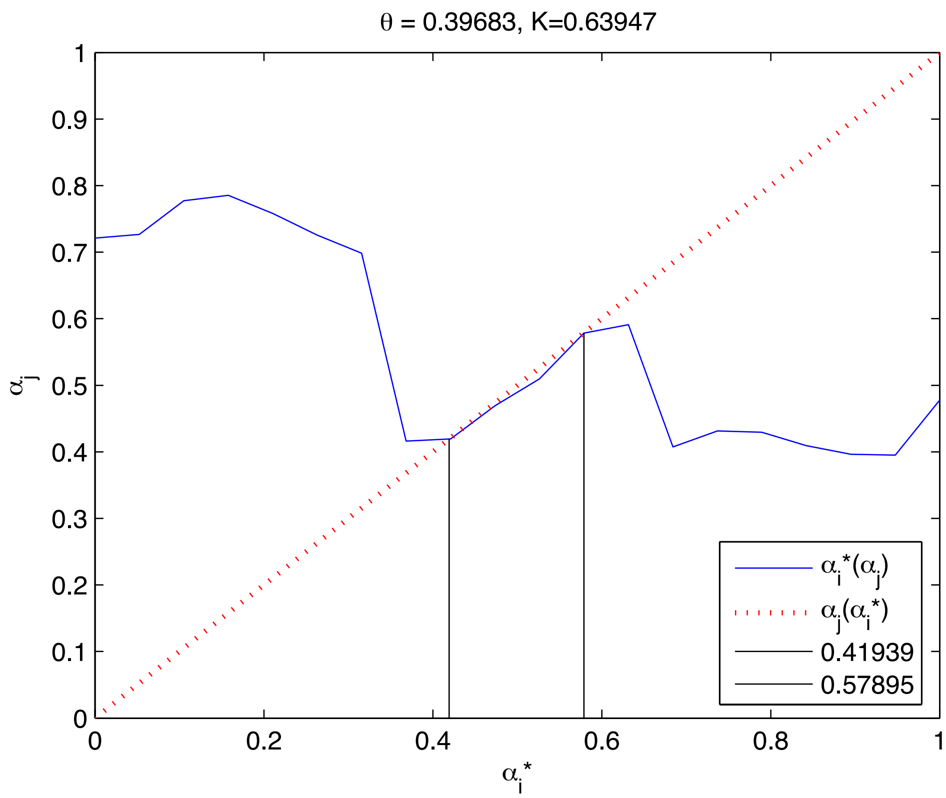

Figure 8. Example of case, where reaction function of individual bank is following the average over a range of decision parameter values.

In the case of Figure 8 practically any value of α_i^* between 0,419 and 0,579 could be a stable equilibrium. This special case can be examined further by calculating the values of total cost function for the possible range of equilibriums. These are presented in Figure 9 below.

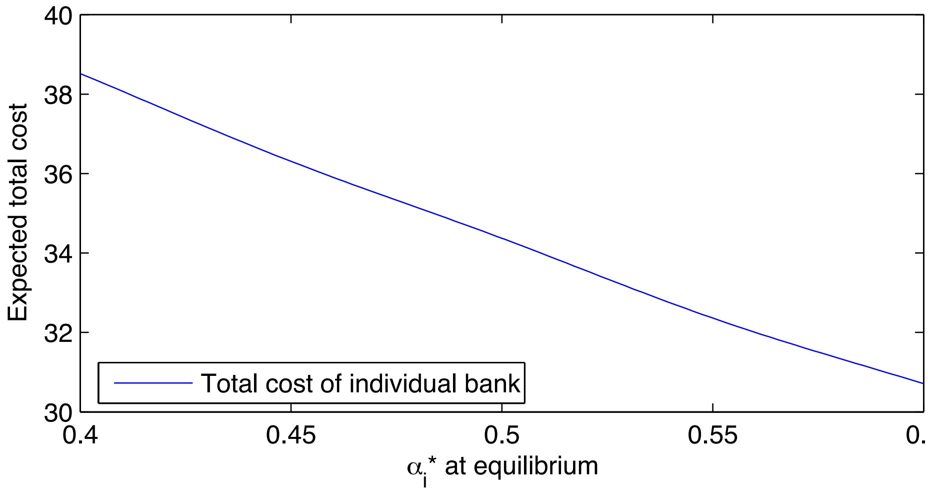

Figure 9. Expected value of total costs for individual bank over the range of possible equilibrium points for game setup presented in Figure 8, where coordination failures are possible.

