Mat-2.4108 - Independent Research Project in Applied Mathematics

MILP models for medium-term production cost optimization in multifuel and multigeneretion combined heat and power systems*

Martti Sutinen 79292F

Aalto University
School of Science
Department of Mathematics and Systems Analysis

December 20, 2011

*The document can be stored and made available to the public on the open internet pages of Aalto University

Contents

1 Introduction

This study examines district heating (DH) in Finland and focuses on developing Mixed Integer Linear Programming (MILP) models for medium-term production planning. MILP is chosen because it has been used in similar problems successfully [1, 2, 3, 4, 5]. Special algorithms have been developed for normal CHP production and trigeneration, but multigeneration complicates efficient formulation especially in a dynamic context [6, 7]. The objective is to minimize costs, which mostly consist of fuel and emission rights procurement.

The modeling principle is quite universal, but especially taxing and demand profiles may change. Also eletricity markets and the main product or production driver, in this case heat, can be different. Often there needs to be multiple periods every day to catch demand and price peaks which affect production. Thousands of periods will be optimized for each year making dynamic problems very heavy in terms of calculation. Medium-term planning is used for a time horizon of one to three years, meaning the model is used for controlling future commodity procurements and their hedging, contrary to daily operation or investment decisions. Sometimes long-term planning is used in the same sense within energy production planning.

Heat production is a significant business in Nordic countries, where heat is used for warming both private and business properties, and for hot water supply. The winters are very cold with the January average temperature being -4,2 °C in Helsinki and -14,1 °C in Sodankylä [8]. Heat consumption is almost directly proportional to temperature and Finland's cold weather resulted in 32,4 TWh of district heat sales in 2009 which equaled 1,82 billion euros, average price being 56,2 €/MWh. For comparison, electricity cost around 70 €/MWh for consumers [9]. District heating had a 47% market share in heating, suppling 2,62 million residents with warm homes [10].

To understand the costs of production we must take into account the prices of the main fuels, coal and natural gas (NG) [10]. According to IMF, the average prices were 77,0 \$/ton for Australian export coal and 318,8 \$/1000m^3 for Russian NG in Germany [11]. Using the average exhange rate and energy contents, this translates as roughly 8 €/MWh and 21 €/MWh, respectively [12, 13, 14]. Adding taxes of 50,5 €/ton and 2,1 €/MWh increases the prices to 16 €/MWh and 23 €/MWh [15, p. 135]. In 2011, the tax is 128,1 €/ton for coal, and rising until 2015, 13,7 €/MWh for NG [16]. We can see that taxation plays a major part in fuel prices. Another source of energy, peat, cost about 14 €/MWh and was not taxed in 2009 [9, 15, p.132].

Combined heat and power (CHP) production differs from the use of traditional heat-only boilers (HOB) in that both heat and power are extracted from the burning process. In separate heat and power (SHP) production power is mostly generated as condensing power at thermal power plants (TPP). CHP's advantage lies in having a higher overall efficiency (many sources report up to 90% [17, p.6], [18, 19, p. 7]), especially when comparing to TPPs whose efficiency is often 30-40 % for nuclear power plants and up to 60 % for new combined cycle gas turbines (CCGT) [20, 21]. The produced power-to-heat ratio of CHP plants varies from 1:3 to 4:3 which means 25-55% of the produced energy is power [22, 17, p.6]. The electrical efficiency of individual turbines is often close to 30% [22]. The higher the ratio, the more we can benefit from producing CHP electricity at a lower cost than with TPPs. Of course, sometimes electricity is not worth producing, as it is a volatile product with a wide price range.

Often a single CHP plant using one fuel for production can be set up to provide heat for a local community and steam for a few industrial clients, and it is simple to operate as production is determined by heat and steam demand. CHP plants can be more cost efficient when they have multiple fuel sources and are used in a large city with high demand, connected by an extensive district heating supply network. Commodity prices tend to be rather volatile in deregulated markets which affects production costs. We can optimize upcoming production by choosing the most affordable combination of operating the plants to satisfy the energy product demands during all periods in the optimization run.

The commodity requirement results can be used for hedging to partly secure a margin. However, then a decision needs to be made whether to use price forecasts or the market forward prices in the model. Using price estimates can produce more stable output but hedging is done with current market prices.

2 District heating and supply networks

District heating (DH) is a popular means of producing heat in densily populated areas, especially cities. Centralizing the production to large plants allows more efficient technology, which has enough benefits to cover the transfer losses. Heat is brought to customers by heating water with boilers in heat plants and circulating it in the supply network where heat exchangers capture the heat before the water returns to the plant. [1]

In a system with multiple production units, the heat plants differ in their configuration. In the simplest CHP plants, a boiler burns fuel to transform the fuel energy F into heat energy Q; hot steam or water, with efficiency η ($Q = \eta F$). The medium is then driven through a turbine with a specific electric efficiency α ($P = \alpha Q$). The remaining heat is circulated in the DH network. Electricity is sold to market. See Figure 1.

A more complex plant has several boilers and turbines using a range of fuels, and has low-, medium- and high pressure steam production for industry clients and district cooling. The boiler steam output can be interconnected and divided between turbines. Steam is extracted from specific points in the process to obtain multiple pressure levels. Industry clients are mostly local whereas heat can be distributed in a widespread area. A fraction of electricity is used for plant functions and the rest is sold to market. See Figure 2.

