

ON INVESTMENT, UNCERTAINTY, AND STRATEGIC INTERACTION WITH APPLICATIONS IN ENERGY MARKETS

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Abstract: The thesis presents dynamic models on investment under uncertainty with the focus on strategic interaction and energy market applications. The uncertainty is modelled using stochastic processes as state variables. The specific questions analyzed include the effect of technological and revenue related uncertainties on the optimal timing of investment, the irreversibility in the choice between alternative investment projects with different degrees of uncertainty, and the effect of strategic interaction on the initiating of discrete investment projects, on the abandonment of a project, and on incremental capacity investments. The main methodological feature is the incorporation of game theoretic concepts in the theory of investment. It is argued that such an approach is often desirable in terms of real applications, because many industries are characterized by both uncertainty and strategic interaction between the firms. Besides extending the theory of investment, this line of work may be seen as an extension of the theory of industrial organization towards the direction that views market stability as one of the factors explaining rational behaviour of the firms.

Keywords: irreversible investment, uncertainty, real options, energy markets, game theory

Academic dissertation

Systems Analysis Laboratory
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On investment, uncertainty, and strategic interaction with applications in energy markets

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Publications

The dissertation consists of the present summary article and the following papers:

- [I] Murto, P., "Timing of investment under technological and revenue related uncertainties", Systems Analysis Laboratory Research Report E11, Helsinki University of Technology, 2003.
- [II] Murto, P., and Nese, G., "Input price risk and optimal timing of energy investment: choice between fossil- and biofuels", Working Paper No. 25/02, Institute for Research in Economics and Business Administration, Bergen, May 2002 (revised January 2003).
- [III] Murto, P., and Keppo, J., "A game model of irreversible investment under uncertainty", International Game Theory Review, Vol. 4, No. 2, 2002.
- [IV] Murto, P., Näsäkkälä, E., and Keppo, J., "Timing of investments in oligopoly under uncertainty: a framework for numerical analysis", European Journal of Operational Research (to appear).
- [V] Murto, P., "Exit in duopoly under uncertainty", RAND Journal of Economics (to appear).
- [VI] Pineau, P.-O., and Murto, P., "An oligopolistic investment model of the Finnish electricity market", Annals of Operations Research (to appear).

Contributions of the author

Papers [II], [III], and [IV] were initiated by the author, who also had the main responsibility in the writing. The development of the models and the analysis of the results were carried out in collaboration with the author and the co-authors. The work for paper [VI] was initiated and led by Dr. Pierre-Olivier Pineau. The development of the model, the analysis of the results, and the writing of the paper were carried out in collaboration. The implementation of the computations was carried out by the author. Papers [I] and [V] are independent research of the author.

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Pauli Murto

1 Introduction

There is an international movement towards the liberalization of energy markets. Particularly in the electricity sector, major structural changes have been taking place since the beginning of the 1990's, some of the pioneering countries being the United Kingdom, Chile, New Zealand, the Nordic countries, and some states of the USA. Also the European Union has adopted directives to open up markets for electricity and natural gas (directives 96/92/EC and 98/30/EC). While the reforms vary across countries, the common tendency is to move from regulated monopolies towards competition and market-based pricing. The main motivation is the effort to improve the economic efficiency.

This restructuring trend has brought out a lot of challenging research topics. In electricity markets, the focus has been on the short-term functioning of the markets.¹ Only recently, the long-term issues, particularly capital investments, have started to gain attention. As the markets are liberalized, also the investment decisions are increasingly made by firms whose objective is to maximize their market values. Some of the topics of interest are whether a sufficient reserve capacity is ensured, and how the competitiveness of different sources of energy supply relative to each other is affected by the changing industry structure.

Concurrent with the liberalization movement, the energy industry faces a major challenge, namely the increasing concern of the environmental impacts of energy production. The most alarming issue is the global warming caused by the emission of greenhouse gases, in particular carbon dioxide. To manage to reduce these emissions without considerably reducing the energy consumption requires a substantial change in the build-up of the existing production capacity. There is pressure on one hand for shutting down old fossil fueled capacity, and on the other hand for building new capacity based on renewable technologies.² Thus, the investment and shut-down decisions of the firms acting in the liberalized energy markets have also a major importance with respect to the achievement of globally acknowledged policy goals.

The literature on energy production investments has traditionally been based on the paradigms of optimization and planning (see, e.g., Chao, 1983, Sherali et al., 1984, He and David, 1995, for individual contributions, or Kahn, 1988, for a broader treatment). However, the relevance of this literature is depreciating as the markets are being liberalized. Instead of optimizing the capacity expansion from the perspective of the society, the new organization of the industry stresses the optimal design of the market rules (see Chao and Huntington, 1998, for topics on the design of electricity markets). In order to account for the long-term objectives in such design, one should have a proper understanding of the investment behavior of the value maximizing market participants. One purpose of this thesis is to shed light on some issues related to investment and the choice of production technologies under such circumstances.

Since capacity investments are irreversible decisions with long-term consequences, different uncertainties are critical determinants of the firms' investment behavior. This is the viewpoint emphasized in the modern economic theory regarding investment. This *real options* approach

¹ Among the main issues of concern have been the possible exercise of market power by dominant companies (e.g. Andersson and Bergman, 1995, Jing-Yuan and Smeers, 1999, and Borenstein et al., 1999), the functioning of the power exchanges (e.g. Bolle, 1992, Green and Newbery, 1992, and von der Fehr and Harbord, 1993), and the effect of the physical network on the trade and incentives (e.g. Chao and Peck, 1996, Hogan, 1997, 1998, Oren, 1997, and Stoff, 1999).

² Another motivation for this is the increasing concern of dependence on imported energy sources, see, e.g., European Commission (2001).

has its foundations in the field of financial economics and it stresses the analogy between financial derivative instruments and physical investment opportunities. In dynamically evolving energy markets, where the values of production assets are contingent on uncertain factors such as input and output prices, the optimal investment decisions and valuation of assets require that kind of methodology.

However, there are also many fundamental differences between physical investment opportunities in energy markets and financial derivative instruments. To have a proper understanding of the investment in realistic settings, the specific properties of the industry must be incorporated into the theory. A characteristic feature of energy production is that there are many alternative technologies available with different properties in terms of the cost structure and associated uncertainties. Some projects are site-specific, e.g., hydro power and transmission capacity investments, in which cases there are rents associated with the investment opportunities. On the other hand, for thermal plants, for instance, the particular site is not the most determining factor, and thus the rents are subject to competition in the output market. Some technologies allow capacity investment in small units (e.g. natural gas and biomass fired plants and wind turbines), while others must be added in large lumps where even a single investment may have an influence on all market participants through the effect in market price (e.g. nuclear plants). The shut-down of production units should also be seen as an investment, where the initial expenditure of the shut-down costs are considered against the payoff of future avoided losses. This is relevant in connection with closing fossil fueled production in response to environmental policy measures.