The exact values of expected cost at the ends on the range are in this case 31.4 and 37,6 and thus the relative difference in the expected cost can be as high as 20% between the extreme cases. This example is clear evidence of possibility of coordination failures in RTGS-system. Here it must be emphasized, however, that these observations are based on purely artificial numerical example. Thus the cost figures and their relative values have to be interpreted primarily as qualitative results.

5.3.1. Impact of external variables on equilibrium outcomes

In stead of examining single separate cases, the effect of external variables on the value and nature of equilibrium can be analyzed. Below the share of morning period demand of payment processing θ is given fixed value slightly over 0,5 and cost ratio K is varied. The plot colours and order are the same as in Figure 7 and Figure 8 above.

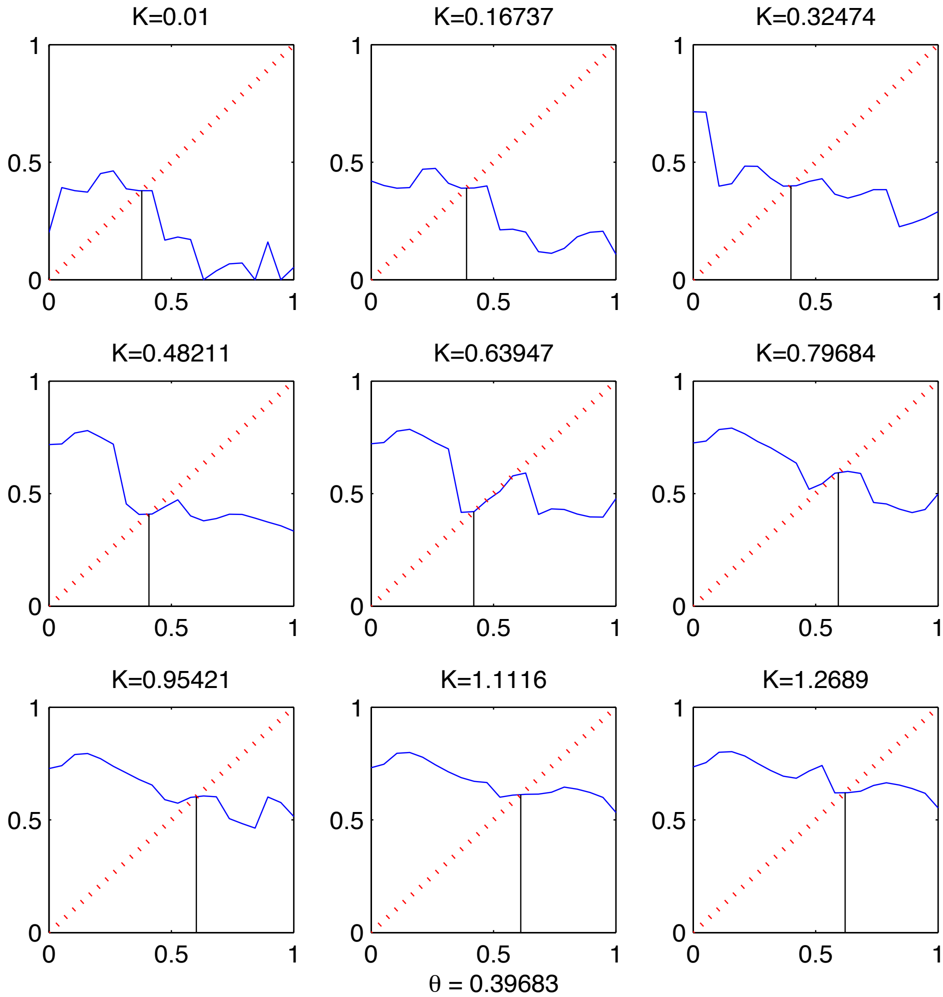

Figure 10. Reaction functions with different cost structures. (K = ratio of coefficients of delaying and liquidity costs). Market structure is fixed so that 40% of demand of payment processing occurs in the morning period.

In top left figure the coefficient of liquidity cost is hundred times as large as coefficient of delaying costs. Reading along the figures until the bottom left one, the ratio increases so that coefficient of delaying costs is eventually 27% bigger than coefficient of liquidity costs. In all presented levels of coefficient ratios the smallest value among the possible equilibrium points is marked with vertical black line. As the ratio of cost coefficients is increased, the equilibrium line can be observed to move towards left. This can be explained in self-evident way: fewer payments become delayed when relative cost of delaying is increased. Similar phenomenon can be observed also in other levels of morning period share of total payments. The individual example analysed in more detail and presented in Figure 8 before was shown in the middle of this current figure.

Changes in reaction function and equilibrium outcomes resulting from altering the market structure in stead of cost coefficients of participants are presented below Figure 11.

