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Preference Programming for Spatial Multiattribute Decision Analysis

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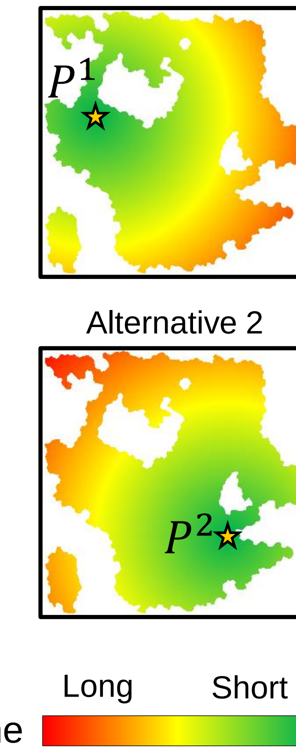
Spatial Decision Analysis

- Consequences of alternatives are distributed across a geographical region
- E.g., select the position of a rescue helicopter base, P^1 or P^2
  - Alternatives imply different response times, i.e., consequences, for each location
  - Locations not equally important? (cf. population density)
- Plenty of other applications
  - Urban, environmental and transportation planning
  - Waste management, hydrology, agriculture, and forestry
  - See, e.g., Malczewski & Rinner 2015, Ferretti & Montibeller 2016



Spatial Value Function

- Value of decision alternative z (Simon, Kirkwood and Keller 2014):

$$V(z) = \int_{s \in S} a(s) v\big(z(s)\big) ds$$

$a(s)$: spatial weight ("importance") of specific location s in region *S*

$z(s)$: consequence for location s when alternative z is chosen

$v(\cdot)$: consequence value function

- Challenges:
  - Specifying spatial weights $a(s)$ for an infinite number of locations s
  - Only a conjecture on the underlying preference assumptions exists

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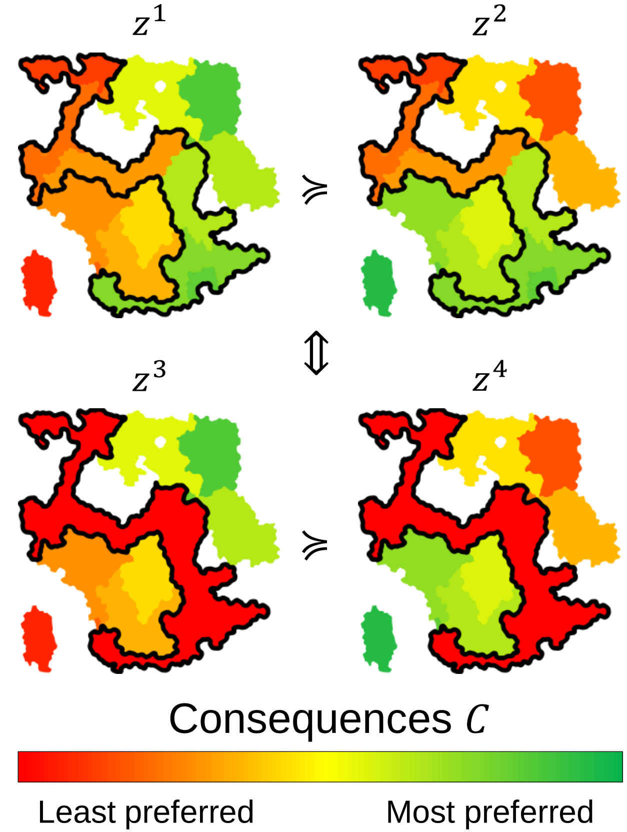
- Our contribution:
  - Axiomatic basis for preferences that can be represented with the spatial value function
  - Spatial preference programming: Determination of dominances among alternatives based on incomplete specification of weights

Preference Assumptions

- Let $\succcurlyeq$ be a binary relation on the set of decision alternatives $Z = \{z: S \rightarrow C\}$
  - S: set of locations
  - C: set of consequences
- Assumptions