Perhaps the most distinctive feature of energy investments in comparison to financial investments is the competitive interaction. All energy investments are at least to some degree interlinked through the output price determined by industry demand and supply. In cases where there are only a relatively small number of actors, strategic interactions must be accounted for. This is also important with network investments, such as power and natural gas transmission. For the modeling of such interactions, game theoretic concepts must be incorporated into the theory of investment.

This thesis has two main themes. First, it contributes to the development of the theory of investment under uncertainty. In particular, a major part is devoted to the incorporation of strategic interactions with this theory. Second, it presents applications that focus on specific settings in energy markets with the emphasis on uncertainties and the timing of investment.

The present introductory article is organized as follows. Section 2 reviews the modern theory of investment under uncertainty and places the articles of this thesis within the literature. Section 3 reviews the thesis and states the contribution of each individual article. Section 4 concludes.

2 Modern theory of investment under uncertainty

2.1 Foundations

The elementary corporate finance theory says that an investment project should be undertaken whenever its net present value is positive. The neoclassical theory of investment in the economics literature is also based on this rule. However, for the rule to be correct, some implicit assumptions must hold: the investment project must be either completely reversible or a "now or never" opportunity. Alternatively, its value must be completely independent of any observable variables that evolve in time. Nevertheless, in most realistic cases investment opportunities are at least to some extent irreversible, and they can be delayed in order to receive more information on the underlying uncertain factors that affect their values. Under

such conditions, the correct decision involves comparing the value of investing today with the value of investing at all possible times in the future. Since the "old" theory ignores this comparison of mutually exclusive alternatives, its relevance has been questioned.

The "new" theory of investment has been developed on the idea of an analogy between investment opportunities and financial derivative instruments. Typically, projects are contingent on input and output prices that fluctuate in time. Therefore, rather than being fixed, the project values evolve dynamically in time. An opportunity to undertake an investment is like a call option. Whereas the value of a financial call option depends on the stochastic security prices, the value of a real investment opportunity depends on such uncertain factors as input and output prices. The main insight is that due to the option value inherent in an investment opportunity, it is not optimal to invest until the present value of the expected cash flows exceeds the cost of investment by a strictly positive amount to compensate for the loss of the option.

Due to the analogy to pricing derivative instruments, this new theory of irreversible investment is also referred to as the *real options* approach. Its foundations are in the field of financial economics. Black and Scholes (1973) and Merton (1973) developed the theory for the valuation of financial derivative instruments whose payoffs are contingent on the values of some underlying assets. This led to the equivalent risk neutral valuation formalized in, e.g., Cox and Ross (1976), Constantinides (1978), and Harrison and Kreps (1979). According to this, any contingent claim on an asset can be valued consistently by replacing its actual growth rate with a certainty equivalent growth rate, and then taking the expectation of the cash flows as if the world were risk neutral. In technical terms, this corresponds to presenting the stochastic processes under the *equivalent martingale measure*.

The first papers applying these insights in physical investment were concerned with individual discrete projects. McDonald and Siegel (1985) consider the valuation of an investment project contingent on stochastic output price and with the possibility to shut down production whenever it is not profitable. Brennan and Schwartz (1985) consider the opening, closing, and abandonment decisions involved with a natural resource mine, where the price of the resource is stochastic. McDonald and Siegel (1986) consider the optimal timing of investment in a project with uncertainty both in the present value and the cost of investment. In these, and in most of the subsequent literature, uncertainty is modeled by state variables that follow the geometric Brownian motion. The optimal investment rule is expressed as a "threshold" or "boundary" in the state space such that the investment occurs when the state variable crosses the threshold for the first time. More generally, in such "Markovian" models the solution takes the form of a "stopping region", i.e., a region in the state space where it is optimal to invest.

The incremental capacity investment from the options perspective was first properly treated by Pindyck (1988). The paper shows that the firm's capacity choice is optimal when the present value of the expected cash flow from a marginal unit of capacity just equals the total cost of that unit. Importantly, the cost must include the cost of exercising the option to buy that unit. The paper illustrates that the firm's market value has two components: the value of installed capacity and the value of the firm's options to add more capacity later. Bertola and Caballero (1994) include firm specific uncertainty and consider the effect of irreversibility on the aggregate investment dynamics of a large number of individual firms. Dixit (1995) considers the effect of scale economies on incremental investment, and analyzes the conditions under which it is optimal to add capacity in discrete lumps rather than incremental units. A related literature considers adjustment costs, where it is assumed that the cost of changing the capital stock depends on the rate of investment. Abel and Eberly (1994) present a unifying model with several types of adjustment costs, which also capture the effect of irreversibility characteristic

to the real options approach. Abel et al. (1996) link the adjustment cost approach and options approach in a simple two period model to clarify their relationship.

A comprehensive summary of the new investment theory is given in the book by Dixit and Pindyck (1994). They emphasize throughout the book that there are two alternative techniques to derive the optimal investment rule and the value of the investment opportunity. The method of contingent claims analysis is more directly based on the financial economics theory, and utilizes replicating portfolios and the absence of arbitrage principle. The dynamic programming methodology, on the other hand, takes a given discount rate and is formulated as the maximization problem of the expected value of discounted cash flows. The methods are linked through the equivalent risk neutral valuation principle, which says that the dynamic programming approach gives the arbitrage-free price for an investment option when the stochastic processes are given under the martingale measure and the risk-free rate of return is used as the discount factor.

From the perspective of the dynamic programming approach, the timing of irreversible investment is an *optimal stopping problem*. There is an extensive literature dealing with the problem from the mathematical point of view (see, e.g., Karatzas and Shreve, 1998, or Øksendal, 2000). Various techniques have been developed to solve different versions of the problem. For example, Insley (2002) applies the linear complementarity formulation, while Alvarez (1998) applies a Greenian approach based on the theory of linear diffusions and potentials.

As emphasized in the new corporate finance literature, the real options methodology is actually applicable more generally in valuing managerial flexibility associated with real assets. Examples include Kulatilaka (1988), who considers the valuing of flexible manufacturing systems, Triantis and Hodder (1990), who develop a methodology for valuing a production system that can be switched to produce different combinations of a number of products, and Kulatilaka (1993), who values the flexibility to switch between two fuels associated with an industrial steam boiler.