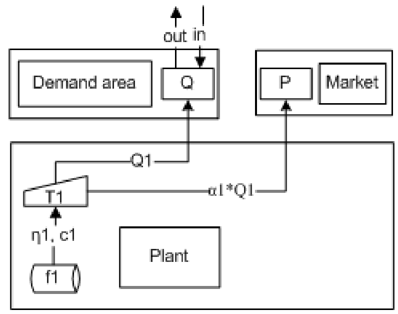

Figure 1: Diagram of a simple CHP plant

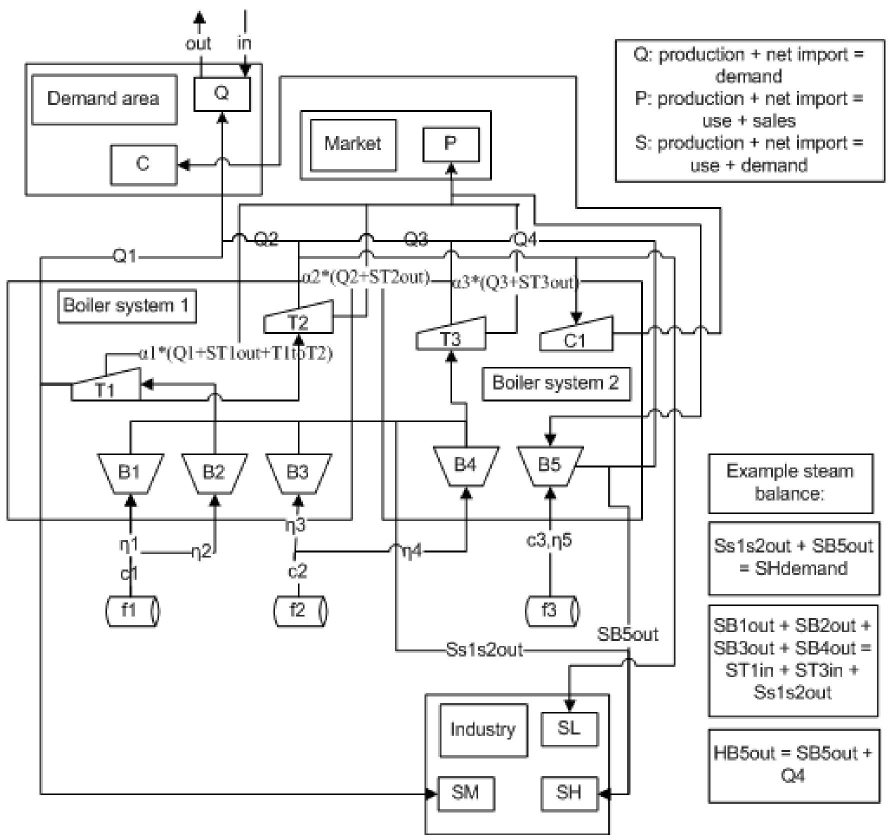

Figure 2: Diagram of a complex CHP plant

The DH networks can extend up to hundreds of kilometers in total pipelength, for example the pipelength of Stockholm's distribution system is 765 kilometers

[23]. In these cases, production and consumption is often centred in a few separate areas. The networks within the areas are designed to operate within local consumption but there are capacity limitations in pipelines connecting the areas. See Figure 3.

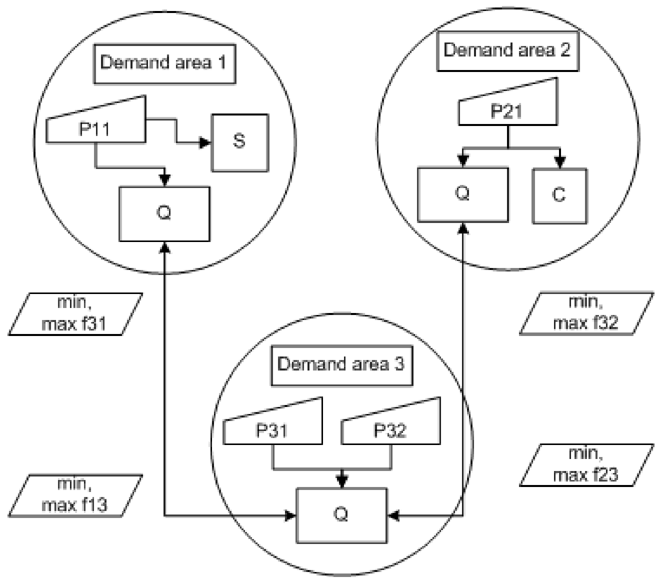

Figure 3: Example of a DH network

3 Modeling combined heat and power systems

Modeling the DH system is based on the idea that the system consists of nodes and arcs. In every node, there must be an energy balance. The sum of incoming energy flows multiplied by efficiencies in the respective arcs must equal the sum of outgoing energy flows. The types of flows in the node are defined by its type. The efficiency can be one for some flows, such as short pipes inside plants, so that no energy is lost in transfer. Nodes can be boilers, junctions, turbines, coolers, storages, plant output, demand areas or waste heat dumping points. For example, boilers require fuel energy and produce an amount hot water or steam which equals the product of boiler efficiency and inputted fuel energy. In a demand area node, the output is simply the estimated demand of the energy product.

Some nodes contain a single input and multiple outputs or vice versa. Those nodes exist purely to clarify the modeling principle and the multiple outputs or inputs should be added as the multiple outputs or inputs of the node that acted as the other end of the single input or output to reduce variables and speed up calculation. This attribute follows from energy balance equalities. The principle is shown in Figure 4.

