# The traveling salesman problem with pickup, delivery and ride-time constraints: Detailed computational results

Enrico Bartolini
*Department of Mathematics and Systems Analysis, Aalto University School of Science, P.O. Box 11100, 00076 Aalto, Finland, enrico.bartolini@aalto.fi*

Lawrence Bodin
*Department of Decisions, Operations and Information Technologies, Van Munching Hall, R. H. Smith School of Business, University of Maryland, College Park, MD, USA, lbodin@rhsmith.umd.edu*

Aristide Mingozzi
*Department of Mathematics, University of Bologna, Via Sacchi 3, 47521 Cesena, Italy, aristide.mingozzi@unibo.it*

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In this document, we report detailed computational results obtained by the two exact algorithms BM1 and BM2 which are described in Sections 4 and 5 of the paper Bartolini et al. [1]. The algorithms are tested on two sets of uncapacitated instances, and three sets of capacitated ones. For a detailed description of the instances characteristics, and how they were generated we refer to Section 7.1 of the paper Bartolini et al. [1]. The algorithms were coded in C, and compiled under Visual Studio 2008. CPLEX 12.1 [3] was used as the LP solver in procedures CG1 and CG2, and as the IP solver for the reduced problem F1 in BM1. All the computational experiments were run on an Intel Xeon X7350 Workstation clocked at 2.93 GHz with 16Gb RAM running Windows Server 2008 x64 Edition. All the computing times reported in this paper are in seconds.

# 1 Detailed computational results

Tables 1 – 4 of this section report the results obtained by each algorithm BM1 and BM2 on the uncapacitated instances *drcl*, and *rbl*, whereas Tables 5 – 10 report the results obtained on the capacitated instances *drcl-Q3*, *rbl-Q3*, and *cord-a*.

The first three columns of Tables 1 – 10 report each instance name, the corresponding ride-time coefficient $\delta$, and the best known upper bound $z_{UB}$ (values in bold indicate that $z_{UB}$ corresponds to the cost of an optimal solution). We also report under column $\%_{LNS}$ the percentage distance of the upper bound $z_{LNS}$ found by the LNS heuristics (see Section 8 of [1]) from the best known upper bound $z_{UB}$ ($z_{UB}$ is the best upper bound found by either the LNS or the exact methods) computed as $(z_{LNS} - z_{UB})/z_{UB} \cdot 100$. For instances *drcl*, and *drcl-Q3*, derived from the instances used in [2], the names are formatted as "$n\alpha$" where $n$ indicates the number of requests involved, and $\alpha \in \{a, b, c, d, e\}$ is used to distinguish each instance among those involving the same number of requests. Each instance "$n\alpha$" is obtained from instance "prob$n\alpha$" used in [2]. For instances *rbl*, and *rbl-Q3* the names have the form "$\Psi n\alpha$" where $\Psi$ is the original TSPLIB name, $n$ is the number of requests involved, and $\alpha$ is a letter used to distinguish the rule that was used to pair the pickup and delivery vertices. Each instance $\Psi n\alpha$ is obtained from the homonym instance used in [2]. The names of the instances *cord-a* have the form "$aK\_n$" where $K$ indicates the number of vehicles in the original DARP instances, and $n$ is the number of requests. Notice that for instances *cord-a*, column "$\delta$" reports the average ride-time coefficient which is computed as the average ratio $L_i/euc(i, n+i)$ over all the requests $i \in O$.

The remaining columns of Tables 1 – 10 report the following data. The lower bounds $z$(LF1), and $LB1$ computed by procedure CG1 (columns "$z$(LF1)" and "$LB1$"). The lower bounds $LH2$, and $LB2$ obtained by H2, and CG2, respectively (columns "$LH2$" and "$LB2$"). The percentage ratios of lower bound $z$(LF1) with respect to the upper bound $z_{UB}$ (columns "%$z$(LF1)"), the percentage ratio of lower bounds $LBx$, $x \in \{1, 2\}$ (columns "%$LBx$", $x \in \{1, 2\}$), and the percentage ratio of lower bound $LH2$ (columns "%$LH2$"). The time spent by procedure H2 to compute the lower bound $LH2$ (columns "$t_{LH2}$"). The time spent by procedure CGx, $x \in \{1, 2\}$ to compute the lower bound $LBx$ (columns "$t_{LB1}$", "$t_{LB2}$"). The total number of partially feasible paths generated by CG1 at the beginning of Stage 2 for the exact algorithm BM1 (columns "$|\tilde{\mathscr{P}}|$"). The total number of clique inequalities separated by CG1 (columns "CLQ"). The total number of subtour elimination constraints separated by CG1 (columns "SEC"). The total number of lifted infeasible path constraints separated by CG2 (columns "LPC"). The total number of branch-and-bound nodes explored for the exact algorithm BM2 (columns "Nds"). The best lower bound $LB^*$ at the end of each exact algorithm (columns "$LB^*$"), and the percentage ratio of $LB^*$ with respect to the upper bound $z_{UB}$ (columns "%$LB^*$"). Finally, columns "*Time*" report the total computing time. The last line (line "Average") of each table reports the average values of the lower bound ratios and of the computing times. Average times for column "*Time*" in row "Average" are computed over all the instances solved to optimality.

All times reported in Tables 1 – 10 include the time spent for computing the upper bound $z_{UB}$ by using the LNS heuristic described in Section 6 of Bartolini et al. [1]. An entry *tl* under columns "$t_{LB1}$", "$t_{LH2}$", or "$t_{LB2}$" indicates that the exact algorithm terminates upon reaching the imposed time limit of three hours. Moreover, an entry "$\Delta_P^{max}$" under column "$|\tilde{\mathscr{P}}|$" indicates that the exact algorithm BM1 terminates prematurely at the beginning of Stage 2 of CG1 because the number of partially feasible paths generated by Genpaths exceeds the maximum allowed $\Delta_P^{max}$. Finally, an entry "*m.o.*" in column "$z$(LF1)" indicates that the exact algorithm terminates prematurely while executing CG1 because the algorithm Genpaths (see Section 4.3.3 of Bartolini et al. [1]) runs out of memory.

We also note that when allowed for a five hours time limit, algorithm BM2 was able to find an additional 10 optima with respect to those reported in Tables 1 – 10. Specifically, the additional instances solved to optimality, and the corresponding total computing times are the following. For data set *drcl*: instance 20$a$ with $\delta = 1.75$ in 11237 seconds, instance 25$d$ with $\delta = 1.75$ in 16830 seconds, instance 30$a$ with $\delta = 1.75$ in 14025 seconds, instance 30$b$ with $\delta = 2.00$ in 27213 seconds, instance 30$c$ with $\delta = 1.75$ in 13714 seconds, instance

30$d$ with $\delta = 1.75$ in 17084 seconds. For data set *drcl-Q3*: instance 35$a$ with $\delta = 1.50$ in 12251 seconds, instance 35$d$ with $\delta = 1.50$ in 16860 seconds, instance 30$a$ with $\delta = 2.00$ in 12855 seconds, and instance 30$b$ with $\delta = 2.00$ in 11063 seconds.

# References


[1] E. Bartolini, L. D. Bodin, and A. Mingozzi. The traveling salesman problem with pickup, delivery and ride-time constraints. *Networks*, (forthcoming).

[2] I. Dumitrescu, S. Røpke, J.-F. Cordeau, and G. Laporte. The traveling salesman problem with pickup and delivery. *Math. Program., Ser. A*, 121:269–305, 2010.

[3] IBM ILOG. *IBM ILOG CPLEX V12.1 User's Manual for CPLEX*. IBM Corporation, 2009.