2.2 Refinements

Real investment opportunities have often physical characteristics that distinguish them from financial derivative instruments. In this section some of the literature that deals with such refinements is reviewed.

Dixit (1989a) extends the problem of timing a discrete investment problem by noting that a project can often be abandoned at some cost if it proves to be unprofitable. An example is entry and exit of a firm to a new market. There are two interlinked option pricing problems that must be solved simultaneously: the firm that is out of the market has an option to enter (or to invest in a project), and the firm that is in the business has an option to exit (or to abandon the project).

Often investment opportunities offer the possibility to choose not only the timing, but also the scale of the project. This is particularly relevant with site-specific projects, such as land development or energy production and transmission. Dixit (1993) allows the choice between a number of mutually exclusive projects. Capozza and Li (1994) allow the intensity of investment to be a free variable, and study the optimal timing and intensity of land development. The papers by Bar-Ilan and Strange (1999) and Dangl (1999) further elaborate the theory. One insight common to all these papers is that the greater the uncertainty, the greater the value of waiting and the eventual intensity of investment.

Physical investment projects are often characterized by considerable lags between initiating the project and getting the production on line. Majd and Pindyck (1987) assume a maximum rate of construction and allow stopping and restarting of the project until the plant is finished. Pindyck (1993a) assumes technical uncertainty over the cost of construction that is resolved as the investment proceeds. Bar-Ilan and Strange (1996) have a fixed time of construction and show that the lag has no real effect on the investment decision as long as there is no abandonment option, but combined with such an option the conventional effect of uncertainty on investment may be weakened or even reversed. Alvarez and Keppo (2002) assume that the investment lag is positively related to the underlying revenue process and show that this may lead to multiple disjoint price regions where waiting is preferable to investing.

Many investment projects consist of several stages. Typical examples include research and development, market analysis, exploration, applying regulatory approval, and the actual construction of the plant. Completing one stage gives an option to proceed to another stage. Then, investment opportunities are compound options. Bar-Ilan and Strange (1998) have two stages, both of which take a fixed time to complete. They show that if it is possible to suspend the project, it may be optimal to carry out "exploratory" investment. Weeds (1999) considers a two-stage model, where in the first stage the firm conducts research and development in order to get a patent, and the second stage is the actual market entry. She shows that it may be optimal to suspend the project after the first stage, which results in a sleeping patent. Technically the model is very similar to Bar-Ilan and Strange (1998) the main distinctive feature being the technological uncertainty over the time to complete the first period.

While most of the real options literature considers uncertainty in input and output prices, other sources of uncertainty are of course possible. Ingersoll and Ross (1992) consider uncertainty in interest rates. Dixit (1989b) considers exchange rate uncertainty. Grenadier and Weiss (1997), Farzin et al. (1998), and article [I] in this thesis consider uncertainty in the technological progress. Depending on the source of uncertainty, other processes than the geometric Brownian motion may be more appropriate models. See for example Cox and Ross (1976) for the valuation of options under jump processes, or Metcalf and Hassett (1995) for investment under mean reverting processes.

2.3 Rational expectations equilibrium

The literature reviewed above considers the optimal investment behavior of a firm in isolation from competitors. In such a case, there is a rent associated with the option that must result from some firm-specific advantages such as patents, ownership of a specific site or natural resources, regulatory rules, reputation, etc. However, one of the main characteristics with real assets in comparison to financial assets is that there is typically competitive interaction in the product market. In the limiting case of completely free entry, the competition should eliminate any rents associated with investment opportunities.

The theory has been extended to the product market equilibrium with industry-wide uncertainty, where the industry is assumed to be composed of a large number of agents who produce output by means of capital. Any firm can enter by investing in capital at a given cost per unit. The price uncertainty is replaced by demand uncertainty, which combined with the firms' actions results in an endogenous stochastic price process. Lucas and Prescott (1971) formalize the rational expectations equilibrium in such a setting, and establish its social optimality. In equilibrium with rational expectations, the resulting probability distribution of the prices must be such that no firm that anticipates this distribution correctly has an incentive to change its investment behavior.

In the framework of irreversible investment, Pindyck (1993b) illustrates the equilibrium in a simple two period setting. Leahy (1993) works in continuous time, and discovers that the equilibrium timing of the next marginal investment is the same as the optimal timing of investment of a *myopic* firm that ignores the presence of competitors. The intuition is that while the competition reduces the value of the investment option, it also reduces the value of the installed capacity, and together these effects leave the optimal timing to exercise the option unaffected. The result facilitates the computation of the competitive equilibrium, since the resulting endogenous price process can be derived by means of the solution of the easier problem of the myopic investor. From the perspective of this thesis, it sets the basis for a possible extension of the articles [I] and [II] to the industry equilibrium level.

Leahy's result links the theory of irreversible investment of an isolated firm to the competitive equilibrium, and therefore complements the link between equilibrium and social optimum established by Lucas and Prescott (1971). The two links together imply also a third link, namely the correspondence of the myopic solution and the social optimum. This illustrates that the competition in essence transfers the option value of waiting from the firm, which maximizes its individual contribution to consumer surplus, to the "social planner", which maximizes aggregate consumer surplus.

Baldursson and Karatzas (1997) generalize and further elaborate the links between social optimum, equilibrium, and optimal entry of a myopic investor utilizing singular stochastic control theory and its connections with the optimal stopping problem. Grenadier (1999) extends the analysis by including investment lags. Tvedt (1999) presents an application in a competitive shipping market.

2.4 Strategic interaction

Many real markets are characterized by a relatively small number of competing firms. In such oligopolistic settings strategic interaction must be explicitly accounted for, which requires a game theoretic approach. The combination of game theoretic analysis with the real options methodology is currently an active area of theoretical work. A recent book edited by Grenadier (2000a) collects some relevant papers. The contributions of this thesis also fall to a large extent on this area.

Since the real options approach to investment under uncertainty is mainly concerned with the *timing* of investment, it is natural that some important background theory can be found in the game theoretic models of timing, such as Reinganum (1981), Fudenberg et al. (1983), Gilbert and Harris (1984), Fudenberg and Tirole (1985, 1986), Ghemawat and Nalebuff (1985). Even more directly related is the literature on stopping games, which are stochastic games where the players' strategies are stopping times (e.g. Huang and Li, 1990, Dutta and Rustichini, 1993). A related formulation of strategies and the Markov perfect equilibrium can also be found in Dutta and Rustichini (1995), who consider a class of continuous time stochastic games in which players make discrete changes in variables that affect their payoffs.