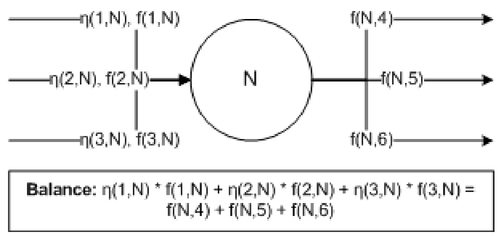

Figure 4: Energy balance in a node

3.1 Assumptions

3.1.1 Linearity

A linear minimization problem can be presented in canonical form but sets are introduced together with parameters to clarify the following formulation:

$$\begin{aligned} \min\ & (c^T x + e^T y) \\ \text{s.t. } & Ax + Dy \leq b \\ & x \geq 0 \\ & y \geq 0 \text{ and integer} \end{aligned} \tag{3.1}$$

The overall cost must be a linear function of the decision variables which include production and flow volumes, and equipment binary states. Increasing marginal costs cannot be included, but the cost function can be estimated as a piecewise linear function. Also the constraints must be linear equations of the variables.

3.1.2 Discrete variables

When there is a dynamic or selective constraint or cost, for example a minimum down time or a start-up cost, integer or binary variables must be used. To speed up the calculation we relax the problem, allowing the integer and binary variables to be continuous, only demanding that binary variables are between 0 and 1. Then we will get a solution in the first optimization run. In practice, when we have a binary variable telling us whether a plant is running during a time period and it gets a value 0.5 in the solution, it is hard to interpret and the result is infeasible. But from the commodity requirement results' point of view, this is not necessarily a problem, because the production amounts and fuel needs can be quite close to the solution with valid integer and binary values. [1]

3.1.3 Subsequent optimizations

In large systems the number of decision variables can be too large and the calculation cannot be completed in a reasonable time. It is possible to divide the optimization into smaller problems covering continuous time intervals. The intervals can range from two weeks to several months or even some years. It is important to execute the optimization consecutively so that the equipment states from the previous optimized time interval can be passed as parameters to the next one. Stored heat levels at beginning and at the end of the intervals should be set to the average level to avoid storage emptying. It needs to be tested what the optimal length is, because shorter intervals provide less optimal solutions as they accelerate calculation.

3.1.4 Time periods

Fixed parameters are used during each time period in the planning horizon. Hourly time periods are often sufficient, as prices or demand do not change too much within them. To simplify the problem, even longer periods may be used. A day might for example be divided in four to twenty four periods of varying lengths. Daily demand and price profiles should be investigated to discover functional periods.

Commodity prices are often received as monthly forward prices or spot estimates, and demand forecasts can be daily or monthly. They can be converted to period compatable input by taking the average from the hourly demand and price profiles. Heat modeling and price estimation are complex tasks, and need to be examined separately [24, 25].

3.1.5 Plants

The power-to-heat ratio for a turbine is the ratio of the produced power and the produced heat. In reality the ratio changes marginally according to the amount of heat produced, but in the model it is assumed constant or as a fixed range. Production level dependent ratios could be included in the model by choosing several points from the curve and formulating the boiler as several mutually exclusive boilers with relevant heat capacity constraints. However, this would require a lot more variables and the effect on the optimal solution would be very small.

3.1.6 Supply network

The model has a simplified network so that we only consider major demand areas and the main pipes connecting them. We can do this because only the demanded heat amount inside a demand area will flow in it and the area's network has capacity for the highest possible demand. The heat that flows through the area to other demand areas does so using the main pipelines, which have capacity constraints in the model. This simplification speeds up the model very much. The demand for an area must then be demand for production, not for delivered heat, because the heat losses inside areas cannot be modeled. Either a transfer efficiency or a cost should be applied for the pipelines to avoid circulation.

3.1.7 Heat and fuel storages

We assume that there are no variable costs in storing heat, because the investments have already been done. Some heat is lost between intervals. Fuel storages are not modeled and all fuel is delivered during its usage. The storages are always assumed to be within limits, when the production is within capacity. The cost of burning fuel is the realized spot price because selling is alternative to burning.

3.1.8 Commodity markets

The revenues for heat and cooling are assumed to be fixed given that the demands must be satisfied because the sales volume is then fixed and price is not affected by production. We do not need to add the revenues to the cost optimization model because they do not affect the optimal solution. In contrast, the produced electricity amount can be adjusted so it must be added as a negative cost to the objective function. The prices which are used as parameters must reflect the real price for buying the commodity (CO2 content cost, freight, taxes etc.) or selling the commodity (electricity area price) at the time of production.

It is recommended that the spot price is used for fuels even though the physical purchase would take place some weeks ahead of time. The physical purchase price risk must be eliminated by executing opposing trades with financial assets. All the markets are assumed to be unrestricted.

3.2 Decision variables

The sets used in the model are introduced in Table 1. The problem could be constructed using only nodes, arcs and their parameters, but sets add readability of the formulation. Parenthesis can be used with sets to pick a corresponding element e.g. $d = D(p)|p \in P$ refers to the demand point d corresponding to the plant output p and $A(D)$ refers to the arcs connecting demand points. The intervals in set I refer to the time periods in a single optimization run spanning a few hours that were discussed in the assumptions above, and should not be confused with the time frames of subsequent optimizations.