Table 1: Detailed results: Exact algorithm BM1 on uncapacitated *drcl* instances

| Instance | | | | CG1 | | | | | branch-and-cut | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $z$(LF1) | $\%z$(LF1) | $LB1$ | $\%LB1$ | $t_{LB1}$ | $\lvert\tilde{\mathscr{P}}_1\rvert$ | CLQ | SEC | $LB^*$ | $\%LB^*$ | *Time* |
| 20a | 1.50 | **9075** | 2.92 | 8426 | 92.85 | 8822 | 97.21 | 10 | 1886 | 146 | - | 9075 | 100.00 | 14 |
| 20b | 1.50 | **8849** | 0.12 | 8849 | 100.00 | 8849 | 100.00 | 6 | 363 | - | 3 | 8849 | 100.00 | 6 |
| 20c | 1.50 | **8983** | 1.81 | 8983 | 100.00 | 8983 | 100.00 | 4 | 248 | - | 5 | 8983 | 100.00 | 4 |
| 20d | 1.50 | **9216** | 6.41 | 9187 | 99.69 | 9216 | 100.00 | 4 | 597 | 10 | 4 | 9216 | 100.00 | 4 |
| 20e | 1.50 | **9335** | 2.40 | 8790 | 94.16 | 9168 | 98.21 | 6 | 897 | 198 | 6 | 9335 | 100.00 | 8 |
| 25a | 1.50 | **10861** | 1.66 | 10559 | 97.22 | 10685 | 98.38 | 6 | 522 | 17 | 12 | 10861 | 100.00 | 7 |
| 25b | 1.50 | **11453** | 0.37 | 11079 | 96.73 | 11392 | 99.47 | 8 | 788 | 161 | 3 | 11453 | 100.00 | 8 |
| 25c | 1.50 | **11360** | 4.08 | 11264 | 99.15 | 11356 | 99.96 | 7 | 1158 | 10 | 10 | 11360 | 100.00 | 8 |
| 25d | 1.50 | **11859** | 1.09 | 11305 | 95.33 | 11596 | 97.78 | 10 | 1337 | 126 | 9 | 11859 | 100.00 | 11 |
| 25e | 1.50 | **10751** | 6.93 | 10268 | 95.51 | 10499 | 97.66 | 10 | 1841 | 248 | 5 | 10751 | 100.00 | 19 |
| 30a | 1.50 | **12089** | 4.59 | 11688 | 96.68 | 11979 | 99.09 | 26 | 6321 | 224 | 1 | 12089 | 100.00 | 36 |
| 30b | 1.50 | **10715** | 3.19 | 10550 | 98.46 | 10679 | 99.66 | 15 | 2345 | 130 | 7 | 10715 | 100.00 | 16 |
| 30c | 1.50 | **13254** | 3.06 | 12758 | 96.26 | 13114 | 98.94 | 13 | 2351 | 212 | 3 | 13254 | 100.00 | 16 |
| 30d | 1.50 | **13517** | 0.83 | 12811 | 94.78 | 13277 | 98.22 | 12 | 1703 | 227 | 2 | 13517 | 100.00 | 16 |
| 30e | 1.50 | **13291** | 6.88 | 12251 | 92.18 | 12972 | 97.60 | 32 | 4712 | 321 | 9 | 13291 | 100.00 | 60 |
| 35a | 1.50 | **14128** | 1.27 | 12549 | 88.82 | 13467 | 95.32 | 141 | 10604 | 683 | 22 | 14128 | 100.00 | 1157 |
| 35b | 1.50 | **13434** | 3.16 | 12595 | 93.75 | 13096 | 97.48 | 27 | 4134 | 231 | 10 | 13434 | 100.00 | 38 |
| 35c | 1.50 | **14055** | 4.73 | 13323 | 94.79 | 13713 | 97.57 | 30 | 6006 | 290 | 43 | 14055 | 100.00 | 211 |
| 35d | 1.50 | 13446 | 2.44 | 12035 | 89.51 | 12649 | 94.07 | 486 | 28995 | 228 | 34 | 13083 | 97.30 | *tl* |
| 35e | 1.50 | **14459** | 2.79 | 13572 | 93.87 | 13973 | 96.64 | 30 | 4475 | 436 | 23 | 14459 | 100.00 | 210 |
| Average | | | | | 95.49 | | 98.16 | 44 | | | | | 99.87 | 97 |
| 20a | 1.75 | **8193** | 0.02 | 6832 | 83.39 | 7359 | 89.82 | 1047 | 31136 | - | - | 7858 | 95.91 | *tl* |
| 20b | 1.75 | **8650** | 7.09 | 8023 | 92.75 | 8386 | 96.95 | 30 | 5387 | 447 | 10 | 8650 | 100.00 | 108 |
| 20c | 1.75 | **8476** | 2.55 | 8077 | 95.29 | 8414 | 99.27 | 7 | 1600 | 113 | 7 | 8476 | 100.00 | 8 |
| 20d | 1.75 | **8943** | 1.08 | 8445 | 94.43 | 8825 | 98.68 | 15 | 2818 | 296 | 9 | 8943 | 100.00 | 23 |
| 20e | 1.75 | **8496** | 6.36 | 7892 | 92.89 | 8253 | 97.14 | 36 | 6930 | 223 | 5000 | 8496 | 100.00 | 111 |
| 25a | 1.75 | **10154** | 0.69 | 9652 | 95.06 | 9954 | 98.03 | 14 | 2892 | 132 | 10 | 10154 | 100.00 | 23 |
| 25b | 1.75 | **10578** | 0.81 | 9795 | 92.60 | 10158 | 96.03 | 41 | 8886 | 293 | 3 | 10578 | 100.00 | 192 |
| 25c | 1.75 | **10942** | 1.34 | 10262 | 93.79 | 10559 | 96.50 | 116 | 20110 | 285 | 22 | 10942 | 100.00 | 1291 |
| 25d | 1.75 | **10697** | 3.00 | 9577 | 89.53 | 10083 | 94.26 | 292 | 15824 | 337 | 11 | 10697 | 100.00 | 1128 |
| 25e | 1.75 | **9704** | 1.94 | 8825 | 90.94 | 9239 | 95.21 | 200 | 22755 | 399 | 4 | 9704 | 100.00 | 3611 |
| 30a | 1.75 | **10825** | 1.46 | 9824 | 90.75 | 9824 | 90.75 | 101 | $\Delta_P^{max}$ | - | 2 | 9824 | 90.75 | 101 |
| 30b | 1.75 | **10100** | 1.63 | 9328 | 92.36 | 9716 | 96.20 | 826 | 38221 | 509 | 14 | 10100 | 100.00 | 3710 |
| 30c | 1.75 | **12205** | 3.74 | 10932 | 89.57 | 11549 | 94.63 | 489 | 30692 | - | - | 11737 | 96.17 | *tl* |
| 30d | 1.75 | **12185** | 4.51 | 11235 | 92.20 | 11625 | 95.40 | 304 | 24739 | 167 | 5 | 12185 | 100.00 | 4506 |
| 30e | 1.75 | 12471 | 0.00 | 10282 | 82.45 | 10282 | 82.45 | 80 | $\Delta_P^{max}$ | - | 7 | 10282 | 82.45 | 80 |
| 35a | 1.75 | 12662 | 0.00 | 10359 | 81.81 | 10359 | 81.81 | 953 | $\Delta_P^{max}$ | - | 8 | 10359 | 81.81 | 953 |
| 35b | 1.75 | 12386 | 0.00 | 11032 | 89.07 | 11032 | 89.07 | 86 | $\Delta_P^{max}$ | - | 6 | 11032 | 89.07 | 86 |
| 35c | 1.75 | 13386 | 0.00 | 11234 | 83.92 | 11234 | 83.92 | 239 | $\Delta_P^{max}$ | - | 11 | 11234 | 83.92 | 239 |
| 35d | 1.75 | 12421 | 0.00 | 10474 | 84.32 | 10474 | 84.32 | 1119 | $\Delta_P^{max}$ | - | 4 | 10474 | 84.32 | 1119 |
| 35e | 1.75 | 13943 | 0.00 | 11884 | 85.23 | 11884 | 85.23 | 96 | $\Delta_P^{max}$ | - | 9 | 11884 | 85.23 | 96 |
| Average | | | | | 89.62 | | 92.28 | 305 | | | | | 94.48 | 1337 |
| 20a | 2.00 | **7429** | 0.00 | 6302 | 84.83 | 6302 | 84.83 | 153 | $\Delta_P^{max}$ | - | 3 | 6302 | 84.83 | 153 |
| 20b | 2.00 | **8398** | 4.08 | 7379 | 87.87 | 7379 | 87.87 | 62 | $\Delta_P^{max}$ | - | 5 | 7379 | 87.87 | 62 |
| 20c | 2.00 | **7913** | 9.05 | 7482 | 94.55 | 7910 | 99.96 | 49 | 11160 | 245 | 7 | 7913 | 100.00 | 50 |
| 20d | 2.00 | **8541** | 7.14 | 7549 | 88.39 | 8123 | 95.11 | 686 | 37443 | 159 | 7 | 8541 | 100.00 | 1555 |
| 20e | 2.00 | **8197** | 0.00 | 7332 | 89.45 | 7332 | 89.45 | 52 | $\Delta_P^{max}$ | - | 5 | 7332 | 89.45 | 52 |
| 25a | 2.00 | **9579** | 0.99 | 8762 | 91.47 | 9133 | 95.34 | 280 | 29231 | 162 | 1 | 9579 | 100.00 | 1138 |
| 25b | 2.00 | **9828** | 0.33 | 8847 | 90.02 | 8847 | 90.02 | 64 | $\Delta_P^{max}$ | - | 4 | 8847 | 90.02 | 64 |
| 25c | 2.00 | **9997** | 0.00 | 9047 | 90.50 | 9047 | 90.50 | 323 | $\Delta_P^{max}$ | - | 5 | 9047 | 90.50 | 323 |
| 25d | 2.00 | 10080 | 4.67 | 8752 | 86.83 | 8752 | 86.83 | 196 | $\Delta_P^{max}$ | - | 4 | 8752 | 86.83 | 196 |
| 25e | 2.00 | 9505 | 1.42 | 8045 | 84.64 | 8045 | 84.64 | 305 | $\Delta_P^{max}$ | - | 4 | 8045 | 84.64 | 305 |
| 30a | 2.00 | 10478 | 0.00 | 9072 | 86.58 | 9072 | 86.58 | 2094 | $\Delta_P^{max}$ | - | 3 | 9072 | 86.58 | 2094 |
| 30b | 2.00 | **9664** | 0.21 | 8374 | 86.65 | 8374 | 86.65 | 666 | $\Delta_P^{max}$ | - | 13 | 8374 | 86.65 | 666 |
| 30c | 2.00 | 11505 | 0.00 | 9508 | 82.64 | 9508 | 82.64 | 593 | $\Delta_P^{max}$ | - | 2 | 9508 | 82.64 | 593 |
| 30d | 2.00 | 11442 | 4.68 | 9737 | 85.10 | 9737 | 85.10 | 570 | $\Delta_P^{max}$ | - | 3 | 9737 | 85.10 | 570 |
| 30e | 2.00 | 11413 | 0.00 | 9072 | 79.49 | 9072 | 79.49 | 1651 | $\Delta_P^{max}$ | - | 5 | 9072 | 79.49 | 1651 |
| 35a | 2.00 | 12138 | 0.00 | *m.o.* | - | - | - | 3677 | - | - | - | - | - | 3677 |
| 35b | 2.00 | 11779 | 0.00 | 9964 | 84.59 | 9964 | 84.59 | 2103 | $\Delta_P^{max}$ | - | 8 | 9964 | 84.59 | 2103 |
| 35c | 2.00 | 12337 | 0.00 | - | - | - | - | *tl* | - | - | - | - | - | *tl* |
| 35d | 2.00 | 11507 | 0.00 | *m.o.* | - | - | - | 3827 | - | - | - | - | - | 3827 |
| 35e | 2.00 | 12722 | 0.00 | 10523 | 82.71 | 10523 | 82.71 | 1610 | $\Delta_P^{max}$ | - | 9 | 10523 | 82.71 | 1610 |
| Average | | | | | 86.84 | | 87.78 | 1490 | | | | | 88.35 | 914 |

*m.o.*: Procedure CG1 terminates prematurely because algorithm Genpaths runs out of memory.