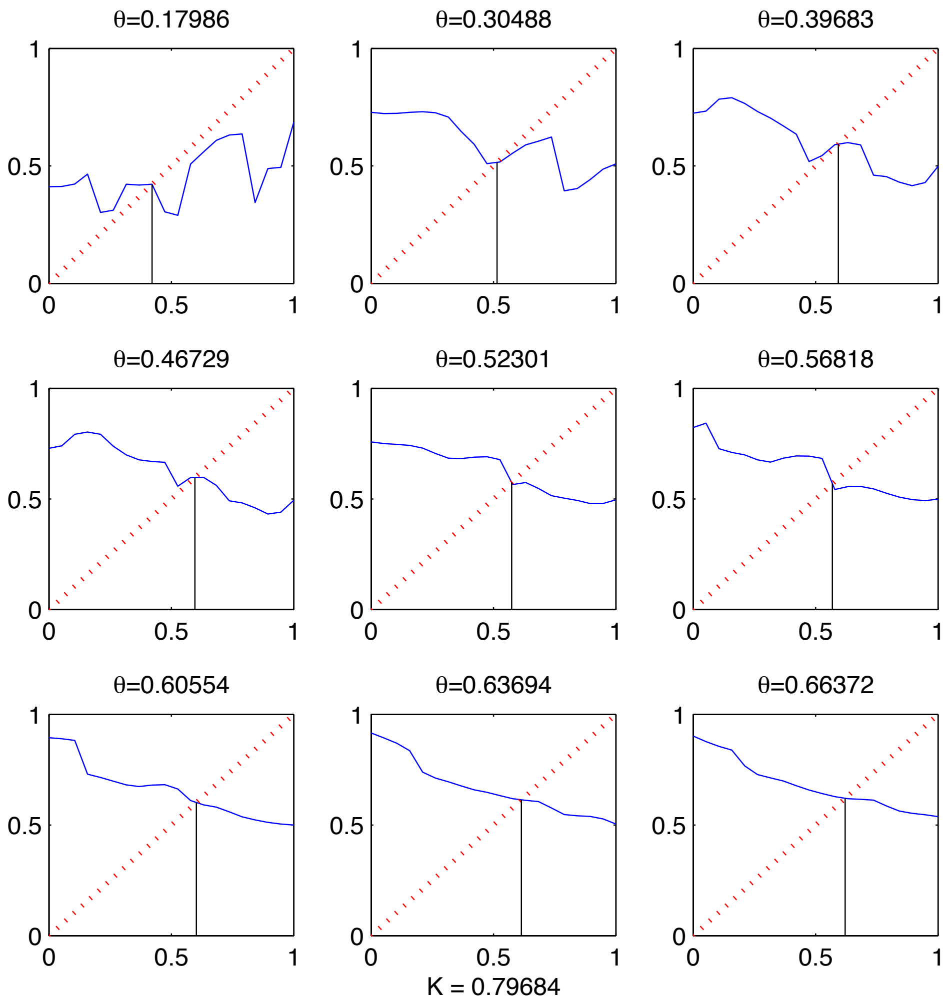

Figure 11. Reaction functions and equilibrium value of instantly processed payments at different levels of θ, the expected share of payments in morning period. Cost structure of participants is fixed so that coefficient of delaying cost is smaller than coefficient of liquidity costs with ratio 0,80.

From Figure 11 it can be noticed that increase in value of parameter θ will also make the equilibrium value of instantly processed payments higher. The impact is not very strong, and after value $\theta = 0{,}39$ the there seems to be no clear change in resulting equilibrium. Another important thing can be noticed however. The shape of reaction functions becomes smoother and monotonically downward sloping as morning period share of expected payments increases. This translates into removal of possibility of multiple equilibriums and coordination failures.

5.3.2. Impact of heterogeneity of participants

Implications on equilibrium of having heterogeneous participants with different cost coefficients interacting in the market under study can also be analyzed from the results. For this purpose reaction functions resulting from several values of cost coefficient ratios are plotted in Figure 12 below. Purpose is to analyze qualitatively the effect of having one bank in the market, which differs from all the others by having either higher or smaller value of cost coefficient ratio.