The decision variables with which the cost is minimized are simply the flows between nodes and the operating statuses of the nodes as well as accumulated storage heats. They involve the produced energy amount in boilers, flows within the plants, flows of the energy products, i.e. heat, steams and cooling, to demand

Table 1: Sets used in the model

Set	Description
I	Intervals
E	Energy products
F	Fuel emission gases
N	All nodes
A	All arcs
$P \subset N$	Plant output points
$B \subset N$	Boilers
$J \subset N$	Junctions in plants
$T \subset N$	Turbines
$C \subset N$	Coolers
$H \subset N$	Heat storages
$D \subset N$	Demand point
$W \subset N$	Waste heat dumping ground
$N = P \cup B \cup J \cup T \cup C \cup H \cup D \cup W$	

points and stored heat amounts. The operating statuses concern only equipment nodes; boilers, turbines and coolers. The used decision variables are shown in Table 2. All demand points are associated with a single energy product and demand. Each flow carries a certain energy product.

Table 2: Decision variables used in the model

Variable	Unit	Description
All the variables are interval dependent, i is omitted		
$f_{n,m}$	MW	Energy flow between nodes n and m
e_h	MWh	Energy in heat storage h
r_n	binary	On-off running state of equipment in node n
s_n	binary	On-off start-up state of equipment in node n

3.3 Parameters

When a boiler can consume multiple fuels, the cheapest fuel mix should be chosen as input so that within consumption constraints, maximum percentages of the most affordable fuels are used. Basically the fuel cost $[fc]_b$ and emission parameters $\lambda_{b,fe}, \overline{[fe]}$ must be updated accordingly. Multiple fuels for a single boiler can be modeled as exclusive boilers with separate fuels but it increases the amount of variables. Analyzing and adjusting input and then solving the simplified problem is quicker than calculation with more decision variables.

Table 3: Parameters used in the model

Parameter	Unit	Description
τ_i	h	Length of interval i
$e_{h,std}$	MWh	Standard heat level in storage h
$r_{n,i_{min}-1}$	binary	Previous running state of equipment in node n
All the following parameters are interval dependent, i is omitted		
$[dm]_d$	MW	Demand in demand point d
$[ep]$	€/MWh	Electricity price
$\eta_{n,m}$	%	Transfer efficiency between nodes n and m
η_n	%	Efficiency of equipment in node n
$\lambda_{b,fe}$	ton/MW	Ratio of gas emission fe from fuel usage in boiler b
η_h	%/h	Hourly efficiency of heat storage h
$[tc]_{n,m}$	€/MWh	Transfer cost between nodes n and m
$[fc]_b$	€/MWh	Fuel cost for boiler b
$[rc]_n$	€/h	Running cost for equipment in node n
$[sc]_n$	€	Start-up cost for equipment in node n
$\underline{[ie]_n}$	MW	Lower bound for equipment n input energy
$\overline{[ie]_n}$	MW	Upper bound for equipment n input energy
$\underline{[de]_n}$	MW	Lower bound for equipment n input energy change
$\overline{[de]_n}$	MW	Upper bound for equipment n input energy change
$\underline{\tau_{n,1}}$	h	Minimum on time for equipment n
$\underline{\tau_{n,0}}$	h	Minimum off time for equipment n
$\underline{\alpha_t}$	%	Lower bound for power-to-heat ratio of turbine t
$\overline{\alpha_t}$	%	Upper bound for power-to-heat ratio of turbine t
$\underline{\beta_c}$	%	Lower bound for cooling-to-heat ratio of cooler c
$\overline{\beta_c}$	%	Upper bound for cooling-to-heat ratio of cooler c
$\underline{f_{n,m}}$	MW	Lower bound for flow between nodes n and m
$\overline{f_{n,m}}$	MW	Upper bound for flow between nodes n and m
$\underline{e_h}$	MWh	Lower bound for heat energy in storage h
$\overline{e_h}$	MWh	Upper bound for heat energy in storage h
$\underline{r_n}$	binary	Lower bound for running state
$\overline{r_n}$	binary	Upper bound for running state
$\underline{s_n}$	binary	Lower bound for start-up state
$\overline{s_n}$	binary	Upper bound for start-up state
$\overline{[fe]}$	ton	Upper bound for fuel emission gas fe

3.4 Problem formulation

Using the above notation and variables, the problem can be formulated as follows. Notice that the interval subscript i is omitted to avoid repetition except in dynamic constraints. The problem can be written in the canonical form presented in Equation (3.1).

$$\min \sum_{I} \left[\sum_{b\in B} \left([fc]_b \frac{\tau \sum_{(b,j)\in A} f_{b,j}}{\eta_b} \right) - [ep]\tau \sum_{(j,p)\in A|p\in P(el)} f_{j,p} + \sum_{j\in B\cup T\cup C} ([rc]_j \tau r_j + [sc]_j s_j) + \tau \sum_{a\in A(D)} [tc]_a f_a + \tau \sum_{(j,w)\in A|w\in W} [tc]_{j,w} f_{j,w} \right] \tag{3.2}$$

s.t.