Table 2: Detailed results: Exact algorithm BM2 on uncapacitated *drcl* instances

| Instance | | | | H2 | | | CG2 | | | branch-and-cut-and-price | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $LH2$ | $\%LH2$ | $t_{LH2}$ | $LB2$ | $\%LB2$ | $t_{LB2}$ | Nds | LPC | $LB^*$ | $\%LB^*$ | $Time$ |
| 20a | 1.50 | **9075** | 2.92 | 8669 | 95.53 | 5 | 8751 | 96.43 | 32 | 113 | 37 | 9075 | 100.00 | 163 |
| 20b | 1.50 | **8849** | 0.12 | 8815 | 99.62 | 4 | 8817 | 99.64 | 49 | 9 | 10 | 8849 | 100.00 | 67 |
| 20c | 1.50 | **8983** | 1.81 | 8983 | 100.00 | 4 | 8983 | 100.00 | 4 | 1 | - | 8983 | 100.00 | 5 |
| 20d | 1.50 | **9216** | 6.41 | 9208 | 99.91 | 4 | 9216 | 100.00 | 26 | 1 | 1 | 9216 | 100.00 | 27 |
| 20e | 1.50 | **9335** | 2.40 | 9194 | 98.49 | 5 | 9194 | 98.49 | 4 | 29 | 3 | 9335 | 100.00 | 22 |
| 25a | 1.50 | **10861** | 1.66 | 10673 | 98.27 | 6 | 10676 | 98.30 | 16 | 21 | 5 | 10861 | 100.00 | 42 |
| 25b | 1.50 | **11453** | 0.37 | 11356 | 99.15 | 6 | 11373 | 99.30 | 21 | 33 | 16 | 11453 | 100.00 | 60 |
| 25c | 1.50 | **11360** | 4.08 | 11210 | 98.68 | 6 | 11220 | 98.77 | 65 | 7 | 2 | 11360 | 100.00 | 87 |
| 25d | 1.50 | **11859** | 1.09 | 11559 | 97.47 | 7 | 11568 | 97.55 | 15 | 171 | 19 | 11859 | 100.00 | 222 |
| 25e | 1.50 | **10751** | 6.93 | 10291 | 95.72 | 7 | 10292 | 95.73 | 7 | 303 | 66 | 10751 | 100.00 | 412 |
| 30a | 1.50 | **12089** | 4.59 | 11674 | 96.57 | 11 | 11679 | 96.61 | 11 | 231 | 84 | 12089 | 100.00 | 879 |
| 30b | 1.50 | **10715** | 3.19 | 10672 | 99.60 | 9 | 10715 | 100.00 | 35 | 1 | 2 | 10715 | 100.00 | 36 |
| 30c | 1.50 | **13254** | 3.06 | 13170 | 99.37 | 10 | 13175 | 99.40 | 26 | 11 | 7 | 13254 | 100.00 | 60 |
| 30d | 1.50 | **13517** | 0.83 | 13206 | 97.70 | 10 | 13209 | 97.72 | 10 | 177 | 14 | 13517 | 100.00 | 598 |
| 30e | 1.50 | **13291** | 6.88 | 12797 | 96.28 | 12 | 12802 | 96.32 | 23 | 1181 | 70 | 13291 | 100.00 | 3704 |
| 35a | 1.50 | **14128** | 1.27 | 13408 | 94.90 | 19 | 13412 | 94.93 | 21 | 1330 | 90 | 13957 | 98.79 | *tl* |
| 35b | 1.50 | **13434** | 3.16 | 13106 | 97.56 | 15 | 13109 | 97.58 | 25 | 1001 | 70 | 13434 | 100.00 | 6339 |
| 35c | 1.50 | **14055** | 4.73 | 13527 | 96.24 | 17 | 13585 | 96.66 | 67 | 1031 | 80 | 14055 | 100.00 | 6428 |
| 35d | 1.50 | 13446 | 2.44 | 12509 | 93.03 | 30 | 12514 | 93.07 | 54 | 871 | 123 | 12946 | 96.28 | *tl* |
| 35e | 1.50 | **14459** | 2.79 | 14095 | 97.48 | 16 | 14096 | 97.49 | 16 | 1425 | 116 | 14459 | 100.00 | 7793 |
| Average | | | | | 97.58 | 10 | | 97.70 | 26 | | | | 99.75 | 1497 |
| 20a | 1.75 | **8193** | 0.02 | 7554 | 92.20 | 10 | 7567 | 92.36 | 18 | 3402 | 998 | 8179 | 99.83 | *tl* |
| 20b | 1.75 | **8650** | 7.09 | 8374 | 96.81 | 6 | 8374 | 96.81 | 5 | 143 | 22 | 8650 | 100.00 | 106 |
| 20c | 1.75 | **8476** | 2.55 | 8179 | 96.50 | 4 | 8206 | 96.81 | 10 | 15 | 18 | 8476 | 100.00 | 24 |
| 20d | 1.75 | **8943** | 1.08 | 8765 | 98.01 | 5 | 8765 | 98.01 | 5 | 9 | - | 8943 | 100.00 | 14 |
| 20e | 1.75 | **8496** | 6.36 | 8345 | 98.22 | 6 | 8346 | 98.23 | 8 | 11 | 6 | 8496 | 100.00 | 23 |
| 25a | 1.75 | **10154** | 0.69 | 9879 | 97.29 | 7 | 9880 | 97.30 | 7 | 129 | 32 | 10154 | 100.00 | 173 |
| 25b | 1.75 | **10578** | 0.81 | 10247 | 96.87 | 8 | 10253 | 96.93 | 14 | 345 | 145 | 10578 | 100.00 | 718 |
| 25c | 1.75 | **10942** | 1.34 | 10613 | 96.99 | 12 | 10614 | 97.00 | 11 | 157 | 13 | 10942 | 100.00 | 297 |
| 25d | 1.75 | **10697** | 3.00 | 10178 | 95.15 | 10 | 10184 | 95.20 | 10 | 3816 | 444 | 10666 | 99.71 | *tl* |
| 25e | 1.75 | **9704** | 1.94 | 9350 | 96.35 | 14 | 9350 | 96.35 | 14 | 229 | 46 | 9704 | 100.00 | 636 |
| 30a | 1.75 | **10825** | 1.46 | 10371 | 95.81 | 44 | 10372 | 95.82 | 45 | 926 | 347 | 10804 | 99.81 | *tl* |
| 30b | 1.75 | **10100** | 1.63 | 9601 | 95.06 | 17 | 9658 | 95.62 | 29 | 955 | 164 | 10100 | 100.00 | 5507 |
| 30c | 1.75 | **12205** | 3.74 | 11496 | 94.19 | 17 | 11592 | 94.98 | 124 | 1379 | 238 | 12173 | 99.74 | *tl* |
| 30d | 1.75 | **12185** | 4.51 | 11534 | 94.66 | 20 | 11556 | 94.84 | 44 | 1365 | 142 | 12133 | 99.57 | *tl* |
| 30e | 1.75 | 12471 | 0.00 | 10923 | 87.59 | 31 | 11040 | 88.53 | 54 | 1031 | 56 | 11477 | 92.03 | *tl* |
| 35a | 1.75 | 12662 | 0.00 | 11353 | 89.66 | 134 | 11354 | 89.67 | 139 | 301 | 32 | 11786 | 93.08 | *tl* |
| 35b | 1.75 | 12386 | 0.00 | 11439 | 92.35 | 37 | 11455 | 92.48 | 103 | 595 | 117 | 11925 | 96.28 | *tl* |
| 35c | 1.75 | 13386 | 0.00 | 12362 | 92.35 | 61 | 12363 | 92.36 | 65 | 401 | 46 | 12737 | 95.15 | *tl* |
| 35d | 1.75 | 12421 | 0.00 | 11170 | 89.93 | 205 | 11173 | 89.95 | 224 | 171 | 128 | 11509 | 92.66 | *tl* |
| 35e | 1.75 | 13943 | 0.00 | 12672 | 90.88 | 42 | 12673 | 90.89 | 44 | 852 | 102 | 13110 | 94.03 | *tl* |
| Average | | | | | 94.34 | 35 | | 94.51 | 49 | | | | 98.09 | 833 |
| 20a | 2.00 | **7429** | 0.00 | 7050 | 94.90 | 36 | 7051 | 94.91 | 38 | 1011 | 538 | 7429 | 100.00 | 4646 |
| 20b | 2.00 | **8398** | 4.08 | 7964 | 94.83 | 33 | 7965 | 94.84 | 33 | 589 | 201 | 8398 | 100.00 | 816 |
| 20c | 2.00 | **7913** | 9.05 | 7770 | 98.19 | 7 | 7777 | 98.28 | 9 | 9 | 7 | 7913 | 100.00 | 24 |
| 20d | 2.00 | **8541** | 7.14 | 8065 | 94.43 | 13 | 8065 | 94.43 | 14 | 299 | 76 | 8541 | 100.00 | 636 |
| 20e | 2.00 | **8197** | 0.00 | 8009 | 97.71 | 17 | 8010 | 97.72 | 16 | 113 | 61 | 8197 | 100.00 | 333 |
| 25a | 2.00 | **9579** | 0.99 | 9272 | 96.80 | 12 | 9277 | 96.85 | 16 | 587 | 151 | 9579 | 100.00 | 1561 |
| 25b | 2.00 | **9828** | 0.33 | 9538 | 97.05 | 23 | 9539 | 97.06 | 23 | 29 | 7 | 9828 | 100.00 | 121 |
| 25c | 2.00 | **9997** | 0.00 | 9828 | 98.31 | 125 | 9831 | 98.34 | 127 | 25 | 6 | 9997 | 100.00 | 378 |
| 25d | 2.00 | 10080 | 4.67 | 9561 | 94.85 | 77 | 9563 | 94.87 | 80 | 2041 | 607 | 9890 | 98.12 | *tl* |
| 25e | 2.00 | 9505 | 1.42 | 8945 | 94.11 | 106 | 8946 | 94.12 | 108 | 1088 | 263 | 9348 | 98.35 | *tl* |
| 30a | 2.00 | 10478 | 0.00 | 9618 | 91.79 | 205 | 9674 | 92.33 | 392 | 176 | 120 | 9835 | 93.86 | *tl* |
| 30b | 2.00 | **9664** | 0.21 | 9229 | 95.50 | 145 | 9230 | 95.51 | 147 | 772 | 121 | 9586 | 99.19 | *tl* |
| 30c | 2.00 | 11505 | 0.00 | 10813 | 93.99 | 109 | 10815 | 94.00 | 115 | 673 | 78 | 11233 | 97.64 | *tl* |
| 30d | 2.00 | 11442 | 4.68 | 10735 | 93.82 | 126 | 10735 | 93.82 | 127 | 763 | 68 | 11263 | 98.44 | *tl* |
| 30e | 2.00 | 11413 | 0.00 | 10059 | 88.14 | 279 | 10069 | 88.22 | 417 | 183 | 54 | 10476 | 91.79 | *tl* |
| 35a | 2.00 | 12138 | 0.00 | 10619 | 87.49 | 1021 | 10620 | 87.49 | 1042 | 53 | 16 | 10823 | 89.17 | *tl* |
| 35b | 2.00 | 11779 | 0.00 | 10599 | 89.98 | 339 | 10600 | 89.99 | 345 | 143 | 38 | 10863 | 92.22 | *tl* |
| 35c | 2.00 | 12337 | 0.00 | 11058 | 89.63 | 569 | 11066 | 89.70 | 706 | 49 | 36 | 11346 | 91.97 | *tl* |
| 35d | 2.00 | 11507 | 0.00 | 10253 | 89.10 | 755 | 10262 | 89.18 | 948 | 42 | 28 | 10403 | 90.41 | *tl* |
| 35e | 2.00 | 12722 | 0.00 | 11402 | 89.62 | 239 | 11418 | 89.75 | 268 | 334 | 64 | 11738 | 92.27 | *tl* |
| Average | | | | | 93.51 | 212 | | 93.57 | 249 | | | | 96.67 | 1064 |