A1	$\succcurlyeq$ is transitive and complete
A2	There exist $z^1, z^2 \in Z$ such that $z^1 \not\succcurlyeq z^2$
A3	“Spatial preference independence”
A4	“Consequence consistency”
A5	“Spatial consistency”
A6	“Divisibility of subregions”
A7	“Monotonicity”

A3: Preference between two alternatives does not depend on locations with equal consequence

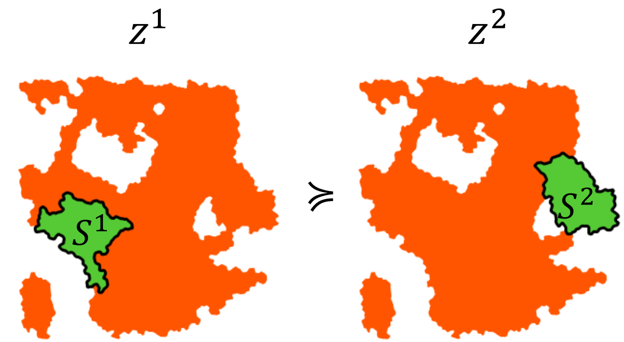


Additive Spatial Value Function $V(z)$

- **Theorem.** $\succcurlyeq$ satisfies A1-A7 iff there exists a non-atomic measure α on S and a bounded function $v: C \to \mathbb{R}$ such that $z \succcurlyeq z' \Leftrightarrow V(z) \geq V(z')$ where
$$V(z) = \int_S v\big(z(s)\big)d\alpha(s)$$
  - Proof based on Savage 1954
- The weighting function $\alpha: 2^S \to \mathbb{R}$
  - Assigns a weight to each subregion $S' \subseteq S$ (cf. relative importance)
  - Connection to Simon's et al. weighting $a(s)$: $\alpha(S') = \int_{S'} a(s)ds$
- v is a cardinal value function for consequences $c = z(s)$
  - I.e., unique up to positive affine scaling
  - E.g., additive multiattribute $v(z) = \sum_{j=1}^{m} b_j v_j\big(z_j\big)$

Incomplete Preference Information

- Small set of feasible weighting functions can be sufficient for ranking alternatives
  - Avoiding the overwhelming task of specifying the exact weighting function α
- Stated preferences between pairs of alternatives $\rightarrow$ Constraints on the spatial weighting function α and the vector b of attribute weights
- Multiple preference statements comparing suitable alternatives $\rightarrow$ System of linear constraints on
  - $\alpha(S^1), \dots, \alpha(S^n)$ where $S^1, \dots, S^n$ is a partition of S
  - $b_1, \dots, b_m$



$$z^1 \succcurlyeq z^2$$
$$\Leftrightarrow V(z^1) \geq V(z^2)$$
$$\Leftrightarrow \alpha(S^1) \geq \alpha(S^2)$$

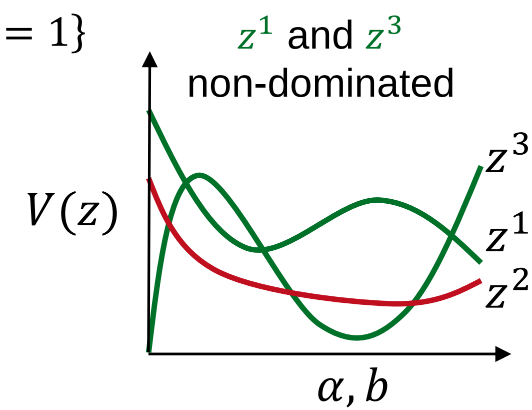
”Subregion S^1 more important than S^2”

Consequences C

Least preferred Most preferred

Dominance

- Constraints from preference statements result in
  - A set of feasible weighting functions $\mathcal{A} \subseteq \{\alpha : 2^S \to \mathbb{R}_+ | \alpha(S) = 1\}$
  - A set of feasible attribute weights $B \subseteq \{b \in \mathbb{R}_+^m | \sum b_j = 1\}$
- Alternative z^1 dominates alternative z^2 if
  - $V(z^1) \geq V(z^2)$ for all $\alpha \in \mathcal{A}$ and $b \in B$
  - $V(z^1) > V(z^2)$ for some $\alpha \in \mathcal{A}$ and $b \in B$