$e_{h,i_{min}} = e_{h,std}$
$e_{h,i_{max}} = e_{h,std}$
$\forall i \in I:$

$$\begin{aligned}
\forall d \in D: & \sum_{(j,d)\in A|j\in(D\setminus d)\cup P} \eta_{j,d} f_{j,d} - \sum_{(d,k)\in A|k\in(D\setminus d)} f_{d,k} = [dm]_d \\
\forall p \in P: & \sum_{(j,p)\in A|j\in B\cup T\cup C\cup J} \eta_{j,p} f_{j,p} = f_{p,D(p)} \\
\forall n \in T\cup C: & \ \eta_n \sum_{(j,n)\in A} \eta_{j,n} f_{j,n} = \sum_{(n,k)\in A} f_{n,k} \\
\forall j \in J: & \sum_{(k,j)\in A} \eta_{k,j} f_{k,j} = \sum_{(j,l)\in A} f_{j,l}
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
\forall n \in B\cup T\cup C: & \ \underline{[ie]_n} r_n \le \frac{\sum_{(n,k)\in A} f_{n,k}}{\eta_n} \le \overline{[ie]_n} r_n \\
\forall t \in T: & \ \underline{\alpha_t} \sum_{(t,k)\in A|k\notin P(el)} f_{t,k} \le f_{t,P(el)} \le \overline{\alpha_t} \sum_{(t,k)\in A|k\notin P(el)} f_{t,k} \\
\forall c \in C: & \ \underline{\beta_c} \sum_{(c,k)\in A|k\notin P(co)} f_{c,k} \le f_{c,P(co)} \le \overline{\beta_c} \sum_{(c,k)\in A|k\notin P(co)} f_{c,k} \\
\forall [fe] \in F: & \sum_{b\in B} \lambda_{b,fe} \frac{\sum_{(b,k)\in A} f_{b,k}}{\eta_b} \le \overline{[fe]} \\
\forall a \in A: & \ \underline{f_a} \le f_a \le \overline{f_a}
\end{aligned} \tag{3.4}$$

$$
\begin{aligned}
&\forall n \in B \cup T \cup C : \\
&\qquad \underline{[de]_n} \leq \frac{1}{\eta_n} \left(\sum_{(n,k)\in A_i} f^i_{n,k} - \sum_{(n,k)\in A_{i-1}} f^{i-1}_{n,k} \right) \leq \overline{[de]_n} \\
&\qquad r_{n,i-1} - r_{n,i} + s_{n,i} \geq 0 \\
&\qquad \underline{r_n} \leq r_n \leq \overline{r_n} \\
&\qquad 0 \leq s_n \leq 1 \text{ (if relaxed)} \\
&\qquad \forall j = i - \max([\underline{\tau_{n,1}} \text{ as intervals at i}], 1) + 1 \ldots i-1 : \\
&\qquad r_{n,j} - r_{n,j-1} \leq r_{n,i} \\
&\qquad \forall j = i - \max([\underline{\tau_{n,0}} \text{ as intervals at i}], 1) + 1 \ldots i-1 : \\
&\qquad r_{n,j-1} - r_{n,j} \leq 1 - r_{n,i} \\
&\quad \forall h \in H : e_{h,i} = \eta_h^{\tau_{i-1}} e_{h,i-1} + \tau_i \left(\sum_{(j,h)\in A} \eta_{j,h} f_{j,h} - \sum_{(h,k)\in A} f_{h,k} \right) \\
&\forall n \in B \cup T \cup C : r_n, s_n \text{ binary (unless relaxed)}
\end{aligned}
\tag{3.5}
$$

3.5 Objective function

$$
\begin{aligned}
\min \sum_I \Bigg[& \sum_{b\in B} \left([fc]_b \frac{\tau \sum_{(b,j)\in A} f_{b,j}}{\eta_b} \right) - [ep]\tau \sum_{(j,p)\in A | p \in P(el)} f_{j,p} + \\
& \sum_{j\in B\cup T\cup C} ([rc]_j \tau r_j + [sc]_j s_j) + \tau \sum_{a\in A(D)} [tc]_a f_a + \tau \sum_{(j,w)\in A|w\in W} [tc]_{j,w} f_{j,w} \Bigg]
\end{aligned}
$$

The objective function in Equation (3.2) is shown above. It consists of five parts: fuel costs, electricity sales revenues, running and start-up costs, transfer costs and heat dumping costs. Fuel costs from each boiler are calculated by dividing the heat produced by the boiler efficiency and multiplying by the fuel cost for the boiler and interval length. The amount of electricity is the sum of incoming flows to plant outputs of electricity type.

Running costs are hourly whereas the much more significant occur only once when a piece of equipment is started. Transfer costs can be used in rented supply networks. Some of the heat might be dumped as waste because of capacity constraints or to get revenues for a fuel with negative costs (waste plants).

More relevant costs can be added by using the decision variables and increasing the amount of parameters. For example storage costs would be easy to add.

3.6 Constraints

The constraints from Equations (3.3), (3.4) and (3.5) are explained below.

3.6.1 Energy balance and demand satisfaction

$$
\begin{aligned}
\forall d \in D : \sum_{(j,d)\in A | j \in (D\setminus d)\cup P} \eta_{j,d} f_{j,d} - \sum_{(d,k)\in A | k\in (D\setminus d)} f_{d,k} &= [dm]_d \\
\forall p \in P : \sum_{(j,p)\in A | j\in B\cup T\cup C\cup J} \eta_{j,p} f_{j,p} &= f_{p,D(p)} \\
\forall n \in T \cup C : \eta_n \sum_{(j,n)\in A} \eta_{j,n} f_{j,n} &= \sum_{(n,k)\in A} f_{n,k} \\
\forall j \in J : \sum_{(k,j)\in A} \eta_{k,j} f_{k,j} &= \sum_{(j,l)\in A} f_{j,l}
\end{aligned}
$$

The main principle in an energy system is the conservation of energy. For demand points, demand equals the net imports from other demand points and plant output points. A connection, or flow, exists only between nodes with similar energy products. Often there is a large supply network for only heat and other products are produced locally.

Plant output points have a simple constraint. Incoming energy from boilers, turbines, coolers and junctions in plants equals the outgoing energy to the corresponding demand point. Plant output points should be eliminated in the final version of the formulation by inserting the expression for flow to the demand point in the demand point energy balance.

