Modelling and optimization of polygeneration systems

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Research interests:
mathematical modelling, simulation, optimization, multicriteria decision support, energy systems

Career
• 81-85 Helsinki U Tech, MSc, Systems & Operations Research
• 85-90 Nokia, Software methodology development
• 90-94 Helsinki U Tech, PhD in Applied Mathematics
• 94-97 U Jyväskylä, associate professor in Maths & SW engineering
• 97-00 VTT (Tech Res C Finland), professor, Energy Markets
• 00-09 U Turku, professor in Computer Science
• 09- Aalto, professor in Energy for Communities, 11-15 head of dept
• 17- Dual affiliation at Systems Analysis Laboratory
Outline of presentation

• Cogeneration
• Long-term planning model
• Efficient reformulation of model
• Efficient solution algorithms
• Reformulating non-convex models
• Experiences
• Selected publications
Cogeneration

- Cogeneration means production of two or more energy products together in an integrated process
  - CHP = combined heat and power generation
  - Trigeneration:
    - district heating + cooling + power
    - high pressure process heat + low pressure heat + power
  - Technologies: backpressure turbines, combined steam&gas turbines, heat pumps ...
  - Much more efficient than producing the products separately
    - Energy efficiency increase 40% -> 90%
  - Cost-efficient way to reduce CO$_2$ emissions
  - Finland is one of the leading countries in cogeneration
Generic backpressure plant for trigeneration

\[ r = v01 + h1 - v12 \]
\[ q = v12 + h2 \]
Cogeneration planning

- Objective is to maximize profit subject to production constraints
- Hourly production of the different products must be planned together
  - Production of heat & cooling must meet the demand (natural monopoly)
  - Power production is planned to maximize the profit from sales to the spot market (liberalized power market)
- A long-term model consists of many hourly models in sequence
  - E.g. an annual model consists of 8760 hourly models
  - Hourly forecasts for demand and power price
- Various advanced analyses, e.g. risk analysis require solving many long-term models based on different scenarios
  - Solution must be fast!
Long-term planning model: Trigeneration

Min

\[ \sum_{u \in U} C_u(x_u) \]

subject to

\[ Hx^t = \begin{bmatrix} P^t \\ Q^t \\ R^t \end{bmatrix}, \quad t \in T \text{ (set of hours)} \]

\[ x_u \in X_u \quad u \in U \text{ (set of plants)} \]

where

- \( U \) is the set of units (plants, contracts, ...)
- \( x \) is the vector of all decision variables
- \( x_u \) is the vector of decision variables for plant \( u \)
- \( x^t \) is the vector of decision variables for hour \( t \)
- \( C_u(x_u) \) is the production cost function for plant \( u \)
- \( X_u \) represent plant-specific constraints
- \( H \) is a transmission matrix
- \( P^t, Q^t, R^t \) are the hourly demands for the three commodities
Decomposition into hourly models

- The long-term model can be decomposed into hourly models

\[
\text{Min} \quad \sum_{u \in U} C_u(x_u^t)
\]

\[
Hx_t^t = \begin{bmatrix}
P_t^t \\
Q_t^t \\
R_t^t
\end{bmatrix}
\]

\[x_u^t \in X_u^t \quad u \in U\]

- The necessary decomposition and co-ordination techniques depend on what kind of dynamic constraints are present
  - Without dynamic constraints, decomposition is trivial
  - Traditionally, the hourly model is a general LP/MILP model
Efficient model reformulation

- The idea is to encode the operating area of each plant as a convex combination of extreme characteristic points \((c_j^t, p_j^t, q_j^t, r_j^t)\)

\[
\begin{align*}
\min \quad & C_u^t = \sum_{j \in J_u} c_j^t x_j^t \\
\text{subject to} \quad & P_u^t = \sum_{j \in J_u} p_j^t x_j^t \\
& Q_u^t = \sum_{j \in J_u} q_j^t x_j^t \\
& R_u^t = \sum_{j \in J_u} r_j^t x_j^t \\
& \sum_{j \in J_u} x_j^t = 1 \\
& x_j^t \geq 0 \quad j \in J_u
\end{align*}
\]

\[V = 0.1V_1 + 0.3V_3 + 0.6V_4\]
Structure of reformulated trigeneration model
Solving the reformulated model