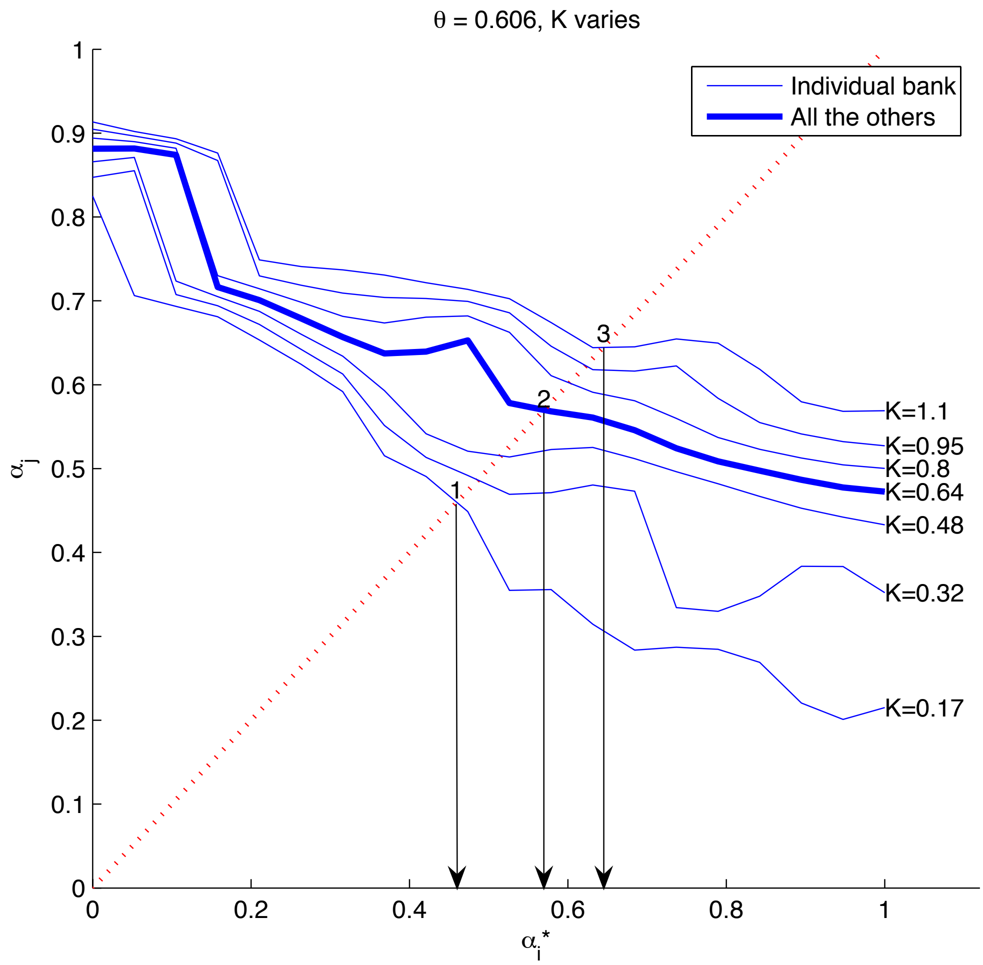

Figure 12. Reaction functions for participants with different cost coefficient ratio in fixed market structure.

Assuming that all the banks in the market would have cost coefficient ratio value *K=0,64* the equilibrium would trivially be placed in value pointed by arrow 2 in Figure 12. Similarly if all players had *K=0,17*or *K=1,1* the equilibriums would be pointed by arrows 1 and 3 respectively.

When only one bank (called bank 1) is having *K=0.17* and reaction function of others remains at the thick line (*K=0,64*), the average market behavior, which bank 1 observes as expected share of payments settled in the morning period, is clearly higher than value at arrow 1, since it is affected by the large number of other banks with different reaction function. As a result the red line representing the average of decision parameters is shifting upwards for the bank1. Similarly the rest of banks observe the impact of bank 1 delaying more payments as them self as downward shift in the average line. The magnitude of movements of average line is reflecting the weight or market share of the counterparty, which is acting differently. If, as an example, it is assumed that bank 1 is a small player, the upward shift of its own average line would be remarkable, while the downward shift on the case of all the others could be negligible. The described mechanisms are illustrated below in Figure 13.