For turbines and coolers, there is an efficiency measure that is used for incoming energy to account for the energy losses in the process. Junctions in the plants only collect energy from several sources and pass it to the next nodes in the energy production process.

3.6.2 Production and flow

$$
\begin{aligned}
\forall n \in B \cup T \cup C : \underline{[ie]_n} r_n &\leq \frac{\sum_{(n,k)\in A} f_{n,k}}{\eta_n} \leq \overline{[ie]_n} r_n \\
\forall t \in T : \underline{\alpha_t} \sum_{(t,k)\in A | k \notin P(el)} f_{t,k} &\leq f_{t,P(el)} \leq \overline{\alpha_t} \sum_{(t,k)\in A | k \notin P(el)} f_{t,k} \\
\forall c \in C : \underline{\beta_c} \sum_{(c,k)\in A | k \notin P(co)} f_{c,k} &\leq f_{c,P(co)} \leq \overline{\beta_c} \sum_{(c,k)\in A | k \notin P(co)} f_{c,k} \\
\forall [fe] \in F : \sum_{b\in B} \lambda_{b,fe} \frac{\sum_{(b,k)\in A} f_{b,k}}{\eta_b} &\leq \overline{[fe]} \\
\forall a \in A : \underline{f_a} &\leq f_a \leq \overline{f_a}
\end{aligned}
$$

Production cannot exceed equipment capacity. Capacity constraints are usually given for inputted energy, so the produced energy is divided by the efficiency. The bounds are multiplied by running status to indicate that the constraint is active only during production. Also, it forces the status on when energy is produced. The operating status variables are not needed for equipment that has no operating costs or minimum up or down times. In turbines power must be produced according to the power-to-heat ratio and in coolers cooling according to the cooling-to-heat ratio. All energy flow except power and cooling is interpreted as heat in turbines and coolers.

Emissions are controlled by adding the emissions from each boiler and demanding the sum to be less than the upper bound for the emitted gas. Each boiler has ratios for how much emissions are generated from burning its fuel. All the flows have their own lower and upper bounds. Instead of having two variables for a single pipeline one can have a negative lower bound for the flow.

3.6.3 Dynamic constraints

$$
\begin{aligned}
&\forall n \in B \cup T \cup C: \\
&\qquad \underline{[de]_n} \leq \frac{1}{\eta_n}\left(\sum_{(n,k)\in A_i} f^i_{n,k} - \sum_{(n,k)\in A_{i-1}} f^{i-1}_{n,k}\right) \leq \overline{[de]_n} \\
&\qquad r_{n,i-1} - r_{n,i} + s_{n,i} \geq 0 \\
&\qquad \underline{r_n} \leq r_n \leq \overline{r_n} \\
&\qquad 0 \leq s_n \leq 1 \text{ (if relaxed)} \\
&\qquad \forall j = i - \max([\underline{\tau_{n,1}} \text{ as intervals at i}], 1) + 1 \ldots i-1: \\
&\qquad r_{n,j} - r_{n,j-1} \leq r_{n,i} \\
&\qquad \forall j = i - \max([\underline{\tau_{n,0}} \text{ as intervals at i}], 1) + 1 \ldots i-1: \\
&\qquad r_{n,j-1} - r_{n,j} \leq 1 - r_{n,i} \\
&\forall h \in H: e_{h,i} = \eta_h^{\tau_{i-1}} e_{h,i-1} + \tau_i \left(\sum_{(j,h)\in A} \eta_{j,h} f_{j,h} - \sum_{(h,k)\in A} f_{h,k}\right) \\
&\forall n \in B \cup T \cup C: r_n, s_n \text{ binary (unless relaxed)}
\end{aligned}
$$

We need dynamic constraints for ramp conditions, start-up costs, minimum up or down time or storages. Dynamic constraints are problematic because the problem cannot be separated into individual period-wise problems which are faster to solve. The first constraint requires the change in production to be within an allowed range. The second one links running and start-up variables. Lower and upper bounds for running can be used to force a maintenance break. The start-up variables must be constrained between zero and one if the problem is relaxed and integer variables are not required (which produces an infeasible but possibly accurate enough solution, discussed in Section 3.1.2).

The minimum up (and down) times are constructed by requiring the piece of equipment to be on (off) if it has been switched on (off) within the given minimum time. Heat storages lose heat based on their hourly efficiency and interval lenghts. The storage heat level equals the previous level adjusted by period efficiency plus net imports from all connected nodes.

3.7 Motivation for the chosen model

A MILP model with straightforward formulation was chosen for the problem because *a)* of its suitability for the problem, *b)* of its successful application in CHP production planning [1, 2, 3, 4, 5] and *c)* the dynamics and multidimensional plant characteristics pose a challenge for optimized formulation [7]. Also dynamic programming (DP) [26, 27], Lagrangian relaxation (LR) [7, 28, 29], genetic algorithms (GA) [30] have been applied to unit commitment problems.

3.8 Some algorithms

The most common algorithm for solving linear problems is Simplex, and its modified form, the Revised Simplex [31, 32, Ch. 3]. There are also algorithms which are developed specifically for optimizing CHP plants, most notably Power Simplex (PS) and Extended Power Simplex (EPS) [33]. They take advantage of the fact that we can often formulate cost as a convex function of heat and power production, and no dynamic constraints are allowed. In non-convex situtations, often due to dynamic or other integer constraints, we can use the Envelope-based Branch & Bound (EBB) [6]. However these algorithms do not allow multigeneration in a sophisticated production network.