However, some of the basic insights have been elaborated in simple two-period settings. Smit and Ankum (1993) model demand evolution with a binomial tree and illustrate different outcomes and associated rents in sub-game perfect equilibrium of the resulting extensive form entry game. Different outcomes are distinguished on the basis of firm asymmetries and values of investment in relation to market uncertainty. Kulatilaka and Perotti (1998) model a duopoly, where in the first period one of the firms can undertake an irreversible investment that reduces its production cost, which is interpreted as a purchase of a growth option. The uncertainty in market demand is resolved in the second period and the firms compete in quantities. It is

shown that due to the strategic interaction in the product market, the effect of uncertainty on the attractiveness of investment is ambiguous.

Continuous time becomes relevant when one wants to focus explicitly on the timing of the investment. Typically, a firm's income flow is modeled as a function of an exogenous state variable and the actions of the other firms. Uncertainty is incorporated by letting the state variable follow a geometric Brownian motion. In most of the cases considered, strategic interaction takes the form of preemptive behavior, where an investment of one firm discourages the investment of other firms creating an incentive to act first. Such interaction is typical in entry to an emerging market or in the adoption of a new technology. Without competition, the investment opportunities would have a positive option value, but preemptive competition reduces such rents. How much the rents are dissipated depends on various factors, for example the degree of uncertainty, whether the investment opportunity is exclusive (i.e. only one firm can invest), whether information is complete or incomplete, and whether firms are symmetric or non-symmetric.

There is a technical matter concerning the possibility of erroneous simultaneous investment. The problem arises in moving to continuous time by taking the limit of the discrete time mixed strategy equilibrium, as shown in a deterministic setting by Fudenberg and Tirole (1985). They propose a certain kind of an extended strategy space to deal with the problem. In the recent real options literature there are different interpretations of an appropriate way to treat the issue. Most papers rule out simultaneous entry by identifying two symmetric equilibria where the firms are indifferent on the role assignment, or by imposing some random mechanism to choose one of the firms in case they try to invest at the same time (e.g. Dixit and Pindyck, 1994, Grenadier, 1996, Lambrecht, 1999, Boyer et al., 2001, Weeds, 2002, Lambrecht and Perraudin, 2003). Exceptions are Grenadier (2000b) and Huisman and Kort (2000a, 2000b), who allow the mixed strategies equilibrium within each infinitesimal time increment, which results in an equilibrium where simultaneous entry occurs at some positive probability.

Some of the models are concerned with an exclusive project such that when one of the firms invests, the opportunity is completely lost for the other firms. Lambrecht and Perraudin (2003) model a duopoly where the firms have incomplete information on each other's investment cost, and find that the Bayesian equilibrium investment timing is between the full rent dissipation and monopoly cases. Article [III] of this thesis considers an oligopoly with firm specific uncertainty with different assumptions on the information that firms have about each other. With complete information and symmetry the rent is fully dissipated, but generally the equilibrium investment strategy of each firm is between the monopoly and rent dissipation cases. Lambrecht (1999) considers a sequential two-stage investment. The first stage is the investment in a patent giving the exclusive right to proceed to the second stage. The model assumes that the firms have incomplete information on each other's profit parameters, and studies the conditions under which a sleeping patent, that is, investment in the patent without proceeding directly to the second stage, occurs in the Bayesian equilibrium.

Models of non-exclusive projects lead usually to sequential investment, or leader-follower models. The literature is restricted to duopoly settings, although the extension to oligopolies would in some cases be straight-forward. In a proper analysis the roles of the leader and follower must be determined endogenously. Dixit and Pindyck (1994, chapter 9) present a simple model, where there are two potential entrants in a market with stochastically evolving demand. In equilibrium one of the firms enters at the moment when both firms are indifferent about being the leader or follower. Joaquin and Butler (1999) generalize the model to asymmetric costs, which leads to the low-cost firm entering first. Baba (2000) presents an application of a similar model to the Japanese loan market.

Some models have an equilibrium with simultaneous investment in addition to sequential investment. In such a case, there is in fact a continuum of such equilibria, but one specific equilibrium with late joint investment Pareto-dominates the others. The existence of such equilibria requires some specific properties of the model, typically that the firms are initially active in the market so that an investment of the leader has a direct negative effect on the follower. This is the case with the adoption of a new technology, which has been studied in the industrial organization literature by, e.g., Reinganum (1981) and Fudenberg and Tirole (1985). Huisman and Kort (2000a) extend the model of Fudenberg and Tirole by introducing uncertainty and identify the two possible equilibrium types, namely, the preemption with sequential investment and simultaneous (joint) investment. They show that the type of equilibrium may depend on the degree of uncertainty. Huisman and Kort (2000b) add technological progress by assuming that a superior technology arrives in an exponentially distributed random time. The new feature in the solution is a third type of equilibrium that appears with some parameter values. In that equilibrium, the firms invest sequentially, but the follower is better off than the leader. In such a situation both firms would like the other firm to invest first, and thus there are actually two asymmetric equilibria, one for each firm being the leader.

Weeds (2002) presents a similar model, but instead of the adoption of a technology, she considers investment in a R&D project, where the time to innovation is random. The first firm to succeed in the innovation gets an exclusive right to the project. Two equilibria, a preemptive sequential R&D investment and a late simultaneous R&D investment are possible depending on the parameter values. The interesting result is that in the latter case, the non-cooperative investment is more delayed than the optimal cooperative sequential investment pattern, because firms hold back from investing in the fear of starting a patent race. A noteworthy point is that in contrast to models of adoption of a new technology, the possibility of simultaneous investment equilibrium is not due to initially active firms, but the fact that the proper preemption is impossible because of the random time to succeed in the innovation.

Grenadier (1996) considers real-estate development in duopoly, where both firms have a single option to redevelop their properties into superior buildings. Specific features of the model include a fixed construction lag and the fact that the development by one firm reduces the value of the undeveloped asset of the competitor. If the initial shock value is low, the Markov perfect equilibrium is characterized by preemption and sequential development. However, if the shock value is initially high, there is an infinite number of equilibria with simultaneous development, and the Pareto-dominating equilibrium has an interesting property that the development may be triggered by a decrease in the shock value.