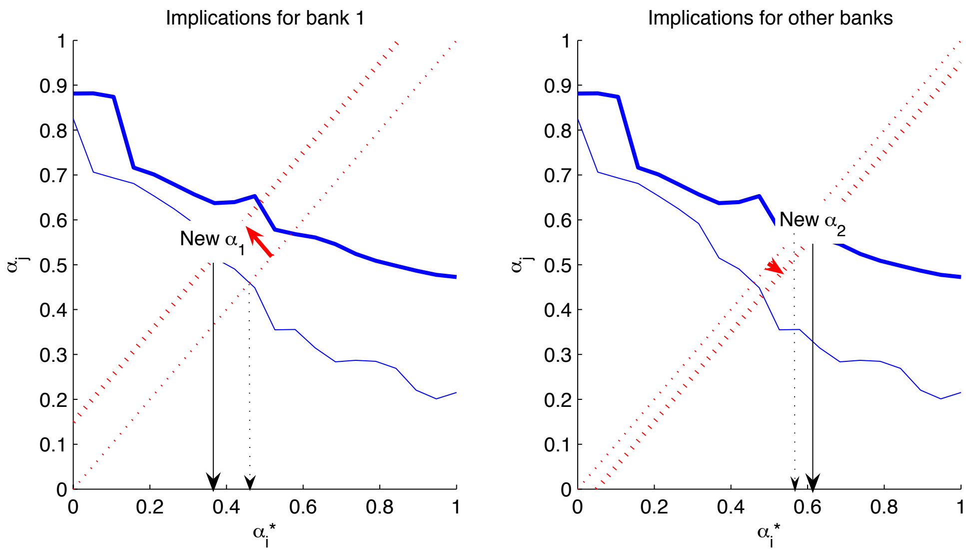

Figure 13. Changes in equilibrium values of actions when a small participant with relatively small value of K i.e. relatively high liquidity costs is entered.

For bank 1, the existence of many participants with relatively lower liquidity costs allows free riding: it self can submit only small share of payments in the morning period and expect to receive larger amount to cover the delayed payments, in comparison to setup where all the counterparties had the same cost structure. Referring to Figure 4 this is likely to decrease the total value of expected costs. For the other market participants impact is clearly smaller. Result will be that these participants are sending out slightly more payments than market average, which is likely to increase the expected costs.

The major observation of this analysis is the lowering of costs from the participant having higher relative liquidity costs, the bank 1. Earlier this possibility was also presented as a possible consequence of economics of scale in intraday liquidity management in section 4.5.3.

Here it was assumed that the reaction function of market average stays as a straight line. In more detailed and realistic analysis the shift in the average curve should be function of distances of compared reaction functions, which clearly is not staying constant. The parallel transition used here will anyway suffice because of the qualitative nature of analysis.

6. Discussion and conclusions

This study analyzed strategic behaviour of banks which are working as an intermediary for processing payments of their customers together with their own payments in real time gross settlement system. Banks were allowed to decide timing of payments in setup with two periods, where demand of payment processing was described as stochastic external variable and strategic interaction with other banks as the value and timing of incoming payments. The revenue of processing payments was assumed to be fixed and optimization task of individual bank was formed from trade-off between cost of liquidity required for immediate processing of payments and cost of delaying payments in hope of free financing of incoming payments from other banks. The delaying cost represented here the possible explicit fees like penalty interest but also, and more importantly, the reputation risk and possible loss of future revenues due to customer dissatisfaction. In the optimization banks were assumed to be risk neutral i.e. minimizing the expected cost of payment processing.

Banks in the model were allowed to have unlimited but increasingly expensive liquidity pool, which was describing the total cost of liquidity management from all possible different sources: intraday credit from central bank against pledged collateral having some opportunity cost, explicitly priced intraday credit from central bank or any form of market funding. Both cost terms, liquidity and delay, were described as quadratic functions to model increasing costs when value of required liquidity or delayed payments is growing.

The game setup consisted of multiple banks with possibly different ratio of coefficients for quadratic cost functions. Strategy alternatives of banks were limited to choosing overall share of payments from morning period to be immediately processed.

The game was solved numerically as a function of several parameters to enable analysis of setups with different market structures, different values of cost coefficients and heterogeneous combinations of participants. Monte Carlo simulations and polynomial spline approximations were used in presenting the expected value of liquidity costs. Best responses of banks were solved with direct numerical optimization of estimated cost function as a reaction to observed average counterparty behaviour.

In the presented results delays in processing the payments emerged in all the cases that were analysed. In previous studies it has been shown that delaying of payments creates dead weight losses in costs of payment system participants and also negative externalities as the quality of information available for banks is decreasing.

Dynamics of intraday liquidity management game were shown to be inverse: delaying payments becomes more profitable for individual bank if others are not doing it. Surprisingly it was also shown that under special conditions intraday liquidity management in RTGS system can form a coordination game, a setup which has several possible equilibrium solutions. This contradicts the general form of inverse dynamics, which tend to create unique equilibrium solutions. One instance with coordination situation was examined further to show that possible equilibriums in such case can be inefficient even for individual participants. Thus possibility of multiple equilibriums means that also market behaviour can become gridlocked in unnecessarily inefficient practices.

As a result from possible heterogeneity of participants it was deducted that competitive advantages from economics of scale in intraday liquidity management are not likely to be large because exploiting this advantage will give the counterparties in payment system better chances to free ride on the liquidity of participant with lower liquidity costs.