- Dominance check: bi-level LP problem

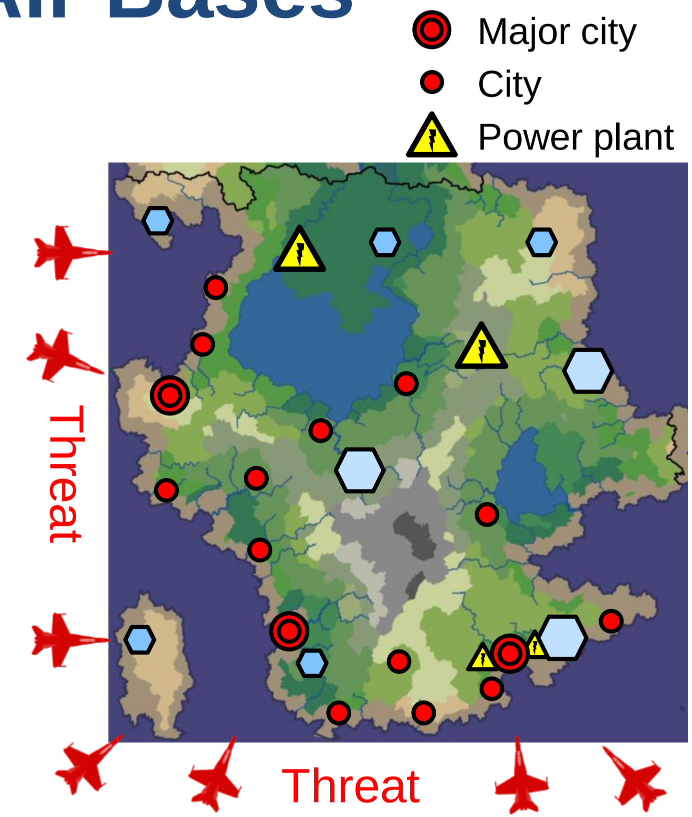
$$\inf_{\alpha \in \mathcal{A}, b \in B} V(z^1) - V(z^2) = \min_{b \in \mathrm{B}} \min_{\alpha \in \mathcal{A}} \sum_{i=1}^{n} \alpha\left(S^i\right) \inf_{s \in S^i} \sum_{j=1}^{m} b_j v_j \left(z_j^1(s)\right) - v\left(z_j^2(s)\right)$$

where $S^1, \dots, S^n$ is a partition of S

  - Solution: Enumerate extreme points of B and solve LP problem in each one

Air Defense Planning: Positioning of Air Bases

- Select positions for 2 main and 3 secondary air bases to maximize air defense capability
  - Main bases: 3 position candidates
  - Secondary bases: 5 position candidates
- Spatial consequences provided by a simulation tool – input parameters:
  - Number of defensive flying units; fuel consumption; weapons consumption; flight speed
  - Positions of air bases; turnaround times; refueling and rearming times; alert, taxi and scramble delays