Boyer et al. (2001) consider lumpy capacity expansion in duopoly. Consistently with the models of technology adoption, also in their model a simultaneous investment may occur only when both firms are initially in the market. However, from the viewpoint of modeling the industry dynamics, the model is restricted in the sense that even if different initial capacities are considered, both firms can expand capacity only once. Article [IV] of this thesis considers the full industry dynamics by allowing each firm to have multiple investment opportunities in a stochastically growing market. However, numerical analysis is necessary to study the dynamics of such a complex setting.

Some papers consider positive externalities such as network benefits, where an investment of a firm has a positive effect on the other firm. Mason and Weeds (2001) consider preemption and externalities, and assume that the joint investment gives a higher payoff flow than acting as a follower. They distinguish different inefficiencies and study the effect of uncertainty on them: due to preemption the leader invests too early, due to the positive (negative) externality on the

leader the follower invests too late (too early), and due to the positive externality on each other with joint investment the firms invest too often sequentially. Moretto (1996) introduces a positive network externality in the adoption of technology assuming that adoption is cheaper for the follower than for the leader. The firms have also incomplete information on the investment costs of each other. Simultaneous and sequential adoption equilibria may occur depending on the parameters. Thijssen et al. (2001) consider information spillovers, and show that either a preemption game or a war of attrition may arise depending on the parameterization.

All of the above mentioned references consider entry, construction of capacity, or adoption of a new technology. The strategic interaction in the opposite case, namely abandonment of an asset, is considered in the article [V] of this thesis. Without strategic interaction, the decision of exit is conceptually similar to entry, but strategic interaction induces a profound difference: while interaction in entry takes the form of preemption, in the case of exit it resembles war of attrition. In the industrial organization literature such settings are considered in Ghemawat and Nalebuff (1985, 1990), and Fudenberg and Tirole (1986). Article [V] shows that the unique equilibrium characteristic to deterministic models may break down if the degree of uncertainty is high enough. More specifically, in addition to the "normal" equilibrium there may be equilibria with strategies that are disconnected "stopping-sets" in the state space. Another recent paper that considers strategic interaction in exit from the real options perspective is Lambrecht (2001). He focuses on the impact of debt financing on entry and exit, but forces the strategies to be connected sets, which ensures a unique equilibrium.

The literature reviewed above is concerned with discrete projects. Also incremental investment has been studied in oligopoly. Williams (1993) develops a model of real estate markets, where there is a fixed number of symmetric investors. In that model, the Markov perfect equilibrium is characterized by a threshold level in income from developed assets above which all investors develop assets at maximum rate, and below which all investors wait. A more general analysis is given in Baldursson (1998), where there is a fixed number of firms who can install and shut down capacity at given costs. He uses a technique of a fictitious social planner, where the problem of finding the Nash equilibrium is converted to the maximization of an artificial objective function. However, the technique requires a linear inverse demand function. It is shown that qualitatively the price process will be similar in oligopoly and competitive equilibrium, although there are positive rents in oligopoly. It is demonstrated numerically that initial non-identical capacities converge to each other, but it may take considerable time.

Also article [VI] of this thesis considers incremental investment in oligopoly under uncertainty. However, the methodology is somewhat different from the above mentioned references. The model is in discrete time and has its background in dynamic game theory (see Basar and Olsder, 1995, for a comprehensive treatment and Haurie et al., 1990, for the information structure adopted in [VI]) The focus is rather on computational analysis than on generality. Strategic interaction appears on two levels: on output strategies with given fixed capacities, and on irreversible investments in new capacity. The model is solved numerically utilizing variational inequality and mixed complementarity problem formulations.

2.5 Applications in energy and resource economics

Natural resource investments were among the first applications of the real options methodology. Brennan and Schwarz (1985) consider opening, closing, and abandonment decisions associated with a mine producing a natural resource whose price fluctuates over time. Paddock et al. (1988) develop a methodology for the valuation of leases for offshore petroleum. Recent applications include Lumley and Zervos (2001), who consider natural resource investments in general, and Insley (2002) who considers tree harvesting under mean-reverting price processes. Conrad (2000) considers the management of a wilderness area,

which can be developed, for example for hydropower production. The model has many linked options: to preserve, to extract the resources (timber or coal for example) and possibly develop later for hydropower, or to develop directly without extracting the resources.

Optimal policies from the society's perspective are considered in several papers. An adoption of an environmental policy, for example, can be seen as an irreversible investment with uncertain costs and benefits. The optimal timing to adopt such a policy has been considered by Conrad (1997), Saphores and Carr (1999), and Pindyck (2000). The real options methodology has also been proposed for assessing other policies. For example, Lund (1992) analyses the effects of petroleum tax changes by looking at the behavior of well-diversified value maximizing firms.

Applications in energy investments include on one hand papers that focus on general insights, and on the other hand papers that develop methodology for practical use. At the more general side, Pindyck (1993a) considers a nuclear power plant investment that takes time to complete. He concentrates on uncertainties associated with the cost of construction sorting out two types of cost uncertainty. The first is technical uncertainty over the difficulty of completing the project, which is only resolved as the construction proceeds, and the second is the uncertainty over the prices of construction inputs, which is external and unfolds continuously. He shows that the two uncertainties affect investment differently: technical uncertainty makes investment more attractive, while input cost uncertainty has the opposite effect. Article [I] in this thesis considers another kind of technical uncertainty, namely exogenous technological progress, which takes the form of randomly arriving innovations. Motivated by wind power, the model assumes that technological innovations reduce the cost of investment. It is shown that the effect of technological uncertainty depends on another uncertainty, namely that over the future revenues that the plant will generate.

The choice between alternative production technologies has been studied in several papers. Kobilá (1990) considers the choice between hydro and thermal power generation under the assumption that hydro production has only completely irreversible capital costs, while natural gas based production has only variable fuel costs. He studies the socially optimal timing to switch from natural gas to hydro in supplying a given demand unit at minimum cost. Martzoukos and Teplitz-Smbitzky (1992) consider a related problem of the optimal timing to switch from decentralized diesel power generation to grid-based power supply by accomplishing an irreversible transmission line investment. Article [II] of this thesis considers the timing of energy investment by a value maximizing investor, when the choice has to be made between fossil fuel and biomass fired production technologies. The focus is on the fossil fuel price uncertainty and the choice between risky and non-risky projects. Brekke and Schieldrop (1999) consider the timing of a power plant investment, when the investor must choose either a pure single-fuel plant or a flexible dual-fuel plant. Kulatilaka (1993) is also concerned with the valuing of the flexibility to switch between two fuels, but does not consider the optimal timing to invest in such a flexible plant.