6.1. Comparison to earlier studies

The model in current study was a mixture of the models used in papers by Angelini (1998) and Bech – Garrat (2002). Common features with model by Angelini were the generalized quadratic cost function structure describing the increase of unit costs of liquidity and delaying and the idea of allowing participants continuously decide their strategy: the value of payments processed immediately in the first period. Also heterogeneity of participants was adopted from Angelini's model. As a simplification the decision of amount of reserves in the beginning of day was not included from the model of Angelini. From the model of Bech and Garratt the structure of the game as Bayesian setup was used, where participants decide their strategy after receiving private information of their morning period payments. Also modelling of liquidity costs was performed according to collateralized intraday credit version of Bech – Garratt model. This included structure, where collateral has to be pledged or the required funding gathered before payments can be settled and also the zero level of returns for excess liquidity held in the central bank account.

Feature of the current model, which was not implemented in either of the mentioned previous studies, was the stochastic multi participant environment. Price of this change was the lost possibility of solving the game analytically.

The overall results of the study and the observed delaying of payments are convergent with the results from all earlier studies on incentives of RTGS systems. Comparing to two earlier studies, which have analyzed or showed the possibility of coordination game outcome in RTGS, both of these[28] have been describing the Fedwire type of system where price of intraday credit is determined according to overdrafts at the end of each period, which in Fedwire are minutes. In that framework it is more evident that coordination games can be emerging. As an ultimate example, when only the cost of liquidity is concerned, it would be best for the participants of such a system to have all their payments settled during one and same minute long period. This would minimize the resulted liquidity costs as only the net value of all payments would be recorded as the overdraft.[29] Current study is first one to pre-

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[28] Bech – Garratt (2002) and McAndrews – Rajan (2000)

[29] In practice this is of course impossible already due to huge number of settled payments and capacity limitations of the processing, but it also ignores time criticality of certain payments, delaying costs, net debit caps limiting the absolute value of intraday credit etc.

sent possibility of coordination failures also under strict RTGS rule – at least among the papers known by the author.

6.2. Assumptions of the model

One of the most critical assumptions in the model is existence of opportunity cost for collateral, which is pledged in order to get intraday liquidity from central bank. If this cost does not exist, trade-off between two opposing costs in the model will collapse and no delays should occur in system with collateralized intraday credit. Zero opportunity cost could be possible in practice only if the participants of payment system would have in any case enough of eligible collateral because of other reasons than liquidity management.

One argument supporting the existence of pledging cost is the work, which is done to enlarge the set of assets eligible for collateral. As an example of this, European Central Bank recently announced decision[30] of including loans from commercial banks to corporations in the list of eligible assets from beginning of year 2007. If the collateral would not be scarce, there would be no need to find new forms of it. There are also some studies on actual value of opportunity cost of collateral. Folkerts-Landau et al[31] mention estimates from United Kingdom pledging cost of 0,25% on yearly bases while more recent study[32] estimated the cost to be at most 0,07% per year.

Referring to the results of current study, very low, but existing cost of pledging collateral would mean that share of immediately processed payments should be high. Also the possibility of coordination game setup emerging would decrease, because these were only observed in cases where relative cost of liquidity was higher than cost of delaying payments. Of course it needs to be assumed here, that delaying costs of payments are also real and quantified by the banks as heavier than the more explicit liquidity costs.

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[30] ECB (2005)

[31] Folkerts-Landau – Garber, P – Schoenmaker, D (1997)

Another critical assumption of the current analysis is that strategic interaction and timing of payments can be simplified into game setup with two periods. In this context, calling the periods morning and afternoon can be slightly misleading since their idea is to also tackle the decision of players: whether to process payments immediately or later. Increasing the number of periods for better describing the opening hours of a payment system and the nature of real time processing would lead to solutions with dynamic programming or ultimately dynamic optimization formulation of the game setup.

The intuitive idea of multiple periods could perhaps be discussed based on the observations of current analysis and especially the impact of expected share payment orders occurring in the morning θ in the numerical results. The difficulty of this approach is whether to interpret this ratio of expected values of morning and afternoon payments as comparison of adjacent periods or cumulative value of expected payments from remaining periods. These questions and the dynamic formulation of the game were left for future studies.

As a central assumption of the analysis, the normality and stability of operating conditions has to be also emphasised. The market was assumed to have reached equilibrium and also the participants had had enough of time to observe the average behaviour of their counterparties in the game. Thus the results do not describe or predict the robustness of the market or behaviour of the participants in changes of the environment.