When there are discrete (integer or binary) variables, Branch & Bound (BB) [34] should be used to obtain the correct solution. Other cutting plane algorithms are also worth investigating [35]. In some situations, the discrete variables can be relaxed to be continuous and the solution will remain accurate enough [1, p. 10]. This should be tested, as BB is a computationally intensive algorithm and significant time savings can be achieved.

3.9 Sensitivity analysis

Simplex provides an easy way to acquire ranges for the decision variables in which the solution is optimal, as well as reduced cost that expresses the value of relaxing constraints. In large systems there can be multiple solutions which are close to the optimal one. The optimal solution range becomes so small that it does not contain enough information for risk analysis. A better approch for sensitivity analysis is to model parameters with simulation, as prices and demands often correlate and have stochastic elements. By optimizing with the different scenarios a discrete distribution of solutions is created for cost and commodity requirements. Of course, calculation must be quick and a more approximate model without dynamics can be used for this purpose if it proves accurate.

4 Conclusions

In this study we have examined how to construct MILP models for production cost optimization in multifuel and multigeneration CHP systems. The model can be used for hedging the commodity requirements of future production. Multiple energy products i.e. heat, power, steam and cooling make the problem difficult to model using efficient formulation and algorithms. Dynamic constraints from heat storages, start-up costs or ramp constraints increase complexity.

In a complex environment the problem must be formulated logically and clearly. A network approach provides intuitiveness to the formulation process. An energy balance is required for each node in the network and the nodes have different constraints based on their type. Production is driven by the energy product demand constraints, except electricity which has a flexible market.

Time complexity becomes a problem in medium-term production planning where the time horizon extends from one to three years. Increasing time period lengths, dividing the problem into several subproblems, relaxing integer variables, simplifying the supply network, replacing some variables with expressions and pre-run parameter optimization are suggested as cures. Especially parameter scenario based sensitivity or risk analysis requires fast performance and rougher models can be constructed for that purpose.

Special care should be taken in choosing the inputs. Markets might overreact to news during the optimization time horizon, and using different prices for a certain period along the way will cause variance in the solution. More stable price estimates will give results that reflect the future production more accurately. However, hedging is executed with current market prices and then the difference between the estimate and the current price should act as a trading trigger. Also, demand input includes losses which are not handled in the model. The cheapest fuel combination for a boiler should be used as input unless boiler fuel selection is modeled.

This study is based on experiences in validating a production cost optimization model for a Nordic energy company. The model had a time horizon of some years and there were 50 to 150 thousand variables for each interval that was optimized. The calculation times ranged from 20 seconds to two minutes on a regular laptop. The General Algebraic Modeling System (GAMS) [36] was used for problem formulation and a compatible commercial solver for calculation.

References

[1] E. Dotzauer, "Experiences in mid-term planning of district heating systems," *Energy*, vol. 28, no. 15, pp. 1545–1555, 2003. [Online]. Available: http://dx.doi.org/10.1016/S0360-5442(03)00151-8

[2] M. Bojic and B. Stojanovic, "MILP optimization of a CHP energy system," *Energy Conversion and Management*, vol. 39, no. 7, pp. 637–642, 1998. [Online]. Available: http://dx.doi.org/10.1016/S0196-8904(97)00042-3

[3] D. T. Gardner and J. S. Rogers, "Joint planning of combined heat and power and electric power systems: An efficient model formulation," *European Journal of Operational Research*, vol. 102, no. 1, pp. 58–72, 1997. [Online]. Available: http://dx.doi.org/10.1016/S0377-2217(96)00221-4

[4] G. Sundberg and D. Henning, "Investments in combined heat and power plants: influence of fuel price on cost minimised operation," *Energy Conversion and Management*, vol. 43, no. 5, pp. 639–650, 2002. [Online]. Available: http://dx.doi.org/10.1016/S0196-8904(01)00065-6

[5] R. Yokoyama, Y. Hasegawa, and K. Ito, “A MILP decomposition approach to large scale optimization in structural design of energy supply systems,” *Energy Conversion and Management*, vol. 43, no. 6, pp. 771–790, 2002. [Online]. Available: http://dx.doi.org/10.1016/S0196-8904(01)00075-9

[6] A. Rong and R. Lahdelma, “An efficient envelope-based Branch and Bound algorithm for non-convex combined heat and power production planning,” *European Journal of Operational Research*, vol. 183, no. 1, pp. 412–431, 2007. [Online]. Available: http://dx.doi.org/10.1016/j.ejor.2006.09.072

[7] A. Rong, R. Lahdelma, and P. B. Luh, “Lagrangian relaxation based algorithm for trigeneration planning with storages,” *European Journal of Operational Research*, vol. 188, no. 1, pp. 240–257, 2008. [Online]. Available: http://dx.doi.org/10.1016/j.ejor.2007.04.008

[8] Ilmatieteen laitos, “Climatological statistics for the normal period 1971-2000,” 2011. [Online]. Available: http://en.ilmatieteenlaitos.fi/normal-period-1971-2000

[9] Statistics Finland, “Preliminary energy statistics 2009 - Tables (Excel),” 2010. [Online]. Available: http://www.stat.fi/til/ehkh/2009/04/ehkh_2009_04_2010-03-24_tau_012.xls

[10] Finnish Energy Industries, “District heat in Finland year 2009,” 2010. [Online]. Available: http://www.energia.fi/en/topical-issues/lehdistotiedotteet/district-heating-year-2009