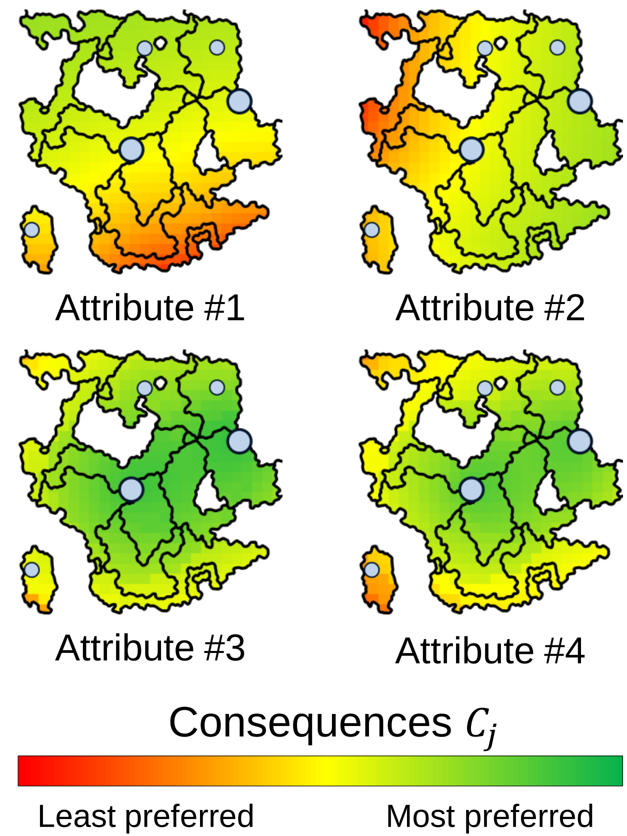


Attributes of Air Defense Capability

Positions of air bases affect…

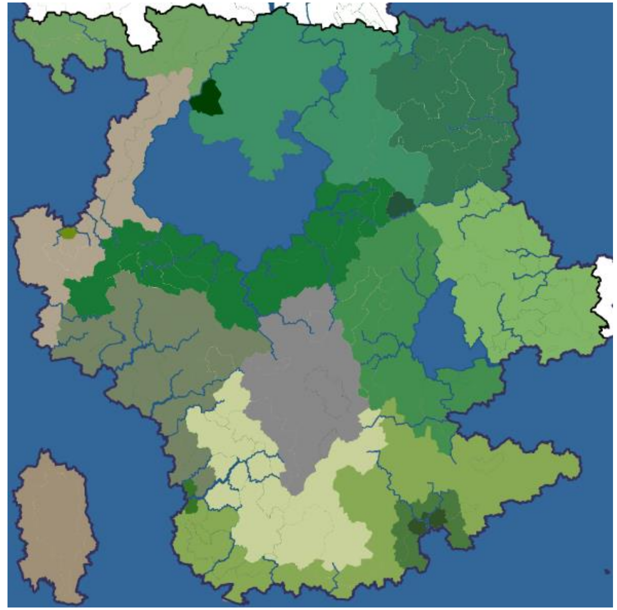
- “Engagement frontier” where hostile aircraft can first be intercepted by defensive flying units
  - Attribute #1: Location’s distance to south frontier
  - Attribute #2: Location’s distance to west frontier
- “Force fulfillment”
  - Attribute #3: Average number of defensive flying units available at the location
  - Attribute #4: As attribute #3 with one secondary base destroyed (cf. combat sustainability)

Alternative with bases at the highlighted positions



Preference Statements

- Spatial preference statements (α)
  - Major cities > SW coastal area
  - Power plants > SW coastal area
  - SW coastal area > NE coastal area
  - NE coastal area > Other areas
- Attribute preference statements (b)
  - Engagement frontier attributes > Force fulfillment attributes



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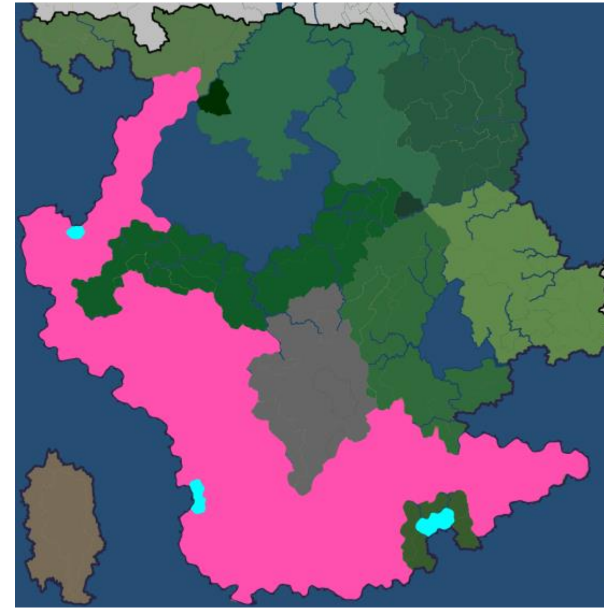