Table 3: Detailed results: Exact algorithm BM1 on uncapacitated *rbl* instances

| Instance | | | | CG1 | | | | | | branch-and-cut | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $z$(LF1) | $\%z$(LF1) | $LB1$ | $\%LB1$ | $t_{LB1}$ | $\lvert\tilde{\mathscr{P}}_1\rvert$ | CLQ | SEC | $LB^*$ | $\%LB^*$ | *Time* |
| EIL51A | 1.50 | **678** | 3.39 | 677 | 99.85 | 677 | 99.85 | 4 | 41 | - | 21 | 678 | 100.00 | 4 |
| EIL51B | 1.50 | **729** | 2.74 | 725 | 99.45 | 725 | 99.45 | 4 | 94 | - | 10 | 729 | 100.00 | 4 |
| EIL51C | 1.50 | **783** | 1.15 | 757 | 96.68 | 772 | 98.60 | 10 | 1752 | 118 | 16 | 783 | 100.00 | 16 |
| EIL75A | 1.50 | **787** | 0.64 | 786 | 99.87 | 786 | 99.87 | 8 | 155 | - | 13 | 787 | 100.00 | 8 |
| EIL75B | 1.50 | **952** | 4.41 | 943 | 99.05 | 950 | 99.79 | 11 | 463 | 2 | 10 | 952 | 100.00 | 11 |
| EIL75C | 1.50 | **1082** | 5.55 | 1041 | 96.21 | 1063 | 98.24 | 24 | 3466 | 247 | 10 | 1082 | 100.00 | 43 |
| EIL101A | 1.50 | **1010** | 3.27 | 1008 | 99.80 | 1008 | 99.80 | 16 | 359 | - | 28 | 1010 | 100.00 | 17 |
| EIL101B | 1.50 | **1093** | 4.85 | 1088 | 99.54 | 1089 | 99.63 | 15 | 220 | 5 | 36 | 1093 | 100.00 | 15 |
| EIL101C | 1.50 | **1299** | 3.70 | 1211 | 93.23 | 1266 | 97.46 | 133 | 13344 | 260 | 9 | 1299 | 100.00 | 973 |
| KROA99A | 1.50 | **34412** | 6.15 | 34108 | 99.12 | 34308 | 99.70 | 19 | 675 | 20 | 34 | 34412 | 100.00 | 19 |
| KROA99B | 1.50 | **42034** | 9.41 | 41224 | 98.07 | 41545 | 98.84 | 42 | 10724 | 54 | 11 | 42034 | 100.00 | 44 |
| KROA99C | 1.50 | 50324 | 0.00 | 39789 | 79.07 | 39789 | 79.07 | 2013 | $\Delta_P^{max}$ | - | 7 | 39789 | 79.07 | 2013 |
| KROC99A | 1.50 | **35188** | 3.83 | 34724 | 98.68 | 34765 | 98.80 | 14 | 211 | 10 | 42 | 35188 | 100.00 | 15 |
| KROC99B | 1.50 | 41948 | 0.00 | 38856 | 92.63 | 38856 | 92.63 | 184 | $\Delta_P^{max}$ | - | 17 | 38856 | 92.63 | 184 |
| KROC99C | 1.50 | 47897 | 0.00 | 40271 | 84.08 | 40271 | 84.08 | 6191 | $\Delta_P^{max}$ | - | 7 | 40271 | 84.08 | 6191 |
| KROD99A | 1.50 | **32926** | 1.38 | 32840 | 99.74 | 32893 | 99.90 | 18 | 627 | 4 | 23 | 32926 | 100.00 | 19 |
| KROD99B | 1.50 | **40242** | 8.56 | 39519 | 98.20 | 40050 | 99.52 | 34 | 4816 | 70 | 13 | 40242 | 100.00 | 35 |
| KROD99C | 1.50 | 49036 | 0.00 | 42103 | 85.86 | 42103 | 85.86 | 561 | $\Delta_P^{max}$ | - | 6 | 42103 | 85.86 | 561 |
| Average | | | | | 95.51 | | 96.17 | 517 | | | | | 96.76 | 87 |
| EIL51A | 1.75 | **649** | 4.01 | 645 | 99.38 | 648 | 99.85 | 4 | 84 | 6 | 19 | 649 | 100.00 | 4 |
| EIL51B | 1.75 | **689** | 3.77 | 677 | 98.26 | 679 | 98.55 | 6 | 224 | 10 | 15 | 689 | 100.00 | 6 |
| EIL51C | 1.75 | **721** | 4.72 | 680 | 94.31 | 705 | 97.78 | 84 | 14103 | 138 | 11 | 721 | 100.00 | 210 |
| EIL75A | 1.75 | **760** | 2.24 | 745 | 98.03 | 753 | 99.08 | 15 | 768 | 52 | 27 | 760 | 100.00 | 15 |
| EIL75B | 1.75 | **883** | 4.87 | 871 | 98.64 | 882 | 99.89 | 20 | 2760 | 73 | 15 | 883 | 100.00 | 20 |
| EIL75C | 1.75 | 1064 | 0.00 | 877 | 82.42 | 877 | 82.42 | 152 | $\Delta_P^{max}$ | - | 1 | 877 | 82.42 | 152 |
| EIL101A | 1.75 | **970** | 5.05 | 962 | 99.18 | 964 | 99.38 | 27 | 2017 | 7 | 28 | 970 | 100.00 | 29 |
| EIL101B | 1.75 | **1033** | 1.84 | 1025 | 99.23 | 1026 | 99.32 | 22 | 852 | 12 | 29 | 1033 | 100.00 | 23 |
| EIL101C | 1.75 | 1196 | 0.00 | 1051 | 87.88 | 1051 | 87.88 | 2709 | $\Delta_P^{max}$ | - | 5 | 1051 | 87.88 | 2709 |
| KROA99A | 1.75 | **33471** | 5.25 | 31908 | 95.33 | 32743 | 97.82 | 50 | 7157 | 145 | 74 | 33471 | 100.00 | 112 |
| KROA99B | 1.75 | 41825 | 0.00 | 38244 | 91.44 | 38244 | 91.44 | 326 | $\Delta_P^{max}$ | - | 14 | 38244 | 91.44 | 326 |
| KROA99C | 1.75 | 44906 | 0.00 | *m.o.* | - | - | - | 1329 | - | - | - | - | - | 1329 |
| KROC99A | 1.75 | **33194** | 3.10 | 32870 | 99.02 | 33033 | 99.51 | 18 | 735 | 40 | 29 | 33194 | 100.00 | 18 |
| KROC99B | 1.75 | 38520 | 0.00 | *m.o.* | - | - | - | 5188 | - | - | - | - | - | 5188 |
| KROC99C | 1.75 | 42032 | 0.00 | *m.o.* | - | - | - | 820 | - | - | - | - | - | 820 |
| KROD99A | 1.75 | **32354** | 0.89 | 31877 | 98.53 | 31955 | 98.77 | 33 | 3984 | 29 | 34 | 32354 | 100.00 | 39 |
| KROD99B | 1.75 | 38003 | 0.00 | 35944 | 94.58 | 35944 | 94.58 | 97 | $\Delta_P^{max}$ | - | 10 | 35944 | 94.58 | 97 |
| KROD99C | 1.75 | 45223 | 0.00 | *m.o.* | - | - | - | 3442 | - | - | - | - | - | 3442 |
| Average | | | | | 95.44 | | 96.16 | 255 | | | | | 96.88 | 48 |

*m.o.*: Procedure CG1 terminates prematurely because algorithm Genpaths runs out of memory.

Table 4: Detailed results: Exact algorithm BM2 on uncapacitated *rbl* instances

| Instance | | | | H2 | | | CG3 | | | branch-and-cut-and-price | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $LH2$ | $\%LH2$ | $t_{LH2}$ | $LB2$ | $\%LB2$ | $t_{LB2}$ | Nds | LPC | $LB^*$ | $\%LB^*$ | $Time$ |
| EIL51A | 1.50 | **678** | 3.39 | 671 | 98.97 | 19 | 671 | 98.97 | 3 | 169 | 169 | 678 | 100.00 | 40 |
| EIL51B | 1.50 | **729** | 2.74 | 722 | 99.04 | 19 | 722 | 99.04 | 25 | 67 | 67 | 729 | 100.00 | 59 |
| EIL51C | 1.50 | **783** | 1.15 | 756 | 96.55 | 19 | 761 | 97.19 | 13 | 132 | 132 | 783 | 100.00 | 177 |
| EIL75A | 1.50 | **787** | 0.64 | 777 | 98.73 | 19 | 782 | 99.36 | 97 | 58 | 58 | 787 | 100.00 | 278 |
| EIL75B | 1.50 | **952** | 4.41 | 943 | 99.05 | 19 | 943 | 99.05 | 13 | 875 | 875 | 952 | 100.00 | 3765 |
| EIL75C | 1.50 | **1082** | 5.55 | 1057 | 97.69 | 38 | 1057 | 97.69 | 29 | 435 | 435 | 1082 | 100.00 | 2859 |
| EIL101A | 1.50 | **1010** | 3.27 | 1000 | 99.01 | 38 | 1002 | 99.21 | 197 | 67 | 67 | 1010 | 100.00 | 639 |
| EIL101B | 1.50 | **1093** | 4.85 | 1083 | 99.09 | 38 | 1083 | 99.09 | 28 | 1619 | 1619 | 1092 | 99.91 | *tl* |
| EIL101C | 1.50 | **1299** | 3.70 | 1241 | 95.54 | 95 | 1242 | 95.61 | 87 | 186 | 186 | 1268 | 97.61 | *tl* |
| KROA99A | 1.50 | **34412** | 6.15 | 33650 | 97.79 | 38 | 33731 | 98.02 | 145 | 459 | 459 | 34412 | 100.00 | 5069 |
| KROA99B | 1.50 | **42034** | 9.41 | 41558 | 98.87 | 38 | 41559 | 98.87 | 135 | 91 | 91 | 42034 | 100.00 | 9532 |
| KROA99C | 1.50 | 50324 | 0.00 | 43616 | 86.67 | 437 | 43621 | 86.68 | 480 | 43 | 43 | 44144 | 87.72 | *tl* |
| KROC99A | 1.50 | **35188** | 3.83 | 34631 | 98.42 | 38 | 34696 | 98.60 | 94 | 2509 | 2509 | 34986 | 99.43 | *tl* |
| KROC99B | 1.50 | 41948 | 0.00 | 38231 | 91.14 | 95 | 38308 | 91.32 | 534 | 444 | 444 | 38782 | 92.45 | *tl* |
| KROC99C | 1.50 | 47897 | 0.00 | 42634 | 89.01 | 437 | 42750 | 89.25 | 646 | 45 | 45 | 43279 | 90.36 | *tl* |
| KROD99A | 1.50 | **32926** | 1.38 | 32283 | 98.05 | 38 | 32378 | 98.34 | 203 | 724 | 724 | 32852 | 99.78 | *tl* |
| KROD99B | 1.50 | **40242** | 8.56 | 39778 | 98.85 | 38 | 39960 | 99.30 | 209 | 55 | 55 | 40242 | 100.00 | 1280 |
| KROD99C | 1.50 | 49036 | 0.00 | 45375 | 92.53 | 228 | 45385 | 92.55 | 276 | 85 | 85 | 45934 | 93.67 | *tl* |
| Average | | | | | 96.39 | 94 | | 96.56 | 179 | | | | 97.83 | 2370 |
| EIL51A | 1.75 | **649** | 4.01 | 642 | 98.92 | 5 | 642 | 98.92 | 4 | 62 | - | 649 | 100.00 | 31 |
| EIL51B | 1.75 | **689** | 3.77 | 673 | 97.68 | 5 | 676 | 98.11 | 15 | 87 | 15 | 689 | 100.00 | 63 |
| EIL51C | 1.75 | **721** | 4.72 | 709 | 98.34 | 12 | 709 | 98.34 | 12 | 129 | 95 | 721 | 100.00 | 411 |
| EIL75A | 1.75 | **760** | 2.24 | 744 | 97.89 | 11 | 746 | 98.16 | 85 | 793 | 34 | 760 | 100.00 | 3115 |
| EIL75B | 1.75 | **883** | 4.87 | 863 | 97.73 | 14 | 865 | 97.96 | 74 | 887 | 63 | 883 | 100.00 | 3181 |
| EIL75C | 1.75 | 1064 | 0.00 | 924 | 86.84 | 67 | 924 | 86.84 | 84 | 349 | 79 | 958 | 90.04 | *tl* |
| EIL101A | 1.75 | **970** | 5.05 | 946 | 97.53 | 31 | 947 | 97.63 | 173 | 722 | 137 | 961 | 99.07 | *tl* |
| EIL101B | 1.75 | **1033** | 1.84 | 1015 | 98.26 | 29 | 1021 | 98.84 | 270 | 413 | 32 | 1033 | 100.00 | 5855 |
| EIL101C | 1.75 | 1196 | 0.00 | 1096 | 91.64 | 734 | 1096 | 91.64 | 846 | 27 | 4 | 1106 | 92.47 | *tl* |
| KROA99A | 1.75 | **33471** | 5.25 | 31141 | 93.04 | 32 | 31158 | 93.09 | 87 | 625 | 86 | 32153 | 96.06 | *tl* |
| KROA99B | 1.75 | 41825 | 0.00 | 38131 | 91.17 | 153 | 38161 | 91.24 | 364 | 219 | 49 | 38499 | 92.05 | *tl* |
| KROA99C | 1.75 | 44906 | 0.00 | 38981 | 86.81 | 2484 | 38986 | 86.82 | 2738 | 7 | - | 39220 | 87.34 | *tl* |
| KROC99A | 1.75 | **33194** | 3.10 | 32654 | 98.37 | 25 | 32725 | 98.59 | 121 | 631 | 37 | 33194 | 100.00 | 8078 |
| KROC99B | 1.75 | 38520 | 0.00 | 35493 | 92.14 | 72 | 35502 | 92.17 | 185 | 173 | 19 | 36157 | 93.87 | *tl* |
| KROC99C | 1.75 | 42032 | 0.00 | 36741 | 87.41 | 2812 | 36745 | 87.42 | 3087 | 4 | 1 | 36822 | 87.60 | *tl* |
| KROD99A | 1.75 | **32354** | 0.89 | 30854 | 95.36 | 29 | 30995 | 95.80 | 246 | 550 | 5000 | 31614 | 97.71 | *tl* |
| KROD99B | 1.75 | 38003 | 0.00 | 36635 | 96.40 | 44 | 36675 | 96.51 | 178 | 235 | 91 | 37211 | 97.92 | *tl* |
| KROD99C | 1.75 | 45223 | 0.00 | 39594 | 87.55 | 1091 | 39598 | 87.56 | 1166 | 15 | 14 | 39968 | 88.38 | *tl* |
| Average | | | | | 94.06 | 425 | | 94.20 | 541 | | | | 95.70 | 2962 |