Papers that study incremental investment under stochastic demand include Benavides (1995) and Kobilá (1993). The former models the electricity production capacity investment and considers the socially optimal expansion rules in the cases of constant and increasing returns to scale. The latter applies stochastic control theory on investment in hydro power capacity expansion under the special assumption that the investments in hydro power are optimized by a single profit maximizing agent, while there is a competitive supply of thermal power capacity.

Papers that aim directly at developing methodology for the practical use include Salahor (1998), Bradley (1998), Laughton (1998), and Baker et al. (1998), who set general guidelines for applying the real options methodology in real cases, and Venetsanos et al. (2002), who

propose a methodology for wind power project appraisal under uncertainty. Smith and McCardle (1999) describe experiences of using the real options methodology to evaluate some real oil and gas investments.

3 Thesis

This thesis consists of six individual articles. In this section, the contributions of each article are described. To clarify the picture, table 1 classifies them according to the main themes of the thesis, that is, *investment*, *uncertainty*, and *strategic interaction*.

Table 1: Classification of articles

ARTICLE	TYPE OF INVESTMENT	SOURCE OF UNCERTAINTY	STRATEGIC INTERACTION
[I]	Single plant	Technological progress and output price	No explicit strategic interaction
[II]	Single plant	Input price	No explicit strategic interaction
[III]	Single project or plant	Value of project or plant in general	Oligopoly, preemption
[IV]	A number of discrete projects or plants	Demand	Oligopoly, preemption
[V]	Shut-down of a single firm or plant	Value of firm or plant in general	Duopoly, war of attrition
[VI]	Incremental capacity investment	Demand	Oligopoly, strategic substitutes

[I] Timing of investment under technological and revenue related uncertainties

Two different uncertainties relate to the investments in developing energy technologies, such as wind power production. First, the revenue stream that the investment would generate evolves unpredictably in time. Second, there is uncertainty over the future technological progress, which determines the cost of investment. There is a conceptual difference between the two uncertainties. Namely, technological innovations that arrive randomly only increase the value of an investment, while revenue is subject to both positive and negative random fluctuations.

Article [I] considers the effects of these two types of uncertainties on the timing of investment. First, their distinction is characterized in the optimal stopping framework. The starting point is an investment opportunity that is contingent on a number of stochastic factors. The uncertainty in technological progress is characterized by the fact that the associated stochastic process is non-decreasing. It is shown that if the value of the investment opportunity is affected only by non-decreasing processes, then the optimal investment rule gets an intuitive form, where the investment is carried out at a moment when the opportunity cost of delaying the project equals the expected change in its net present value.

Then, a more specific model motivated by wind power investments with both types of uncertainties is presented. The technological progress is modeled as a Poisson arrival process, where innovations that reduce the cost of investment arrive at random times. The revenue stream that the investment would generate follows a geometric Brownian motion. The investor observes these two processes, and considers when the investment cost is low enough and revenue stream is high enough to carry out the investment. It is shown that the optimal investment rule is to invest when the ratio of the output price to the investment cost exceeds a certain threshold level.

The uncertainty in the technological progress gives rise to some interesting findings. In the absence of revenue uncertainty, the technological uncertainty as such does not matter. The investor can act as if the actual stochastic process for the investment cost were replaced by its expected path. However, when the revenue uncertainty is added in the model, the technological uncertainty starts to matter as well. Namely, keeping the expected path of the investment cost fixed, the higher the uncertainty in the process, the more reluctant the investor is to invest. It is perhaps against common intuition that the effect of technological uncertainty depends crucially on whether the revenue stream is deterministic or stochastic.

The extension of the article to the previous literature is the inclusion of the technological uncertainty in an otherwise standard real options model. Technological uncertainty has been considered in several papers (e.g. Grenadier and Weiss, 1997, Farzin et al. 1998), but the new feature is the interaction of the two types of uncertainties. Even if motivated by wind power investments, the results are applicable more generally. Possible applications include the development and market introduction of a new product and adoption of a new technology in different contexts.

[II] Input price risk and optimal timing of energy investment: choice between fossil- and biofuels

A characteristic feature of heat and power production is that there are many alternative technologies available. Different technologies have different properties in terms of the cost structure and associated uncertainties. Concerning production costs, there are significant differences in the uncertainties associated with prices of alternative fuel types. Natural gas, for example, being typically an export product with few sources of supply, is subject to considerable price fluctuations, while biomass is typically domestically produced, and its price is based on such factors as the costs of growing, harvesting, and transporting. Nevertheless, in the literature the comparisons of the total production costs using different technologies are typically carried out using fixed estimates for the fuel prices.

Article [II] takes a dynamic perspective on the competitiveness of alternative production technologies. It is pointed out that the irreversibility in the choice of technology combined with uncertainties in the input prices have important implications on the optimal investment behavior. This is done by looking at a representative investor who is considering an energy production investment, and faces the choice between fossil fuel and biomass fired plants. To make the point as clear as possible, the fossil fuel price is assumed stochastic, while the biomass price is constant. The investment creates a given payoff stream, either as an explicit revenue from selling the output, or as a flow of avoided cost of purchasing the demanded energy service from elsewhere. It is assumed that the investment can be delayed without constraints, thus both the timing and the type of the plant must be chosen.

The optimal investment behavior is driven by the stochastic fossil fuel price development. The solution consists of two threshold levels that trigger an investment: if the fossil fuel price falls to the lower threshold it becomes optimal to invest in the fossil fueled plant while at the higher

threshold it becomes optimal to invest in the biomass fired plant. Between the thresholds it is optimal to wait.

The results indicate that as the choice of technology is irreversible, it is optimal to postpone the investment at a wide range of prices even if it would otherwise be optimal to invest in one or both of the plant types. This means that the fuel price uncertainty associated with one plant type may postpone investments in other plant types as well if the technologies are alternatives to each other and the choice of technology is irreversible. The result is due to the flexibility inherent in the real option that allows the choice between the two technologies. This increases the value of the option, and because the investor has to give this option up at the moment of investment, it induces additional reluctance to invest. Even in the case where biomass plant is chosen, the investment is triggered by the fossil fuel price development, because the two alternatives are interrelated through the real option.

A numerical example is given based on rough cost estimates of two types of power plants applicable in Finland. It is demonstrated that increased input price volatility widens the waiting range, while increased value of the low-risk biomass project reduces it.

Even if the article is restricted to a specific setting, the general idea can be seen from a broader perspective by noting that a similar irreversibility associated with the choice between different actions may appear in other context. An example could be the optimal choice and timing of an environmental policy. If there are uncertainties specific to different policies, then this may induce a value of waiting for further information delaying the optimal timing to implement the policy.