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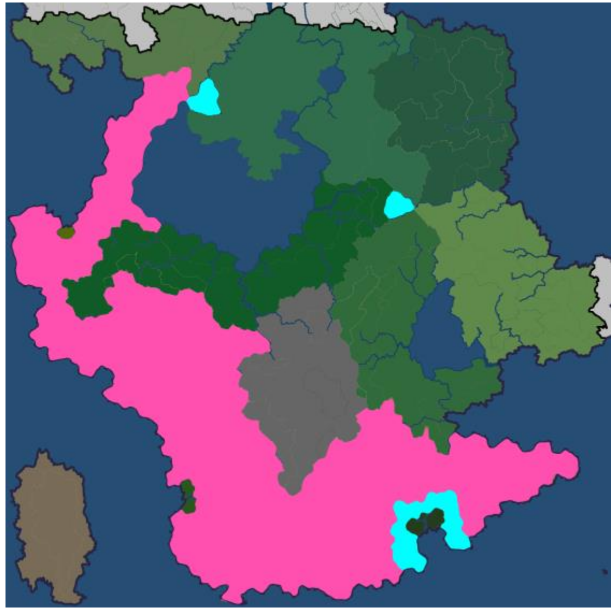

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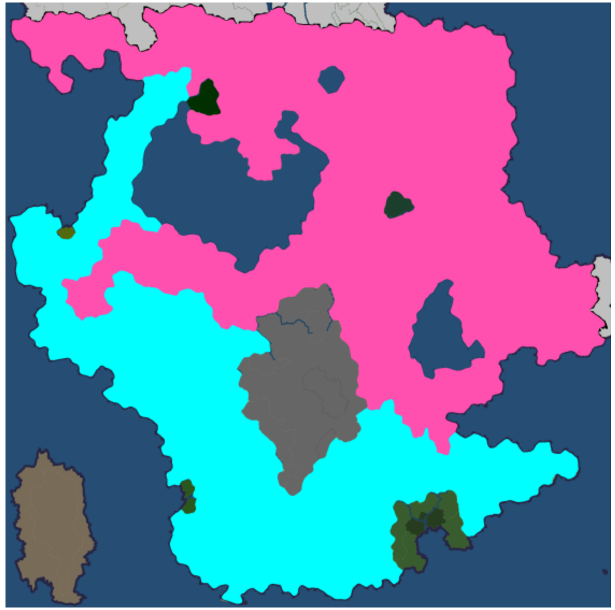

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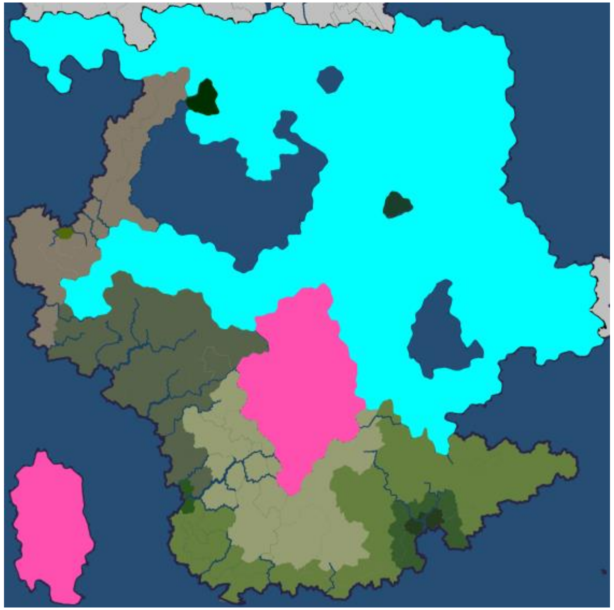