Table 5: Detailed results: Exact algorithm BM1 on capacitated *drcl-Q3* instances

| Instance | | | | CG1 | | | | | branch-and-cut | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $z$(LF1) | $\%z$(LF1) | $LB1$ | $\%LB1$ | $t_{LB1}$ | $\lvert\tilde{\mathscr{P}}_1\rvert$ | CLQ | SEC | $LB^*$ | $\%LB^*$ | *Time* |
| 20a | 1.50 | **9262** | 5.16 | 8471 | 91.46 | 8871 | 95.78 | 8 | 1689 | 83 | 7 | 9262 | 100.00 | 17 |
| 20b | 1.50 | **8851** | 0.01 | 8851 | 100.00 | 8851 | 100.00 | 6 | 351 | - | 3 | 8851 | 100.00 | 6 |
| 20c | 1.50 | **8983** | 1.81 | 8983 | 100.00 | 8983 | 100.00 | 3 | 237 | - | 5 | 8983 | 100.00 | 3 |
| 20d | 1.50 | **9545** | 7.78 | 9394 | 98.42 | 9545 | 100.00 | 4 | 553 | 34 | 4 | 9545 | 100.00 | 4 |
| 20e | 1.50 | **9483** | 5.18 | 8878 | 93.62 | 9341 | 98.50 | 5 | 859 | 190 | 6 | 9483 | 100.00 | 7 |
| 25a | 1.50 | **10861** | 1.66 | 10559 | 97.22 | 10685 | 98.38 | 6 | 522 | 17 | 12 | 10861 | 100.00 | 7 |
| 25b | 1.50 | **11750** | 0.28 | 11213 | 95.43 | 11594 | 98.67 | 9 | 871 | 292 | 4 | 11750 | 100.00 | 10 |
| 25c | 1.50 | **11862** | 2.89 | 11321 | 95.44 | 11797 | 99.45 | 9 | 1120 | 118 | 7 | 11862 | 100.00 | 9 |
| 25d | 1.50 | **11860** | 0.38 | 11331 | 95.54 | 11598 | 97.79 | 10 | 1145 | 113 | 13 | 11860 | 100.00 | 12 |
| 25e | 1.50 | **10970** | 2.43 | 10320 | 94.07 | 10625 | 96.86 | 10 | 1425 | 108 | 8 | 10970 | 100.00 | 20 |
| 30a | 1.50 | **13038** | 3.87 | 11905 | 91.31 | 12374 | 94.91 | 24 | 4950 | 163 | 29 | 13038 | 100.00 | 212 |
| 30b | 1.50 | **11327** | 1.39 | 10719 | 94.63 | 11112 | 98.10 | 15 | 2082 | 288 | 17 | 11327 | 100.00 | 32 |
| 30c | 1.50 | **13575** | 1.43 | 12839 | 94.58 | 13202 | 97.25 | 12 | 2165 | 212 | 7 | 13575 | 100.00 | 23 |
| 30d | 1.50 | **13539** | 0.73 | 12829 | 94.76 | 13385 | 98.86 | 12 | 1598 | 198 | 2 | 13539 | 100.00 | 16 |
| 30e | 1.50 | **13468** | 3.00 | 12451 | 92.45 | 13176 | 97.83 | 26 | 3740 | 246 | 5000 | 13468 | 100.00 | 71 |
| 35a | 1.50 | **15147** | 4.50 | 12932 | 85.38 | 14240 | 94.01 | 176 | 7500 | 683 | 40 | 15147 | 100.00 | 4174 |
| 35b | 1.50 | **13434** | 2.11 | 12634 | 94.04 | 13150 | 97.89 | 23 | 3490 | 188 | 9 | 13434 | 100.00 | 30 |
| 35c | 1.50 | **14953** | 2.71 | 13781 | 92.16 | 14542 | 97.25 | 30 | 4684 | 228 | 15 | 14953 | 100.00 | 164 |
| 35d | 1.50 | **13917** | 1.55 | 12374 | 88.91 | 13029 | 93.62 | 114 | 15993 | 137 | 46 | 13917 | 100.00 | 6978 |
| 35e | 1.50 | **14624** | 1.92 | 13674 | 93.50 | 14155 | 96.79 | 30 | 3795 | 263 | 30 | 14624 | 100.00 | 150 |
| Average | | | | | 94.15 | | 97.60 | 27 | | | | | 100.00 | 597 |
| 20a | 1.75 | **8738** | 0.00 | 7007 | 80.19 | 7561 | 86.53 | 79 | 14058 | 122 | 18 | 8738 | 100.00 | 8125 |
| 20b | 1.75 | **8668** | 4.14 | 8066 | 93.05 | 8455 | 97.54 | 19 | 4511 | 301 | 9 | 8668 | 100.00 | 45 |
| 20c | 1.75 | **8767** | 0.83 | 8242 | 94.01 | 8702 | 99.26 | 7 | 1136 | 182 | 6 | 8767 | 100.00 | 9 |
| 20d | 1.75 | **8990** | 5.32 | 8549 | 95.09 | 8973 | 99.81 | 12 | 2682 | 146 | 8 | 8990 | 100.00 | 12 |
| 20e | 1.75 | **9120** | 1.50 | 8073 | 88.52 | 8540 | 93.64 | 22 | 5144 | 155 | 11 | 9120 | 100.00 | 241 |
| 25a | 1.75 | **10285** | 3.03 | 9740 | 94.70 | 10063 | 97.84 | 13 | 2883 | 84 | 10 | 10285 | 100.00 | 20 |
| 25b | 1.75 | **11057** | 2.91 | 9948 | 89.97 | 10379 | 93.87 | 28 | 7070 | 292 | 5 | 11057 | 100.00 | 307 |
| 25c | 1.75 | **11232** | 2.88 | 10351 | 92.16 | 10730 | 95.53 | 58 | 14396 | 106 | 19 | 11232 | 100.00 | 601 |
| 25d | 1.75 | **11013** | 2.54 | 9649 | 87.61 | 10366 | 94.13 | 78 | 11356 | 338 | 17 | 11013 | 100.00 | 793 |
| 25e | 1.75 | **10327** | 2.31 | 9035 | 87.49 | 9678 | 93.72 | 82 | 12480 | 113 | 35 | 10327 | 100.00 | 3787 |
| 30a | 1.75 | 12021 | 0.00 | 10143 | 84.38 | 10143 | 84.38 | 71 | $\Delta_P^{max}$ | - | 3 | 10143 | 84.38 | 71 |
| 30b | 1.75 | **10696** | 2.13 | 9419 | 88.06 | 9946 | 92.99 | 282 | 23962 | 430 | 8 | 10418 | 97.40 | *tl* |
| 30c | 1.75 | **12847** | 1.22 | 11130 | 86.64 | 12007 | 93.46 | 282 | 20831 | 176 | 13 | 12660 | 98.54 | *tl* |
| 30d | 1.75 | **12598** | 1.86 | 11362 | 90.19 | 11925 | 94.66 | 196 | 18359 | 149 | 3 | 12598 | 100.00 | 3113 |
| 30e | 1.75 | 13359 | 0.16 | 10853 | 81.24 | 11922 | 89.24 | 847 | 35271 | 611 | 8 | 12278 | 91.91 | *tl* |
| 35a | 1.75 | 14877 | 0.00 | 11054 | 74.30 | 11054 | 74.30 | 331 | $\Delta_P^{max}$ | - | 11 | 11054 | 74.30 | 331 |
| 35b | 1.75 | 12730 | 0.97 | 11166 | 87.71 | 11714 | 92.02 | 414 | 37591 | 136 | 7 | 12111 | 95.14 | *tl* |
| 35c | 1.75 | 14786 | 0.00 | 11600 | 78.45 | 11600 | 78.45 | 160 | $\Delta_P^{max}$ | - | 14 | 11600 | 78.45 | 160 |
| 35d | 1.75 | 14104 | 0.00 | 10970 | 77.78 | 10970 | 77.78 | 278 | $\Delta_P^{max}$ | - | 3 | 10970 | 77.78 | 278 |
| 35e | 1.75 | 14419 | 0.00 | 12089 | 83.84 | 12869 | 89.25 | 919 | 32731 | - | - | 13123 | 91.01 | *tl* |
| Average | | | | | 86.77 | | 90.92 | 209 | | | | | 94.45 | 1550 |
| 20a | 2.00 | **8627** | 1.67 | 6563 | 76.08 | 6563 | 76.08 | 46 | $\Delta_P^{max}$ | - | 1 | 6563 | 76.08 | 46 |
| 20b | 2.00 | **8481** | 4.42 | 7455 | 87.90 | 7746 | 91.33 | 138 | 32995 | 170 | 9 | 8481 | 100.00 | 2667 |
| 20c | 2.00 | **8602** | 0.77 | 7681 | 89.29 | 8165 | 94.92 | 26 | 8086 | 79 | 10 | 8602 | 100.00 | 281 |
| 20d | 2.00 | **8762** | 7.26 | 7775 | 88.74 | 8480 | 96.78 | 180 | 19349 | 398 | 5 | 8762 | 100.00 | 314 |
| 20e | 2.00 | **8792** | 5.05 | 7616 | 86.62 | 8111 | 92.25 | 288 | 28171 | 328 | 19 | 8560 | 97.36 | *tl* |
| 25a | 2.00 | **10061** | 0.48 | 8917 | 88.63 | 9445 | 93.88 | 129 | 18953 | 131 | 19 | 10061 | 100.00 | 2396 |
| 25b | 2.00 | **10355** | 1.94 | 9009 | 87.00 | 9530 | 92.03 | 465 | 47038 | - | - | 9862 | 95.24 | *tl* |
| 25c | 2.00 | **10958** | 6.67 | 9247 | 84.39 | 9247 | 84.39 | 101 | $\Delta_P^{max}$ | - | 5 | 9247 | 84.39 | 101 |
| 25d | 2.00 | **10425** | 4.31 | 8864 | 85.03 | 8864 | 85.03 | 69 | $\Delta_P^{max}$ | - | 4 | 8864 | 85.03 | 69 |
| 25e | 2.00 | **9911** | 5.48 | 8438 | 85.14 | 8438 | 85.14 | 100 | $\Delta_P^{max}$ | - | 7 | 8438 | 85.14 | 100 |
| 30a | 2.00 | **11568** | 4.68 | 9393 | 81.20 | 9393 | 81.20 | 502 | $\Delta_P^{max}$ | - | 2 | 9393 | 81.20 | 502 |
| 30b | 2.00 | **10374** | 2.03 | 8657 | 83.45 | 8657 | 83.45 | 300 | $\Delta_P^{max}$ | - | 11 | 8657 | 83.45 | 300 |
| 30c | 2.00 | **12348** | 1.93 | 9868 | 79.92 | 9868 | 79.92 | 224 | $\Delta_P^{max}$ | - | 1 | 9868 | 79.92 | 224 |
| 30d | 2.00 | **11867** | 7.77 | 10028 | 84.50 | 10028 | 84.50 | 265 | $\Delta_P^{max}$ | - | 3 | 10028 | 84.50 | 265 |
| 30e | 2.00 | 12720 | 0.00 | 9789 | 76.96 | 9789 | 76.96 | 370 | $\Delta_P^{max}$ | - | 7 | 9789 | 76.96 | 370 |
| 35a | 2.00 | **13919** | 6.02 | 10229 | 73.49 | 10229 | 73.49 | 4357 | $\Delta_P^{max}$ | - | 7 | 10229 | 73.49 | 4357 |
| 35b | 2.00 | **11985** | 0.00 | 10200 | 85.11 | 10200 | 85.11 | 494 | $\Delta_P^{max}$ | - | 7 | 10200 | 85.11 | 494 |
| 35c | 2.00 | 14355 | 0.00 | 10435 | 72.69 | 10435 | 72.69 | 1907 | $\Delta_P^{max}$ | - | 13 | 10435 | 72.69 | 1907 |
| 35d | 2.00 | 14093 | 0.00 | 10073 | 71.48 | 10073 | 71.48 | 3492 | $\Delta_P^{max}$ | - | 2 | 10073 | 71.48 | 3492 |
| 35e | 2.00 | 13524 | 0.00 | 10908 | 80.66 | 10908 | 80.66 | 569 | $\Delta_P^{max}$ | - | 11 | 10908 | 80.66 | 569 |
| Average | | | | | 82.41 | | 84.06 | 701 | | | | | 85.63 | 1415 |

