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Ranking and selection (R&S) with multiple performance measures using incomplete preference information

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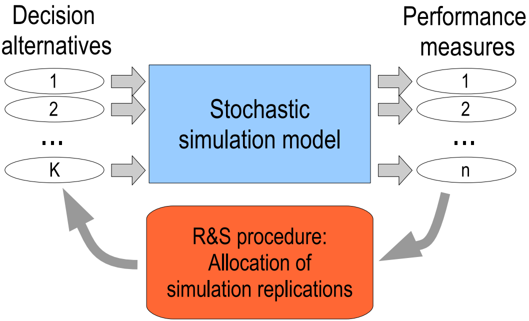
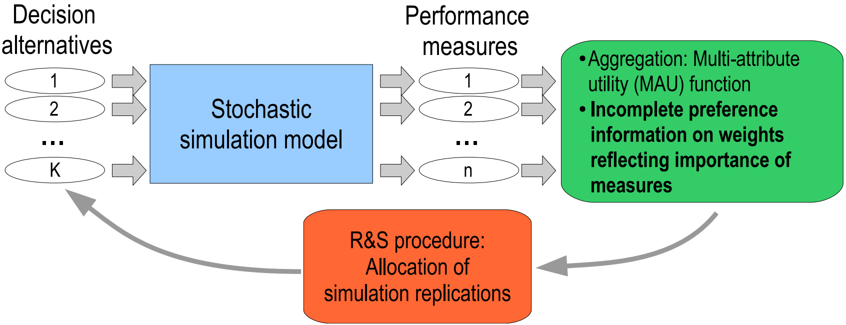
R&S procedures in simulation-optimization



→ Best decision alternative(s) efficiently and with a high level of confidence

New procedure [Mattila and Virtanen, 2013]



→ Advantages over existing procedures:

- Ease of giving preference information
- Savings in simulation effort
- Increased confidence in correct selection

The R&S problem

$$\min_{k \in \{1,\ldots,K\}} \left(E\left(X_{k1}\right), \ldots, E\left(X_{kn}\right) \right)$$

- K decision alternatives, *designs*
- $\mathbf{X}_k = (X_{k1}, \ldots, X_{kn})$, n performance measures of a stochastic simulation model for design k
- $E\left(X_{ki}\right)$ estimated from samples of X_{ki} obtained through simulation replications of the model
- *Computing budget*, i.e., number of available simulation replications limited

Existing approaches

- Optimal computing budget allocation (OCBA) [Chen et al., 2000]
  - Performance measures aggregated with MAU function
  - Maximizes probability of correctly selecting design with highest expected utility
  - Requires complete preference information
- Multi-objective OCBA (MOCBA) [Lee et al., 2004]
  - Dominance:

    $k \succ l$ if $E\left(X_{ki}\right) \leq E\left(X_{li}\right)\ \forall i = 1, \ldots, n$ and at least one inequality is strict
  - Maximizes probability of correctly selecting non-dominated designs
  - May be tedious, several designs may remain

Incomplete preference information

- Additive MAU function: $U(\mathbf{X}_k) = \sum_{i=1}^{n} w_i u_i\,(X_{ki})$
  - $u_i, w_i \in [0,1]\ \forall i = 1, \ldots, n, \sum w_i = 1$
- Incomplete preference information
  - Linear constraints for the weights $\mathbf{w} = (w_1, \ldots, w_n)$

  $\rightarrow$ Feasible set of weights a bounded convex polyhedron with extreme points $\{\mathbf{w}_1, \ldots, \mathbf{w}_H\}$
- Pairwise dominance:

  $k \succ_p l$ if $E\,(U(\mathbf{X}_k)) \geq E\,(U(\mathbf{X}_l))\ \forall \mathbf{w} \in \{\mathbf{w}_1, \ldots, \mathbf{w}_H\}$ and at least one inequality is strict
- Similarity to dominance $\rightarrow$ MOCBA applied for maximizing probability of correctly selecting pairwise non-dominated designs

New procedure: MOCBA-p

0. Determine $u_i, i = 1, \ldots, n$ and $\{\mathbf{w}_1, \ldots, \mathbf{w}_H\}$

Perform initial replications to all designs

Estimate $E(U(\mathbf{X}_k))$ and $Var(U(\mathbf{X}_k))$ for all $\mathbf{w} \in \{\mathbf{w}_1, \ldots, \mathbf{w}_H\}$

1. Perform additional replications according to *allocation rules*

Dominated designs: proportional to uncertainty about dominance
Dominating designs: proportional to allocations of such designs that the one in question dominates most likely