• The reformulated model is an LP (or MILP) model with a special structure
  – The special structure allows the model to be solved extremely efficiently using tailored algorithms
    • Power Simplex (PS) for CHP (two-generation) models
    • Extended Power Simplex (EPS) for multi-site CHP problems
    • Three Commodity Simplex (TCS) for trigeneration problems
    • On-line and off-line envelope construction algorithms (ECON, ECOFF) for CHP problems under the liberalized market
  – The specialized algorithms can be 20-2000 times faster than generic commercial LP algorithms, such as CPLEX
    • Speed is comparable to specialized network algorithms
## Speed of different algorithms

- **CHP planning against power market**
  - Test models (A1-A6)
    - 3 to 8 convex plants
    - Yearly (8760 h) planning models without dynamic constraints
    - Constraints: 43 800 to 87 600, variables: 420 480 to 1 077 480
  - Solution time on a 2.2 GHz Pentium 4 PC
    - Speedup against CPLEX: 400 to 2300 times

<table>
<thead>
<tr>
<th>CPU (ms)</th>
<th>From scratch</th>
<th>Reusing previous results</th>
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<tbody>
<tr>
<td>Model</td>
<td>CPLEX  PS ECON ECOFF</td>
<td>CPLEX  PS ECON ECOFF</td>
</tr>
<tr>
<td>A1</td>
<td>22548 127 30 13</td>
<td>24136 44 25 10</td>
</tr>
<tr>
<td>A2</td>
<td>24638 191 47 21</td>
<td>29937 52 36 10</td>
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<tr>
<td>A3</td>
<td>30373 191 60 27</td>
<td>34967 60 49 16</td>
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<tr>
<td>A4</td>
<td>33410 260 75 37</td>
<td>38090 77 60 20</td>
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<td>A5</td>
<td>37134 315 88 40</td>
<td>46619 80 66 21</td>
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<td>A6</td>
<td>40933 403 105 50</td>
<td>51797 96 78 24</td>
</tr>
<tr>
<td>Average</td>
<td>31506 247 68 31</td>
<td>37591 68 52 17</td>
</tr>
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</table>
Non-convex CHP model

• Can be formulated as a Mixed Integer Linear Programming (MILP) model

• Necessary when either (or both)
  – The cost function is non-convex
  – P-Q the characteristic is non-convex

• Idea
  – Partition the objective function into convex parts
  – Partition the characteristic into convex parts
  – Use **binary variables** to choose in which area to operate
  – A binary variable can only have value 0 or 1
Sample non-convex cogeneration model

<table>
<thead>
<tr>
<th>Area</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
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<tbody>
<tr>
<td>A1</td>
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</table>
Non-convex cogeneration model

- Characteristic is partitioned into 3 convex parts
- $A_j$ is set of areas to which $x_j$ belongs
- Define zero-one variables $y_1, y_2, y_3$, and allow exactly one of them to have value 1.
- $y$-variables select which corner points are allowed in convex combination
- $U^*$ is set of non-convex plants

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<tr>
<th>Area</th>
<th>P1</th>
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\[
x_j \leq \sum_{a \in A_j} y_a, \quad j \in J_u, \quad u \in U^*,
\]

\[
\sum_{a \in A_u} y_a = 1, \quad u \in U^*,
\]

\[
y_a \in \{0, 1\}, \quad a \in A_u, \quad u \in U^*.
\]
Experiences

- Experiments with other plant models have showed that the number of extreme points typically varies:
  - from 10 to 70 for trigeneration plants
  - from 5 to 20 for CHP plants
- This number of extreme points is very reasonable.
- The method works well in the case of 1, 2 or 3 commodities.
- In more complex models the combinatorial explosion may become a problem.
- Similarly, the method can handle efficiently only a fairly small number of 0/1 variables.
Selected publications

Selected publications