Table 6: Detailed results: Exact algorithm BM2 on capacitated *drcl-Q3* instances

| Instance | | | | H2 | | | CG2 | | | branch-and-cut-and-price | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $LH2$ | $\%LH2$ | $t_{LH2}$ | $LB2$ | $\%LB2$ | $t_{LB2}$ | Nds | LPC | $LB^*$ | $\%LB^*$ | *Time* |
| 20a | 1.50 | **9262** | 5.16 | 9049 | 97.70 | 5 | 9056 | 97.78 | 12 | 189 | 26 | 9262 | 100.00 | 154 |
| 20b | 1.50 | **8851** | 0.01 | 8817 | 99.62 | 4 | 8846 | 99.94 | 18 | 5 | 2 | 8851 | 100.00 | 27 |
| 20c | 1.50 | **8983** | 1.81 | 8983 | 100.00 | 4 | 8983 | 100.00 | 3 | 1 | - | 8983 | 100.00 | 4 |
| 20d | 1.50 | **9545** | 7.78 | 9520 | 99.74 | 4 | 9526 | 99.80 | 15 | 5 | 1 | 9545 | 100.00 | 24 |
| 20e | 1.50 | **9483** | 5.18 | 9293 | 98.00 | 5 | 9298 | 98.05 | 7 | 31 | 4 | 9483 | 100.00 | 29 |
| 25a | 1.50 | **10861** | 1.66 | 10673 | 98.27 | 6 | 10676 | 98.30 | 13 | 21 | 5 | 10861 | 100.00 | 38 |
| 25b | 1.50 | **11750** | 0.28 | 11600 | 98.72 | 6 | 11603 | 98.75 | 12 | 101 | 13 | 11750 | 100.00 | 109 |
| 25c | 1.50 | **11862** | 2.89 | 11394 | 96.05 | 6 | 11485 | 96.82 | 45 | 51 | 4 | 11862 | 100.00 | 138 |
| 25d | 1.50 | **11860** | 0.38 | 11715 | 98.78 | 7 | 11717 | 98.79 | 10 | 55 | 11 | 11860 | 100.00 | 79 |
| 25e | 1.50 | **10970** | 2.43 | 10452 | 95.28 | 7 | 10462 | 95.37 | 11 | 1299 | 81 | 10970 | 100.00 | 1630 |
| 30a | 1.50 | **13038** | 3.87 | 12171 | 93.35 | 11 | 12172 | 93.36 | 11 | 3614 | 241 | 12793 | 98.12 | *tl* |
| 30b | 1.50 | **11327** | 1.39 | 11004 | 97.15 | 10 | 11008 | 97.18 | 12 | 149 | 25 | 11327 | 100.00 | 249 |
| 30c | 1.50 | **13575** | 1.43 | 13225 | 97.42 | 11 | 13226 | 97.43 | 11 | 171 | 18 | 13575 | 100.00 | 421 |
| 30d | 1.50 | **13539** | 0.73 | 13425 | 99.16 | 11 | 13426 | 99.17 | 11 | 3 | 1 | 13539 | 100.00 | 17 |
| 30e | 1.50 | **13468** | 3.00 | 13163 | 97.74 | 11 | 13165 | 97.75 | 10 | 197 | 20 | 13468 | 100.00 | 422 |
| 35a | 1.50 | **15147** | 4.50 | 14659 | 96.78 | 20 | 14660 | 96.78 | 20 | 1597 | 137 | 15134 | 99.91 | *tl* |
| 35b | 1.50 | **13434** | 2.11 | 13154 | 97.92 | 15 | 13157 | 97.94 | 44 | 745 | 55 | 13434 | 100.00 | 4940 |
| 35c | 1.50 | **14953** | 2.71 | 14266 | 95.41 | 15 | 14310 | 95.70 | 39 | 3485 | 103 | 14805 | 99.01 | *tl* |
| 35d | 1.50 | **13917** | 1.55 | 13484 | 96.89 | 19 | 13485 | 96.90 | 20 | 1930 | 98 | 13885 | 99.77 | *tl* |
| 35e | 1.50 | **14624** | 1.92 | 14275 | 97.61 | 16 | 14276 | 97.62 | 16 | 1772 | 101 | 14624 | 100.00 | 8241 |
| Average | | | | | 97.58 | 10 | | 97.67 | 17 | | | | 99.84 | 1033 |
| 20a | 1.75 | **8738** | 0.00 | 8517 | 97.47 | 7 | 8518 | 97.48 | 6 | 103 | 22 | 8738 | 100.00 | 96 |
| 20b | 1.75 | **8668** | 4.14 | 8493 | 97.98 | 4 | 8497 | 98.03 | 4 | 33 | 8 | 8668 | 100.00 | 32 |
| 20c | 1.75 | **8767** | 0.83 | 8542 | 97.43 | 4 | 8546 | 97.48 | 6 | 47 | 21 | 8767 | 100.00 | 35 |
| 20d | 1.75 | **8990** | 5.32 | 8990 | 100.00 | 6 | 8990 | 100.00 | 5 | 1 | - | 8990 | 100.00 | 6 |
| 20e | 1.75 | **9120** | 1.50 | 8736 | 95.79 | 6 | 8736 | 95.79 | 5 | 133 | 20 | 9120 | 100.00 | 96 |
| 25a | 1.75 | **10285** | 3.03 | 10034 | 97.56 | 7 | 10034 | 97.56 | 7 | 107 | 27 | 10285 | 100.00 | 122 |
| 25b | 1.75 | **11057** | 2.91 | 10543 | 95.35 | 9 | 10544 | 95.36 | 8 | 2887 | 251 | 11057 | 100.00 | 4900 |
| 25c | 1.75 | **11232** | 2.88 | 11011 | 98.03 | 11 | 11012 | 98.04 | 10 | 33 | 7 | 11232 | 100.00 | 80 |
| 25d | 1.75 | **11013** | 2.54 | 10557 | 95.86 | 9 | 10557 | 95.86 | 9 | 915 | 74 | 11013 | 100.00 | 1938 |
| 25e | 1.75 | **10327** | 2.31 | 9939 | 96.24 | 10 | 9940 | 96.25 | 12 | 1065 | 72 | 10327 | 100.00 | 2084 |
| 30a | 1.75 | 12021 | 0.00 | 11340 | 94.33 | 18 | 11341 | 94.34 | 19 | 2796 | 192 | 11801 | 98.17 | *tl* |
| 30b | 1.75 | **10696** | 2.13 | 10226 | 95.61 | 14 | 10227 | 95.62 | 14 | 1133 | 103 | 10696 | 100.00 | 3267 |
| 30c | 1.75 | **12847** | 1.22 | 12553 | 97.71 | 17 | 12555 | 97.73 | 36 | 215 | 41 | 12847 | 100.00 | 1016 |
| 30d | 1.75 | **12598** | 1.86 | 12029 | 95.48 | 19 | 12069 | 95.80 | 32 | 1229 | 93 | 12598 | 100.00 | 6566 |
| 30e | 1.75 | 13359 | 0.16 | 12296 | 92.04 | 20 | 12318 | 92.21 | 47 | 1889 | 83 | 12748 | 95.43 | *tl* |
| 35a | 1.75 | 14877 | 0.00 | 13812 | 92.84 | 47 | 13813 | 92.85 | 49 | 850 | 45 | 14146 | 95.09 | *tl* |
| 35b | 1.75 | 12730 | 0.97 | 12121 | 95.22 | 25 | 12122 | 95.22 | 26 | 1333 | 90 | 12512 | 98.29 | *tl* |
| 35c | 1.75 | 14786 | 0.00 | 13288 | 89.87 | 36 | 13291 | 89.89 | 46 | 891 | 100 | 13697 | 92.63 | *tl* |
| 35d | 1.75 | 14104 | 0.00 | 13026 | 92.36 | 48 | 13027 | 92.36 | 50 | 940 | 32 | 13313 | 94.39 | *tl* |
| 35e | 1.75 | 14419 | 0.00 | 13260 | 91.96 | 24 | 13265 | 92.00 | 33 | 1336 | 105 | 13739 | 95.28 | *tl* |
| Average | | | | | 95.46 | 17 | | 95.49 | 21 | | | | 98.46 | 1557 |
| 20a | 2.00 | **8627** | 1.67 | 8301 | 96.22 | 10 | 8302 | 96.23 | 10 | 943 | 58 | 8627 | 100.00 | 962 |
| 20b | 2.00 | **8481** | 4.42 | 8258 | 97.37 | 7 | 8259 | 97.38 | 6 | 93 | 32 | 8481 | 100.00 | 114 |
| 20c | 2.00 | **8602** | 0.77 | 8350 | 97.07 | 6 | 8350 | 97.07 | 5 | 71 | 26 | 8602 | 100.00 | 63 |
| 20d | 2.00 | **8762** | 7.26 | 8709 | 99.40 | 7 | 8709 | 99.40 | 7 | 7 | 1 | 8762 | 100.00 | 18 |
| 20e | 2.00 | **8792** | 5.05 | 8616 | 98.00 | 9 | 8617 | 98.01 | 9 | 89 | 41 | 8792 | 100.00 | 121 |
| 25a | 2.00 | **10061** | 0.48 | 9588 | 95.30 | 10 | 9589 | 95.31 | 10 | 2335 | 316 | 10061 | 100.00 | 4048 |
| 25b | 2.00 | **10355** | 1.94 | 9982 | 96.40 | 13 | 9983 | 96.41 | 13 | 123 | 35 | 10355 | 100.00 | 289 |
| 25c | 2.00 | **10958** | 6.67 | 10673 | 97.40 | 18 | 10674 | 97.41 | 18 | 167 | 15 | 10958 | 100.00 | 626 |
| 25d | 2.00 | **10425** | 4.31 | 10072 | 96.61 | 14 | 10073 | 96.62 | 14 | 679 | 40 | 10425 | 100.00 | 1631 |
| 25e | 2.00 | **9911** | 5.48 | 9651 | 97.38 | 19 | 9652 | 97.39 | 19 | 147 | 23 | 9911 | 100.00 | 474 |
| 30a | 2.00 | **11568** | 4.68 | 11206 | 96.87 | 59 | 11207 | 96.88 | 66 | 1364 | 41 | 11550 | 99.84 | *tl* |
| 30b | 2.00 | **10374** | 2.03 | 10001 | 96.40 | 33 | 10002 | 96.41 | 33 | 2516 | 79 | 10368 | 99.94 | *tl* |
| 30c | 2.00 | **12348** | 1.93 | 11923 | 96.56 | 33 | 11924 | 96.57 | 34 | 963 | 67 | 12348 | 100.00 | 5328 |
| 30d | 2.00 | **11867** | 7.77 | 11581 | 97.59 | 35 | 11582 | 97.60 | 37 | 181 | 17 | 11867 | 100.00 | 1465 |
| 30e | 2.00 | 12720 | 0.00 | 12130 | 95.36 | 59 | 12131 | 95.37 | 61 | 1073 | 33 | 12382 | 97.34 | *tl* |
| 35a | 2.00 | **13919** | 6.02 | 13706 | 98.47 | 149 | 13707 | 98.48 | 155 | 69 | 3 | 13919 | 100.00 | 2024 |
| 35b | 2.00 | **11985** | 0.00 | 11836 | 98.76 | 55 | 11838 | 98.77 | 58 | 349 | 28 | 11985 | 100.00 | 4471 |
| 35c | 2.00 | 14355 | 0.00 | 12822 | 89.32 | 119 | 12823 | 89.33 | 121 | 328 | 40 | 13130 | 91.47 | *tl* |
| 35d | 2.00 | 14093 | 0.00 | 12808 | 90.88 | 303 | 12810 | 90.90 | 309 | 416 | 10 | 13051 | 92.61 | *tl* |
| 35e | 2.00 | 13524 | 0.00 | 12643 | 93.49 | 73 | 12644 | 93.49 | 77 | 874 | 53 | 12943 | 95.70 | *tl* |
| Average | | | | | 96.24 | 52 | | 96.25 | 53 | | | | 98.85 | 1545 |