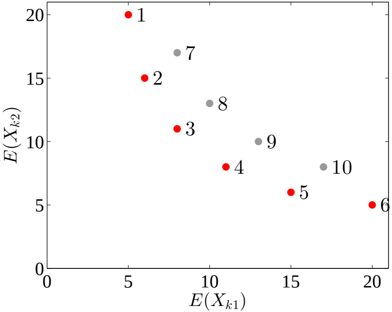
2. Update estimates for $E(U(\mathbf{X}_k))$ and $Var(U(\mathbf{X}_k))$

3. If computing budget has not been consumed, return to step 1

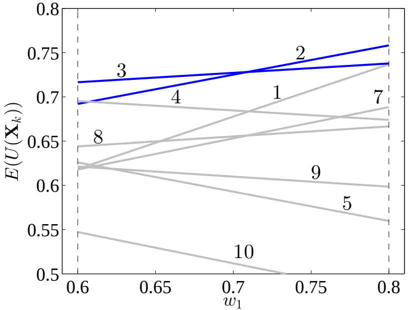
Else, select pairwise non-dominated designs based on estimates for $E(U(\mathbf{X}_k))$

Example

- X_{ki} normally distributed
- Linear, decreasing u_i
- $w_1 \in [0.6, 0.8] \rightarrow \mathbf{w}_1 = (0.6, 0.4), \mathbf{w}_2 = (0.8, 0.2)$



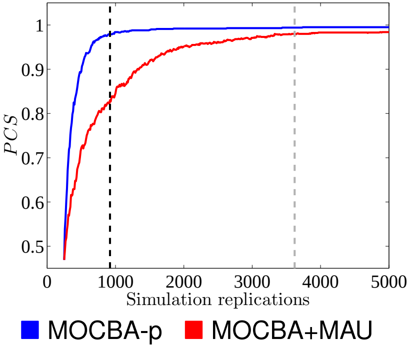
■ Non-dominated



■ Pairwise non-dominated

Example: Probability of correct selection

- MOCBA-p
- Reference procedure, MOCBA+MAU
    1. Non-dominated designs using MOCBA
    2. Pairwise non-dominated using same MAU function as MOCBA-p
- MOCBA-p reaches higher probability with given budget or requires smaller budget for given probability

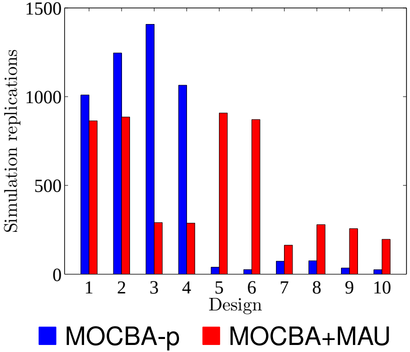


Example: Allocated replications

- MOCBA-p allocates more replications to pairwise non-dominated designs

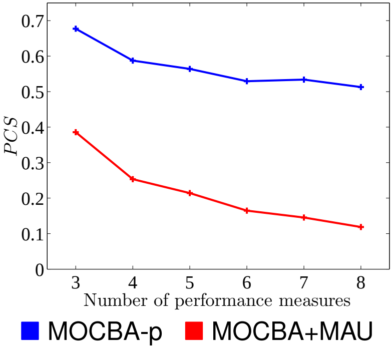
→ Evaluated with greater accuracy

→ Compared with higher degree of confidence, e.g., to select most preferred one



Increasing number of performance measures

- Setting
  - 100 randomly generated test problems with 50 designs
  - $w_1 \geq w_i, i > 1$
  - Average probability of correct selection over the test problems
- MOCBA-p reaches higher probabilities
- Difference between procedures slightly increases with number of measures



Conclusions

- New procedure for R&S with multiple performance measures
  - Complete preference information not required (vs. MAU+OCBA)
  - Smaller set of designs remain to be compared after the simulations (vs. MOCBA)
  - Pairwise non-dominated designs selected correctly with higher probability or smaller computing budget (vs. MOCBA+MAU)
  - Pairwise non-dominated designs evaluated with greater accuracy (vs. MOCBA+MAU)
- Similar procedure developed based on *absolute* dominance and OCBA
  - Returns a higher number of designs compared with MOCBA-p
  - Allows non-additive MAU functions

References

C.-H. Chen, J. Lin, E. Yücesan, and S. E. Chick . Simulation Budget Allocation for Further Enhancing the Efficiency of Ordinal Optimization.
Discrete Event Dynamic Systems: Theory and Aplications, 10:251–270, 2000.

L. H. Lee, E. P. Chew, S. Teng, and D. Goldsman. Optimal Computing Budget Allocation for Multi-objective Simulation Models.
Proc. 2004 Winter Sim. Conf., 586–594, 2004.

V. Mattila and K. Virtanen. Ranking and Selection with Multiple Performance Measures Using Incomplete Preference Information.
Manuscript, 2013.