6.3. Empirical testing of result

For empirical testing of theoretical results presented in this study it would be necessary to know exact moment when individual payment orders were originally sent from customers to banks in certain payment system. This information is usually not available to anyone else besides the individual customer and bank included. In addition the eventual moments when payments were settled should be known. Also the cost structure creating the incentives for

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[32] James – Willison (2004)

the banks should be quantifiable for both types of cost: cost of liquidity and cost of delaying payments.

In case one part of these data requirements would not be filled, it could be possible to estimate its value by assuming rationality and risk neutrality of the banks and by examining which value of the missing parameter would explain observed behaviour.

If data of a real payment system could be collected, it raises some problems however. Being under observation could affect the behaviour of participants, as their possible decisions of delaying payments and free riding could be noticed. In the model this would translate into increased value of reputation risk and thus increased delaying cost, which would shift the equilibrium. Since the implications of this change could be positive in the social point of view, the possibility of collecting payment system data or alternatively requiring that banks inform their customers of the actual moment when payments are eventually settled could be discussed. Disadvantage of such requirement would naturally be increase in IT costs for banks and processing cost of payments.

Also it can be noticed that there do exist real payment systems which suite the model structure used in this study rather seamlessly. The particular example is PMJ, the retail payment system in Finland, which is used to clearing of customer payments between commercial banks[33]. It has two batch runs per day, where net values of individual customer payments are covered with bilateral fund transfers between the RTGS accounts of participating banks at Bank of Finland. For a given pair of banks, say Bank A and B, there are two payments in each batch run: one for the net value of transactions between A and B originated by customers of bank A and other one including the net value of payments originated by customers of bank B. Participating banks can at least in some of the cases select on which period they settle individual transactions. Most of the payments also fulfil the requirement of being stochastic as they are originated by consumer demand activities, with the exception of recurrent payments such as salaries and pensions.

[33] Koskenkylä (2003)

6.4. Further research

Many simplifications were made in the current study which allows interesting possibilities for generalization and further developments of the model. In following some of these are addressed.

Analysis of the equilibrium outcomes could be done better by including the a priori probabilities for value of payment instructions observed in the morning period. In the presented results only the expected value of this parameter $\hat{x}_i$ was used.

Increasing the number of decision variables available for a bank would enable analysis of large number of strategy alternatives as it was shortly discussed in section 4.5.4. This would also allow full utilization of possibilities of modelling heterogeneous participants. In such case the payment probabilities in the model could also be estimated from data of some real payment system. Increase in the variables would however make the analysis more complex.

Parallel to the previous idea is the possibility of increasing the number of periods in the model. This also would raise the number of possible strategies, since for each period the decision of a participant would be repeated. Both of these would quickly create a problem with exponential amount of calculations required for numerical solution.

More straightforward extension would be the comparison of dynamic and precautionary collateral management. For this purpose the amount of available intraday credit would be limited by decision made before arrival of any payments. This would change the decision of delaying payments into constrained optimization problem and also necessitate definition of extra cost for payments that would get delayed until the end of day, since funding these payments would require different ad hoc actions.

Yet another challenging possibility would be to alter the information structure of the game so that payments, which are already known by both participants, would be included in the strategic decisions.

Taking the viewpoint of behavioural game theory, assumption of having the stable equilibrium could be removed. This could be especially interesting in the situations with multiple possible equilibriums, because trough different possible processes of learning and adapting to observed market behaviour it could be analysed, what is the possibility of ending up in inefficient equilibrium in such case. This question could perhaps be tackled even easier by examining the suitability of different equilibrium selection criteria in current setup with incomplete and imperfect information.

As features of technical implementation of the analysis, the numerical solution Monte Carlo simulations could be boosted with some variance reduction technique. Also using smoothening splines instead of exact cubic splines in the approximation could be worthwhile. Some replacement for the spline approximation in larger multidimensional problems was also noticed to be necessary, since this step was one of the most time consuming, when the game was solved numerically.

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Annex 1: Matlab code used in solving the game numerically