[III] A game model of irreversible investment under uncertainty

Some investment projects are exclusive in the sense that even if the opportunity is available to many firms, only one can take the project. Once one of the firms invests, the opportunity is lost for the other firms. An example of such a situation could be the opportunity to build a natural gas or electricity network to connect two price regions in international energy markets. There is a rent associated with such an opportunity, which would be fully realized by waiting until the present value of the project exceeds the cost of investment by the value of the investment option. However, since only one investor gets the project, the rent is subject to competitive interaction. In the case of preemptive competition, it is even possible that the rent is completely dissipated.

Article [III] considers a setting where there is a single investment opportunity and a number of non-cooperative firms who can invest in it. It is assumed that the present value of the project to each individual firm follows a geometric Brownian motion. The uncertainty can be firm specific or common to all firms. This is modeled by allowing the value to be a separate process for each firm, and letting the processes be correlated to an arbitrary degree. If the processes are non-correlated, uncertainty is completely firm specific, and if they are perfectly correlated, uncertainty is industry-wide. Different assumptions on the symmetry properties and information that the firms have about each other's project values are considered.

When the firms have constantly perfect information on the project values of each other, the equilibrium is simple. In the special case where the firms are symmetric and the uncertainty is industry-wide so that each firm has the same present value for the project, the rent is fully dissipated by the preemption. As soon as the present value of the project reaches the cost of investment, one of the firms invests. All firms are indifferent between investing and not investing, but if all firms would wait until the present value would rise above that level there would be a positive rent available and it would be profitable for any of the firms to preempt the

others. If the firms are asymmetric, for example so that one firm has a lower investment cost than the others, the "best firm" gets a positive rent, because it can wait until preemption becomes attractive for the "second best" firm. Finally, if the uncertainty is at least to some extent firm specific, then there is a positive rent for each firm. There is some probability that the project value exceeds the cost of investment for any firm before the others. In such a case, the lucky firm can wait until the same happens for some other firm without fearing preemption.

If the firms do not know each other's valuation for the project, then an additional assumption is adopted to describe their expectations of their rivals' actions. The firms are assumed to use a hazard rate to model the likelihood that their competitors invest within the next short time interval. The firms understand that the probability that a competitor invests depends on its strategy, and thus the hazard rates are functions of the competitor's strategies. Under such an assumption, the equilibrium is characterized by a private threshold level for each firm such that an investment is undertaken by the firm whose private project value first reaches its own threshold level. Even if the exact equilibrium investment threshold levels depend on the somewhat arbitrary hazard rate functions, the equilibrium gives a reasonable idea of rational behavior under such conditions of incomplete information. The investment threshold for each firm is between the case where the present value equals the cost of investment and the case of optimal monopoly investment. Thus, the rent is only partly dissipated.

[IV] Timing of investments in oligopoly under uncertainty: a framework for numerical analysis

The existing literature either considers a single investment project, or incremental divisible investment. However, in many cases investments are individual, discrete projects, but with the important property that successive projects affect the values of each other. As the industry develops, there will be many such projects taken by firms that must account for the strategic interaction. An example could be a growing market for energy or telecommunications, where the consecutive network investments affect the values of past and future investments.

The full dynamics of such a setting is difficult to study analytically. Article [IV] proposes a general framework for studying it numerically. The market consists of a fixed number of firms who produce a homogenous output the demand for which evolves according to a discrete time stochastic process. The firms carry out discrete investment projects in order to adjust their production cost functions or capacities in response to the growing demand.

The modeling approach consists of two stages. In the first stage, the Markov perfect Nash equilibrium is solved numerically by backward induction. To ensure a unique equilibrium, it is assumed that within each period, only one of the firms may act. For this purpose, an extended state space is introduced that enables investment turns to be determined randomly. In the second stage, the market is simulated by picking samples of the stochastic demand process and using the equilibrium strategies to determine the firms' investments. This two-stage approach can be seen as an extension of the standard Monte Carlo simulation methods to oligopolistic settings. It allows one to make conclusions on how the primary uncertainty in the exogenous demand process is transmitted through the strategic interaction to the probability distribution of the resulting cash-flows.

An example of the model is given in a market, where the demand grows slowly in the beginning, then goes through a period of strong growth, and eventually stabilizes. This kind of dynamics is typical for emerging markets. The three stages of growth are modeled by using different parameters in the binomial stochastic process used to describe the demand growth. An asymmetric case, where one of the firms must increase capacity in larger lumps than the other firm, illustrates the trade-off between the value of flexibility and scale economies under

competition. The firm with the possibility to make small investments is more flexible, but the firm that makes larger investments is more cost-efficient. The simulation results reveal an interesting division of the resulting total payoffs into different groups according to the investment patterns that occur along different realizations of the stochastic process. This kind of patterns resulting from strategic interaction may have important effects on the statistical properties of the firms' payoff distributions.

[V] Exit in duopoly under uncertainty

A decision to shut down a plant or other operations is actually an irreversible investment decision. The initial expenditure of the "exit investment" consists typically of severance payments to labor and other similar costs, and the expected rewards are the future losses avoided by the exit. Therefore, in an industry characterized by uncertainty, it is natural to model exit using the same methodology as irreversible investment (e.g. Dixit, 1989a, Alvarez, 1998, 1999).

However, strategic interaction induces a profound difference between the decisions of entry and exit. When the firms are strategic substitutes, i.e., the profitability of a firm is negatively affected by the presence of a competitor, the interaction in entry takes the form of preemption, while in exit it resembles war of attrition. The reason is that in a game of entry firms would like their competitors to enter as *late* as possible, whereas in a game of exit firms would like their competitors to exit as *early* as possible.

Article [V] studies a market with two active firms, where the general profitability in the industry is modeled by a state variable that follows a geometric Brownian motion. Since being active in the market induces a cost flow, it becomes optimal for the firms to exit if the state variable decreases low enough. Once one of the firms exits, the remaining firm enjoys monopoly profits until it exits as well. The extension of the model in relation to earlier literature can be seen from two perspectives. On one hand, it extends the real options literature to consider strategic interaction associated with abandonment options in oligopoly, and on the other hand, it extends the industrial organization literature on exit (in particular Ghemawat and Nalebuff, 1985) to include uncertainty.