[11] International Monetary Fund, “Actual market prices for non-fuel and fuel commodities, 2008-2011,” 2011. [Online]. Available: http://www.imf.org/external/np/res/commod/Table3.pdf

[12] Internal Revenue Service, “Yearly average currency exchange rates,” 2011. [Online]. Available: http://www.irs.gov/businesses/small/international/article/0,,id=206089,00.html

[13] Wikipedia, “Coal,” 2011. [Online]. Available: http://en.wikipedia.org/wiki/Coal

[14] ——, “Natural gas,” 2011. [Online]. Available: http://en.wikipedia.org/wiki/Natural_gas

[15] Ministry of Finance, “Taxation in Finland 2009,” 2009. [Online]. Available: http://www.vm.fi/vm/en/04_publications_and_documents/01_publications/075_taxation/20090504Taxati26547/name.jsp

[16] Finnish Energy Industries, “Energy taxation in Europe, Japan and the United States,” 2010. [Online]. Available: http://www.energia.fi/julkaisut/energy-taxation-europe-japan-and-united-states

[17] S. Au, S. McPhail, N. Woudstra, and K. Hemmes, "The influence of operating temperature on the efficiency of a combined heat and power fuel cell plant," *Journal of Power Sources*, vol. 122, no. 1, pp. 37–46, 2003. [Online]. Available: http://dx.doi.org/10.1016/S0378-7753(03)00345-8

[18] Fortum, "Reducing emissions," 2011. [Online]. Available: http://www.fortum.com/en/sustainability/low-carbon-society/reducing-emissions/pages/default.aspx

[19] S. Nesheim and I. Ertesvåg, "Efficiencies and indicators defined to promote combined heat and power," *Energy Conversion and Management*, vol. 48, no. 3, pp. 1004–1015, 2007. [Online]. Available: http://dx.doi.org/10.1016/j.enconman.2006.08.001

[20] World Nuclear Association, "Nuclear power reactors," 2011. [Online]. Available: http://www.world-nuclear.org/info/inf32.html

[21] Power Technology, "Baglan Bay CCGT power plant," 2011. [Online]. Available: http://www.power-technology.com/projects/baglan/

[22] Department of Energy and Climate Change, "Plant selection," 2011. [Online]. Available: http://chp.decc.gov.uk/cms/plant-selection-2/

[23] C. Lindroth, "The heat supply of Stockholm," 1996. [Online]. Available: http://www.energy.rochester.edu/se/stockholm/heatsupply.htm

[24] A. J. Heller, "Heat-load modelling for large systems," *Applied Energy*, vol. 72, no. 1, pp. 371–387, 2002. [Online]. Available: http://dx.doi.org/10.1016/S0306-2619(02)00020-X

[25] K. Difs, M. Danestig, and L. Trygg, "Increased use of district heating in industrial processes - Impacts on heat load duration," *Applied Energy*, vol. 86, no. 11, pp. 2327–2334, 2009. [Online]. Available: hhttp://dx.doi.org/10.1016/j.apenergy.2009.03.011

[26] M. Korpaas, A. T. Holena, and R. Hildrumb, "Operation and sizing of energy storage for wind power plants in a market system," *International Journal of Electrical Power and Energy Systems*, vol. 25, no. 8, pp. 599–606, 2003. [Online]. Available: http://dx.doi.org/10.1016/S0142-0615(03)00016-4

[27] W. Hobbs, G. Hermon, S. Warner, and G. Shelbe, "An enhanced dynamic programming approach for unit commitment," *IEEE Transactions on Power Systems*, vol. 3, no. 3, pp. 1201–1205, 1988. [Online]. Available: http://dx.doi.org/10.1109/59.14582

[28] F. Zhuang and F. Galiana, "Towards a more rigorous and practical unit commitment by Lagrangian relaxation," *IEEE Transactions on Power Systems*, vol. 3, no. 2, pp. 763–773, 1988. [Online]. Available: http://dx.doi.org/10.1109/59.192933

[29] J. M. Ngundam, F. Kenfack, and T. T. Tatietse, "Optimal scheduling of large-scale hydrothermal power systems using the Lagrangian relaxation technique," *International Journal of Electrical Power and Energy Systems*, vol. 22, no. 4, pp. 237–245, 2000. [Online]. Available: http://dx.doi.org/10.1016/S0142-0615(99)00054-X

[30] C.-P. Cheng, C.-W. Liu, and C.-C. Liu, "Unit commitment by Lagrangian relaxation and genetic algorithms," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 707–714, 2000. [Online]. Available: http://dx.doi.org/10.1109/59.867163

[31] G. Dantzig, *Linear Programming and Extensions.* Princeton University Press, 1963.

[32] D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd ed. Springer, 2008.

[33] A. Rong, H. Hakonen, and R. Lahdelma, "An efficient linear model and optimisation algorithm for multi-site combined heat and power production," *European Journal of Operational Research*, vol. 168, no. 2, pp. 612–632, 2006. [Online]. Available: http://dx.doi.org/10.1016/j.ejor.2004.06.004

[34] A. H. Land and A. G. Doig, "An automatic method of solving discrete programming problems," *Econometrica*, vol. 28, no. 3, pp. 497–520, 1960. [Online]. Available: http://dx.doi.org/10.2307%2F1910129

[35] A. Letchford and A. Lodi, "Primal cutting plane algorithms revisited," *Mathematical Methods Of Operations Research*, vol. 56, no. 1, pp. 67–82, 2002.

[36] GAMS Development Corporation, "GAMS home page," 2011. [Online]. Available: http://www.gams.com/