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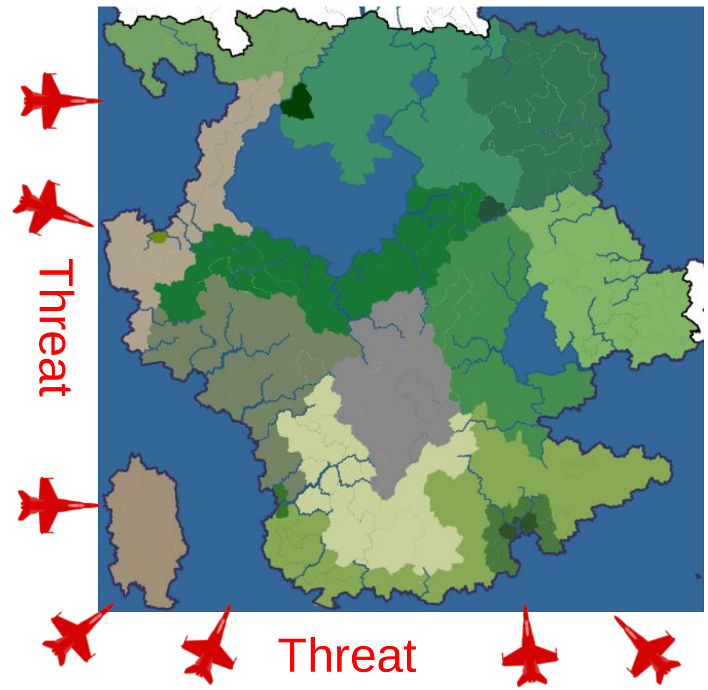

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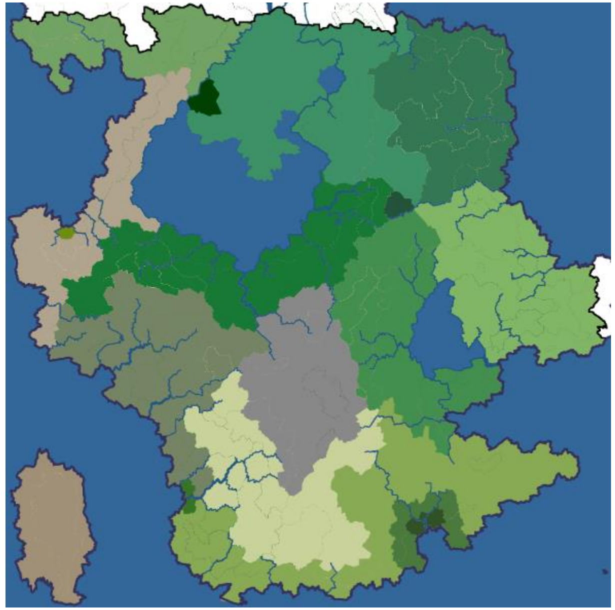
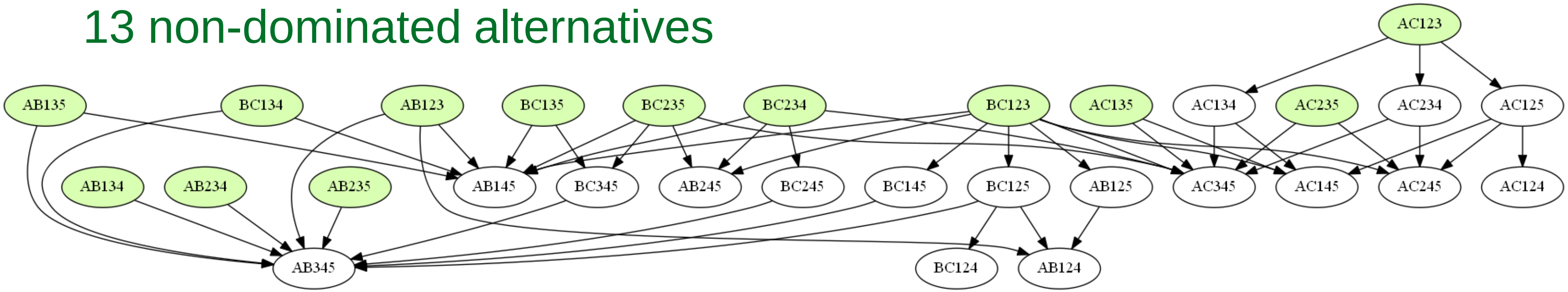
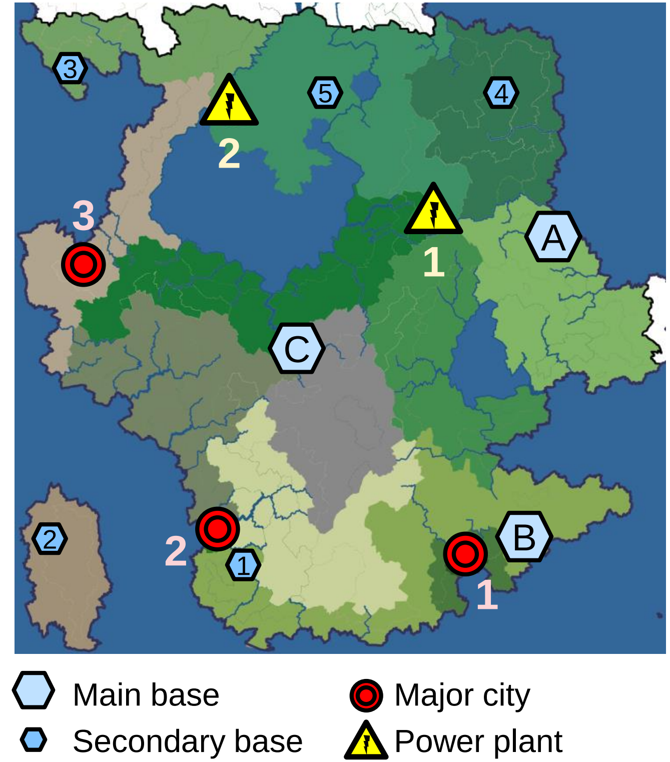