Table 7: Detailed results: Exact algorithm BM1 on capacitated *rbl-Q3* instances

| Instance | | | | CG1 | | | | | branch-and-cut | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $z$(LF1) | $\%z$(LF1) | $LB1$ | $\%LB1$ | $t_{LB1}$ | $\lvert\tilde{\mathscr{P}}_1\rvert$ | CLQ | SEC | $LB^*$ | $\%LB^*$ | *Time* |
| EIL51A | 1.50 | **678** | 3.39 | 677 | 99.85 | 677 | 99.85 | 4 | 41 | - | 21 | 678 | 100.00 | 4 |
| EIL51B | 1.50 | **729** | 2.74 | 725 | 99.45 | 725 | 99.45 | 4 | 94 | - | 10 | 729 | 100.00 | 4 |
| EIL51C | 1.50 | **802** | 5.11 | 765 | 95.39 | 796 | 99.25 | 10 | 1712 | 300 | 8 | 802 | 100.00 | 13 |
| EIL75A | 1.50 | **807** | 0.99 | 788 | 97.65 | 801 | 99.26 | 7 | 162 | 40 | 23 | 807 | 100.00 | 8 |
| EIL75B | 1.50 | **960** | 6.35 | 945 | 98.44 | 957 | 99.69 | 10 | 444 | 20 | 10 | 960 | 100.00 | 10 |
| EIL75C | 1.50 | **1120** | 2.41 | 1054 | 94.11 | 1089 | 97.23 | 24 | 2853 | 258 | 16 | 1120 | 100.00 | 130 |
| EIL101A | 1.50 | **1010** | 3.27 | 1008 | 99.80 | 1008 | 99.80 | 16 | 359 | - | 28 | 1010 | 100.00 | 16 |
| EIL101B | 1.50 | **1093** | 4.85 | 1088 | 99.54 | 1089 | 99.63 | 14 | 220 | 5 | 36 | 1093 | 100.00 | 15 |
| EIL101C | 1.50 | **1332** | 1.35 | 1236 | 92.79 | 1300 | 97.60 | 112 | 10871 | 186 | 18 | 1332 | 100.00 | 928 |
| KROA99A | 1.50 | **35422** | 2.54 | 34896 | 98.52 | 35057 | 98.97 | 18 | 653 | 40 | 33 | 35422 | 100.00 | 19 |
| KROA99B | 1.50 | **42114** | 3.04 | 41336 | 98.15 | 41576 | 98.72 | 47 | 9546 | 58 | 10 | 42114 | 100.00 | 48 |
| KROA99C | 1.50 | 55341 | 0.00 | 40893 | 73.89 | 40893 | 73.89 | 605 | $\Delta_P^{max}$ | - | 6 | 40893 | 73.89 | 605 |
| KROC99A | 1.50 | **35188** | 3.83 | 34724 | 98.68 | 34765 | 98.80 | 14 | 211 | 10 | 42 | 35188 | 100.00 | 14 |
| KROC99B | 1.50 | 41897 | 0.00 | 38938 | 92.94 | 38938 | 92.94 | 123 | $\Delta_P^{max}$ | - | 13 | 38938 | 92.94 | 123 |
| KROC99C | 1.50 | 55430 | 0.00 | 42036 | 75.84 | 42036 | 75.84 | 1671 | $\Delta_P^{max}$ | - | 8 | 42036 | 75.84 | 1671 |
| KROD99A | 1.50 | **32926** | 1.38 | 32840 | 99.74 | 32893 | 99.90 | 19 | 639 | 2 | 23 | 32926 | 100.00 | 19 |
| KROD99B | 1.50 | **40443** | 5.55 | 39573 | 97.85 | 40096 | 99.14 | 34 | 4781 | 50 | 23 | 40443 | 100.00 | 38 |
| KROD99C | 1.50 | 54872 | 0.00 | 43391 | 79.08 | 43391 | 79.08 | 291 | $\Delta_P^{max}$ | - | 8 | 43391 | 79.08 | 291 |
| Average | | | | | 93.98 | | 94.95 | 168 | | | | | 95.65 | 90 |
| EIL51A | 1.75 | **649** | 4.01 | 645 | 99.38 | 648 | 99.85 | 4 | 84 | 6 | 19 | 649 | 100.00 | 4 |
| EIL51B | 1.75 | **689** | 3.77 | 677 | 98.26 | 679 | 98.55 | 6 | 224 | 10 | 15 | 689 | 100.00 | 6 |
| EIL51C | 1.75 | **753** | 3.85 | 694 | 92.16 | 724 | 96.15 | 45 | 10174 | 260 | 25 | 753 | 100.00 | 969 |
| EIL75A | 1.75 | **771** | 0.13 | 746 | 96.76 | 764 | 99.09 | 13 | 684 | 103 | 29 | 771 | 100.00 | 14 |
| EIL75B | 1.75 | **894** | 3.80 | 871 | 97.43 | 889 | 99.44 | 20 | 2483 | 99 | 23 | 894 | 100.00 | 23 |
| EIL75C | 1.75 | 1130 | 0.00 | 908 | 80.35 | 983 | 86.99 | 848 | $\Delta_P^{max}$ | 505 | 0 | 998 | 88.32 | *tl* |
| EIL101A | 1.75 | **970** | 5.05 | 962 | 99.18 | 964 | 99.38 | 26 | 2010 | 7 | 34 | 970 | 100.00 | 29 |
| EIL101B | 1.75 | **1036** | 4.15 | 1028 | 99.23 | 1032 | 99.61 | 21 | 850 | 19 | 28 | 1036 | 100.00 | 21 |
| EIL101C | 1.75 | 1334 | 0.00 | 1092 | 81.86 | 1092 | 81.86 | 1020 | $\Delta_P^{max}$ | - | 5 | 1092 | 81.86 | 1020 |
| KROA99A | 1.75 | **33471** | 5.43 | 32139 | 96.02 | 32958 | 98.47 | 48 | 6826 | 150 | 30 | 33471 | 100.00 | 64 |
| KROA99B | 1.75 | 43762 | 0.00 | 38440 | 87.84 | 38440 | 87.84 | 153 | $\Delta_P^{max}$ | - | 9 | 38440 | 87.84 | 153 |
| KROA99C | 1.75 | 53056 | 0.00 | - | - | - | - | *tl* | - | - | - | - | - | *tl* |
| KROC99A | 1.75 | **33194** | 3.10 | 32870 | 99.02 | 33033 | 99.51 | 18 | 732 | 50 | 26 | 33194 | 100.00 | 18 |
| KROC99B | 1.75 | 39104 | 0.00 | 35722 | 91.35 | 35722 | 91.35 | 680 | $\Delta_P^{max}$ | - | 0 | 35722 | 91.35 | 680 |
| KROC99C | 1.75 | 54066 | 0.00 | - | - | - | - | *tl* | - | - | - | - | - | *tl* |
| KROD99A | 1.75 | **32354** | 0.91 | 31877 | 98.53 | 31955 | 98.77 | 40 | 4026 | 19 | 39 | 32354 | 100.00 | 48 |
| KROD99B | 1.75 | 39311 | 0.00 | 35974 | 91.51 | 35974 | 91.51 | 94 | $\Delta_P^{max}$ | - | 11 | 35974 | 91.51 | 94 |
| KROD99C | 1.75 | 54119 | 0.00 | - | - | - | - | *tl* | - | - | - | - | - | *tl* |
| Average | | | | | 93.93 | | 95.22 | 202 | | | | | 96.06 | 120 |