Without strategic interaction, the optimal strategy would be simply a threshold level such that it is optimal to exit whenever the state variable is below it. However, with strategic interaction, a more formal definition of the strategies is essential. In [V], the strategies are defined as arbitrary stopping sets in the state space. This implies that the equilibrium must be Markov perfect. To allow arbitrary stopping sets is important, because it turns out that there may be equilibria with stopping sets of disconnected nature, referred to as "gap equilibria" in the article. If the strategies were simply defined as threshold levels, as is typical for preemption models, such equilibria could not be detected.

It is shown that due to the strategic interaction, the value function of the firm that stays in the market longer in equilibrium has a kink that corresponds to the exit of its competitor. On the other hand, uncertainty has an effect on the strategic interaction: it reduces the payoff difference across the firms caused by the asymmetry and strategic interaction.

However, the most important result is that only when the degree of uncertainty is sufficiently low or asymmetry between the firms is sufficiently large, a unique equilibrium exists. In that case one of the firms is doomed to exit before the other, because the other firm can credibly commit to staying in the market longer in case it is left alone. However, when the degree of uncertainty is high enough and asymmetry small enough, then a gap equilibrium, where the exit order of the firms is reversed, appears. Then there is no unique equilibrium outcome, and it is

no longer possible to predict which of the firms would exit first. This is a new finding, and in contrast to the presumption stated in Ghemawat and Nalebuff (1990) according to which there is a unique equilibrium with stochastic demand as long as it changes smoothly. It is also interesting to compare the result to Fine and Li (1989). They have a model with many equilibria, but they assert that the multiple equilibria result is driven by *jumps* in the demand process, independent of whether the demand process is stochastic or deterministic. In this thesis it is shown that multiple equilibria result may be driven by uncertainty even if the demand decline path is continuous.

[VI] An oligopolistic investment model of the Finnish electricity market

Article [VI] addresses incremental production capacity expansion in an oligopolistic market. For the purpose of numerical analysis, a dynamic, stochastic game model is presented and applied on a rough representation of the Finnish electricity market.

An important concept in dynamic game models is the information structure (see Basar and Olsder, 1995, for a summary). Whereas the Markov perfect equilibrium in article [IV] is based on feedback information structure, the model in article [VI] has a so-called sample-path adapted (S-adapted) information structure adopted from Haurie et al. (1990). This information structure is similar to the open-loop one, except that the strategies of the players adapt to the sample path of the stochastic variable. Using the terminology of extensive form games, the corresponding Nash equilibrium is not sub-game perfect. However, it is time-consistent (as a normal open-loop equilibrium) in the terminology of dynamic game theory. An example of comparison between open-loop and feedback models of dynamic oligopoly is Karp and Perloff (1993).

The source of uncertainty in the model is the demand growth. This is described as an event tree, where at each period there are two possible growth rates represented by separate nodes. Because there are many periods, these nodes form an event tree, which branches at each period resulting in a growing number of nodes per period. A certain path through the tree corresponds to a scenario of events. Strategic interaction appears at two levels. Within each node, the firms decide both the outputs with given capacities and the investments in new production capacity. The output decisions only influence the current period, but the investment decisions are irreversible and carry over to the forthcoming periods.

As shown in Haurie et al. (1990), the S-adapted open-loop equilibrium can be numerically solved using stochastic equilibrium programming techniques. This means that in principle the computation is not different from computing a static Nash-Cournot equilibrium. Two methods were used to solve the model. One of them is to solve directly the first order optimality conditions simultaneously for all players. This results in a nonlinear complementarity problem, which was solved using GAMS modeling software and MILES solver (see Rutherford, 1993). Another method is the diagonalization algorithm, which is based on iterated optimization (see Harker, 1984).

Real data of the main actors in the Finnish electricity market is used to sketch stylized scenarios of possible future market structures. Using these scenarios, possible market outcomes are analyzed. Sensitivity analysis is carried out on the number of actors in the market, on the investment costs, and on demand elasticity. A welfare maximizing competitive benchmark solution is also established to isolate the effect of the market power. The main result is that the firms can effectively use their market power to rise the price. The investments are very limited. Particularly in the case of strong demand growth, high prices result.

When considering the relevance of the results, it should be kept in mind that they rest on the strong assumption that entry by new firms is not possible. This assumption may not be very realistic in all cases, in particular in the long-run. Nevertheless, the results can be seen as a cautionary example of the effect of entry barriers. If such barriers exist, the market power coupled with uncertain demand growth strongly restrains capacity investments resulting in high prices.

4 Concluding remarks

This thesis presents six separate articles on investment under uncertainty with the focus on strategic interaction and energy market applications. The first two articles consider optimal investment in isolation from competition. While [I] focuses on the interaction between different uncertainties associated with one investment opportunity, [II] focuses on the irreversibility in the choice between alternative investment opportunities with different risk characteristics. The four remaining articles include strategic interactions in different forms. While [III] and [IV] consider the initiating of discrete projects, [V] considers the abandonment of a project, and [VI] focuses on incremental capacity investment.

The emphasis has been on theoretical development and general insights rather than on practical tools. Articles [III], [V], and [VI] are purely descriptive in nature, while [I], [II], and [IV] contain also prescriptive elements. In any case, further work aimed at strengthening the links with the reality would be important. With descriptive results, this would require linking to empirical observations, while with the prescriptive results, more tailored modeling is necessary to get applicable tools for the real decision making. For instance, the geometric Brownian motion used in many articles of this thesis serves as a convenient characterization of uncertainty in general, but in practical applications other stochastic processes would be appropriate to capture some characteristics of energy prices, such as seasonality and mean reversion. This kind of refinements would also require further development of numerical solving methodologies.

The liberalization of energy markets is a recent trend, and its real implications on capital investments will be observed with delay. For example, the Nordic electricity industry was characterized by excess capacity before liberalization, and therefore the capital investments since the restructuring have been minor. However, as the demand is growing, the surplus in capacity is reducing. It is likely that capital investments will in the near future gain increasing interest also from the practical point of view, when it will be observed directly how the price mechanism signals the scarcity of supply to investment decisions of the market participants. Meanwhile, it is important to develop appropriate methodologies for the understanding and analyzing of the firms' investment behavior.

Concerning the theoretical contributions of this thesis, the main effort has been on the incorporation of strategic interaction with the real options methodology. As reviewed in section 2.4, this is currently an active area of research. However, this line of work can also be seen from another perspective, namely as an extension of the theory of industrial organization to incorporate uncertainty. Since most industries are in reality characterized by uncertainties, this extension towards the direction that views market stability as one of the factors explaining rational behavior of the firms is important. An example of such an extension is the article [V] of this thesis, which shows that the degree of uncertainty may determine whether a particular pattern of exit from a declining market is supported by a unique equilibrium.

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