Main function of Monte Carlo sampling

```
% Main level function for Monte carlo sampling of nonlinear combination of
% random variables. Setup according to intraday liquidity management game.
%
% This version calculates two first moments for resulting random variable
% with all combinations of alphas in setup with given fixed
% distribution parameters.
%
% [Mean,Var,Flag]=FixMtCarlo(n,m,r,alphas,P,Q,subfunc,savename)
% Arguments:
%   n       = Number of participants in the game
%   m       = Number of sampling intervals on each axis
%   r       = Number of Monte Carlo samples in each point of the grid
%   alphas  = Values of decision variable alphas of which the combinations
        % are created as arguments for approximated stochastic function.
        % n*m matrix, m values have to be given for each alpha.
        % Negative values can be used to mark combinations, which are
        % not executed in this sampling (possibility for parallel
        % computing.)
%   P       = Fixed row vector of distribution parameters for first period, size (1,2n-2)
        % First n-1 cells refer to outgoing payments
        % Last n-1 cells refer to incomming payments
%   Q       = Probability parameters for outgoing payments of 2.nd period.
%   subfunc = Function handle for the function generating samples
%   Savename= Filename for saving the results.

function [Mean,Var,Flag]=FixMtCarlo(n,m,r,alphas,P,Q,subfunc,savename)
   Mean=zeros(m*ones(1,n));
   Var=Mean;
   Flag=Mean; % Which alpha combinations have been calculated
   % in which alpha value the sampling is currently conducted on each axis
   counter=ones(n,1);
   for(i=1:m^n) % Work trough all combinations of alphas
      alpha_now=zeros(n,1);
      for(j=1:n)
         alpha_now(j)=alphas(j,counter(j));
      end
      if min(alpha_now>=0) % No negative values =>
         % all alpha values were meant to be evaluated here
         [Mean_temp,Var_temp]=subfunc(n,m,r,alpha_now,P,Q); % sampling
         % Linear indexing for storing the results in matrix form
         Mean(1 + ((m*ones(1,n)).^(0:(n-1)))*(counter-ones(n,1)))=Mean_temp;
         Var(1 + ((m*ones(1,n)).^(0:(n-1)))*(counter-ones(n,1)))=Var_temp;
         %Update Flag
         Flag(1 + ((m*ones(1,n)).^(0:(n-1)))*(counter-ones(n,1)))=1;
         % This equals Flag(counter(1),counter(2),...,counter(n))=1;
      end
      % update counters for next round
      temp_i=i+1;
```

```
    for(j=n:-1:1)
      counter(j)=1+max(0,floor((temp_i-1)/m^(j-1)));
      temp_i=1+mod(temp_i-1,m^(j-1));
    end
    if mod(i,1000)==0
      i/m^n % "progress reporting"
    end
  end
  save(['FixMtCarlo_' savename],'n','m','r','alphas','P','Mean','Var');
end
```

Sampler function for Monte Carlo with four parameters

```
% Simple Bernoulli sampler for two period game.
% Liquidity cost structure is assumed to be quadratic

function [SpMean,SpVar]=Bernoulli_4arguments(z,n,m,r,args,P,Q)
  % Arguments:
  alpha_i =args(1);   %  Proportion of immediately processed, outgoing
  X_i     =args(2);   %  Observed amount of payment orders from morning period
  alpha_j =args(3);   %  Average of decisions by counterparties, incoming
  theta   =args(4);   %  Proportion of morning period of total demand
  % P = parameters for total value od leaving payments (p_ij + q_ij)
  % Q = Parameters for total value of incomings

  % Realizations for incoming payments for first period.
  x_j=sum(binornd(ones(r,n-1),ones(r,1)*theta*Q)')';
  % Realizations for outgoing payments for second period.
  y_i=sum(binornd(ones(r,n-1),ones(r,1)*(1-theta)*P)')';

  % Overall required liquidity
  Sample= X_i*alpha_i+ max(0,(1-alpha_i)*X_i+ y_i-x_j*alpha_j);

  % Quadratic value of liquidity requirement.
  Sample=Sample.^2;

  % Estimators for two first moments
  SpMean=mean(Sample);
  SpVar=cov(Sample);
end
```

Solving the best responces of the game

```
% Direct minimizing of cost function
th=0.7;
n1=20;  % X_i values
n2=20;  % K values
n3=20;   % Alpha_j values
```

```
X_i=linspace(0,9,n1);
K_k=linspace(0.01,3,n2);
wl=1;
cl=wl./K_k;
alpha_J=linspace(0.2,1,n3);

Minimit20=zeros(n1,n2,n3);
for count1=1:n1
   for(count2=1:n2)
      for(count3=1:n3)
         Minimit20(count1,count2,count3) =minbnd(@(ai)…
 totalcost(ai,X_i(count1),alpha_J(count3),th,cl(count2),wl,spl4),0,1);
      end
   end
   count1
end
```

The total cost function for best response optimization

```
function result=totalcost(alphaI,xI,alphaJ,theta,cI,wI,splineCost)
%function result=totalcost(alphaI,xI,alphaJ,theta,cI,wI,splineCost)
result=fnval(splineCost,[alphaI,xI,alphaJ,theta]')*cI+ wI*((1-alphaI)*xI)^2;

% (The spline approximation on liquidity cost estimates was calculated with spline toolbox function
% csape)
```