Table 8: Detailed results: Exact algorithm BM2 on capacitated *rbl-Q3* instances

| Instance | | | | H2 | | | CG2 | | | branch-and-cut-and-price | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | *LH*2 | %*LH*2 | $t_{LH2}$ | *LB*2 | %*LB*2 | $t_{LB2}$ | Nds | LPC | $LB^*$ | $\%LB^*$ | *Time* |
| EIL51A | 1.50 | **678** | 3.39 | 671 | 98.97 | 4 | 671 | 98.97 | 3 | 169 | - | 678 | 100.00 | 42 |
| EIL51B | 1.50 | **729** | 2.74 | 722 | 99.04 | 4 | 722 | 99.04 | 25 | 67 | 5 | 729 | 100.00 | 61 |
| EIL51C | 1.50 | **802** | 5.11 | 780 | 97.26 | 6 | 785 | 97.88 | 11 | 42 | 17 | 802 | 100.00 | 57 |
| EIL75A | 1.50 | **807** | 0.99 | 785 | 97.27 | 10 | 785 | 97.27 | 12 | 2884 | 9 | 804 | 99.63 | *tl* |
| EIL75B | 1.50 | **960** | 6.35 | 947 | 98.65 | 11 | 947 | 98.65 | 12 | 2306 | 33 | 959 | 99.90 | *tl* |
| EIL75C | 1.50 | **1120** | 2.41 | 1088 | 97.14 | 19 | 1088 | 97.14 | 31 | 1447 | 77 | 1120 | 100.00 | 7936 |
| EIL101A | 1.50 | **1010** | 3.27 | 1000 | 99.01 | 27 | 1002 | 99.21 | 183 | 67 | 20 | 1010 | 100.00 | 619 |
| EIL101B | 1.50 | **1093** | 4.85 | 1083 | 99.09 | 25 | 1083 | 99.09 | 28 | 1669 | 3 | 1092 | 99.91 | *tl* |
| EIL101C | 1.50 | **1332** | 1.35 | 1294 | 97.15 | 57 | 1294 | 97.15 | 63 | 320 | 56 | 1315 | 98.72 | *tl* |
| KROA99A | 1.50 | **35422** | 2.54 | 34242 | 96.67 | 22 | 34282 | 96.78 | 124 | 999 | 50 | 34923 | 98.59 | *tl* |
| KROA99B | 1.50 | **42114** | 3.04 | 41579 | 98.73 | 25 | 41580 | 98.73 | 192 | 59 | 32 | 42114 | 100.00 | 4302 |
| KROA99C | 1.50 | 55341 | 0.00 | 49190 | 88.89 | 141 | 49194 | 88.89 | 161 | 149 | 20 | 49783 | 89.96 | *tl* |
| KROC99A | 1.50 | **35188** | 3.83 | 34631 | 98.42 | 22 | 34696 | 98.60 | 92 | 2545 | 7 | 34987 | 99.43 | *tl* |
| KROC99B | 1.50 | 41897 | 0.00 | 38456 | 91.79 | 41 | 38605 | 92.14 | 305 | 558 | 92 | 39046 | 93.20 | *tl* |
| KROC99C | 1.50 | 55430 | 0.00 | 50470 | 91.05 | 142 | 50485 | 91.08 | 219 | 159 | 12 | 51076 | 92.15 | *tl* |
| KROD99A | 1.50 | **32926** | 1.38 | 32283 | 98.05 | 22 | 32378 | 98.34 | 199 | 748 | 22 | 32853 | 99.78 | *tl* |
| KROD99B | 1.50 | **40443** | 5.55 | 39898 | 98.65 | 30 | 40037 | 99.00 | 172 | 107 | 32 | 40443 | 100.00 | *tl* |
| KROD99C | 1.50 | 54872 | 0.00 | 50299 | 91.67 | 99 | 50304 | 91.68 | 134 | 195 | 29 | 50939 | 92.83 | *tl* |
| Average | | | | | 96.53 | 39 | | 96.65 | 109 | | | | 98.00 | 2138 |
| EIL51A | 1.75 | **649** | 4.01 | 642 | 98.92 | 5 | 642 | 98.92 | 4 | 62 | - | 649 | 100.00 | 31 |
| EIL51B | 1.75 | **689** | 3.77 | 673 | 97.68 | 5 | 676 | 98.11 | 15 | 87 | 15 | 689 | 100.00 | 63 |
| EIL51C | 1.75 | **753** | 3.85 | 743 | 98.67 | 11 | 743 | 98.67 | 10 | 59 | 13 | 753 | 100.00 | 66 |
| EIL75A | 1.75 | **771** | 0.13 | 748 | 97.02 | 11 | 748 | 97.02 | 53 | 2411 | 56 | 767 | 99.48 | *tl* |
| EIL75B | 1.75 | **894** | 3.80 | 869 | 97.20 | 13 | 871 | 97.43 | 43 | 1511 | 149 | 894 | 100.00 | 6326 |
| EIL75C | 1.75 | 1130 | 0.00 | 1003 | 88.76 | 40 | 1003 | 88.76 | 41 | 534 | 66 | 1027 | 90.88 | *tl* |
| EIL101A | 1.75 | **970** | 5.05 | 946 | 97.53 | 28 | 947 | 97.63 | 195 | 635 | 116 | 961 | 99.07 | *tl* |
| EIL101B | 1.75 | **1036** | 4.15 | 1018 | 98.26 | 27 | 1026 | 99.03 | 228 | 284 | 30 | 1036 | 100.00 | 2912 |
| EIL101C | 1.75 | 1334 | 0.00 | 1238 | 92.80 | 187 | 1238 | 92.80 | 201 | 77 | 13 | 1250 | 93.70 | *tl* |
| KROA99A | 1.75 | **33471** | 5.43 | 31920 | 95.37 | 29 | 31923 | 95.38 | 39 | 938 | 61 | 32654 | 97.56 | *tl* |
| KROA99B | 1.75 | 43762 | 0.00 | 39365 | 89.95 | 92 | 39426 | 90.09 | 236 | 308 | 26 | 40150 | 91.75 | *tl* |
| KROA99C | 1.75 | 53056 | 0.00 | 47864 | 90.21 | 667 | 47868 | 90.22 | 758 | 38 | 1 | 48177 | 90.80 | *tl* |
| KROC99A | 1.75 | **33194** | 3.10 | 32654 | 98.37 | 27 | 32725 | 98.59 | 121 | 631 | 37 | 33194 | 100.00 | 7662 |
| KROC99B | 1.75 | 39104 | 0.00 | 36491 | 93.32 | 72 | 36506 | 93.36 | 184 | 325 | 39 | 36911 | 94.39 | *tl* |
| KROC99C | 1.75 | 54066 | 0.00 | 48490 | 89.69 | 618 | 48494 | 89.69 | 654 | 59 | - | 48796 | 90.25 | *tl* |
| KROD99A | 1.75 | **32354** | 0.91 | 30854 | 95.36 | 30 | 30995 | 95.80 | 228 | 617 | 105 | 31599 | 97.67 | *tl* |
| KROD99B | 1.75 | 39311 | 0.00 | 36940 | 93.97 | 37 | 36952 | 94.00 | 156 | 322 | 93 | 37560 | 95.55 | *tl* |
| KROD99C | 1.75 | 54119 | 0.00 | 48254 | 89.16 | 250 | 48258 | 89.17 | 276 | 99 | 6 | 48649 | 89.89 | *tl* |
| Average | | | | | 94.57 | 119 | | 94.70 | 191 | | | | 96.17 | 2843 |

Table 9: Detailed results: Exact algorithm BM1 on capacitated *cord-a* instances

| Instance | | | | CG1 | | | | | branch-and-cut | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $z$(LF1) | $\%z$(LF1) | *LB*1 | %*LB*1 | $t_{LB1}$ | $\lvert\tilde{\mathscr{P}}_1\rvert$ | CLQ | SEC | $LB^*$ | $\%LB^*$ | *Time* |
| a2_16 | 2.26 | **214** | 2.80 | 213 | 99.53 | 214 | 100.00 | 4 | 510 | 44 | 7 | 214 | 100.00 | 4 |
| a4_16 | 2.38 | **192** | 1.56 | 188 | 97.92 | 192 | 100.00 | 4 | 495 | 20 | 9 | 192 | 100.00 | 4 |
| a3_18 | 2.35 | **225** | 1.33 | 211 | 93.78 | 218 | 96.89 | 8 | 1287 | 99 | 7 | 225 | 100.00 | 15 |
| a2_20 | 2.73 | **246** | 0.00 | 235 | 95.53 | 242 | 98.37 | 7 | 1429 | 148 | 12 | 246 | 100.00 | 12 |
| a2_24 | 2.30 | **275** | 0.36 | 258 | 93.82 | 269 | 97.82 | 18 | 3923 | 189 | 15 | 275 | 100.00 | 83 |
| a3_24 | 2.74 | **255** | 1.96 | 246 | 96.47 | 251 | 98.43 | 25 | 5852 | 225 | 13 | 255 | 100.00 | 33 |
| a4_24 | 2.38 | **252** | 4.37 | 247 | 98.02 | 252 | 100.00 | 11 | 1937 | 93 | 8 | 252 | 100.00 | 11 |
| a3_30 | 2.39 | **341** | 1.17 | 324 | 95.01 | 335 | 98.24 | 41 | 4232 | 102 | 13 | 341 | 100.00 | 59 |
| a4_32 | 2.40 | **313** | 3.19 | 293 | 93.61 | 303 | 96.81 | 49 | 6676 | 319 | 17 | 313 | 100.00 | 911 |
| a3_36 | 2.35 | **402** | 3.48 | 377 | 93.78 | 389 | 96.77 | 74 | 8235 | 317 | 50 | 402 | 100.00 | 2965 |
| a4_40 | 2.45 | 387 | 1.55 | 345 | 89.15 | 360 | 93.02 | 283 | 14557 | 341 | 30 | 365 | 94.32 | *tl* |
| a5_40 | 2.54 | 367 | 0.54 | 325 | 88.56 | 344 | 93.73 | 663 | 27038 | - | - | 347 | 94.55 | *tl* |
| Average | | | | | 94.60 | | 97.51 | 99 | | | | | 99.07 | 410 |

Table 10: Detailed results: Exact algorithm BM2 on capacitated *cord-a* instances

| Instance | | | | H2 | | | CG2 | | | branch-and-cut-and-price | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | $\delta$ | $z_{UB}$ | $\%_{LNS}$ | $LH2$ | $\%LH2$ | $t_{LH2}$ | $LB2$ | $\%LB2$ | $t_{LB2}$ | Nds | LPC | $LB^*$ | $\%LB^*$ | *Time* |
| a2_16 | 2.26 | **214** | 2.80 | 214 | 100.00 | 4 | 214 | 100.00 | 4 | 2 | 1 | 214 | 100.00 | 6 |
| a4_16 | 2.38 | **192** | 1.56 | 188 | 97.92 | 3 | 189 | 98.44 | 7 | 53 | 12 | 192 | 100.00 | 26 |
| a3_18 | 2.35 | **225** | 1.33 | 216 | 96.00 | 4 | 217 | 96.44 | 3 | 199 | 57 | 225 | 100.00 | 115 |
| a2_20 | 2.73 | **246** | 0.00 | 239 | 97.15 | 5 | 240 | 97.56 | 10 | 577 | 70 | 246 | 100.00 | 309 |
| a2_24 | 2.30 | **275** | 0.36 | 265 | 96.36 | 10 | 265 | 96.36 | 24 | 447 | 94 | 275 | 100.00 | 732 |
| a3_24 | 2.74 | **255** | 1.96 | 250 | 98.04 | 11 | 251 | 98.43 | 15 | 40 | 22 | 255 | 100.00 | 108 |
| a4_24 | 2.38 | **252** | 4.37 | 250 | 99.21 | 7 | 252 | 100.00 | 70 | 11 | 11 | 252 | 100.00 | 113 |
| a3_30 | 2.39 | **341** | 1.17 | 335 | 98.24 | 19 | 335 | 98.24 | 30 | 43 | 25 | 341 | 100.00 | 233 |
| a4_32 | 2.40 | **313** | 3.19 | 304 | 97.12 | 19 | 304 | 97.12 | 46 | 752 | 141 | 313 | 100.00 | 6987 |
| a3_36 | 2.35 | **402** | 3.48 | 383 | 95.27 | 36 | 384 | 95.52 | 99 | 461 | 147 | 394 | 98.01 | *tl* |
| a4_40 | 2.45 | 387 | 1.55 | 355 | 91.73 | 72 | 355 | 91.73 | 115 | 214 | 104 | 363 | 93.80 | *tl* |
| a5_40 | 2.54 | 367 | 0.54 | 340 | 92.64 | 81 | 342 | 93.19 | 163 | 219 | 88 | 348 | 94.82 | *tl* |
| Average | | | | | 96.64 | 23 | | 96.92 | 49 | | | | 98.89 | 959 |