13 non-dominated alternatives

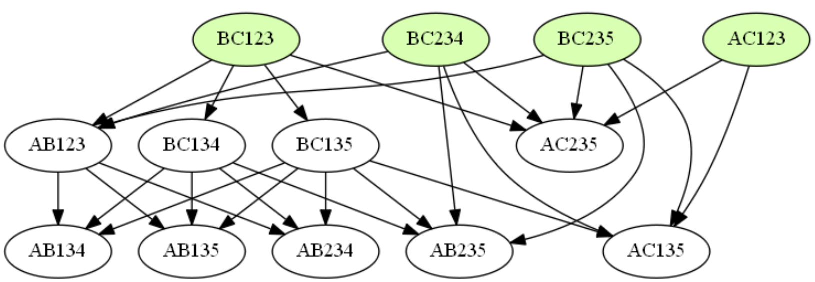


Additional Preference Statements

- Spatial preference statements (α)
  - Power plants > Major cities
  - Power plant #1 > Power plant #2
  - City #1 > City #2 > City #3
  - ...
- Attribute preference statements (b)
  - West engagement frontier > South engagement frontier
  - Force sustainability > Initial force fulfillment



4 non-dominated alternatives



Conclusion: Spatial Decision Analysis Benefits from Preference Programming

- The additive spatial value function
  - Axiomatic basis
  - Weighting subregions rather than locations
- Preference programming for spatial decision analysis
  - Incomplete preference information & non-dominated decision alternatives

    ⇨
    - Burden of DM eased considerably by not requiring unique spatial weighting
    - Global sensitivity analysis: Effect of spatial weighting on ranking of alternatives
- Future development
  - Practices and behavioral issues of eliciting the weighting function
  - Spatial decision support systems: Graphical user interface, utilization of GIS data

References

- Ferretti, V. and Montibeller, G., 2016. Key challenges and meta-choices in designing and applying multi-criteria spatial decision support systems. *Decision Support Systems*, *84*
- Malczewski, J. and Rinner, C., 2015. *Multicriteria decision analysis in geographic information science*. New York: Springer
- Salo, A. and Hämäläinen, R.P., 1992. Preference assessment by imprecise ratio statements. *Operations Research*, *40*(6)
- Savage, L.J., 1954. *The foundations of statistics*. New York: John Wiley and Sons
- Simon, J., Kirkwood, C.W. and Keller, L.R., 2014. Decision analysis with geographically varying outcomes: Preference models and illustrative applications. *Operations Research*, *62*(1)